Mixing of Active and Sterile Neutrinos

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Contents

§1

Introduction

Neutrino oscillations

- Neutrino mass scales
  - Atmospheric: $\Delta m_{\mathrm{atm}} \simeq 2.5 \times 10^{-3} \mathrm{eV}^2$
    - Atmospheric neutrino exps. (⋯, SuperK)
    - Long-baseline accelerator exps. (K2K, MINOS)
  - Solar : $\Delta m_{\mathrm{sol}} \simeq 8.0 \times 10^{-5} \mathrm{eV}^2$
    - Solar neutrino exps. (⋯, SuperK, SNO)
    - Reactor exp. (KamLand)
- Need for physics beyond the MSM !

Extension by RH neutrinos

- Introduce three RH neutrinos $\nu_{R1}, \nu_{R2}, \nu_{R3}$

$$\delta L = i\overline{\nu_{RI}}\partial_\mu \gamma^\mu \nu_{RI} - F_{\alpha I}\overline{L_\alpha}\nu_{RI}\Phi - \frac{M_I}{2}\overline{\nu_{RI}}\nu_{RI}^c + \mathrm{h.c.}$$

$$I = 1, 2, 3$$

$$\alpha = e, \mu, \tau$$

- Mass terms of neutrinos

$$-L = \frac{1}{2}(\overline{\nu_L}, \overline{N^c})\begin{pmatrix} 0 & M_D \\ M_D^T & M_M \end{pmatrix}\begin{pmatrix} \nu_L^c \\ N \end{pmatrix} + h.c.$$

  - Dirac masses $M_D = F\langle\Phi\rangle$
  - Majorana masses $M_M = diag(M_1, M_2, M_3)$

- We shall assume:

$$|[M_D]_{\alpha I}| \ll M_I \quad \Rightarrow \text{Seesaw mechanism works}$$

Seesaw mechanism

Minkowsi (77),
Yanagida (79),Gell-Mann, Ramond,
Slansky (79), Glashow (79),...

- If Majorana masses » Dirac masses,

$$\begin{pmatrix} 0 & M_D \\ M_D^T & M_M \end{pmatrix} \Rightarrow \begin{pmatrix} M_\nu & 0 \\ 0 & M_M \end{pmatrix}$$

  - **Active neutrinos**

    $\boldsymbol{\nu_1, \nu_2, \nu_3}$

$$\begin{cases} M_\nu = -M_D^T \dfrac{1}{M_M} M_D \\ U^T M_\nu U = diag(m_1, m_2, m_3) \end{cases}$$

  - **Sterile neutrinos**

    $\boldsymbol{N_1, N_2, N_3}$

$$\begin{cases} N_I \simeq \nu_{RI} \\ M_M = diag(M_1, M_2, M_3) \end{cases}$$

- Flavor mixing in CC current

$$\nu_{L\alpha} = U_{\alpha i}\, \nu_i + \Theta_{\alpha I}\, N_I$$

active-sterile mixing

$$\Theta_{\alpha I} = [M_D]_{\alpha I} / M_I$$

Scale for Majorana mass

- Neutrino oscillations are explained by flavor mixing between active neutrinos !
  - Masses of active neutrinos

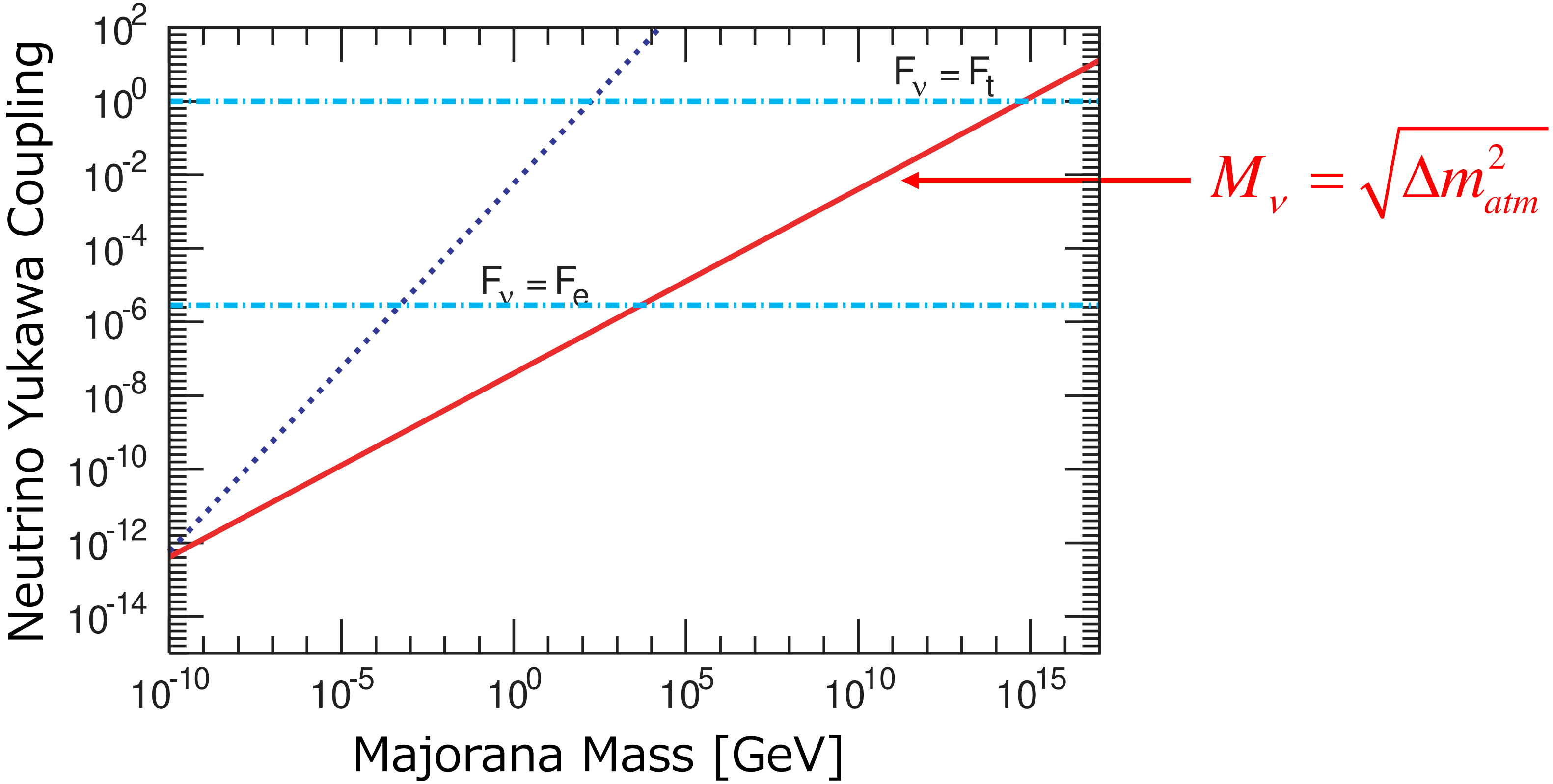
$$M_\nu = -M_D \frac{1}{M_M} M_D^T \quad \Rightarrow \quad \begin{array}{l} \Delta m_{\rm atm}^2 \simeq 2.5 \times 10^{-3}\ {\rm eV}^2 \\ \Delta m_{\rm sol}^2 \simeq 8.0 \times 10^{-5}\ {\rm eV}^2 \end{array}$$

- **Where is the scale of Majorana mass ??**
  - Two “natural” options
    - The conventional seesaw scenario
    - The νMSM

M_M ???

Scale of Majorana mass

$$M_\nu = -M_D^T \frac{1}{M_M} M_D \quad \Rightarrow F^2 = M_M M_\nu / \langle H \rangle^2$$



$$M_\nu = \sqrt{\Delta m^2_{atm}}$$

Conventional seesaw scenario:

- Neutrino Yukawa couplings are comparable to those of quarks and charged leptons
  - $M_M \gg 100\ \mathrm{GeV}$

$$M_M \simeq 6 \times 10^{14}\mathrm{GeV}\, F^2 \left(\frac{2.5 \times 10^{-3}\mathrm{eV}^2}{m_\nu^2}\right)^{\frac{1}{2}}$$

- Explain "naturally" smallness of neutrino masses

[Minkowski, Yanagida; Gell-Mann, Ramond, Slansky]

- Decays of RH neutrino(s) can account for BAU through leptogenesis

[Fukugita, Yanagida]

- Physics of RH neutrinos can **NOT** be tested directly by experiments

The νMSM:

[TA, Blanchet, Shaposhnikov '05;
TA, Shposhnikov '05]

- No new mass scale is introduced
  - $M_M < 100\ \mathrm{GeV}$

$$F \simeq 4 \times 10^{-7} \left(\frac{M_M}{100\mathrm{GeV}}\right)^{\frac{1}{2}} \left(\frac{m_\nu^2}{2.5 \times 10^{-3}\mathrm{eV}^2}\right)$$

- One keV sterile neutrino can be DM

[Dodelson, Widrow '93,⋯]

