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Mixed PD-Type Iterative Learning Control Algorithm for a Class of Parabolic Singular Distributed Parameter Systems

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ABSTRACT In this paper, the iterative learning control (ILC) problem is investigated for a class of time-invariant parabolic singular distributed parameter systems. Initially, the singular distributed parameter systems is decomposed into infinite number of singular systems based on the separation principle. Meanwhile, the slow-fast subsystems are introduced via singular value decomposition method. Then, a novel mixed PD-type ILC algorithm with finite dimension is designed for the low dimensional slow part and the corresponding convergence conditions are manifested. With the proposed controller, the output error of high dimensional fast complement can satisfy the given value instead of neglecting the effect of high dimensional modes. Furthermore, under the aforesaid ILC law and the appropriate number of the low dimensional slow part, the resulting tracking error of parabolic singular distributed parameter systems can converge to any small tracking accuracy. Finally, simulation results on the distributed building automatic temperature system verify the convergence and effectiveness of the mixed PD-type ILC algorithm.

INDEX TERMS Iterative learning control, singular distributed parameter systems, separation principle, low dimensional slow part, convergence analysis.

I. INTRODUCTION

Many complex systems can be modeled by singular distributed parameter systems (SDPSs, see Refs. [1]–[4]). On one hand, the singular distributed parameter systems has the characteristics of singular systems, such as the systems includes the dynamic part described by differential equations and the static part governed by algebra equations. On the other hand, the SDPSs, inherently distributed in space, which has the characteristics of distributed parameter systems with infinite-dimensional. Although there exists plenty of research results about singular systems [5], [6] and distributed parameter systems [7], [8]. There are only some theoretical results on SDPSs have been investigated over the past decades. For example, the solution of singular distributed parameter systems is studied by using the Fourier approach and separation principle in [9]. In [10], the effective research on a boundary-value problem for linear partial differential-algebraic equations was solved based on the Kronecker-Weierstrass form of the matrix pencil. Meanwhile, there are still also other research attentions on SDPSs, such as decoupling [11], controllability and observability [12], passivity criteria [13], exponential stability [14], solvability [15], and so forth.

Iterative learning control (ILC) is an important branch of the intelligence control field. The ILC scheme, originating from the industrial robotic [16], has considerable practical systems and broader engineering applications which include transport systems [17], rehabilitation robots [18], multiagent systems [19]–[22], digital networks [23] and so on. Compared with the existing intelligence control methods, the learning capability of the algorithm is the most prominent characteristic of ILC. In addition, for a given desired output, ILC can find the desired input by the repetitive learning process in a finite time interval. Over the past four decades, the ILC has been successfully developed to many kinds of systems, for example, singular systems [24], distributed parameter systems (see, e.g., [25]–[29], [33]), stochastic

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systems [34], [35], switched systems [36], impulsive systems [37], [38], fractional-order systems [39], etc..

The following facts motivate the study of this paper. Firstly, up to now, to the best authors' knowledge, there are few results reported on the ILC of singular distributed parameter systems. Due to the variables of the DPSs are related to infinite dimensional space, the research in this area should be theoretically interesting and more challenging [29]–[31]. Secondly, many industrial and engineering processes can be established on the SDPSs model which governed by partial differential equations (PDE) with a singular matrix coefficient ([1]–[4], [9]–[13]). Since the SDPSs is an infinite-dimensional system and even for the known distributed parameter systems (DPSs), in this process, the reduction to ordinary differential equations (ODE) with finite-dimensional is needed because only a finite number of actuators/sensors can be used in practice. In [40], an ILC controller is designed via the model reduction methods for a class of quasi-linear parabolic DPSs, which is simplified into lumped parameter systems via the eigenspectrum of DPSs. The work in [41] utilized the Karhunen-Loeve decomposition and singular perturbation theory to reduce the nonlinear DPSs and implement the data-driven control technique. In view of Galerkin's theory, the DPSs is simplified into the finite-dimensional slow system and an infinite-dimensional fast system in reference [42], and the guaranteed cost sampled-data fuzzy controller is designed. An incremental spatiotemporal learning scheme is applied to the online modeling of DPSs for time-space separation in [43]. Thirdly, owing to the singular form is useful to represent and handle the systems problem in considerable control fields, the iterative learning control of singular systems have been extensively studied in recent years (see, [5], [6], [44], [45]). The convergence analysis methods are mainly divided into two classes: (i) the constrained equivalent system method, which means that the dynamic equivalent system is obtained based on the regularity hypothesis of matrix parameters and the singular value decomposition theory [24], (ii) direct analysis, i.e., through the new matrix which contains a singular matrix is invertible by the learning gain matrix to be determined and then the learning convergence of output error is guaranteed [38], [46].

In association with the aforesaid observations, the iterative learning tracking control for singular distributed parameter systems is studied. The contributions of this paper are summarized as three folds:

(1) The iterative learning control problem is firstly investigated for multi-input and multi-output (MIMO) parabolic singular distributed parameter systems, which can be applied to track the desired trajectory both in space and time.

(2) A novel mixed PD-type ILC law based on the low dimensional slow part is proposed. Since the reduced SDPSs includes dynamic part described by differential equations and static part described by algebra equations, the differential of state error from dynamic part and the proportion of state error from static part are integrated into the mixed PD-type ILC law. Moreover, the inherent mechanism of finite dimension ILC law guarantees the convergence property of the output error.

(3) The convergence of the closed-loop learning system is rigorously proved via the separation principle, model reduced technology, and contraction mapping approach. With the aforementioned analysis, not only the overall performance of the output tracking which takes into account the fast modes of the system can be enhanced, but also the uniformity of the state tracking can be improved on the basis of the dynamics of whole modes.

This paper has the following structure. In Section II, problem formulation and system description are firstly given under some assumptions. We present the ILC algorithm and details to analyze the convergence conditions of output tracking error for the repetitive MIMO SDPSs in Section III. In consequence, based on the derivation in Section III, we focus on numerical simulations about the 2-storey office building temperature system in Section IV. At last, Section V concludes the paper and further discussions are shown.

Notations: Throughout the paper, let $\mathbb{C}$ denotes complex plane, $\mathbb{C}^+$ is the right half plane of $\mathbb{C}$. $\mathbb{R}$ denotes the set of real numbers. $\mathbb{R}^m$ and $\mathbb{R}^{j\times l}$ describe the set of m-dimensional real vectors and the set of $j \times l$ real matrices, respectively. The superscript $'T'$ denotes the matrix transposition; $A > 0$ (respectively, $A < 0$) represents a symmetric positive (respectively, negative) definite matrix. Define $V = (v_1, v_2, \cdots, v_j)$ as a vector, and its Euclidean norm is $\| V \|= \sqrt{\sum_{i=1}^{j} v_i^2}$. For V is a matrix equipped with the matrix norm $\| V \|= \sqrt{\lambda_{\max}(V^T V)}$, where $\lambda_{\max}(\cdot)$ is the maximum eigenvalue. Let $z_i(\xi) : \Omega \to \mathbb{R}$ is a Lebesgue square integrable function on the bounded open set Ω, that is $z_i(\xi) \in \mathbf{L}^2(\Omega)$ $(i = 1, 2, \cdots, j)$, and define $\|z_i\|_{\mathbf{L}^2}= \{\int_\Omega z_i^2(\xi)\mathrm{d}\xi\}^{\frac{1}{2}}$. If $z(\xi) = (z_1(\xi), z_2(\xi), \cdots, z_j(\xi))^T \in \mathbb{R}^j \cap \mathbf{L}^2(\Omega)$, then $\| z \|_{\mathbf{L}^2}= \{\int_\Omega z^T(\xi)z(\xi)\mathrm{d}\xi\}^{\frac{1}{2}}$. Let $f(t) : [0, T] \to \mathbb{R}^j$, its λ-norm is defined as $\|f\|_\lambda = \sup_{0\leqslant t\leqslant T} \{\| f(t) \| e^{-\lambda t}\}$ with a given positive constant λ. Besides, $\triangle$ denotes the Laplace operator.

