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QUALITY PAPER

Mixed two- and four-level experimental designs for interchangeable parts with different degrees of assembly difficulty

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Abstract

Purpose – The purpose of this paper is to show how to properly use the method of replacement to construct mixed two- and four-level minimum setup split-plot type designs to accommodate the presence of hard-to-assemble parts.

Design/methodology/approach – Split-plot type designs are economical approaches in industrial experimentation. These types of designs are particularly useful for situations involving interchangeable parts with different degrees of assembly difficulties. Methodologies for designing and analyzing such experiments have advanced lately, especially for two-level designs. Practical needs may require the inclusion of factors with more than two levels. Here, the authors consider an experiment to improve the performance of a Baja car including two- and four-level factors.

Findings – The authors find that the direct use of the existing minimum setup maximum aberration (MSMA) catalogs for two-level split-plot type designs may lead to inappropriate designs (e.g. low resolution). The existing method of replacement for searching exclusive sets of the form (α, β, $\alpha\beta$) available in the literature is suitable for completely randomized designs, but it may not provide efficient plans for designs with restricted randomization.

Originality/value – The authors provide a general framework for the practitioners and have extended the algorithm to find out the number of generators and the number of base factor at each stratum, which guide the selection of mixed two-level and four-level MSMA split-plot type designs.

Keywords Hard-to-change factor, Minimum setup, Prototype testing, Regular design, Restricted randomization, Split-plot fractional factorial design

Paper type Case study

1. Introduction

Two-level factorial experiments have been the most widely employed type of design in industrial settings (Ilzarbe *et al.*, 2008; Prvan and Street, 2002). However, in many practical situations we are faced with mixed-level or multi-level experiments (e.g. Bagheri, Sarajia and Naderia, 2000; Bagheri, Saraji, Chitsazan, Mousavi and Naderi, 2000; Chee *et al.*, 1995; Chee *et al.*, 1996; Hunter and Naylor, 1970). This is especially true when the experimental factors are naturally comprised of different number of levels, e.g., type of solvent (ethyl acetate, acetonitrile, methanol, hexane) in a chemical process. Taguchi matrices can be used to select designs with various combinations of factors and levels. These matrices

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represent orthogonal arrays based on the theory of factorial designs (Kacker *et al.*, 1991). However, Taguchi matrices lack explicit account for hard-to-change factors (Simonovic and Kalin, 2016, p. 12, Section 2.2).

In the presence of hard-to-change factors, split-plot type designs are a common approach to execute an experiment (e.g. Bailey, 1983; Huang *et al.*, 1998; Ju and Lucas, 2002; Goos and Vandebroek, 2003; Vining *et al.*, 2005; Jones and Goos, 2007; Anbari and Lucas, 2008; Cheng and Tsai, 2009; Jones and Nachtsheim, 2009). Although interest in such experiments has been growing, most of the cases consider factors with only two degrees of difficulty for changing levels: hard-to-change and easy-to-change factors (e.g. Box and Jones, 1992; Bisgaard *et al.*, 1996; Vivacqua and Bisgaard, 2004, 2009; Verma and Jha, 2015). However, there are some few exceptions, for example, Paniagua-Quiñones and Box (2008, 2009). In this paper, we consider experiments involving three and four interchangeable parts with different degrees of assembly difficulties. The concept of interchangeability is based on the notion that all parts of the same type are identical (Bisgaard, 1997). Generally, during prototype testing, only one part of each type is available for experimentation.

We assume that the possible configurations of each part can be expressed by the combination of the levels of one or more factors with a total of $k+j$ factors, in which k of them are four-level factors and the remaining j have two levels. The simplest way to construct mixed two- and four-level completely randomized designs is to consider a fractional factorial design 2^{q-p}, where $q=2k+j$. Each four-level factor is replaced by two two-level pseudo-factors and their interactions using a three-column-system representation, labeled as $(\alpha, \beta, \alpha\beta)_i$, $i=1, \ldots, k$. This procedure is known as method of replacement (see e.g. Wu and Hamada, 2009). Catalogs for up to three four-level factors and 13 two-level factors of completely randomized designs considering the minimum aberration (MA) criterion (see e.g. Fries and Hunter, 1980; Bingham and Sitter, 1999a, b, 2003; Bingham *et al.*, 2004) are available for 8, 16, and 32 runs (see e.g. Ankenman, 1999; Wu and Hamada, 2009). A completely randomized design is useful when all parts have the same degree of assembly difficulty. When the effort to assemble each part or a group of parts is different, split-plot type designs can lead to convenient savings in the execution of the experiment. Moreover, it makes sense to have a lower number of setups associated with the harder-to-assemble parts. With this objective, Ho *et al.* (2015) introduced the minimum setup (MS) criterion. In this paper we use the method of replacement to construct mixed two- and four-level split-split-plot (here denoted by split^3-plot to simplify the notation) and split-split-split-plot (split^4-plot) designs from 2^{q-p} designs according to the MS criterion. Trinca and Gilmour (2015) provide a general way of creating multistratum designs. Their approach takes into account the optimality criteria based on theoretical design properties and usually leads to irregular fractional factorial designs. Here, however, considering a practitioner's point of view, we have decided to emphasize a more practical criterion (MS) and for the sake of simplicity on the analysis, we restrict our construction method to regular designs. The paper is organized as follows: the motivating experiment and its executed design are described in Sections 2 and 3, respectively; alternative designs and their comparisons with the executed one are presented in Section 4; a general framework for MS two-and four-level split-plot type designs is presented in Section 5 and conclusions are outlined in Section 6.