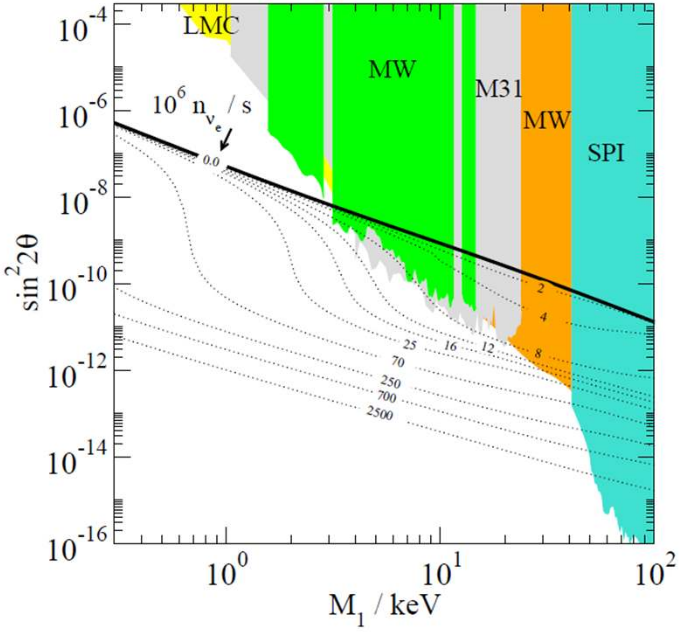
- Oscillation of quasi degenerate sterile neutrinos can account for BAU

[Akhmedov, Rubakov, Smirnov '98]

- Physics of RH neutrinos can be tested directly by experiments !

Roles of three sterile (RH) neutrinos

- N_1 : **Dark Matter** Candidate
  - $M_1 = 4\text{-}50$ keV
    $|F_{\alpha 1}| = 5 \cdot 10^{-15}\text{-}4 \cdot 10^{-13}$
  - Negligible contribution to M_ν
  - Test by X-rays from $N_1 \longrightarrow \nu\,\gamma$



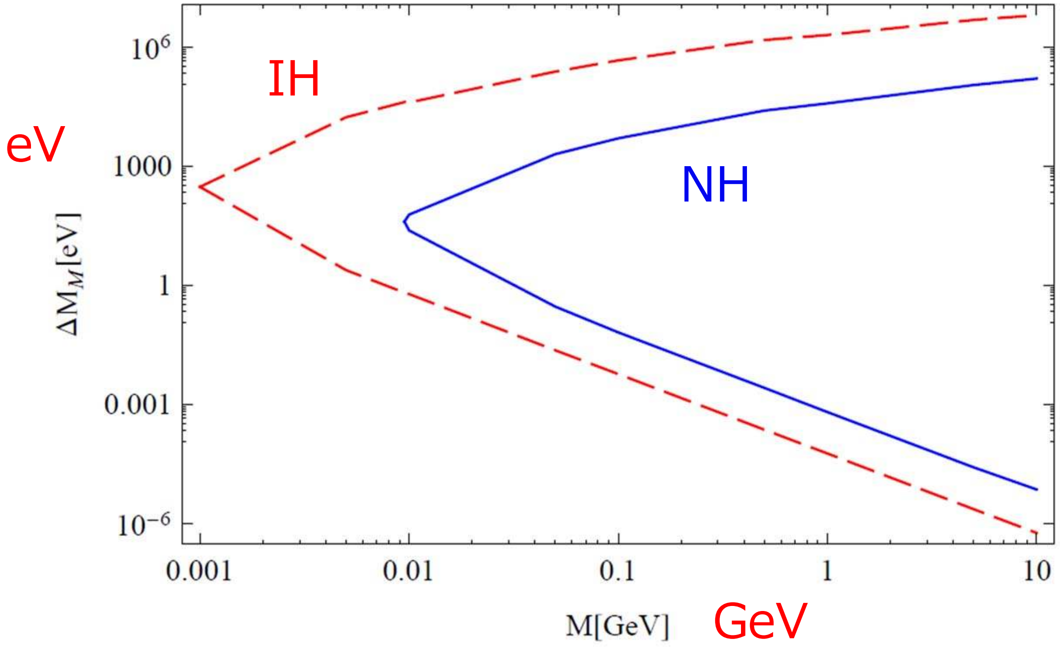
[Laine, Shaposhnikov '08]

- N_2 and N_3 :
  - Neutrino Oscillation data
    - Masses and mixings of active neutrinos
  - Baryon Asymmetry of the Universe (BAU)
    - Mechanism via RH neutrino oscillation

Mass region of N_2 and N_3

- BAU requires quasi-degenerate N_2 and N_3

[Canetti, Shaposhnikov '10]



- Experimental test of N_2 and N_3 is crucial to reveal the origin of masses and BAU !

Purpose of this talk

- Study the mixing elements $\Theta_{\alpha I}$ for N_2 and N_3
    - Interactions of sterile neutrinos
        - Yukawa int. $F_{\alpha I} \propto \Theta_{\alpha I}$
        - Weak gauge int. $g_{\mathrm{W}} \Theta_{\alpha I}$

        $\Longrightarrow$ $\Theta_{\alpha I}$ determine the strength of int.

- Discuss
    - How $\boldsymbol{\Theta_{\alpha I}}$ depend on neutrino parameters?
    - Impacts on
        - Neutrinoless double beta decay
        - Search for N_2 and N_3 $(M_{2,3} < m_\pi)$

§2

Mixing elements of sterile neutrinos

Mixing elements of $N_{2,3}$

- Flavor mixing in the CC interaction

$$\nu_{L\alpha} = U_{\alpha i}\,\nu_i + \Theta_{\alpha I}\,N_I$$

$$\boxed{\Theta_{\alpha I} = \frac{[M_D]_{\alpha I}}{M_I} = \frac{F_{\alpha I}\langle\Phi\rangle}{M_I}}$$

  - Model with one pair of active and sterile neutrinos

$$|\Theta|^2 = \frac{|M_D|^2}{M_M^2} = \frac{m_\nu}{M_M} = 5\times10^{-11}\left(\frac{m_\nu^2}{m_{\mathrm{atm}}^2}\right)^{\frac{1}{2}}\left(\frac{1\mathrm{GeV}}{M_M}\right)$$

  - In the νMSM, $|\Theta_{\alpha I}|$ can be much larger / smaller !

- Parameterization of $\Theta_{\alpha I}$ $(F_{\alpha I})$ for $N_{2,3}$

$$[M_\nu]_{\alpha\beta} = -\sum_{I=1,2,3}\frac{[M_D]_{\alpha I}[M_D]_{\beta I}}{M_I} = -\sum_{I=2,3}\frac{[M_D]_{\alpha I}[M_D]_{\beta I}}{M_I}$$

Essentially no contribution from
Dark matter N_1 $\Rightarrow$ "$F_{\alpha 1}=0$"

Neutrino Yukawa couplings for $N_{2,3}$

$$\boxed{F = U_{\rm PMNS}\, D_\nu^{1/2}\, \Omega\, D_N^{1/2} / \langle \Phi \rangle}$$

(in NH)

[Casas, Ibarra '01]

- Parameters of active neutrinos

$D_\nu^{1/2} = {\rm diag}(\sqrt{m_1} = 0, \sqrt{m_2}, \sqrt{m_3})$: active ν masses

Dirac phase δ

$$U_{\rm PMNS} = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -c_{23}s_{12} - s_{23}c_{12}s_{13}e^{i\delta} & c_{23}c_{12} - s_{23}s_{12}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{23}s_{12} - c_{23}c_{12}s_{13}e^{i\delta} & -s_{23}c_{12} - c_{23}s_{12}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix} \begin{pmatrix} 1 & & \\ & e^{i\eta} & \\ & & 1 \end{pmatrix}$$

Majorana phase η

- Parameters of sterile neutrinos

$D_N^{1/2} = {\rm diag}(\sqrt{M_2}, \sqrt{M_3})$: sterile ν masses

$$\Omega = \begin{pmatrix} 0 & 0 \\ \cos\omega & -\sin\omega \\ \xi\sin\omega & \xi\cos\omega \end{pmatrix}$$

ω: complex number

$\xi = \pm 1$

Imω

Enhancement of $\Theta_{\alpha I}$

- **In the νMSM, $\Theta_{\alpha I}$ can be larger!**
  - Imω in Ω matrix is important

    $[\cos\omega = \cos\mathrm{Re}\omega \cosh\mathrm{Im}\omega - i\sin\mathrm{Re}\omega\sinh\mathrm{Im}\omega]$
  - **For $\mathrm{Im}\omega \gg 1$**, $\Omega \propto e^{\mathrm{Im}\omega}$

$$\boxed{F, \Theta \propto e^{\mathrm{Im}\omega} \equiv X_\omega}$$

Enhancement of Θ by large X_ω

Cf. U(1) flavor symmetry model [Shaposhnikov '06]