II. SYSTEM DESCRIPTION AND PROBLEM FORMULATION

A. PROBLEM FORMULATION

Consider the following time-invariant singular distributed parameter systems with parabolic type

$$\begin{cases} E\dfrac{\partial z(\xi,t)}{\partial t} = D\dfrac{\partial^2 z(\xi,t)}{\partial \xi^2} + Az(\xi,t) + Bu(\xi,t), \\ y(\xi,t) = Cz(\xi,t), \end{cases} \tag{1}$$

where $(\xi, t) \in [0, 1] \times [0, T]$ describes time and space coordinates, respectively. $z(\xi, t) \in \mathbb{R}^j$,$u(\xi, t) \in \mathbb{R}^m$, $y(\xi, t) \in \mathbb{R}^l$ denote the system state, control input and the output of system, respectively. $E \in \mathbb{R}^{j\times j}$ is singular constant matrix with $rank(E) = r < j$, $D \in \mathbb{R}^{j\times j}$ is diagonal positive definite constant matrix, $A \in \mathbb{R}^{j\times j}$, $B \in \mathbb{R}^{j\times m}$, $C \in \mathbb{R}^{l\times j}$, are time-invariant bounded matrices.

The initial and boundary conditions of (1) are given as,

$$z(0,t) = z(1,t) = 0,\ t \in [0,T], \tag{2}$$
$$z(\xi,0) = \phi(\xi),\ \xi \in [0,1]. \tag{3}$$

Remark 1: The singular distributed parameter systems (1) is a parabolic DPSs with singular matrix E, which can be applied to describe a wide family of problems in practical applications including the temperature description of an acetylene reactor, the transdermal application of drugs, etc., has been extensively studied (see e.g., [47], [48]). For example, as a model for molten carbonate fuel cells, the dynamics consist of second-order temperature equations and transportation equations were studied in [48].

For the singular distributed parameter systems (1) repeatedly operates on a finite time interval $[0,T]$, that is

$$\begin{cases} E\dfrac{\partial z^k(\xi,t)}{\partial t} = D\dfrac{\partial^2 z^k(\xi,t)}{\partial \xi^2} + Az^k(\xi,t) + Bu^k(\xi,t), \\ y^k(\xi,t) = Cz^k(\xi,t), \end{cases} \tag{4}$$

where k denotes the iteration number, $k = 1, 2, \cdots$.

The iterative learning control goal is to find the desired control input $u_d(\xi,t)$ such that when $k \to \infty$, the output of learning systems $y_k(\xi,t)$ can track the reference trajectory $y^d(\xi,t)$ as follows

$$\begin{cases} E\dfrac{\partial z^d(\xi,t)}{\partial t} = D\dfrac{\partial^2 z^d(\xi,t)}{\partial \xi^2} + Az^d(\xi,t) + Bu^d(\xi,t), \\ y^d(\xi,t) = Cz^d(\xi,t), \end{cases} \tag{5}$$

Remark 2: With the generalized operator semigroup theory and differential inequalities, Ge et. and his work team make several insight results on well-posedness of SDPSs [14], [15], [51]. The existence, uniqueness and constructive expression for the strong solution $z_d(\xi,t)$ in (5) can be found in ([51], Thm. 9).

Before considering the ILC issue of SDPSs (1), the following basic assumptions that would be useful in the derivation of our main result.

Assumption 1. For a desired output $y^d(\xi,t)$, there exists an unique $u^d(\xi,t)$ to meet the equations in the learning systems described by (5).

Assumption 2. The SDPSs (1) is regular, that is, there exists a complex number $s_0 \in \mathbb{C}$ such that matrix pencil $\sigma(E,D) \neq 0$, i.e. $\det(s_0E - D) \neq 0$.

Assumption 3. Assume that, for all $k = 1, 2, \cdots$, the boundary and initial conditions are given as follows:

$$z^k(0,t) = z^k(1,t) = 0,\ t \in [0,T], \tag{6}$$
$$z^k(\xi,0) = \varphi(\xi) = z^d(\xi,0),\ \xi \in [0,1]. \tag{7}$$

Remark 3: For Assumption 1, it is a necessary condition for the iterative learning algorithm (see, e.g., [16], [24], [26], [36], [40]). Meanwhile, the constructive expression, existence and uniqueness for the solution of parabolic SDPSs are confirmed in the light of the separation principle [9], [10]. The requirement of Assumption 2 is also the general assumption in the control theory of singular system and SDPSs (e.g., [5], [6], [9], [12]). In standard ILC, the identical initial condition $z^k(\xi,0) = z^d(\xi,0)$ is a common assumption [25], [26], [29]. Moreover, most industrial processes often start from the same position and the process is repeatable from a practical point of view. Therefore, those proposed assumptions are rational.

B. SYSTEMS SIMPLIFICATION VIA THE SEPARATION PRINCIPLE

According to the time-space separation principle, the spatiotemporal state $z(\xi,t)$ can be represented as $z(\xi,t) = X(t)\Xi(\xi)$. Then, the eigenvalue problem of parabolic SDPS (1) is concerned as follows:

$$\begin{cases} -\Delta\Xi(\xi) = \lambda\Xi(\xi),\ \xi \in (0,1), \\ \Xi(0) = \Xi(1) = 0, \end{cases}$$

and the aforesaid eigenfunctions $\Xi(\xi)$ are $\{\sin(n\pi\xi)\}, n = 1, 2, \cdots$ and $\lambda_n = \{n^2\pi^2\}, n = 1, 2, \cdots$.

Furthermore, the following decoupled forms for systems (4) can be implemented

$$z^k(\xi,t) = \sum_{n=1}^{\infty} X_n^k(t)\sin(n\pi\xi),\ X_n(t) \in \mathbb{R}^j, \tag{8}$$
$$u^k(\xi,t) = \sum_{n=1}^{\infty} U_n^k(t)\sin(n\pi\xi),\ U_n^k(t) \in \mathbb{R}^m, \tag{9}$$
$$y^k(\xi,t) = \sum_{n=1}^{\infty} Y_n^k(t)\sin(n\pi\xi),\ Y_n^k(t) \in \mathbb{R}^l, \tag{10}$$
$$\varphi^k(\xi) = \sum_{n=1}^{\infty} \varphi_n\sin(n\pi\xi),\ \varphi_n \in \mathbb{R}^j. \tag{11}$$

Remark 4: In views of the separation principle and PDE spectrum theory, then $\{\sin(n\pi\xi)\}_{n=1}^{\infty}$ develops an orthonormal eigenfunction family, which makes efforts for the following infinite decomposition of SDPSs. For more discussions on the aforementioned theories, please see [2], [3], [7].