2. Baja SAE racing experiment

Baja SAE racing is a competition promoted by the Society of Automotive Engineers (SAE) in which teams of students from universities all over the world design and build small off-road vehicles. Besides the performance of the car, the evaluation process considers detailed reports and presentations of the engineering and design process used in developing each system of the team´s vehicle, which should be supported with rigorous engineering principles.

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One team of students who traditionally participate in the Baja SAE competition is faced again with the challenge of building an improved Baja car prototype. Based on the successful experience of employing statistically planned experiments to build the car for a previous race (Ho *et al.*, 2015), the team understands the benefits of continuously applying the methodology to guide their decision-making process in the design phase of the project. Similar to the previous experiment, the objective is to maximize the performance of the vehicle on two tests carried out on a paved street with an asphalt layer. The first one, called acceleration test, evaluates the time that the vehicle takes to cover a distance of 30 meters starting from a complete stop. The second one, called velocity test, measures the final velocity reached by the car between the 99 and 100 meters mark. The ideal setup is the one that simultaneously provides the maximum final velocity and the minimum time to cover the first 30 meters.

In this experiment, the focus is the transmission system. The following parts related to the transmission system are considered: gear, driven clutch springs, driven clutch, and gearbox. Table I summarizes the seven factors identified for the study by the student team. Two different types of gears are available for testing. Four different types of drive clutch springs are tested; four levels of pre-compression of the driven clutch springs are used; four levels of the driven clutch masses are available; two different geometries of ramp are employed; four levels of engine speed are considered; and two levels of the gear ratio are used. The levels for each factor are left undisclosed due to confidentiality. There are a total of 2,048 possible combinations to assemble the transmission system of the Baja vehicle. The team has a short time period to run tests to design the transmission system. In addition, there is only one piece available of each part type. With the next race approaching and based on past experience, the team reached a consensus that would be able to execute an experiment with up to 32 runs.

Moreover, assembling different transmission parts is associated with distinct levels of difficulty and also distinct times to complete the task. It is suffice to say that the setup of the gearbox, switching engine speed (factor F), and gear ratio (factor G), is a lot easier than assembling the masses (factor D) and geometry of the ramp (factor E) of the driven clutch, which turns out to be easier than assembling the driven clutch springs (factors B and C). Finally, changing the type of gear (factor A) takes the longest time due to its complexity. In summary, the Baja experiment involves seven factors, which can be

Degree of Assembly difficulty	Transmission Part	Factor Label	Description of the Factor	Number of Levels
Difficult	Gear	A	Type of gear	2
↓	Driven clutch springs	B	Type of springs	4
↓		C	Pre-compression of the springs	4
↓	Driven clutch	D	Masses	4
↓		E	Geometry of ramp	2
↓	Gearbox	F	Engine speed	4
Easy		G	Gear ratio	2

Table I. Degree of assembly difficulty of transmission parts and factors for the Baja SAE racing experiment

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associated to four parts with different degrees of assembling difficulty, as shown in first column of Table I.

The design is given by a split[3]-plot plan represented by $(2)\times(4^2)\times(4\times2)\times(4\times2)$. The specific design of the Baja competition experiment is described on Section 4.

3. About the minimum setup minimum aberration (MSMA) criterion

To select a design, we should specify appropriate criteria. As pointed out in Ho *et al.* (2015), the MS criterion is driven by the restrictions present in real applications, especially due to hard-to-change factors. Thus, the MSMA criterion is chosen as the baseline to select designs in the Baja experiment. The authors have shown that the MS criterion not only can be used to differentiate designs with the same word length pattern (WLP), but also is independent from the MA criterion. To illustrate these aspects let us consider the designs, d_1 and d_2, both with six factors, three whole-plot factors (one four-level factor, namely, Factor A, and two two-level factors, Factors B and C) and three sub-plot factors with the same configuration (two two-level factors, namely, factors D and E and one four-level factor F). By the method of replacement, each four-level factor can be replaced by two two-level pseudo-factors. Consequently, two two-level pseudo-factors K_1 and K_2 are assigned for a generic four-level factor K. The generators of designs d_1 and d_2 are put in Table II.

Both designs have the same WLP. Therefore, from the WLP criterion, both d_1 and d_2 are indistinguishable. Nevertheless, taking into account the number of setups at each stratum (setup pattern) d_1 and d_2 are no longer equivalent, and so the MS criterion can be used to differentiate designs with the same WLP, whenever hard-to-change factors are present. The independency between MS and MA criteria is illustrated by the design d_3 with three generators at the sub-plot level (and other information are also put in Table II). The resolution of d_3 is IV (as its WLP presents seven words of length 4) better than d_2 (which has Resolution III). Nonetheless, from the perspective of MS criterion, the number of setups in each stratum in d_3 is (16, 32), whereas in d_2 is (8, 32). So, in terms of number of setups, d_2 is better than d_3. Thus, when the experiment is driven by hard-to-change factors, one approach is to first look up for MS designs and then within this class find a subclass of designs with MA and clear effects. A key step in the search of MS designs is the determination of the number of generators for each stratum.

One approach to find generators is to use the catalogs for MSMA split-plot type designs available in Ho *et al.* (2015). These catalogs are built for two-level factors and so caution must be taken when using to four-level factors. This issue is addressed in Section 4.