- Masses of active neutrinos do not change

- Large $\Theta_{\alpha I}$ is crucial for
  - **larger production/detection rates of $N_{2,3}$**
  - **shorter lifetimes of $N_{2,3}$**

Mixing elements in NH case

- Leading $O(X_\omega^2)$ terms

$$|\Theta_{eI}|^2 = X_\omega^2 \frac{m_{\rm atm}}{4\,M_I} \cos^2\theta_{13}\,[\tan^2\theta_{13} + 2\,\xi\,\sin(\delta+\eta)\sqrt{r_m}\,\sin\theta_{12}\tan\theta_{13} + r_m \sin^2\theta_{12}]$$

$$\left|\Theta_{\mu I}\right|^2 = X_\omega^2 \frac{m_{\rm atm}}{4\,M_I} \cos^2\theta_{13}\,\sin^2\theta_{23}\,[1 + O(\sqrt{r_m})]$$

$$|\Theta_{\tau I}|^2 = X_\omega^2 \frac{m_{\rm atm}}{4\,M_I} \cos^2\theta_{13}\,\cos^2\theta_{23}\,[1 + O(\sqrt{r_m})]$$

$$r_m = \frac{m_{\rm atm}}{m_{\rm sol}} \simeq 0.18$$

- μ and τ types
  - $\theta_{23} \simeq \pi/4$ and $\theta_{13} \ll 1$

$$\left|\Theta_{\mu I}\right|^2 \simeq |\Theta_{\tau I}|^2 \simeq X_\omega^2 \frac{m_{\rm atm}}{8\,M_I} = 6\times 10^{-11} X_\omega^2 \left(\frac{100\text{MeV}}{M_I}\right)^2$$

  - $\boldsymbol{\left|\Theta_{\mu I}\right|^2 \simeq |\Theta_{\tau I}|^2 \propto X_\omega^2}$ **!!**

e type behaves differently !

Mixing elements in NH case

- e **type**

$$|\Theta_{eI}|^2 = X_\omega^2 \frac{m_{\rm atm}}{4\,M_I} \cos^2\theta_{13}\left[\tan^2\theta_{13} + 2\,\xi\sin(\delta+\eta)\sqrt{r_m}\sin\theta_{12}\tan\theta_{13} + r_m\sin^2\theta_{12}\right]$$

- $O(X_\omega^2)$ and $O(X_\omega^0)$ terms in $|\Theta_{eI}|^2$ vanish, when

$$\boxed{\begin{gathered}\tan\theta_{13}^{cr} = \sqrt{r_m}\sin\theta_{12} \\ \xi\sin(\delta+\eta) = -1\end{gathered}}$$

$\sin^2\theta_{13}^{cr} = 0.041\sim0.070\ (3\sigma),\ 0.046\sim0.065\ (2\sigma)$

$\sin^2\theta_{13} < 0.053\ \ (3\sigma),\ 0.039\ (2\sigma)$

[Schwetz, Tortola, Valle '08]

$$|\Theta_{eI}|^2 \simeq 3\times10^{-11}\frac{1}{X_\omega^2}\left(\frac{100\mathrm{MeV}}{M_I}\right)^2$$

- In this case, $|\boldsymbol{\Theta}_{\mu I}|^2 \simeq |\boldsymbol{\Theta}_{\tau I}|^2 \propto \boldsymbol{X}_\omega^2 \gg |\boldsymbol{\Theta}_{eI}|^2 \propto \boldsymbol{X}_\omega^{-2}$!!

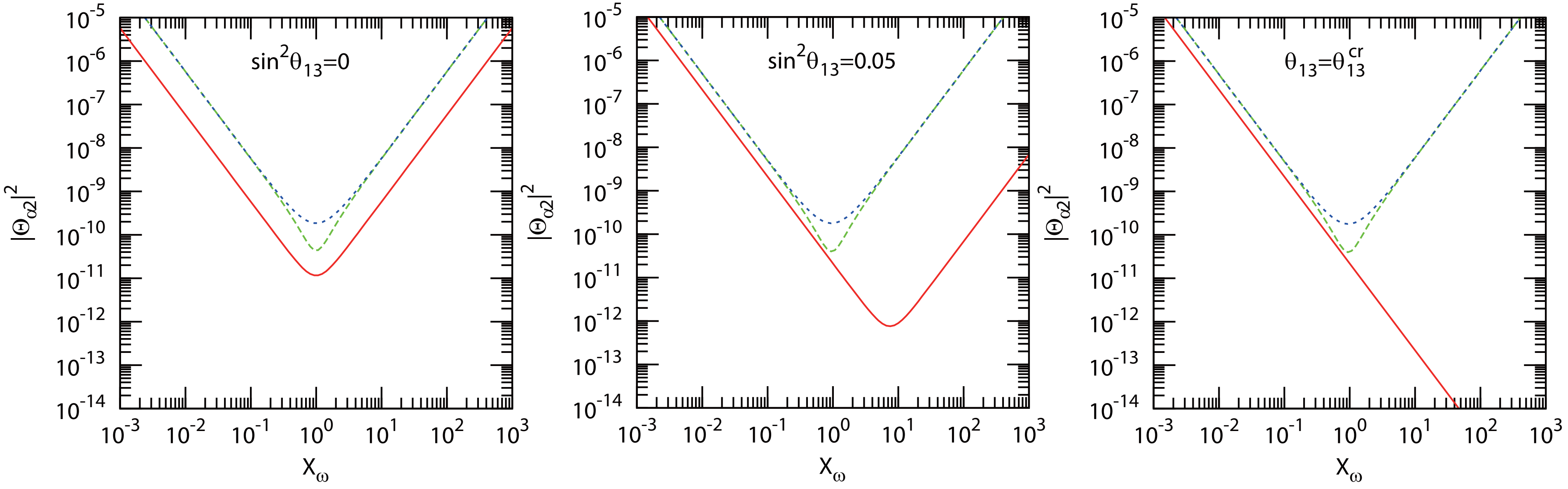
- When $\theta_{13} = 0$, $\dfrac{|\Theta_{eI}|^2}{|\Theta_{\mu I}|^2} \simeq \dfrac{|\Theta_{eI}|^2}{|\Theta_{\tau I}|^2} \simeq 2r_m\sin^2\theta_{12} \simeq 0.11$

[Shaposhnikov '08]

NH case

[TA, Eijima, Ishida ’11]

θ_{13} and CP phases are important !



- For Im$\omega \gg 1$, we can obtain
  - ☐ Very Large $|\Theta_{\mu I}|^2 \simeq |\Theta_{\tau I}|^2$
  - ☐ Very suppressed $|\Theta_{eI}|^2$

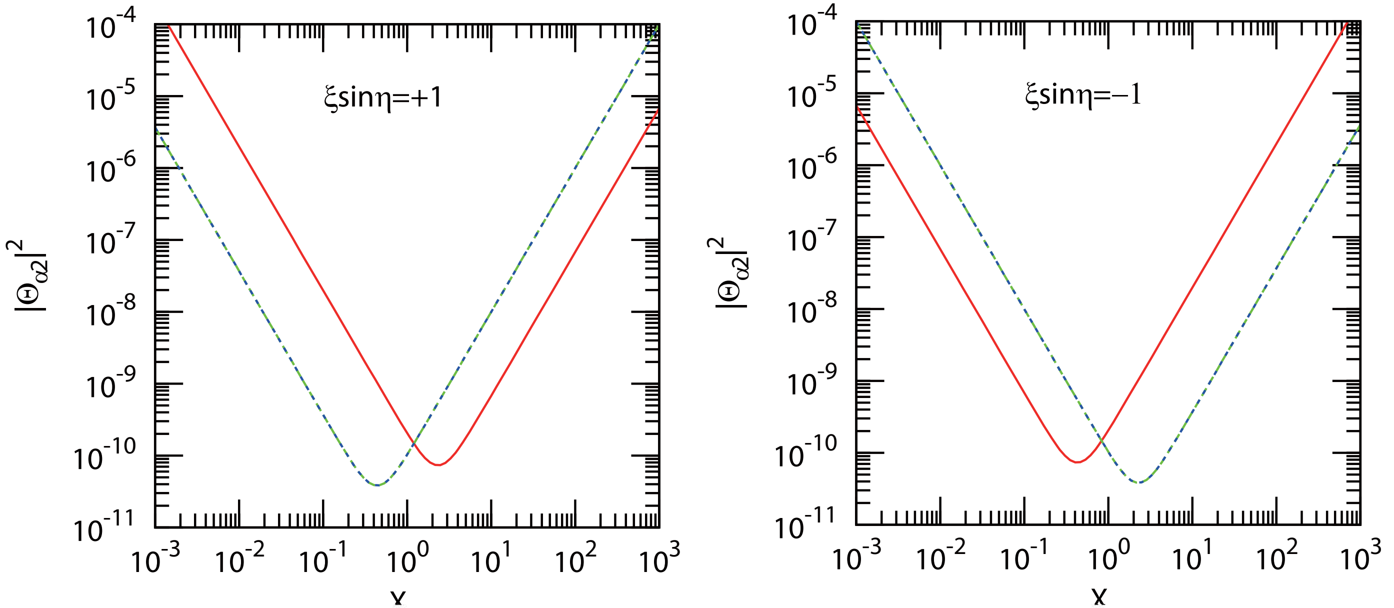
Cancellation in $|\Theta_{eI}|$ occurs if

$$\tan\theta_{13}^{cr} = \sqrt{r_m}\sin\theta_{12}$$
$$\xi \sin(\delta+\eta) = -1$$