Then making the infinite decomposition of (4) via Eqs.(8)∼ (11) yields the following singular system family.

$$\begin{aligned} \sum_{n=1}^{\infty} E\dot{X}_n^k(t)\sin(n\pi\xi) &= \sum_{n=1}^{\infty} -(n\pi)^2DX_n^k(t)\sin(n\pi\xi) \\ &\quad + \sum_{n=1}^{\infty} AX_n^k(t)\sin(n\pi\xi) \\ &\quad + \sum_{n=1}^{\infty} BU_n^k(t)\sin(n\pi\xi), \end{aligned} \tag{12}$$
$$\sum_{n=1}^{\infty} Y_n^k(t)\sin(n\pi\xi) = \sum_{n=1}^{\infty} CX_n^k(t)\sin(n\pi\xi), \tag{13}$$
$$\sum_{n=1}^{\infty} X_n^k(0)\sin(n\pi\xi) = \sum_{n=1}^{\infty} \varphi_n\sin(n\pi\xi). \tag{14}$$

The above expressions can be arranged in a compact form as follow:

$$\begin{cases} E\dot{X}_n^k(t) = -(n\pi)^2 DX_n^k(t) + AX_n^k(t) + BU_n^k(t), \\ Y_n^k(t)=CX_n^k(t), \\ X_n^k(0)=\varphi_n, \end{cases} \tag{15}$$

where $X_n^k(t)\in\mathbb{R}^j$, $U_n^k(t)\in\mathbb{R}^m$,$\varphi_n\in\mathbb{R}^j$, $n = 1, 2, \cdots$.

On the basis of the singular value matrix theory [5] and Assumption 2, there exists two nonsingular matrices $P \in \mathbb{R}^{j\times j}$, $Q \in \mathbb{R}^{j\times j}$ such that

$$PEQ = \begin{bmatrix} I_r & 0 \\ 0 & 0 \end{bmatrix}, PDQ = \begin{bmatrix} D_1 & 0 \\ 0 & I_{n-r} \end{bmatrix},$$
$$PAQ = \begin{bmatrix} A_1 & A_2 \\ A_3 & A_4 \end{bmatrix}, PB = \begin{bmatrix} B_1 \\ B_2 \end{bmatrix}, CQ = \begin{bmatrix} C_1 & C_2 \end{bmatrix},$$

where I_r is an unit matrix with $r = rank(E)$ order.

By introducing the following state transformation,

$$Q^{-1}X_n^k(t)=\begin{bmatrix} X_{n_1}^k(t) \\ X_{n_2}^k(t) \end{bmatrix},$$

with $X_{n1}^k(t) \in \mathbb{R}^r$, $X_{n2}^k(t) \in \mathbb{R}^{j-r}$.

Then, the system (15) can be rewritten as

$$\begin{cases} \begin{bmatrix} I_r & 0 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} \dot{X}_{n_1}^k(t) \\ \dot{X}_{n_2}^k(t) \end{bmatrix} = -(n\pi)^2\begin{bmatrix} D_1 & 0 \\ 0 & I_{n-r} \end{bmatrix}\begin{bmatrix} X_{n_1}^k(t) \\ X_{n_2}^k(t) \end{bmatrix} \\ + \begin{bmatrix} A_1 & A_2 \\ A_3 & A_4 \end{bmatrix}\begin{bmatrix} X_{n_1}^k(t) \\ X_{n_2}^k(t) \end{bmatrix} + \begin{bmatrix} B_1 \\ B_2 \end{bmatrix} U_n^k(t), \\ Y_n^k(t) = C_1X_{n_1}^k(t) + C_2X_{n_2}^k(t), \\ X_n^k(0) = \varphi_n. \end{cases} \tag{16}$$

It follows from (16) that

$$\begin{cases} \dot{X}_{n_1}^k=-(n\pi)^2D_1X_{n_1}^k(t)+A_1X_{n_1}^k(t)+A_2X_{n2}^k(t)+B_1U_n^k(t), \\ 0=-(n\pi)^2X_{n_2}^k(t)+A_3X_{n_1}^k(t)+A_4X_{n_2}^k(t)+B_2U_n^k(t), \\ Y_n^k(t) = C_1X_{n_1}^k(t) + C_2X_{n_2}^k(t), \\ X_n^k(0) = \varphi_n, \end{cases} \tag{17}$$

where $U_n^k(t) \in \mathbb{R}^m$, $Y_n^k(t) \in \mathbb{R}^l$, $n = 1, 2, \cdots$.

Likewise, in view of the reduced steps of learning systems (4), the desired systems (5) can be simplified as

$$\begin{cases} \dot{X}_{n_1}^d=-(n\pi)^2D_1X_{n_1}^d(t)+A_1X_{n_1}^d(t)+A_2X_{n2}^d(t)+B_1U_n^d(t), \\ 0=-(n\pi)^2X_{n_2}^d(t)+A_3X_{n_1}^d(t)+A_4X_{n_2}^d(t)+B_2U_n^d(t), \\ Y_n^d(t) = C_1X_{n_1}^d(t) + C_2X_{n_2}^d(t), \\ X_n^d(0) = \varphi_n, \end{cases} \tag{18}$$

with $X_{n1}^d(t) \in \mathbb{R}^r$, $X_{n2}^d(t) \in \mathbb{R}^{j-r}$, $U_n^d(t) \in \mathbb{R}^m$, $Y_n^d(t) \in \mathbb{R}^l$, $n = 1, 2, \cdots$.

Remark 3: The learning systems (15) has infinite number of singular systems. The transformed systems (17) is the limited equivalence to singular systems (15), which consists of fast subsystems (the systems state is $X_{n1}^k(t)$) and slow subsystems (the systems state is $X_{n2}^k(t)$).

III. ITERATIVE LEARNING CONTROL ALGORITHM AND CONVERGENCE ANALYSIS

In practice, it is difficult to design the controller for infinite number of singular systems. Thus, the singular systems (17) is taken into account divides into two parts as follows: low dimensional mode (19) and high dimensional mode (20).

Low dimensional mode:

$$\begin{cases} \dot{X}_{n_1}^k=-(n\pi)^2D_1X_{n_1}^k(t)+A_1X_{n_1}^k(t)+A_2X_{n2}^k(t)+B_1U_n^k(t), \\ 0=-(n\pi)^2X_{n_2}^k(t)+A_3X_{n_1}^k(t)+A_4X_{n_2}^k(t)+B_2U_n^k(t), \\ Y_n^k(t) = C_1X_{n_1}^k(t) + C_2X_{n_2}^k(t), \\ X_n^k(0) = \varphi_n, (n = 1, 2, \cdots, N), \end{cases} \tag{19}$$

which contanis N number finite dimensional singular systems.

High dimensional mode:

$$\begin{cases} \dot{X}_{n_1}^k=-(n\pi)^2D_1X_{n_1}^k(t)+A_1X_{n_1}^k(t)+A_2X_{n2}^k(t)+B_1U_n^k(t), \\ 0=-(n\pi)^2X_{n_2}^k(t)+A_3X_{n_1}^k(t)+A_4X_{n_2}^k(t)+B_2U_n^k(t), \\ Y_n^k(t) = C_1X_{n_1}^k(t) + C_2X_{n_2}^k(t), \\ X_n^k(0) = \varphi_n, (n = N+1, N+2, \cdots), \end{cases} \tag{20}$$

where consists of infinite number of finite-dimensional singular systems.

A. LOW DIMENSIONAL MODE ALGORITHM DESIGN AND CONVERGENCE ANALYSIS

For low dimensional mode part (19), $n = 1, 2, \cdots, N$, the mixed PD-type ILC algorithm is proposed as follow:

$$U_n^{k+1}(t) = U_n^k(t) + \Gamma_1\dot{e}_{n_1}^k(t) + \Gamma_2 e_{n_2}^k(t), \tag{21}$$

with Γ_i, $i = 1, 2$ are the learning gain matrices and

$$e_{n_1}^k(t) = Y_{n_1}^d(t) - Y_{n_1}^k(t) = C_1(X_{n_1}^d(t) - X_{n_1}^k(t))$$
$$e_{n_2}^k(t) = Y_{n_2}^d(t) - Y_{n_2}^k(t) = C_2(X_{n_2}^d(t) - X_{n_2}^k(t)).$$

Remark 4: Based on the singular value decomposition and state transformation for singular parameter distributed systems, a new structure of the ILC algorithm for SDPSs is obtained, that is the mixed PD-type ILC law which is incorporated with D-type and P-type algorithms. In view of the different characteristics of differential description and algebraic description in the restricted equivalent transformation form of SDPSs, D-type learning gain matrix Γ_1 and P-type learning gain matrix Γ_2 are adopted, respectively.