4. The executed and alternative designs for the Baja competition experiment

This section presents the executed design for the Baja experiment. Alternative plans are also discussed in this section. This car prototype study is a 32-run experiment with seven

	Designs		
	d_1	d_2	d_3
Split-plot design	$(4^1\times2^2)\times(2^{2-1}\times4^{1-1})$	$(4^1\times2^{2-1})\times(2^2\times4^{1-1})$	$(4^1\times2^2)\times(2^{2-1}\times4^{1-1})$
Written as 2-level factors	$2^4\times2^{4-3}$	$2^{4\text{-}1}\times2^{4-2}$	$2^4\times2^{4-3}$
Whole-plot generator		$C=A_1B$	
Sub-plot generators	$E=A_1B$; $F_1=A_1C$; $F_2=BCD$	$F_1=A_1BD$; $F_2=BDE$	$E=BCD$; $F_1=A_1BC$; $F_2=A_2BD$
WLP	(3, 3, 1)	(3, 3, 1)	(0, 7, 0)
Setup pattern	(16, 32)	(8, 32)	(16, 32)
Resolution	III	III	IV

Table II. Properties of the designs d_1, d_2, and d_3

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mixed two- and four-level factors. The approach considered is to exchange each four-level factor (e.g. factor B) by two two-level pseudo-factors (e.g. B_1 and B_2) and to use three columns labeled as (B_1, B_2, B_1B_2) to generate one four-level factor (Wu, 1989). This procedure is repeated four times to accommodate the four four-level factors in the Baja experiment. In terms of run size, the experiment would be equivalent to a 2^{11-6} fractional factorial design. In the literature catalogs of MA designs up to three four-level factors and up to thirteen two-level factors are available (see Ankenman, 1999).

Considering that there is only one piece of each auto part and that the different prototypes are assembled and tested sequentially a completely randomized design was, then, discarded as a plausible option, since it requires 32 complete independent assemblies and disassemblies of the vehicle. A natural choice is a split^3-plot design, because the seven factors from the Baja experiment are classified in four groups based on a decreasing degree of difficulty in changing their levels. Hence, it is more rational to execute all runs under the same gear (factor A from the first group, the most difficult to change) and then switch the other one to carry out the remaining runs, thus requiring only two setups of the gear. Moreover, it is more economical to execute a small number of setups of the driven clutch springs, and so on.

For the Baja experiment, from now on, let us refer to this design, executed by the engineering student team, as d_4. According to these catalogs, the MS design (after replacement of each four-level factor into two two-level factors) is $2^{(1-0)} \times 2^{(4-2)} \times 2^{(3-2)} \times 2^{(3-2)}$ which results in (2, 8, 16, 32) setups for each stratum. In this case, B_1 and B_2 (two-level factors) are replaced for the four-level factor B. Likewise, we proceed to the remaining three four-level factor, that is, C_1 and C_2 for the four-level factor C; analogously, D_1 and D_2 for the four-level factor D and; in the same fashion, F_1 and F_2 for the four-level factor F. Properties of this design are put at the first block of columns in Table III as the number of four- and two-level factors, the base factors and the generators of each stratum, the setup pattern, WLP, and the resolution.

The design d_4 should have Resolution III and would be optimum, in terms of MSMA criterion, if all factors were two-levels. The employment of the catalogs available for split-plot-type designs (Ho *et al.*, 2015), for experiments with mixed two- and four-level factors, requires special attention and care. According to these catalogs, the setup pattern of design d_4 is (2, 8, 16, 32). After the replacement of four-level factors by two two-level pseudo-factors, the factors labels were as follow: A, B_1, B_2, C_1, C_2, D_1, D_2, E, F_1, F_2, and G. Considering also the six generators in Table III, some interesting results are observed. The generators $B_2 = AB_1$ and $C_2 = AC_1$, chosen in the second stratum, yield the aliases $B_1B_2 = A = C_1C_2$. However, $B_1B_2 = B_3$ and $C_1C_2 = C_3$ represent part of the main effects of the four-level factors B and C, respectively, since each four-level factor has three degrees of freedom (three columns or contrasts) associated with it. In the third stratum the generators assigned are $D_2 = AD_1$ and $E = B_1C_1D_1$, so part of the main effect of the four-level factor D is also confounded with the main effect of the two-level factor A ($D_1D_2 = D_3 = A$). So, considering the four-level factors, this design has six words of length two, leading to a Resolution II instead of III, as for a two-level design. The WLP of design d_4 (including words of length two) is WLP (d_4) = (6; 3; 18; 23; 11; 2). Specifying WLP(d_4) according to the type of words, in the sense of Wu (1989), Wu and Zhang (1993) and Wu and Hamada (2009), we get: {(3; 3; 0; 0); (0; 1; 2; 0); (0; 3; 13; 2); (0; 2; 14; 7); (0; 0; 4; 7); (0; 0; 0; 2)}, where each integer A_{ij} denotes the number of i four-level factors included in the composition of a word of length j; $j = 2, \ldots, 7$ and $i = 0, 1, 2, 3$. For example, for the six words of length 2, three of them are type 0 (involves only two-level factors) and three of them are type 1 (involve components of one four-level factor).