IH case

[TA, Eijima, Ishida ’11]

- No cancellation (suppression) for large X_ω
- “ξsinη” is important !



$$\frac{|\Theta_{eI}|^2}{|\Theta_{\mu I}|^2} \simeq \frac{|\Theta_{eI}|^2}{|\Theta_{\tau I}|^2} \simeq 2\,\frac{1-\xi\sin\eta\sin 2\theta_{12}}{1+\xi\sin\eta\sin 2\theta_{12}} = \begin{cases} 0.071 & \text{for } \xi\sin\eta=+1 \\ 56 & \text{for } \xi\sin\eta=-1 \end{cases}$$

[Shaposhnikov ’08]

§3

Neutrinoless Double Beta Decay

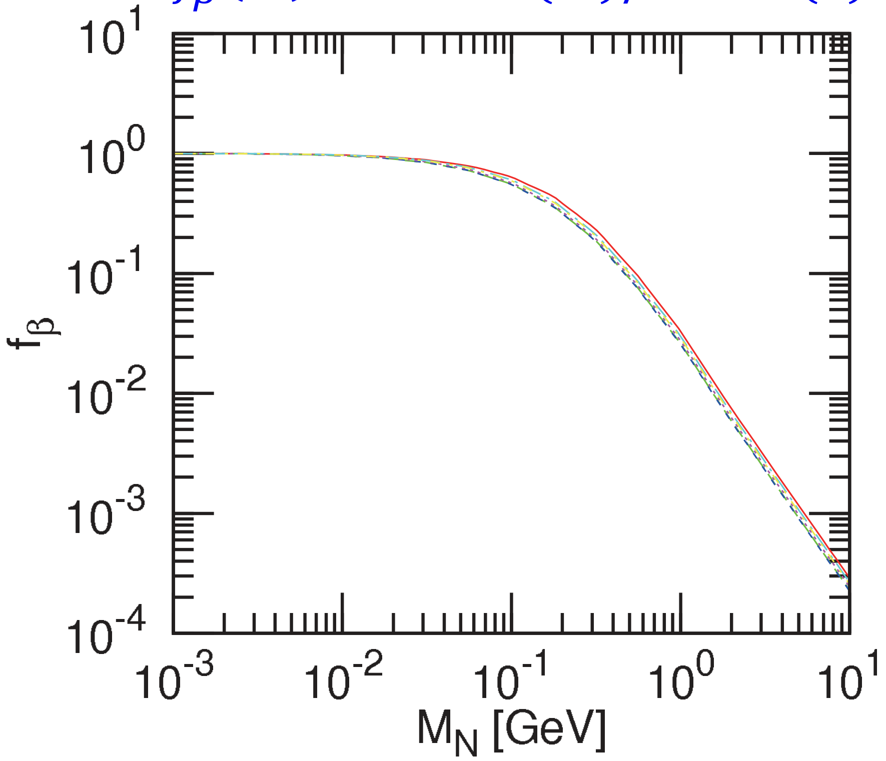
Effective neutrino mass

- $$m_{\rm eff} = \sum_{i=1,2,3} m_i\, U_{ei}^2 + \sum_{I=1,2,3} f_\beta(M_I)\, M_I\ \Theta_{eI}^2$$

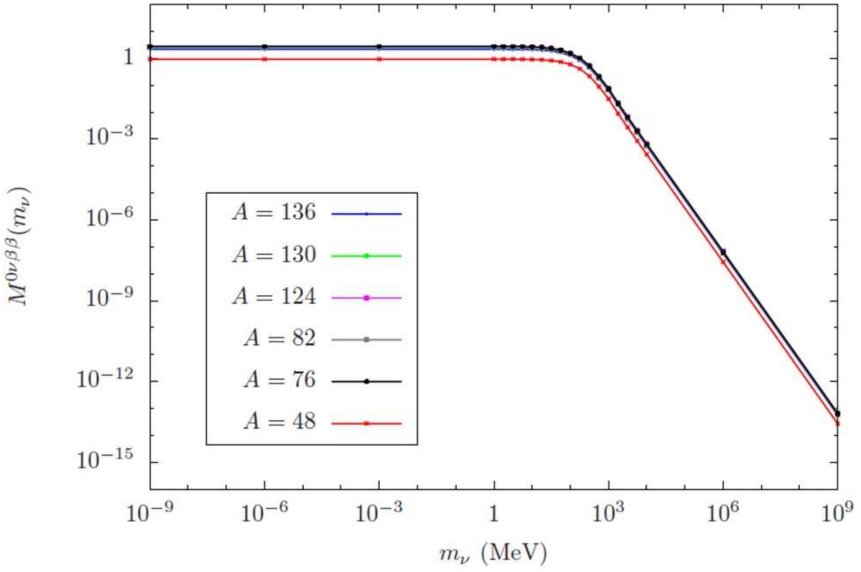
active neutrinos

sterile neutrinos

$f_\beta(M) = M^{0\nu\beta\beta}(M)/M^{0\nu\beta\beta}(0)$

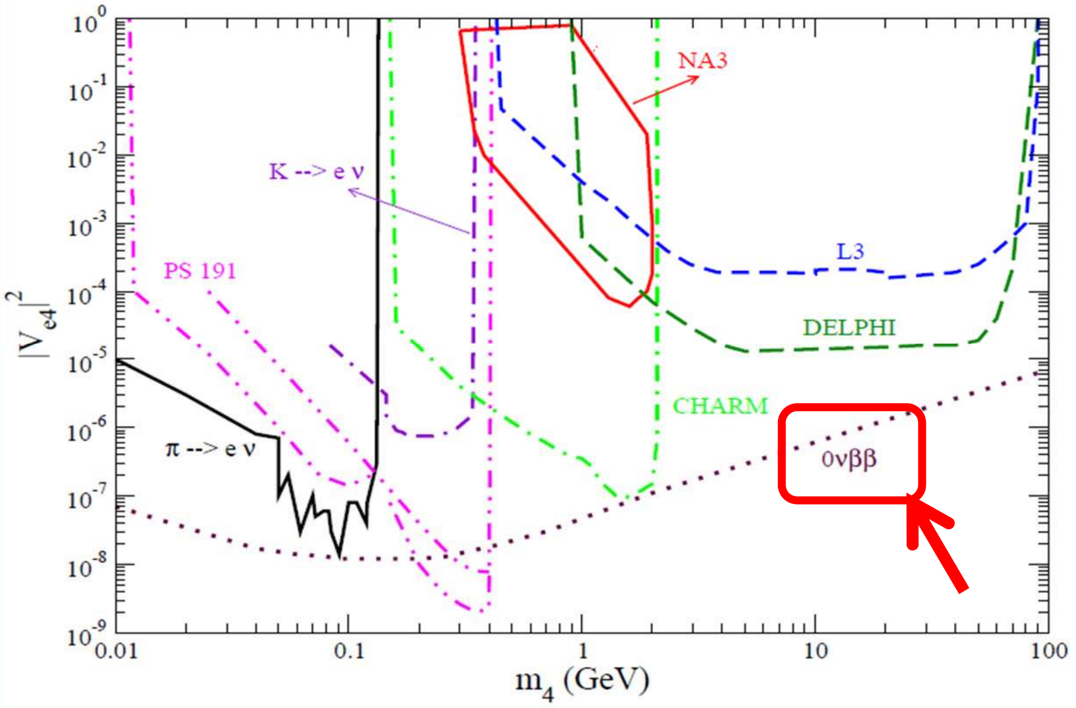


[Blennow, Frenandez-Martinez, Pavon, Mnendez ’10]



Constraint on Θ_{eI} ??