With the aforesaid development, the following theorem results about the low dimensional mode is proposed:

Theorem 1: Consider repetitive parabolic SDPSs (19), the mixed PD-type ILC algorithm (21) and Assumptions 1-3 hold. If the following requirements satisfy

(1) $((n\pi)^2I - A_4) \in \mathbb{R}^{(j-r)\times(j-r)}$ is invertible,

(2) $\rho = ||I - \Gamma_1C_1\bar{B}_1 - \Gamma_2C_2\bar{B}_2|| < 1$, where $\bar{B}_1 = B_1 + A_2\bar{B}_2$, $\bar{B}_2 = ((n\pi)^2I - A_4)^{-1}B_2$, for $n = 1, 2, \cdots, N$.

Then the output of systems (19) converges to the desired output as k goes to infinity, that is

$$\lim_{k\to\infty} Y_n^k(t) = Y_n^d(t), \forall t \in [0, T], n = 1, 2, \cdots, N.$$

Proof: The proof of this Theorem is divided into three major steps. First step, the state error $\| \bar{X}^k_{n1}(t) \|$ is estimated under the mixed PD-type ILC law (21), and Second step, the convergence of input error $\| \bar{U}^{k+1}_n(t) \|$ is demonstrated. The convergence of output error is proven in the last step.

Step 1: The estimation of the state error $\| \bar{X}^k_{n1}(t) \|$.

For the simplicity of presentation,the following notations are introduced:

$$\begin{aligned}\bar{X}^k_{ni} &\triangleq X^d_{ni} - X^k_{ni}, i = 1, 2\\ \bar{U}^k_n(t) &\triangleq U^d_n(t) - U^k_n(t),\\ e^k_n(t) &\triangleq Y^d_n(t) - Y^k_n(t).\end{aligned}$$

According to the desired systems (18) and learning systems (19), one has

$$\begin{cases}\dot{\bar{X}}^k_{n_1}=-(n\pi)^2D_1\bar{X}^k_{n_1}(t)+A_1\bar{X}^k_{n_1}(t)+A_2\bar{X}^k_{n2}(t)+B_1\bar{U}^k_n(t),\\ 0=-(n\pi)^2\bar{X}^k_{n2}(t)+A_3\bar{X}^k_{n_1}(t)+A_4\bar{X}^k_{n_2}(t)+B_2\bar{U}^k_n(t),\\ e^k_n(t) = C_1\bar{X}^k_{n_1}(t)+C_2\bar{X}^k_{n_2}(t),\\ X^k_n(0) = \varphi_n,\end{cases} \tag{22}$$

where $n = 1, 2, \cdots, N$.

Combing the second equation of (22) with the first condition of Theorem 1, i.e., $((n\pi)^2I–A_4)$ is invertible, one induces

$$\bar{X}^k_{n2}(t) = ((n\pi)^2I - A_4)^{-1}[A_3\bar{X}^k_{n1}(t) + B_2\bar{U}^k_n(t)]. \tag{23}$$

Let $\bar{A}_3 = ((n\pi)^2I - A_4)^{-1}A_3$, then Eq.(23) is rewritten as

$$\bar{X}^k_{n2}(t) = \bar{A}_3\bar{X}^k_{n1}(t) + \bar{B}_2\bar{U}^k_n(t)]. \tag{24}$$

where $\bar{B}_2 = ((n\pi)^2I - A_4)^{-1}B_2$.

Substituting (24) into the first equation of (22), one has

$$\begin{aligned}\dot{\bar{X}}^k_{n1}(t) &= [A_2\bar{A}_3 - (n\pi)^2D_1 + A_1]\bar{X}^k_{n1}(t)\\ &\quad+[B_1 + A_2\bar{B}_2]\bar{U}^k_n(t)\\ &= \bar{A}_1\bar{X}^k_{n1}(t) + \bar{B}_1\bar{U}^k_n(t),\end{aligned} \tag{25}$$

where $\bar{A}_1 = A_2\bar{A}_3 - (n\pi)^2D_1 + A_1, \bar{B}_1 = B_1 + A_2\bar{B}_2$.

Integrating both sides of (25) with respect to t and by Assumption 3 yields

$$\bar{X}^k_{n1}(t) = \int_0^t \bar{A}_1\bar{X}^k_{n1}(\tau) + \bar{B}_1\bar{U}^k_n(\tau)d\tau. \tag{26}$$

Taking Euclidean norms on both sides of the Equation (27), one implies

$$\begin{aligned}\| \bar{X}^k_{n1}(t) \| &\leqslant \int_0^t \left(\|\bar{A}_1\| \| \bar{X}^k_{n1}(\tau) \| + \| \bar{B}_1 \| \| \bar{U}^k_n(\tau) \| \right)d\tau\\ &\leqslant \int_0^t (a_1 \| \bar{X}^k_{n1}(\tau) \| + b_1 \| \bar{U}^k_n(\tau) \| d\tau,\end{aligned} \tag{27}$$

where $a_1 =\| \bar{A}_1 \|, b_1 =\| \bar{B}_1 \|$.

By applying the Bellman-Gronwall inequality in [49], one can obtain

$$\| \bar{X}^k_{n1}(t) \| \leqslant b_1 \int_0^t e^{a_1(t-\tau)} \| \bar{U}^k_n(\tau) \| d\tau. \tag{28}$$

Multiplying by $e^{-\lambda t}$ on both sides of (28), one gets

$$\| \bar{X}^k_{n1}(t) \| e^{-\lambda t} \leqslant b_1 \int_0^t e^{-(\lambda-a_1)(t-\tau)} \| \bar{U}^k_n(\tau) \| e^{-\lambda\tau}d\tau,$$

which implies that

$$\| \bar{X}^k_{n1} \|_\lambda \leqslant \frac{b_1}{\lambda - a_1} \| \bar{U}^k_n \|_\lambda . \tag{29}$$

Step 2: The convergence of input error $\| \bar{U}^{k+1}_n(t) \|$

By adopting the mixed ILC algorithm (21), one has

$$\begin{aligned}\bar{U}^{k+1}_n(t) &= \bar{U}^k_n(t) - \Gamma_1\dot{e}^k_{n1}(t) - \Gamma_2e^k_{n2}(t)\\ &= \bar{U}^k_n(t) - \Gamma_1C_1\dot{\bar{X}}^k_{n1}(t) - \Gamma_2C_2\bar{X}^k_{n2}(t)\end{aligned} \tag{30}$$

Substituting (25) and (24) into (30), one gets

$$\begin{aligned}\bar{U}^{k+1}_n(t) &= (I - \Gamma_1C_1\bar{B}_1 - \Gamma_2C_2\bar{B}_2)\bar{U}^k_n(t)\\ &\quad- (\Gamma_1C_1\bar{A}_1 + \Gamma_2C_2\bar{A}_3)\bar{X}^k_{n1}(t),\end{aligned} \tag{31}$$

which indicates that

$$\| \bar{U}^{k+1}_n(t) \| \leqslant \rho \| \bar{U}^k_n(t) \| +h \| \bar{X}^k_{n1}(t) \|, \tag{32}$$

with $h =\| \Gamma_1C_1\bar{A}_1 + \Gamma_2C_2\bar{A}_3 \|$.