Note that by the setup pattern, up to the second stratum in the Baja executed design d_4, eight setups are considered. Nonetheless, eight runs are insufficient to estimate the main effects without confounding with other main effects. Wu and Hamada (2009) show that a

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Table III. The executed and alternative designs of Baja experiment

		Designs											
		d_4 Stratum				d_5 Stratum				d_6 Stratum			
		1	2	3	4	1	2	3	4	1	2	3	4
4 level factors	No.	0	2	1	1	0	2	1	1	0	2	1	1
	Labels	–	B1, B2, C1, C2	D1, D2	F1, F2	–	B1, B2, C1, C2	D1, D2	F1, F2	–	B1, B2, C1, C2	D1, D2	F1, F2
2 level factors	No.	1	0	1	1	1	0	1	1	1	0	1	1
	Labels	A	–	E	G	A	–	E	G	A	–	E	G
Base factor		A	B1, C1	D1	F1	A	B1, B2, C1	D1		A	B1, B2, C1	D1	G
Generators			B2 = AB1	D2 = AD1	F2 = B1C1F1		C2 = AC1	D1 = AB2	F1 = AB3		C2 = AC1	D1 = AB2	F1 = AB3
			C2 = AC1	E = BC1D1	G = B2D1F1			D2 = AC1	F2 = AB2C1			D2 = AC1	F2 = AB2C1
								E = AB3C1	G = B3C1			E = AB3C1	
Split plot design				$2^{(1-0)} \times 2^{(4-2)} \times 2^{(3-2)} \times 2^{(3-2)}$				$2^{(1-0)} \times 2^{(4-1)} \times 2^{(3-3)} \times 2^{(3-3)}$				$2^{(1-0)} \times 2^{(4-1)} \times 2^{(3-3)} \times 2^{(3-2)}$	
Set up pattern				2,8,16,32				2,16,16,16				2,16,16,32	
WLP				(6,3,18,23,11,2)				(0,31,33,30,30,3)				(14; 19; 14; 12; 4)	
Resolution				II				III				III	
No. of runs				32				16				32	

maximum of one four-level factor can be assigned with this quantity of setups if one desires a design of Resolution III, since exists only a single triple of the form (α, β, $\alpha\beta$). However, in the Baja executed design d_4, there are two four-level factors in the second stratum, thus two mutually exclusive triples of the form (α, β, $\alpha\beta$) are needed to assign the two four-level factors to result a design of Resolution III. So the number of runs up to the second stratum needs to be increased to 16 runs. But due to operational or cost restrictions if one really needs to use only eight runs up to second stratum, then the resolution of the design is II instead of III, since some main effects are confounded with other main effects. This feature negatively affects the analysis of the executed experiment. Therefore, the results of the actual experiment on the Baja vehicle are omitted here. Instead, our approach in this paper is to call attention to this fact and to show ways to avoid this situation.

Let us consider an alternative procedure based on the method of replacement (see Wu, 1989; Mukerjee and Wu, 1995) and an algorithm to search exclusive sets of the form (α, β, $\alpha\beta$) to find out the setup pattern of an alternative design d_5. The number of main effect contrasts to be estimated in a mixed j two-level and k four-level factor design is $t=3k+j$ effects (higher than designs with all two-level factors). Let $r=int$ [$\log_2$(t)] and for $2^r \leqslant t \leqslant 2^{(r+1)}-1$, the minimum number of runs necessary to execute the experiments is $N=2^m$; $m=r+1$; if $k \leqslant ((2^m-1)/3)$, for m being even or $k \leqslant ((2^m-5)/3)$, for m being odd otherwise, $m=r+2$. As $N=2^m=2^{(q-p)}=2^{(2k+j-p)}$ then $p=(2k+j-m)$ is the number of the generators and m is the total number of base factors. The total number of four-level factors to be assigned is limited by the total number of mutually exclusive sets of columns of the form (α, β, $\alpha\beta$) in $2^{(q-p)}$ runs. Table IV presents the minimum value of N for $0 \leqslant k, j \leqslant 15$.

First, we address the MS for a completely randomized mixed two-level and four-level design employing Table IV. In the Baja experiment, we have four four-level factors and three two-level factors. From Table IV, a minimum of 16 runs is needed.

Additionally, it seems reasonable to apply the same procedure to determine the number of generators (or the number of setups) at each stratum. Similarly, the number of four-level factors to be assigned at each stratum is limited by the number of mutually exclusive sets of columns of the form (α, β, $\alpha\beta$) in $2^{\sum_1^i q_i-p_i}$ runs with $q_i=2k_i+j_i$; $k_i=$ no. of four-level factors; $j_i=$ no. of two-level factors; $p_i=$ no. of generators at stratum i if one wishes the estimation of the main effects not confounded with other main effects.

No. of two-level factor	No. of four-level factor (k)															
(j)	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
0	–	4	16	16	16	16	32	32	32	32	64	64	64	64	64	64
1	2	8	16	16	16	32	32	32	32	32	64	64	64	64	64	64
2	4	8	16	16	16	32	32	32	32	32	64	64	64	64	64	64
3	4	8	16	16	16	32	32	32	32	32	64	64	64	64	64	64
4	8	8	16	16	32	32	32	32	32	32	64	64	64	64	64	64
5	8	16	16	16	32	32	32	32	32	64	64	64	64	64	64	64
6	8	16	16	16	32	32	32	32	32	64	64	64	64	64	64	64
7	8	16	16	32	32	32	32	32	32	64	64	64	64	64	64	64
8	16	16	16	32	32	32	32	32	64	64	64	64	64	64	64	64
9	16	16	16	32	32	32	32	32	64	64	64	64	64	64	64	64
10	16	16	32	32	32	32	32	32	64	64	64	64	64	64	64	64
11	16	16	32	32	32	32	32	64	64	64	64	64	64	64	64	64
12	16	16	32	32	32	32	32	64	64	64	64	64	64	64	64	64
13	16	32	32	32	32	32	32	64	64	64	64	64	64	64	64	64
14	16	32	32	32	32	32	64	64	64	64	64	64	64	64	64	64
15	16	32	32	32	32	32	64	64	64	64	64	64	64	64	64	64