- Benes, Faessler, Simkovic, Kovalenko ('05) derived a stringent limit on Θ_{eI}



[Atre, Han, Pascoli, Zhang '09]

- We will show
  the νMSM receives no such a limit !
  - Bezrukov ('05) considered $M_{2,3} \gg 100$ MeV
  - We also consider $M_{2,3} \lesssim 100$ MeV

$m_{\rm eff}$ in the νMSM

$$m_{\rm eff} = \sum_{i=1,2,3} m_i \, U_{ei}^2 + f_\beta(M_1) M_1 \Theta_{e1}^2 + \sum_{I=2,3} f_\beta(M_I) \, M_I \; \Theta_{eI}^2$$

- DM Sterile Neutrino N_1
  - Relic density requires

    $M_1 = 4\text{-}50$ keV

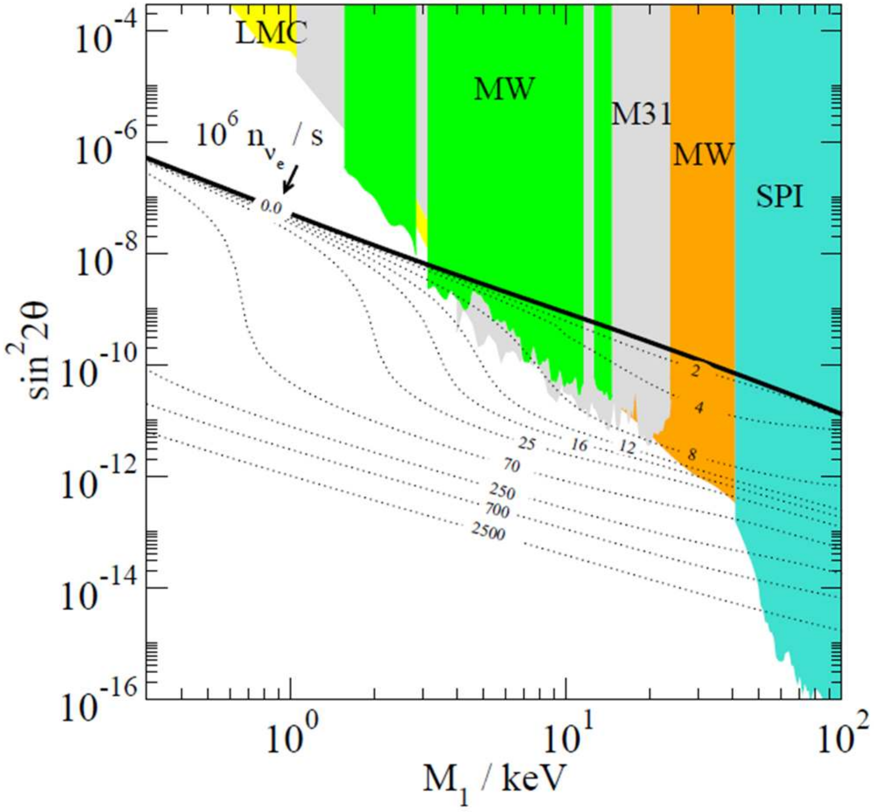
    $|F_{\alpha 1}| = 5 \cdot 10^{-15}\text{-}4 \cdot 10^{-13}$

$$m_{\rm eff}^{N_1} \simeq M_1 \, \Theta_{e1}^2$$
$$= O(10^{-11} \sim 10^{-6}) \text{ eV}$$

negligible contribution !

[Bezrukov '05]

[Laine, Shaposhnikov '08]



m_{eff} in the νMSM

$$m_{\text{eff}} = \sum_{i=1,2,3} m_i\, U_{ei}^2 + \cancel{f_\beta(M_1) M_1 \Theta_{e1}^2} + \sum_{I=2,3} f_\beta(M_I)\, M_I\;\Theta_{eI}^2$$

- Sterile Neutrinos N_2 and N_3
  - BAU requires mass degeneracy

$$\begin{cases} M_3 = M_N + \dfrac{\Delta M}{2} \\ M_2 = M_N - \dfrac{\Delta M}{2} \end{cases} \qquad \Delta M \ll M_N$$

$$m_{\text{eff}}^{N_{2,3}} = \bar{m}_{\text{eff}}^{N_{2,3}} + \delta m_{\text{eff}}^{N_{2,3}}$$

$$\bar{m}_{\text{eff}}^{N_{2,3}} = f_\beta(M_N) \sum_{I=2,3} M_I \Theta_{eI}^2$$

$$\delta m_{\text{eff}}^{N_{2,3}} = \sum_{I=2,3} [f_\beta(M_I) - f_\beta(M_N)] M_I \Theta_{eI}^2 \propto \Delta M / M_N$$

$\delta m_{\text{eff}}^{N_{2,3}}$ gives negligible contribution !

[TA, Eijima, Ishida ’11]

m_{eff} in the νMSM

$$m_{\text{eff}} = \sum_{i=1,2,3} m_i \, U_{ei}^2 + \cancel{f_\beta(M_1) M_1 \Theta_{e1}^2} + \overline{m}_{\text{eff}}^{N_{2,3}} + \cancel{\delta m_{\text{eff}}^{N_{2,3}}}$$

- Active neutrinos and sterile neutrinos N_2 and N_3
    - 6x6 neutrino mass matrix $\widehat{M}_\nu = \begin{pmatrix} 0 & M_D \\ M_D^T & M_M \end{pmatrix}$

$$0 = \left[\widehat{M}_\nu\right]_{ee} = \left[\widehat{U} \widehat{M}_\nu^{\,diag} \widehat{U}^T\right]_{ee} = \sum_{i=1,2,3} m_i \, U_{ei}^2 + \sum_{I=2,3} M_I \Theta_{eI}^2$$

$$\Downarrow$$

$$\overline{m}_{\text{eff}}^{N_{2,3}} = f_\beta(M_N) \sum_{I=2,3} M_I \Theta_{eI}^2 = -f_\beta(M_N) \sum_{i=1,2,3} m_i \, U_{ei}^2$$

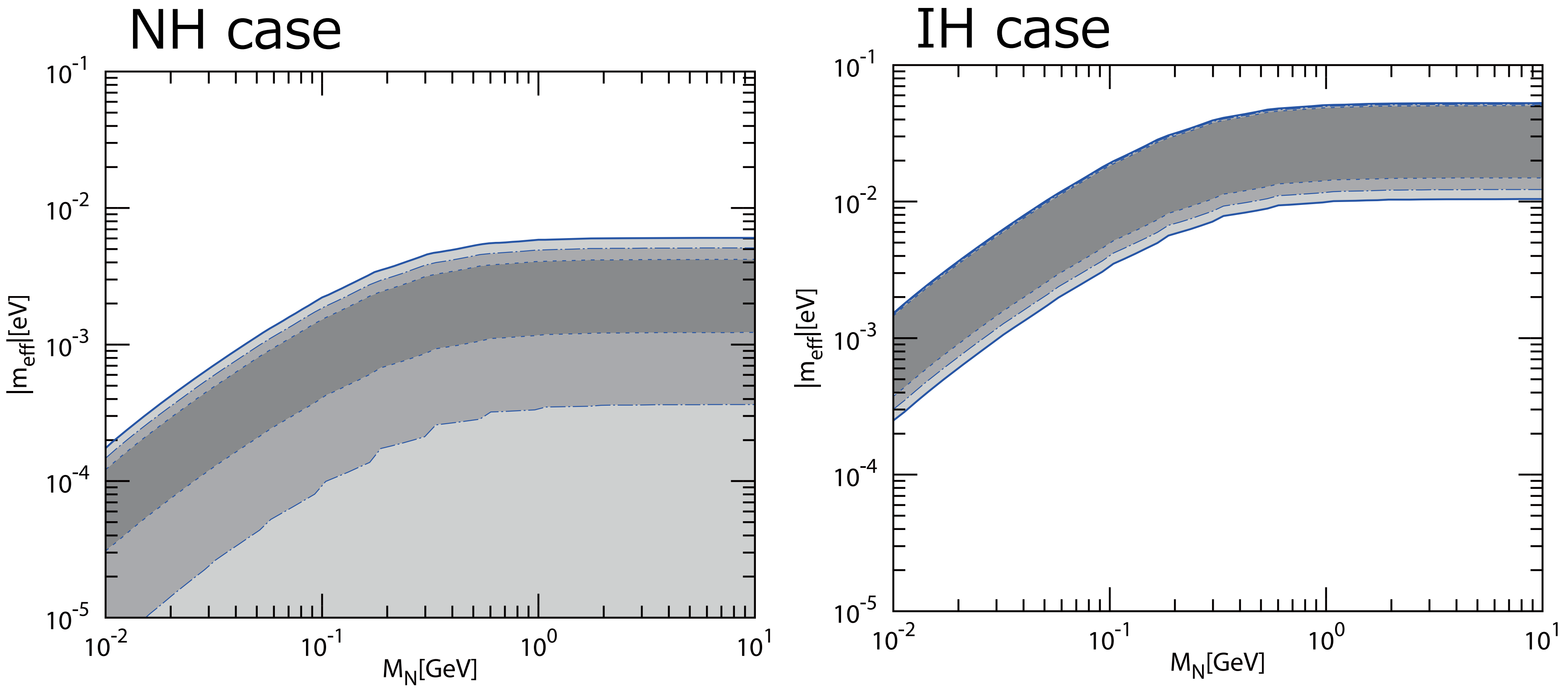
$$\boxed{m_{\text{eff}} \simeq [1 - f_\beta(M_N)] \sum_{i=1,2,3} m_i \, U_{ei}^2}$$

independent on Θ_{eI} !

[TA, Eijima, Ishida ’11]

m_{eff} in the νMSM

[TA, Eijima, Ishida '11]



- m_{eff} in the νMSM is smaller than active ν's one
- No significant constraint on Θ_{eI} in the νMSM !