Taking λ-norm on both sides of (32), and using (29), we can conclude that

$$\begin{aligned}\| \bar{U}^{k+1}_n \|_\lambda &\leqslant \rho \| \bar{U}^k_n \|_\lambda + \frac{hb_1}{\lambda - a_1} \| \bar{U}^k_n \|_\lambda\\ &\leqslant (\rho + \frac{hb_1}{\lambda - a_1}) \| \bar{U}^k_n \|_\lambda\end{aligned} \tag{33}$$

Since $\rho < 1$, there chooses λ sufficiently large which also make

$$\rho + \frac{hb_1}{\lambda - a_1} < 1.$$

It follows from (33) that

$$\lim_{k\to\infty} \| \bar{U}^k_n \|_\lambda = 0. \tag{34}$$

Step 3: The convergence analysis of output error.

Clearly, from (24), (29) and (34), it follows that

$$\lim_{k\to\infty} \| \bar{X}^k_{n1} \|_\lambda = 0, \lim_{k\to\infty} \| \bar{X}^k_{n2} \|_\lambda = 0. \tag{35}$$

By noting (22), it implies that

$$e^k_n(t) = Y^d_n(t) - Y^k_n(t) = C_1\bar{X}^k_{n1}(t) + C_2\bar{X}^k_{n2}(t), \tag{36}$$

combining with (35), one derives that

$$\begin{aligned}\lim_{k\to\infty} \|e^k_n\|_\lambda &= \|C_1\| \lim_{k\to\infty} \| \bar{X}^k_{n1} \|_\lambda + \| C_2 \| \lim_{k\to\infty} \| \bar{X}^k_{n2} \|_\lambda\\ &= 0.\end{aligned} \tag{37}$$

By nothing that

$$\| e^k_n(t) \| = \| e^k_n(t) \| e^{-\lambda t}e^{\lambda t} \leqslant \| e^k_n \|_\lambda e^{\lambda t} \leqslant \| e^k_n \|_\lambda e^{\lambda T}, \tag{38}$$

then, the following result can be obtained from (37) and (38)

$$\lim_{k\to\infty} \| e^k_n(t) \| = \lim_{k\to\infty} \| Y^d_n(t) - Y^k_n(t) \| = 0. \tag{39}$$

Thus,

$$\lim_{k\to\infty} Y^k_n(t) = Y^d_n(t), \forall t \in [0, T], n = 1, 2, \cdots, N. \tag{40}$$

This completes the proof of Theorem 1.

B. THE CONVERGENCE OF HIGH MODE

Since the high-dimensional mode (20) has infinite modes, it is possible to choose N large enough and let $U_n^k(t) = 0$. Then, similar to the Low dimensional case (22), one gets

$$\begin{cases} \dot{\bar{X}}_{n_1}^k = -(n\pi)^2 D_1 \bar{X}_{n_1}^k(t) + A_1 \bar{X}_{n_1}^k(t) + A_2 \bar{X}_{n2}^k(t) + BU_n^d(t), \\ 0 = -(n\pi)^2 \bar{X}_{n_2}^k(t) + A_3 \bar{X}_{n_1}^k(t) + A_4 \bar{X}_{n_2}^k(t) + B_2 U_n^d(t), \\ e_n^k(t) = C_1 \bar{X}_{n_1}^k(t) + C_2 \bar{X}_{n_2}^k(t), \\ X_n^k(0) = \varphi_n, \end{cases} \tag{41}$$

where $n = N+1, N+2, \cdots$.

Based on error systems of high mode (41), we have following result.

Theorem 2: For the high-dimensional modes (20), if Assumptions (1)-(3) and the conditions of Theorem 1 hold. In addition, $\sigma(E,D) \in \mathbb{C}^+$. Then, for any $\varepsilon > 0$, there exists large enough N, when $n > N$, we have

$$\| y_n^d(t) - y_n^k(t) \| < \varepsilon, \forall t \in [0, T].$$

Proof: From the second equation of (41), one has

$$\bar{X}_{n2}^k(t) = ((n\pi)^2 I - A_4)^{-1}[A_3 \bar{X}_{n1}^k(t) + B_2 U_n^d(t)]. \tag{42}$$

Thus,

$$\begin{aligned} \| \bar{X}_{n2}^k(t) \| &\leq \| ((n\pi)^2 I - A_4)^{-1} \| \\ &\quad \cdot (\| A_3 \| \| \bar{X}_{n1}^k(t) \| + \| B_2 \| U_n^d(t) \|), \\ &= \frac{1}{(n\pi)^2} \| (I - (n\pi)^{-2} A_4)^{-1} \| \\ &\quad \cdot (a_3 \| \bar{X}_{n1}^k(t) \| + b_2 \| U_n^d(t) \|), \end{aligned} \tag{43}$$

where $a_3 = \| A_3 \|$, $b_2 = \| B_2 \|$.

Substituting (42) into the first equation of (41), we can induce that

$$\begin{aligned} \dot{\bar{X}}_{n1}^k(t) &= \big(A_2((n\pi)^2 I - A_4)^{-1} A_3 - (n\pi)^2 D_1 + A_1\big) \bar{X}_{n1}^k(t) \\ &\quad + \big(B_1 + ((n\pi)^2 I - A_4)^{-1} B_2\big) U_n^d(t). \end{aligned} \tag{44}$$

Introducing the following new notations

$$\tilde{A}_1(n) \triangleq \big(A_2((n\pi)^2 I - A_4)^{-1} A_3 - (n\pi)^2 D_1 + A_1\big),$$
$$\tilde{B}_1(n) \triangleq \big(B_1 + ((n\pi)^2 I - A_4)^{-1} B_2\big),$$

the Eq.(44) can be rewritten as

$$\dot{\bar{X}}_{n1}^k(t) = \tilde{A}_1(n) \bar{X}_{n1}^k(t) + \tilde{B}_1(n) U_n^d(t). \tag{45}$$

Integrating both sides of (45) from 0 to t,

$$\bar{X}_{n1}^k(t) = \int_0^t e^{\tilde{A}_1(n)(t-\tau)} \tilde{B}_1(n) U_n^d(t) d\tau, \tag{46}$$

and taking Euclidean norm on both sides of (46), one has

$$\| \bar{X}_{n1}^k(t) \| \leq \int_0^t e^{\|\tilde{A}_1(n)(t-\tau)\|} \| \tilde{B}_1(n) \| \| U_n^d(\tau) \| d\tau. \tag{47}$$

Since

$$\begin{aligned} &\exp\{\| \tilde{A}_1(n) t \|\} \\ &= \exp\{\| [-(n\pi)^2 D_1 + A_2((n\pi)^2 I - A_4)^{-1} A_3 + A_1] t \|\} \\ &\leq \exp\big(\|(-n\pi)^2 D_1 \| t\big) \exp\big(\|A_2((n\pi)^2 I - A_4)^{-1} A_3 + A_1 \| t\big), \end{aligned} \tag{48}$$

and by $\sigma(E,D) = \sigma(D_1) \in \mathbb{C}^+$, which means that exists $\lambda(\lambda = \lambda_{\min}(D_1)) > 0$, such that

$$\begin{aligned} &\exp\big(\| \tilde{A}_1(n) t \| \big) \\ &\leq \exp\big(-\lambda(n\pi)^2 t\big) \exp\big(a_2 \| ((n\pi)^2 I - A_4)^{-1} \| a_3 + a_1\big) t. \end{aligned} \tag{49}$$