Table IV.
Minimum number of runs for mixed two-level and four-level factors design

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Repeating this procedure at each stratum in the Baja experiment:

- First stratum: only one two-level factor and a minimum of two runs → it corresponds to the design $2^{(1-0)}$.
- Second stratum: cumulative there are two four-level factors and one two-level factor. From Table IV, a minimum of 16 runs is needed, so one generator is needed → results the design $2^{(1-0)} \times 2^{(4-1)}$.
- Up to third stratum: there are three four-level factors and two two-level factors. From Table IV, again a minimum of 16 runs is enough, so three generators are required → provides the design $2^{(1-0)} \times 2^{(4-1)} \times 2^{(3-3)}$.
- Up to fourth stratum: again a minimum of 16 runs is enough and consequently three more generators are needed yielding the following design $2^{(1-0)} \times 2^{(4-1)} \times 2^{(3-3)} \times 2^{(3-3)}$.
- Applying such procedure, the setup pattern becomes: (2, 16, 16, 16).

On the assignments of the generators, one possible design (d_5) is:

- Stratum 1: factor A is the first base factor.
- Stratum 2: factors B and C are both four-level factors; replacing the factor B by two two-level factors B_1 and B_2 and letting both being the second and the third base factors, respectively, the interaction $B_1B_2 = B_3$ is also a main effect; the factor C replaced by two two-level factors C_1 and C_2 with C_1 as the fourth base factor, C_2 has three possibilities of assignment: $C_2 = AB_1$ (and $C_3 = AB_1C_1$) or $C_2 = AB_2$ (and $C_3 = AB_2C_1$) or $C_2 = AB_3$ (and $C_3 = AB_3C_1$).
- Stratum 3: considering the first choice at Stratum 2 (i.e. $C_2 = AB_1$ and $C_3 = AB_1C_1$) the four possibilities are: $D_1 = AB_2$; $D_2 = AC_1$; (and $D_3 = B_2C_1$); $E = AB_3C_1$ or $D_1 = AB_2$; $D_2 = B_1C_1$; (and $D_3 = AB_3C_1$); $\mathrm{E} = AB_2C_1$ or $D_1 = AB_3$; $D_2 = AB_2C_1$; (and $D_3 = B_1C_1$); $E = B_3C_1$ or $D_1 = AC_1$; $D_2 = AB_3$; (and $D_3 = B_3C_1$); $E = B_2C_1$.
- Stratum 4: considering chosen the first possibility (i.e. $D_1 = AB_2$; $D_2 = AC_1$; (and $D_3 = B_2C_1$); $E = AB_3C_1$) then there is a single option: $F_1 = AB_3$; $F_2 = AB_2C_1$; (and $F_3 = B_1C_1$); $G = B_3C_1$ which is exactly the third alternative of the Stratum 3. In the design d_5, generators of factors D and F can be each other switched; similarly with generators of the factors E and G.
- All 24 possible designs are MA of Resolution III, and present the same WLP(d_5) = (31, 33, 30, 30, 3), considering words of length three and higher. Specifying by type of defining relations, we have WLP(d_5) = {(1; 18; 12; 0), (0; 18; 12; 3), (0; 0; 12; 18), (0; 0; 12; 18), (0; 0; 0; 3)}.

Properties of design d_5 are put in the second block of columns of Table IV. Another alternative design d_6 can also be proposed. It is a slight modification of previous design d_5 but built with 32 setups in the last stratum. This design is also of Resolution III and may be written as $2^{(1-0)} \times 2^{(4-1)} \times 2^{(3-3)} \times 2^{(3-2)}$ with the setup pattern (2, 16, 16, 32). Among more than 4 thousand possibilities, the WLP of MA design is WLP(d_6) = (14; 19; 14; 12; 4). Specifying by type of defining relations, we have W(d_6) = {(0; 8; 6; 0), (0; 8; 10; 1), (0; 2; 6; 6), (0; 0; 2; 10), (0; 0; 0; 4)} (see the third block of columns in Table III).

Now we are faced with a problem: which design plan should be chosen? The three proposals have positive and negative aspects. Design d_4 provides less number of setups in the second stratum; but main effects are confounded with other main effects. Design d_5 requires less resources (cost and time) since it uses less number of total runs. It is a Resolution III split-plot design but presents a greater number of main effects confounded with second-order interactions than d_6 and also a larger number of setups at the second stratum than d_4. The third option d_6 is also a Resolution III design but the number of setups for the last stratum is greater than d_5, but less number of main effects is confounded with second-order interactions.

In practical situations, the three proposals, depending on the point of view, present advantages and disadvantages. However, if operational restrictions really exist and/or it is known that there is not effect of the most-hard-to-change factor assigned, the first option could be chosen in some situations. The reason is based on the fact that it would be unlikely that the three main factors in a four-level factor would be simultaneously all active.