§4

Search for N_2 and N_3 with $M_{2,3} < m_\pi$

Limits on light sterile neutrinos

- Direct search experiments put
  - Upper bounds on $\Theta_{\alpha I}$

- Big Bang Nucleosynthesis puts
  - Upper bounds on lifetimes
    $\Rightarrow$ Lower bounds on $\Theta_{\alpha I}$

- Allowed range of $\Theta_{\alpha I}$
  - Gorbunov, Shaposhnikov ('07) claimed
    - No allowed region for $M_{2,3} < m_\pi$!

  - Let us reconsider this point

Search for light sterile neutrinos

- Production by meson decays

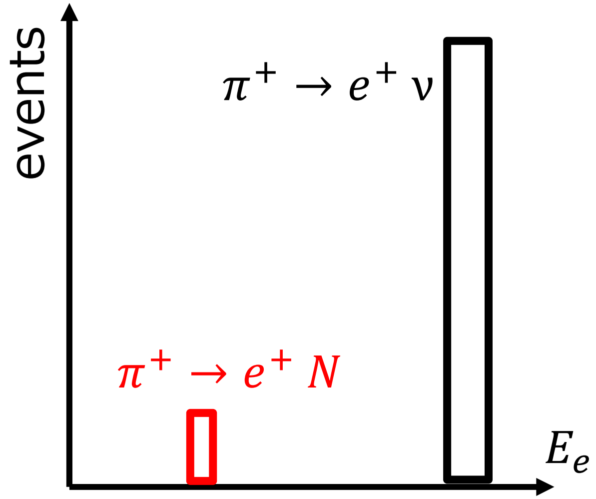
  $\pi^+ \to e^+ N$

  $K^+ \to e^+ N,\ K^+ \to \mu^+ N$

▫ Peak search [Shrock '80]

- Measure E_e in $\pi^+ \to e^+ N$

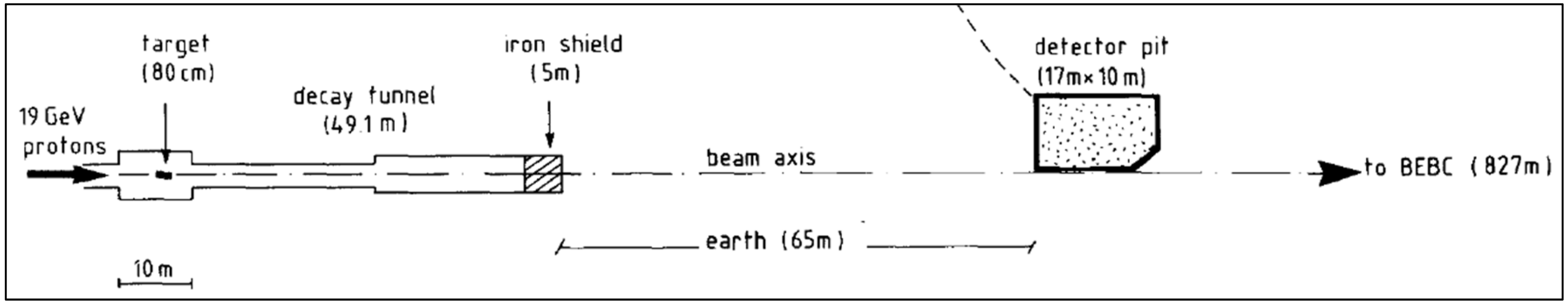
$$E_e = \frac{m_\pi^2 - m_e^2 - M_N^2}{2\, m_\pi}$$



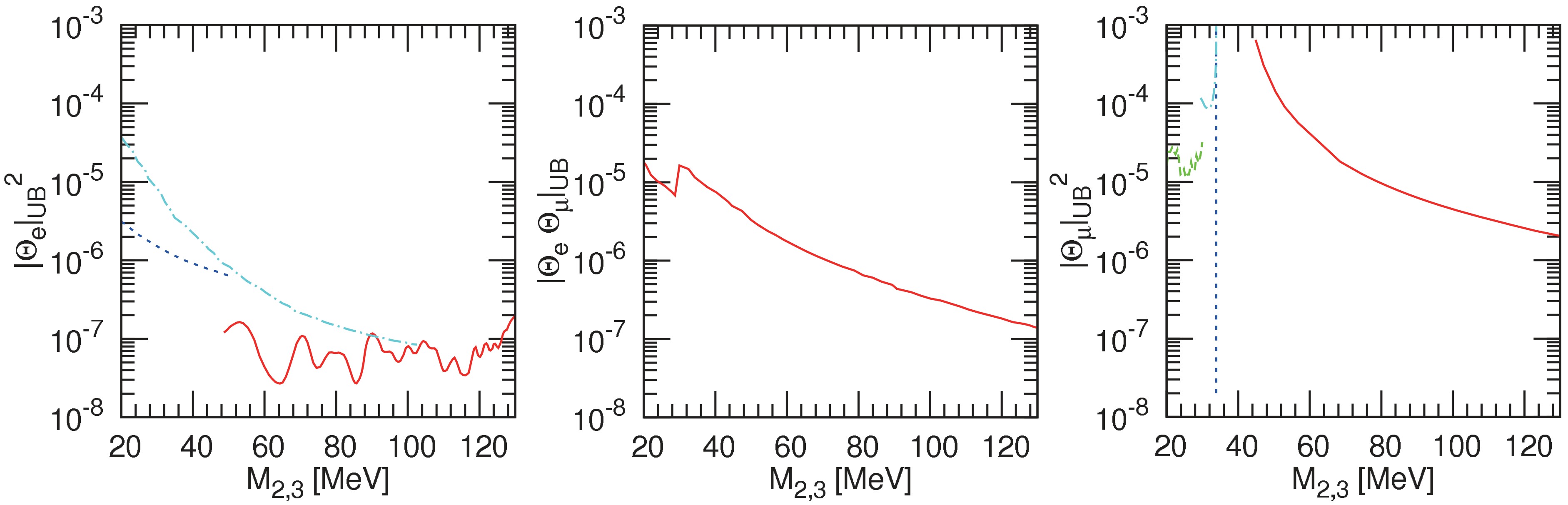
▫ Decays inside the detector

$\pi^+ \to e^+ N$ $\qquad$ $N \longrightarrow \ell^+ \ell^- \nu + c.c.$

CERN PS191



Experimental Upper bounds $\Theta_{\alpha I}$

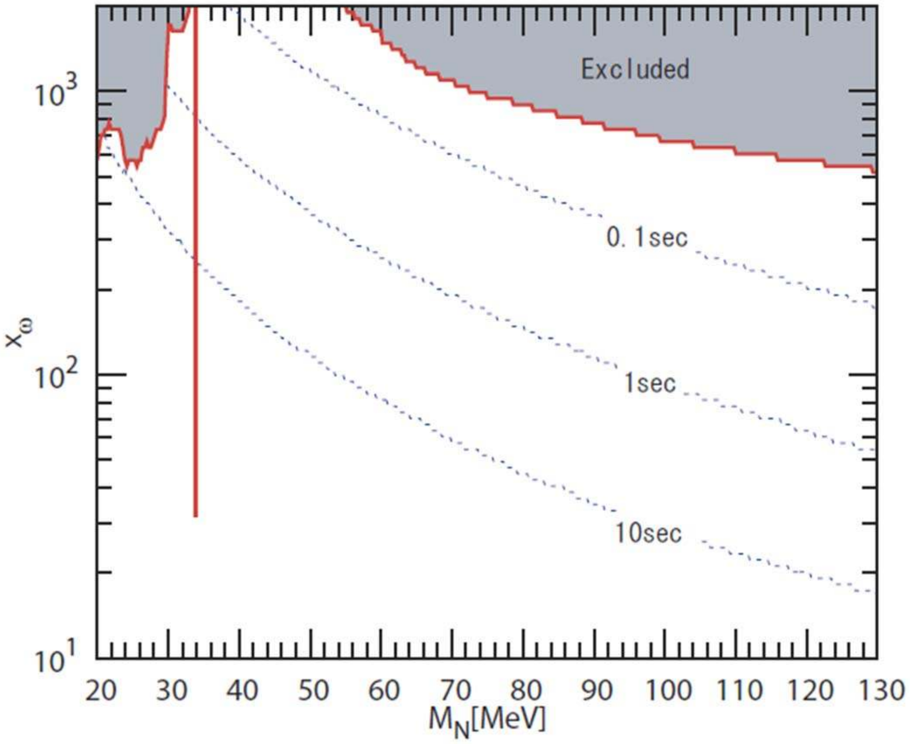


- Bounds on $\Theta_{\tau I}$ are much weaker and irrelevant
- Bound on $|\Theta_{eI}|^2$ is severer than others