On the other hand, note that

$$\begin{aligned} \| (I - (n\pi)^{-2} A_4)^{-1} \| &\leq \frac{\| I \|}{\| I \| - \| (n\pi)^{-2} A_4 \|} \\ &\leq \frac{1}{1 - a_4 (n\pi)^{-2}}, \end{aligned} \tag{50}$$

When $n > N_1$ in the High mode, it is possible to choose N_1 sufficiently large so that $1 - \frac{a_4}{(n\pi)^2} > \frac{1}{2}$. Subsequently, in views of (49) and (50), we can derive that

$$\begin{aligned} &\exp\big(\| \tilde{A}_1(n) t \| \big) \\ &\leq \exp\{-\lambda n^2 [\pi^2 - \frac{1}{\lambda n^2}(a_1 + \frac{2a_2 a_3}{(n\pi)^2})] t\}, \end{aligned} \tag{51}$$

on the other hand, there exists N_2, for $n > N_2$,

$$\pi^2 - \frac{1}{\lambda n^2}\big(a_1 + \frac{2a_2 a_3}{(n\pi)^2}\big) > 1. \tag{52}$$

Clearly, the inequality (51) indicates

$$e^{\|\tilde{A}_1(n)(t-\tau)\|} \leq e^{-\lambda n^2 (t-\tau)}, (n > \max\{N_1, N_2\}). \tag{53}$$

Then, together with (47), (50) and (53) becomes

$$\begin{aligned} &\| \bar{X}_{n1}^k(t) \| \\ &\leq \int_0^t e^{-\lambda n^2 (t-\tau)} \|B_1 + ((n\pi)^2 I - A_4)^{-1} B_2\| \|U_n^d(\tau)\| d\tau, \\ &\leq \frac{1}{\lambda n^2} \|B_1 + ((n\pi)^2 I - A_4)^{-1} B_2\| \| U_n^d(t) \|, \\ &\leq \frac{1}{\lambda n^2}(b_1 + 2b_2) \| U_n^d(t) \| . \end{aligned} \tag{54}$$

Due to the boundedness of the desired input $U_n^d(t)$, then it can find a positive constant $\delta > 0$ so that

$$\| U_n^d(t) \| \leq \delta, (n = N+1, \cdots). \tag{55}$$

As a result

$$\| \bar{X}_{n1}^k(t) \| \leq \frac{1}{\lambda n^2}(b_1 + 2b_2)\delta. \tag{56}$$

Let us return to the inequality (43), for $n > N_2$, one can conclude that

$$\| \bar{X}_{n2}^k(t) \| \leq \frac{2}{(n\pi)^2}(a_3 \| \bar{X}_{n1}^k(t) \| + b_2 \| U_n^d(t) \|). \tag{57}$$

Substituting (56) into (57), one gives

$$\begin{aligned} \| \bar{X}_{n2}^k(t) \| &\leq \frac{2}{(n\pi)^2}[\frac{a_3}{\lambda n^2}[b_1 + 2b_2]\delta + b_2 \delta] \\ &\leq \frac{2}{(n)^2}[\frac{a_3(b_1 + 2b_2)}{\lambda \pi^2} + \frac{b_2}{\pi^2}]\delta \\ &= \frac{1}{n^2} \alpha_1 \delta, \end{aligned} \tag{58}$$

where $\alpha_1 = 2[\frac{a_3(b_1+2b_2)}{\lambda n^2} + \frac{b_2}{\pi^2}]$.

Finally, selecting N large enough such that $N > \max\{N_1, N_2\}$, one obtains

$$\begin{aligned}\sum_{n=N+1}^{\infty}\|e_n^k(t)\| &\leqslant \sum_{n=N+1}^{\infty}\{\|C_1\|\|\bar{X}_{n1}^k(t)\|+\|C_2\|\|\bar{X}_{n2}^k(t)\|\},\\ &\leqslant c_1\sum_{n=N+1}^{\infty}\|\bar{X}_{n1}^k(t)\|+c_2\sum_{n=N+1}^{\infty}\|\bar{X}_{n2}^k(t)\|\},\\ &\leqslant c_1\delta\frac{b_1+2b_2}{\lambda}\sum_{n=N+1}^{\infty}\frac{1}{n^2}+c_2\alpha_1\delta\sum_{n=N+1}^{\infty}\frac{1}{n^2},\\ &\leqslant M\sum_{n=N+1}^{\infty}\frac{1}{n^2}, \end{aligned} \tag{59}$$

where $M = \left[\frac{b_1+2b_2}{\lambda}\left(c_1+\frac{2a_3c_2}{\pi^2}\right)+\frac{2b_2c_2}{\pi^2}\right]\delta$ and $c_i = ||C_i||, i = 1, 2$.

Based on the convergence analysis of series $\sum_{n=1}^{\infty}\frac{1}{n^2}$, it is possible for any small $\varepsilon > 0$ to choose N sufficiently enough so that when $n > N$, $\| e_n^k(t) \| < \varepsilon$ holds. Therefore, in consideration of the aforementioned analysis and inequality (59), $\forall\varepsilon > 0, \exists N, \forall n > N$, one yields

$$\| e_n^k(t) \| = \| Y_n^d(t) - Y_n^k(t) \| < \varepsilon, t \in [0, T]. \tag{60}$$

The proof of Theorem 2 is completed.

Now it comes to the convergence of output tracking errors for all modes. The following theorem provides the convergence conditions of the tracking errors in learning systems (4).

Theorem 3: For learning systems (4) and the mixed PD-type ILC algorithm (21), assume that Assumptions 1-3 hold and the parameter matrices $\sigma(E, D) \in \mathbb{C}^+$ meets. If the learning gain matrices Γ_1, Γ_2 in (21) satisfies $\| I-\Gamma_1 C_1\bar{B}_1-\Gamma_2 C_2\bar{B}_2 \| < 1$, then for an arbitrarily small tracking accuracy ε, there exits the number of mode N large enough such that when $n > N$,

$$\lim_{k\to\infty} \| e^k(\cdot, t) \|_{\mathbf{L}^2}^2 < \varepsilon. \tag{61}$$

Proof: By noting that

$$\begin{aligned}\| e^k(\cdot, t) \|_{\mathbf{L}^2}^2 &= \int_0^1 (e^k(\xi, t))^T e^k(\xi, t)d\xi,\\ &= \int_0^1 (y^d(\xi, t) - y^k(\xi, t))^T(y^d(\xi, t) - y^k(\xi, t))d\xi,\\ &= \sum_{n=1}^{\infty}\int_0^1 ([Y_n^d(t) - Y_n^k(t)]sin(n\pi\xi))^T([Y_n^d(t)\\ &\quad -Y_n^k(t)]\sin(n\pi\xi))d\xi,\\ &\leqslant \frac{1}{2}\sum_{n=1}^{\infty}\| Y_n^d(t) - Y_n^k(t) \|^2,\\ &\leqslant \frac{1}{2}\sum_{n=1}^{N}\| e_n^k(t) \|^2 + \frac{1}{2}\sum_{n=N+1}^{\infty}\| e_n^k(t) \|^2, \end{aligned} \tag{62}$$

Furthermore, from the conclusions of Theorem 1 and Theorem 2 (or from inequalities (39) and (50)), it can be seen that

$$\begin{aligned}\lim_{k\to\infty}\|e^k(\cdot, t)\|_{\mathbf{L}^2}^2 &\leqslant \lim_{k\to\infty}\frac{1}{2}\{\sum_{n=1}^{N}\|e_n^k(t)\|^2+\sum_{n=N+1}^{\infty}\|e_n^k(t)\|^2\}\\ &\leqslant \lim_{k\to\infty}\sum_{n=N+1}^{\infty}\frac{1}{2}\| e_n^k(t) \|^2\\ &\leqslant M^2\sum_{n=N+1}^{\infty}n^{-4}, \end{aligned} \tag{63}$$

Therefore, for any small tracking accuracy ε, there exits the number of N large enough such that

$$\lim_{k\to\infty} \| e^k(\cdot, t) \|_{\mathbf{L}^2}^2 < \varepsilon. \tag{64}$$

This completes the proof of Theorem 3.