However, the above procedure of allocation of four-level factors and the determination of the setup pattern may not be always efficient to provide a plan of Resolution III. Let us illustrate with an example. Consider another design d_7 based on a slight modification of the Baja experiment. The description of the factors and the corresponding number of levels at each stratum is in Table V.

From Table III, a minimum of 32 runs is enough for five four-level factors and four two-level factors. After the replacement of one four-level factor by two two-level pseudo-factors, the previous procedure to find out the setup pattern is applied, obtaining (8, 16, 16, 32). The first four base factors are: A; B_1; B_2; C_1. The following generators up to third stratum are considered: $C_2 = AB_1$; $D_1 = AB_2$; $D_2 = AC_1$; $E = AB_3C_2$; $F_1 = AB_3$; $F_2 = AB_2C_1$; $G = B_3C_1$. Note that up to third stratum, 16 effects are to be estimated. To assign generators for the factors in the fourth stratum, a fifth base factor needs to be included, for example H_1. However, there is not any triple mutually exclusive set of columns of the form (H_1, β, $H_1\beta$) to be assigned for the main effects of the factor H (which allows at least the estimation of the main effects without confounding with other main effects) since up to the third stratum, all 15 possible effects (related to the four base factors) are assigned to some main effects leaving any effect free to compose with the fifth base factor. So, if one desires a design of Resolution III, some adjustments are needed. In this case, the setup pattern should be (8, 16, 32, 32) to accommodate all five four-level factors that corresponds to the design: $2^{(3-0)} \times 2^{(5-4)} \times 2^{(3-2)} \times 2^{(3-3)}$.

The previous procedure worked for designs d_5 and d_6. In design d_5, since up to third stratum, four effects are left to be assigned for next stratum: the four-level factor F and one two-level factor G. And in the design d_6 the four free effects may be combined with the fifth base factor to compose the triple of the form (5, β, 5β) for assignment of the factor F. So the previous procedure needs some adjustments or additional restrictions to find out the setup pattern in order to provide a design at least of Resolution III. The general framework is the subject of Section 5.

5. A general framework

We now introduce the notation for mixed two-level and four-level split-plot type designs with minimum number of setups in each stratum. Suppose that there are a total of

Table V.
Design d_7

Stratum	Factor	No. of level	No. of effects
1	A	2	1
	B	4	3
2	C	4	3
	D	4	3
	E	2	1
3	F	4	3
	G	2	1
4	H	4	3
	J	2	1

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$k+j$ factors; k four-level factors and j two-level factors. These factors can be divided into s groups according to the degree of difficulty in changing their levels. The groups of factors are arranged in decreasing order according to the degree of difficulty in such a way that the first group is the most difficult to change and the last group is the easiest to change.

Each group has k_i four-level factors and j_i two-level factors such that $\sum_1^s k_i = k$; $\sum_1^s j_i = j$; $\sum_1^s (k_i+j_i) = k+j$. Note that up to stratum (group) i there are $t_i = \sum_1^i 3k_i + j_i$ contrasts associated to main effects to be estimated: $3k_i$ related to the four-level factors; j_i to the two-level factors.

Split-plot type experiments involve several strata, leading to different kinds of experimental units. The number of strata is the same number of groups of factors (s). Here, we denote the design as a split^{s-1}-plot to simplify notation. The number of experimental units in each stratum, in an unreplicated design, is equal to the number of treatment changes considered in the respective stratum. Furthermore, the total number of runs of the experiment is equal to the number of treatment setups in the last stratum.

Hence, we need to distinguish among the number of treatment setups for each stratum, N_i and the total number of runs N involved in a single replicate of the experiment. In general, in unreplicated experiments, N_i, $i=1, 2, \ldots, s$ also represents the number of experimental units associated with the ith stratum and N, the number of experimental units associated with the combined design.

For the first stratum, there are k_1 four-level factors; j_1 two-level factors and a total of $t_1 = 3k_1 + j_1$ main effect contrasts to estimate. Let $r_1 = int\ [\log_2(t_1)]$ and for $2^{r_1} \leqslant t_1 < 2^{r_1+1}-1$, $m_1 = r_1+1$ if $k_1 \leqslant ((2^{m_1}-1)/3)$ for even m_1 or $k_1 \leqslant ((2^{m_1}-5)/3)$ for odd m_1 otherwise $m_1 = r_1+2$.

For $t_1 = 2^{r_1+1}-1$, $m_1 = r_1+2$ if $k_1 \leqslant ((2^{m_1}-1)/3)$ for even m_1 or $k_1 \leqslant ((2^{m_1}-5)/3)$ for odd m_1 and k_2 0, otherwise $m_1 = r_1+1$.

The minimum number of runs is $N_1 = 2^{m_1}$. We then use a $2^{q_1-p_1}$ fractional factorial design with $q_1 = 2k_1 + j_1$ and $N_1 = 2^{q_1-p_1}$ treatments; $p_1 = q_1 - m_1$, the number of generated factors.

For a general ith stratum, we have k_i four-level factors; j_i two-level factors, and $t_i = \sum_1^i 3k_i + j_i$ effects to estimate up to ith stratum. Let $r_i = int\ [\log_2(t_i)]$ and for $2^{r_i} \leqslant t_i < 2^{r_i+1}-1$; $m_i = r_i+1$; if $\sum_1^i k_i \leqslant ((2^{m_i}-1)/3)$, for even m_i or $\sum_1^i k_i \leqslant ((2^{m_i}-5)/3)$ for odd m_i; otherwise $m_i = r_i+2$.