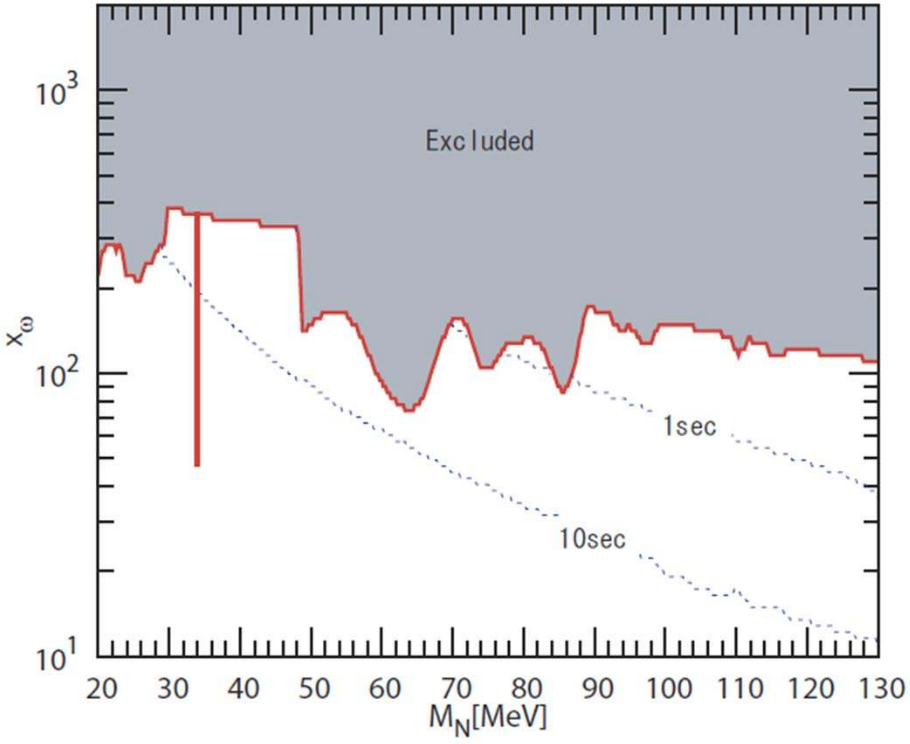
- $|\Theta_{\alpha I}| \propto X_\omega \quad \Longrightarrow$ Upper bound on X_ω

Upper bounds on Xω

NH case



IH case



- Due to cancellation in $|\Theta_{eI}|$ for the NH case, bound in NH is much weaker than IH
- $N_{2,3}$ are long-lived particles $\Longrightarrow$ BBN constraint!

BBN constraint on lifetime

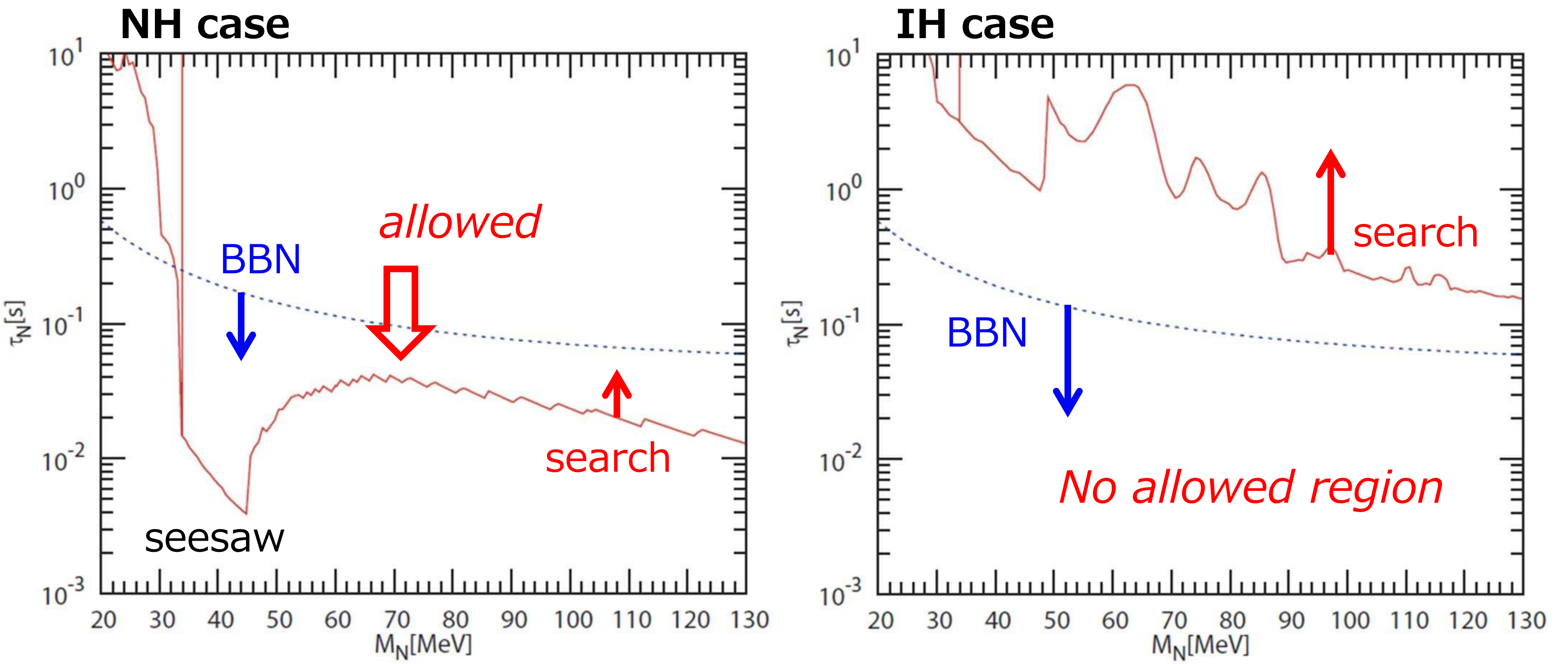
- Long-lived $N_{2,3}$ may spoil the success of BBN
  - Speed up the expansion of the universe
    - $\rho_{\mathrm{tot}} = \rho_{\mathrm{MSM}} + \rho_{N_{2,3}} \Rightarrow \quad H^2 = \frac{\rho_{\mathrm{tot}}}{3\, M_P^2}$
    - p-n conv. decouples earlier ⇒ overproduction of $^4\mathrm{He}$

      $n + \nu \;\leftrightarrow\; p + e^-, \ldots$
  - Distortion of spectrum of active neutrinos
    - $N_{2,3} \;\rightarrow \nu\, \bar{\nu}\, \nu,\;\; e^+\, e^-\, \nu, \ldots$
    - Additional neutrinos may not be thermalized

  ⇒ Upper bound on lifetime

- Dolgov, Hansen, Rafflet, Semikoz ('00)
  - One family case: $\tau_{\mathrm{BBN}}[\mathrm{sec}] = 128.7(M_N/\mathrm{MeV})^{0.04179} - 1.828$