IV. NUMERICAL SIMULATION

In this section, in order to verify the effectiveness of the mixed PD-type ILC scheme, the building temperature control system equipped with air conditioning is employed.

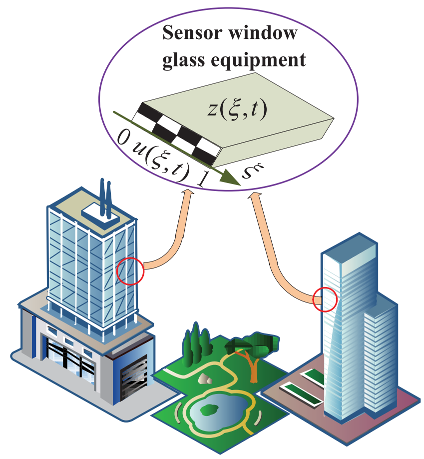

FIGURE 1. Multi-functional office building with sensor window glass equipment.

As shown in Figure 1, concern a n-storey corporate office building is equipped by multiple officers who have different temperature preferences. According to different temperature settings, the air conditioning is applied to optimal thermal management for obtaining collaborative energy [3], [50]. Moreover, the air conditioning system is composed of two operating subsystems, one is the cooling system driven by deep wells pumps, and another is the heating system which is installed in the window glass by collecting the solar radiation. Assume that the coupling is only affected between two adjacent floors, and the horizontal dimension of the solar collector is much great than the vertical size. For simplicity, the developed ILC strategy is utilized to control the temperature profile of a two storey building by means of air conditioning. With above standard assumptions and considerations [3], the following temperature control of 2-storey office building is presented.

$$E\frac{\partial z(\xi,t)}{\partial t} = D\frac{\partial^2 z(\xi,t)}{\partial \xi^2} + Az(\xi,t) + Bu(\xi,t),$$

where $(\xi, t) \in [0, 1] \times [0, 0.8]$, $z(\xi, t) \in \mathbb{R}^2$ represents the temperature spatiotemporal distribution of the system. $u(\xi, t) \in \mathbb{R}^2$ describes the control input-temperature of air conditioning.

The controlled output of the system is created as:

$$y(\xi,t) = Cz(\xi,t),$$

The process parameters are selected as [3]:

$$E = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, D = \begin{bmatrix} 8.65 & 0 \\ 0 & 8.65 \end{bmatrix}, A = \begin{bmatrix} 28.74 & -28.74 \\ -28.74 & 36.44 \end{bmatrix},$$
$$B = \begin{bmatrix} 50 & 0 \\ 40 & -60 \end{bmatrix}, C = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix},$$

subject to the initial and boundary conditions

$$z^k(0,t) = z^k(1,t) = 0, t \in [0,1],$$
$$z(\xi,0) = 0, \xi \in [0,1].$$

The desired temperature evolution both in space and time is given as

$$\begin{bmatrix} y^{1d}(\xi,t) \\ y^{2d}(\xi,t) \end{bmatrix} = \begin{bmatrix} 8(1-e^{-0.03t})(\sin 2\pi\xi) \\ 5\sin(0.6t)\sin(4\pi\xi) \end{bmatrix},$$

with $(\xi, t) \in [0, 1] \times [0, 0.8]$.

By using the same decomposed way in (15), the following simplified systems can be derived

$$\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}\dot{X}_n(t) = -(n\pi)^2 \begin{bmatrix} 8.65 & 0 \\ 0 & 8.65 \end{bmatrix} X_n(t) + \begin{bmatrix} 28.74 & -28.74 \\ -28.74 & 36.44 \end{bmatrix} X_n(t) + \begin{bmatrix} 50 & 0 \\ 40 & -60 \end{bmatrix} U_n(t),$$
$$Y_n(t) = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} X_n(t).$$

with $n = 1, 2, \cdots$.

The number of the eigenvalues is selected as 4 and the eigenfunction $\sin n\pi\xi$ is utilized to obtain the desired states of the SDPSs as follows:

$$X_{n1}^d(t) = 2\int_0^1 z^{1d}(\xi,t)\sin(n\pi\xi)d\xi, n = 1, 2, \cdots$$
$$X_{n2}^d(t) = 2\int_0^1 z^{2d}(\xi,t)\sin(n\pi\xi)d\xi, n = 1, 2, \cdots.$$

The tracking accuracy is defined as 1×10^{-3} and two desired states of the finite modes are listed as below

$$\begin{bmatrix} X_{11}^d(t) \\ X_{21}^d(t) \\ X_{31}^d(t) \\ X_{41}^d(t) \end{bmatrix} = \begin{bmatrix} 2\int_0^1 8(1-e^{-0.03t}) sin(2\pi\xi)\sin(\pi\xi)d\xi \\ 2\int_0^1 8(1-e^{-0.03t}) sin(2\pi\xi)\sin(2\pi\xi)d\xi \\ 2\int_0^1 8(1-e^{-0.03t}) sin(2\pi\xi)\sin(3\pi\xi)d\xi \\ 2\int_0^1 8(1-e^{-0.03t}) sin(2\pi\xi)\sin(4\pi\xi)d\xi \end{bmatrix}$$
$$= \left[0, 8(1-e^{-0.03t}), 0, 0\right]^{\mathrm{T}},$$

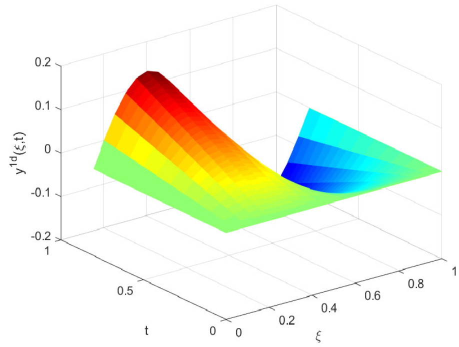

FIGURE 2. Desired surface $y^{1d}(\xi, t)$.

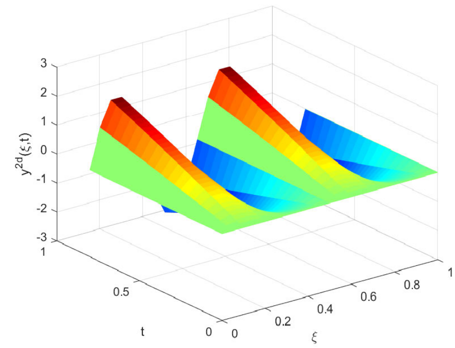

FIGURE 3. Desired surface $y^{2d}(\xi, t)$.