For $t_i = 2^{r_i+1}-1$, $m_i = r_i+2$, if $\sum_1^i k_i \leqslant ((2^{m_i}-1)/3)$, for even m_i or $\sum_1^i k_i \leqslant ((2^{m_i}-5)/3)$, for odd m_i and $k_{i+1} > 0$; $i+1 \leqslant s$; otherwise $m_i = r_i+1$. The minimum number of runs up to ith stratum is $N_i = 2^{m_i}$. So, a $2^{q_i-p_i}$ fractional factorial design with $q_i = 2k_i + j_i$ and $q_i - p_i = m_i - m_{i-1}$ can be used ($m_0 = 0$) and the combined design up to the ith stratum is a $2^{(\sum_1^i q_i - p_i)}$ fractional factorial design with $N_i = 2^{(q_1-p_1)} \times \cdots \times 2^{(q_i-p_i)} = 2^{m_1} \times 2^{(m_2-m_1)} \times \cdots \times 2^{(m_i-m_{i-1})}$ treatments.

Note that $\sum_1^s p_i = p$ represents the number of generators of the fractionated design. The total number of runs of the complete combined design is $N = N_s = 2^{(q_1-p_1)} \times \cdots \times 2^{(q_s-p_s)} = 2^{m_1} \times 2^{(m_2-m_1)} \times \cdots \times 2^{(m_s-m_{s-1})}$.

The same number of runs can be obtained, for example, by a 2^{q-p} completely randomized fractional factorial design or a $2^{(q_1-p_1)} \times 2^{(\sum_2^s q_i - p_i)} = 2^{(q_1-p_1)} \times 2^{(q-q_1)-(p-p_1)}$ fractional factorial split-plot design. However, the first design involves only one randomization step, and therefore each replicate of the experiment needs $N = 2^{q-p}$ treatment setups. The second has two randomizations steps with (k_1+j_1) hard-to-change factors and $(k+j)-(k_1+j_1)$ easy-to-change factors, and therefore each replicate of the experiment needs $N_1 = 2^{(q_1-p_1)}$ setups of the k_1+j_1 hard-to-change factors and $N = N_2 = 2^{(q-p)}$ setups of the $(k+j)-(k_1+j_1)$ easy-to-change factors, $N_1 < N_2 = N$. So, there are fewer setups associated with the hard-to-change factors, which is desirable in practice to reduce

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experimental effort. Therefore, the notation $2^{(q_1-p_1)} \times 2^{(q_2-p_2)} \times \cdots \times 2^{(q_s-p_s)}$ is employed to emphasize the number of randomization steps. The combined design can be constructed using regular fractional factorial designs. Known the number of strata, the task is to determine the number of generators in each stratum that would provide the MS. The following algorithm describes this task:

Algorithm for the determination of no. of generators at each stratum for experiments with the minimum number of setups.

INPUT:

$m_0 = 0$;

$k = \#$ four-level factors;

$j = \#$ two-level factors;

$N =$ Total # runs;

$s = \#$ strata;

$k_i = \#$ four-level factors at stratum i: $k_1, k_2, \ldots, k_s$; $0 \leqslant k_i \leqslant k$; $\sum_1^s k_i = k$; $i = 1, \ldots, s$

$j_i = \#$ four-level factors at stratum i: $j_1, j_2, \ldots, j_s$; $0 \leqslant j_i \leqslant j$; $\sum_1^s j_i = j$; $i = 1, \ldots, s$

RESTRICTIONS:

$t = 3k + j$; the total number of effects;

$r = int\ [log_2(t)]$;

if $2^r + 1 \leqslant t \leqslant 2^{r+1}-1$ then $m = r + 1$ if $k \leqslant ((2^m-1)/3)$ for m even *or* if $k \leqslant ((2^m-5)/3)$ for m odd; other wise $m = r + 2$;

$N \geqslant 2^m$

CALCULATE

$t_i =$ the number of effects to be estimated up to stratum i;

$q_i =$ the number of factors in full factorial 2^{q_i} at the stratum i;

$2^{m_i} =$ the minimum number of runs needed at stratum i to estimate the t_i main effects;

$m_i =$ the minimum number of base factors at stratum i;

For $i = 1$ to s by 1

$t_i = \sum_1^i 3k_i + j_i$;

$q_i = 2k_i + j_i$;

$r_i = int\ [log_2(t_i)$;

if $2^{r_i} \leqslant t_i < 2^{r_i+1}-1$ then $m_i = r_i + 1$ if $\sum_1^i k_i \leqslant ((2^{m_i}-1)/3)$ for m_i even or if $\sum_1^i k_i \leqslant ((2^{m_i}-5)/3)$ for m_i odd; otherwise $m_i = r_i + 2$;

if $t_i = 2^{r_i+1}-1$ then $m_i = r_i + 2$ if $\sum_v^i k_i \leqslant ((2^{m_i}-1)/3)$ for m_i even or if $\sum_1^i k_i \leqslant ((2^{m_i}-5)/3)$ for m_i odd and $k_{i+1} > 0$; $i + 1 \leqslant s$; otherwise $m_i = r_i + 1$;

$N_i = 2^{m_i}$.