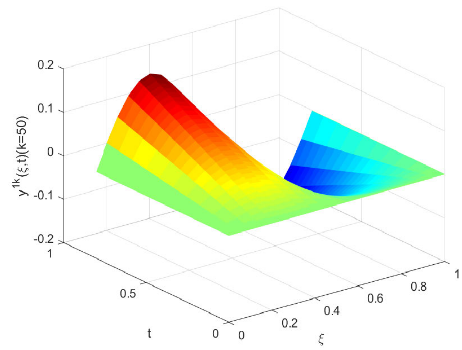

FIGURE 4. Actual output surface $y^{1k}(\xi, t)$ (k=50).

and the second desired state of the finite modes

$$\begin{bmatrix} X_{12}^d(t) \\ X_{22}^d(t) \\ X_{32}^d(t) \\ X_{42}^d(t) \end{bmatrix} = \begin{bmatrix} 2\int_0^1 5\sin(0.6t)\sin(4\pi\xi)\sin(\pi\xi)d\xi \\ 2\int_0^1 5\sin(0.6t)\sin(4\pi\xi)\sin(2\pi\xi)d\xi \\ 2\int_0^1 5\sin(0.6t)\sin(4\pi\xi)\sin(3\pi\xi)d\xi \\ 2\int_0^1 5\sin(0.6t)\sin(4\pi\xi)\sin(4\pi\xi)d\xi \end{bmatrix}$$
$$= \left[0, 0, 0, 5\sin(0.6t)\right]^{\mathrm{T}}.$$

The specific mixed PD-type ILC steps for SDPSs can be summarized as follows:

Step 1: By virtue of the variable separation principle in Eqs. (8)-(11), the infinite number of singular systems (15) can be obtained.

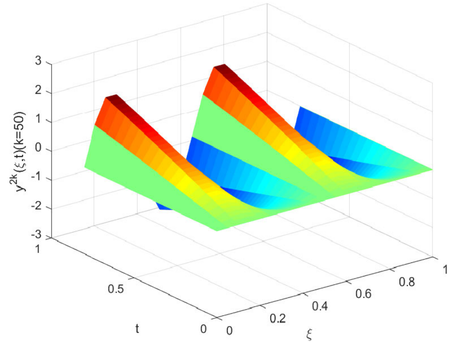

FIGURE 5. Actual output surface $y^{2k}(\xi, t)$ (k=50).

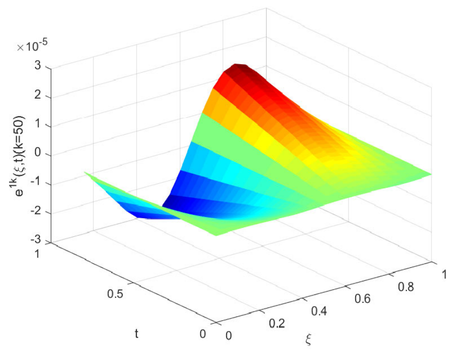

FIGURE 6. Error surface $e^{1k}(\xi, t)$ (k=50).

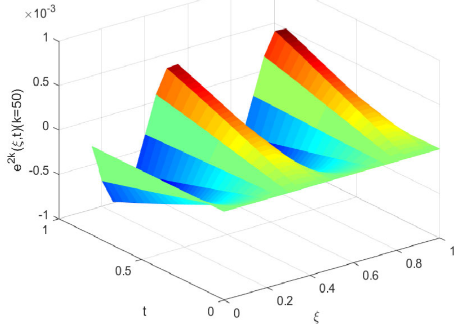

FIGURE 7. Error surface $e^{2k}(\xi, t)$ (k=50).

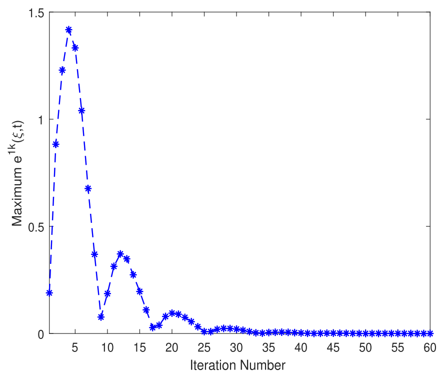

FIGURE 8. Maximum error $e^{1k}(\xi, t)$-iteration number curve.

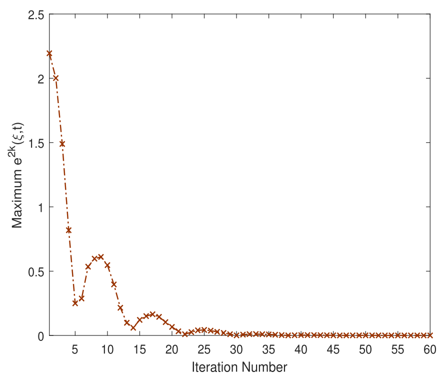

FIGURE 9. Maximum error $e^{2k}(\xi, t)$-iteration number curve.

*Step 2:*For the generalized matrix E, utilizing the nonsingular matrix $P = I$ and $Q = I$ to transform the singular systems (15) into the restricted equivalent form (17).

Step 3: Setting the tracking accuracy as $\varepsilon = 1 \times 10^{-3}$, in order to meet the tracking accuracy requirements, the number of finite-dimensional modes is chosen as four.

Step 4: Selecting the following learning gain matrices so that the convergence condition (14) for the finite dimensional mode can be satisfied in Theorem 3,

$$\Gamma_1 = \begin{bmatrix} 0.6 & 0 \\ 0 & -0.3 \end{bmatrix}, \quad \Gamma_2 = \begin{bmatrix} -0.8 & 0 \\ 0 & -0.9 \end{bmatrix}.$$

Step 5: The initial state value $X_n(0)$ and the input value $U_n(0)$ for control at the beginning of learning are set to be 0, then start iteration by applying the mixed PD-type learning law (21). Subsequently, calculating state error $e^{ik}(\xi, t)$, $i = 1, 2$.

Step 6: Checking the termination conditions such that until the specified error precision $|e^{ik}(\xi, t)| < 1 \times 10^{-3}$, $i = 1, 2$ meets. Otherwise, the iteration number will be incremented by one, which is $k = k + 1$ and the systems (17) will operate repetitively.

Through the aforementioned steps, the following simulation results can be obtained, that is, Fig. 2 to Fig. 9.

The two desired output surfaces $y^{1d}(\xi, t)$ and $y^{2d}(\xi, t)$ are presented in Fig. 2 and Fig. 3. The actual output surfaces $y^{1k}(\xi, t)$ and $y^{2k}(\xi, t)$ are given in Figs. 4-5, respectively.Combining Figs. 2–3 with Figs. 4–5, it discovers that the desired profile approach consistently to corresponding actual output in the 50th iteration. Besides, in view of Fig. 6 and Fig. 7 which depict the tracking error both in space and time, it can observe that, the max tracking error of surfaces are 2.242×10^{-5} and 6.995×10^{-4}, respectively, in 50th iteration.

As the number of iteration increases, the mixed PD-type ILC law becomes more and more effective that demonstrated in the last two figures. In the 40th iteration, the control performance of system is good enough. More specifically, the maximum of the absolute values $e^{1k}(\xi, t)$ and $e^{2k}(\xi, t)$ converge to the specified precision ($<1 \times 10^{-3}$) after the

40th iteration, which confirms the effectiveness of the proposed control laws.

V. CONCLUSION

This paper has adopted the model reduction method and designed a mixed PD-type learning law. In contrast to the design finite-dimensional controllers for distributed parameter systems in other literature, the eigenvalues of systems after a given mode were not assumed to be less than zero. Instead, the parabolic SDPSs with infinite-dimensional has transformed into the infinite number of singular systems based on the separation principle, and then the dynamic equivalent form has been obtained by the nonsingular transformation. A mixed PD type learning control law has been designed for the low-dimensional modes. The convergence condition of tracking error has been presented in the light of the contraction mapping principle. Through the selection of the number of low-dimensional modes, the convergence of the output errors in high-dimensional modes has been guaranteed, and the accuracy of the overall output errors can also been adjusted. In the simulation study, by the appropriate control parameters, the tracking process for the desired temperature evolution with the predetermined error accuracy in the temperature control of 2-storey office building has been achieved.

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