End

OUTPUT:

$p_i = \#$ generators at stratum i;

$$p_i = q_i - (m_i - m_{i-1}); \quad p_1; p_2; \cdots; p_s; 0 \leqslant p_i \leqslant p; \sum_1^s p_i = p.$$

The algorithm provides the number of generators for each stratum and the setup pattern: $2^{(q_1-p_1)}: 2^{(q_1-p_1)+(q_2-p_2)}: \cdots : 2^{\sum_1^s (q_i-p_i)}$. For illustrative purposes, consider the design d_5, d_8 and one modified based on the design d_8 (d_{8A}).

Case 1: design d_5 with the inputs: $k = 4$ four-level factors; $j = 3$ two-level factors, and $s = 4$ strata. According to Table IV, $N = 16$ runs are enough. The number of four-level factors (k_i) and two-level factors (j_i) in each stratum and outputs (p_i, m_i, t_i, r_i) after running the algorithm are put together in the first block of rows of Table VI. By the values of q_i and p_i, it is possible to identify the MS split-plot design $(2^{(1-0)}) \times (2^{(4-1)}) \times (2^{(3-3)}) \times (2^{(3-3)})$which provides the setup pattern (2, 16, 16, 16).

Design			Stratum 1	2	3	4	Properties $s=4$
d_5	Input	k_i	0	2	1	1	$k=4$
		j_i	1	0	1	1	$j=3$
		q_i	1	4	3	3	MS split-plot design: $(2^{(1-0)})\times(2^{(4-1)})\times(2^{(3-3)})\times(2^{(3-3)})$
	Output	p_i	0	1	3	3	
		m_i	1	4	4	4	Setup pattern: $(2^1; 2^4; 2^4; 2^4)$
		t_i	1	7	11	15	$n=16$ runs
		r_i	0	2	3	3	
d_8	Input	k_i	1	1	2	0	$k=4$
		j_i	1	0	2	3	$j=6$
		q_i	3	2	6	3	MS split-plot design: $(2^{(3-0)})\times(2^{(2-1)})\times(2^{(6-6)})\times(2^{(3-2)})$
	Output	p_i	0	1	6	2	
		m_i	3	4	4	5	Setup pattern: $(2^3; 2^4; 2^4; 2^5)$
		t_i	4	7	15	18	$n=32$ runs
		r_i	2	2	3	4	
d_{8a}	Input	k_i	1	1	2	1	$k=5$
		j_i	1	0	2	0	$j=3$
		q_i	3	2	6	3	MS split-plot design: $(2^{(3-0)})\times(2^{(2-1)})\times(2^{(6-5)})\times(2^{(3-3)})$
	Output	p_i	0	1	5	3	
		m_i	3	4	5	5	Setup pattern: $(2^3; 2^4; 2^5; 2^5)$
		t_i	4	7	15	18	$n=32$ runs
		r_i	2	2	3	4	

Table VI. Examples of application of the algorithm: Designs d_5; d_8; d_{8A}

Case 2: designs d_8 with inputs: $s=4$ strata; $k=4$ four-level factors; $j=6$ two-level factors; according to Table IV, at least $N=32$ runs are needed. The outputs and inputs (the number of four-level and two-level factors at each stratum) of algorithm of the Case 2 are in second block of rows of Table VI. It is a MS split-plot design $(2^{(3-0)})\times(2^{(2-1)})\times(2^{(6-6)})\times(2^{(3-2)})$ with the setup pattern (8, 16, 16, 32) setups.

Case 3: This example is similar to Case 2. The main different is at fourth stratum: one four-level factor instead of three two-level factors. Inputs: $s=4$ strata; $k=5$ (four four-level factors); $j=3$ (six two-level factors). The outputs and inputs (the number of four-level and two-level factors at each stratum) are summarized in third block of rows of Table VI. Running the algorithm, it yields the design $(2^{(3-0)})\times(2^{(2-1)})\times(2^{(6-5)})\times(2^{(3-3)})$ with setup pattern (8, 16, 32, 32).

6. Conclusions

The planning stage of an experiment is the most critical phase. The choices made will have both theoretical and practical implications, since it will directly affect the properties of the design, the cost of the experiment, its analysis, and the quality of the results. Therefore, a proper design selection is extremely important in practice. A poor design selection may lead to a total waste of the data collected during the execution of the experiment. Consequently, it can jeopardize the whole effort for process or product improvement, resulting in a disbelief of the usefulness of DOE applicability to problem-solving.

In this paper, we dealt with mixed-level or multi-level experiments specifically mixed two and four-level designs. Due to the nature of the experiment (only one piece of each auto part and the presence of hard-to-change factors), completely randomized designs were discarded as plausible options. Thus, split-plot type designs have been a common approach to execute an experiment with these features.

In this paper, we show how to properly use the method of replacement to construct mixed two-level and four-level split-plot type designs from 2^{q-p} fractional factorial designs and

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choose designs according to MS and MA criteria. We have illustrated that the direct use of the existing MSMA catalogs for two-level split-plot type designs may lead to inappropriate designs (e.g. low resolution). The existing method of replacement for searching exclusive sets of the form (α, β, $\alpha\beta$) available in the literature is suitable for completely randomized designs, but it may not provide efficient plans for designs with restricted randomization. Therefore, to avoid the inadvertent choice of an undesirable design, we have extended the algorithm to find out the number of generators and the number of base factor at each stratum, which guide the selection of appropriate mixed two-level and four-level MSMA split-plot type designs.

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