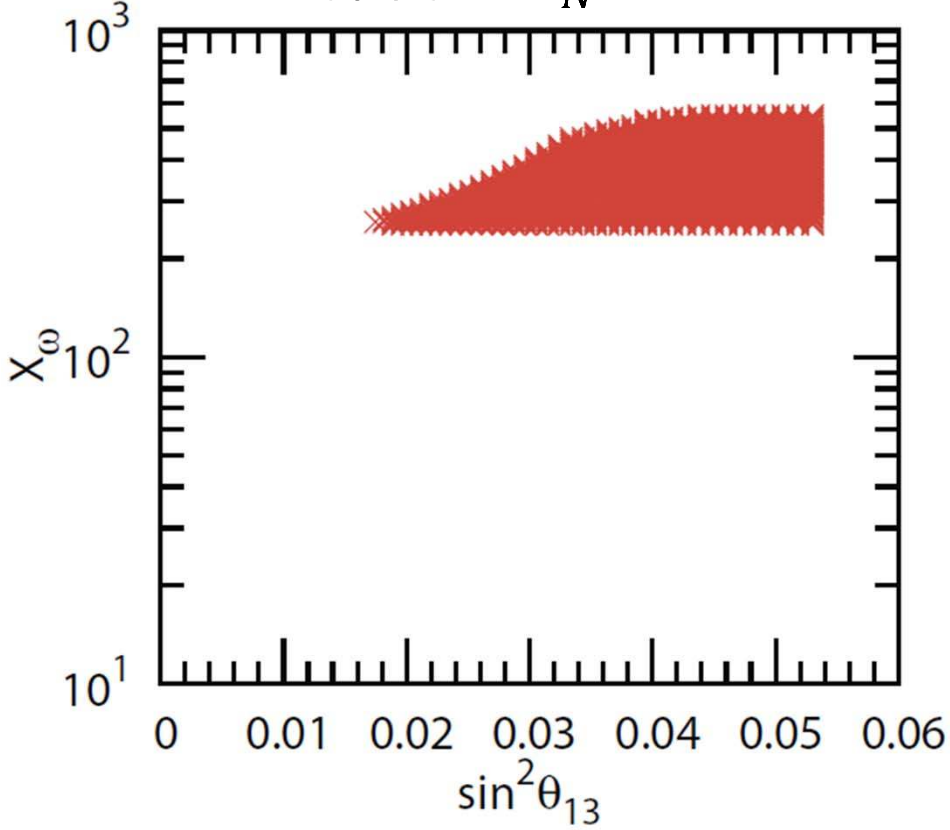
Bounds on lifetime

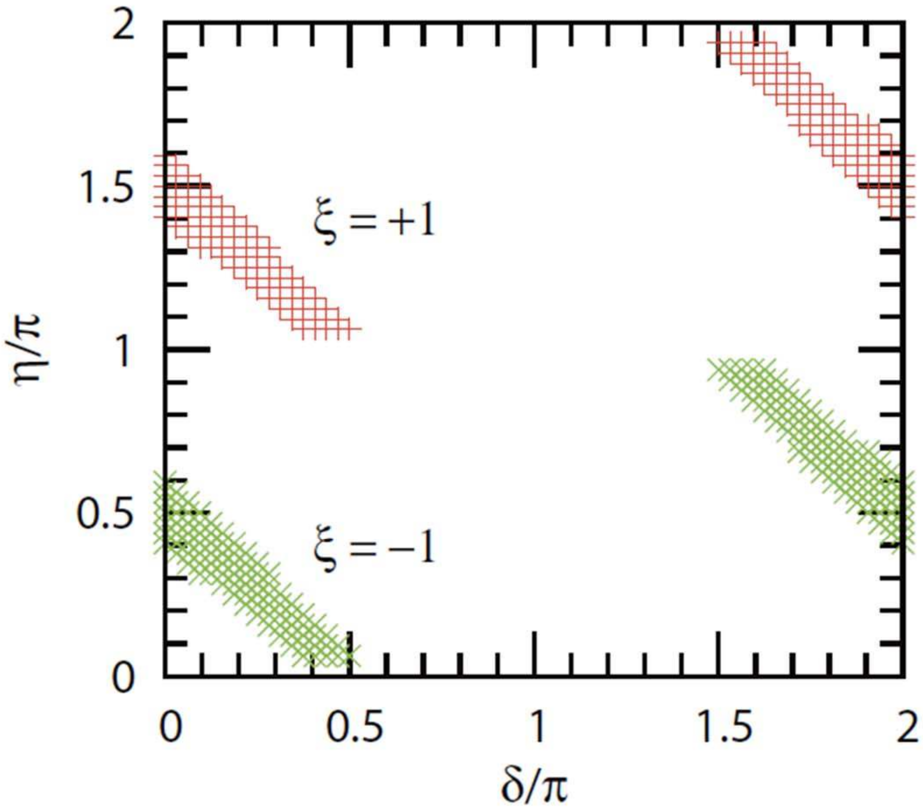


- Allowed region $M_N = 34\mathrm{MeV} \sim m_\pi$ in the NH case.
- The suppression of Θ_{eI} in the NH is crucial !

Allowed regions (search & BBN)

NH case $M_N = 120$ MeV





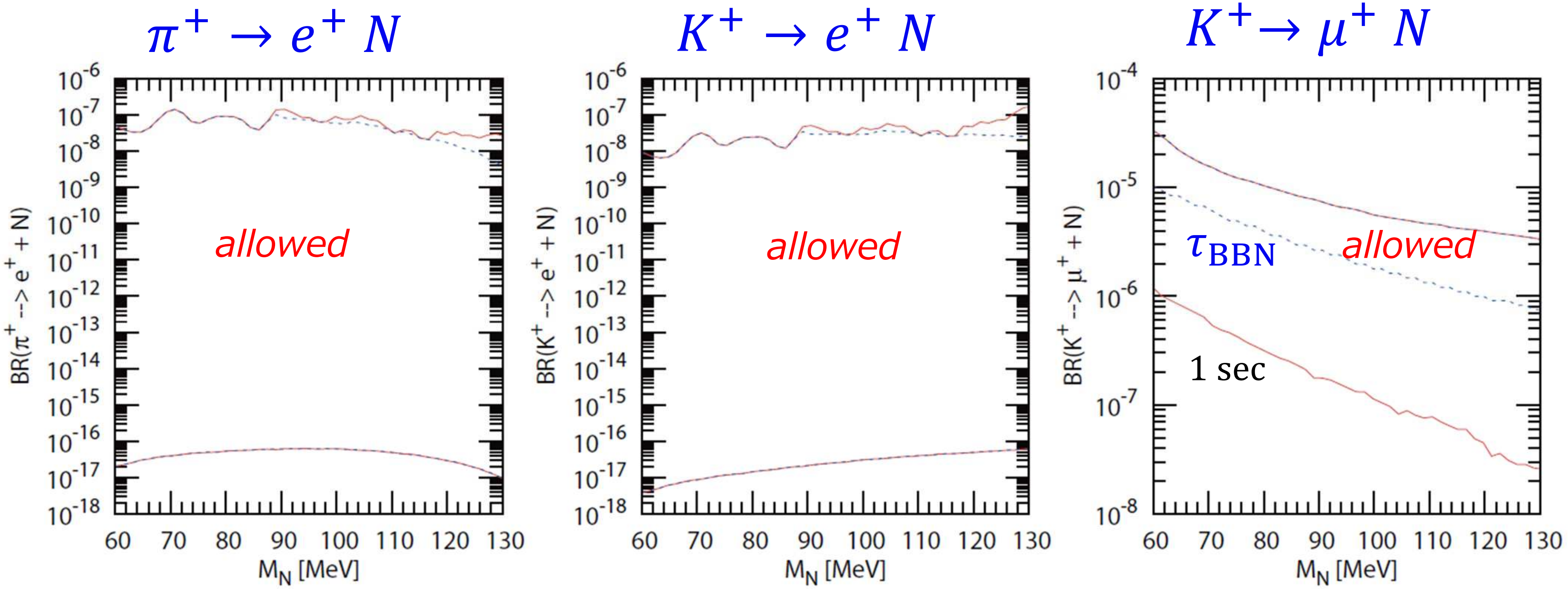
Large X_ω

Large θ_{13}

CP phases are aligned as

$$\xi \sin(\delta + \eta) \approx -1$$
$$\xi \sin\eta < 0$$

Branching ratios in NH case



§5

Summary

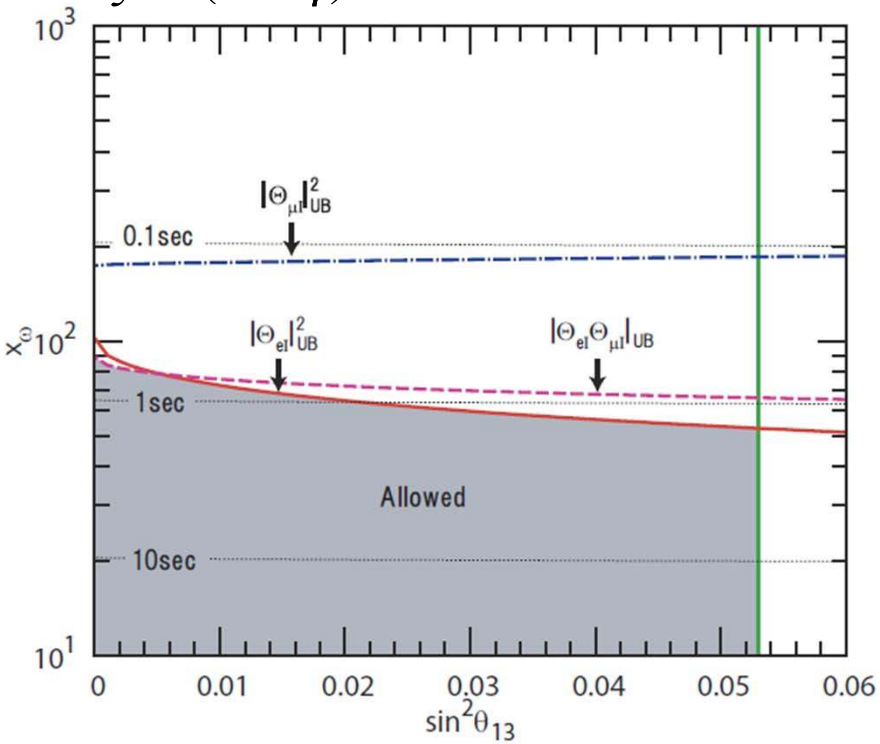
Summary

- **We considered the νMSM**
  - The MSM + 3 RH neutrinos with $M_N < O(10^2)$ GeV
    - N_1 : dark matter
    - N_2 and N_3 : seesaw + BAU
  - Mixing elements $\Theta_{\alpha 2}$ and $\Theta_{\alpha 3}$
    - $|\Theta_{\alpha I}|$ can be very large for large X_ω
    - Cancellation of $|\Theta_{eI}|$ is possible in NH
- **Neutrinoless double beta decay**
  - Effective neutrino mass is smaller than that of active neutrinos
  - No significant constraint on Θ_{eI}
- **Search for N_2 and N_3 with $M_{2,3} < m_\pi$**
  - Allowed region ($M_{2,3} \simeq 34\mathrm{MeV} \sim m_\pi$) in NH
  - $K^+ \longrightarrow \mu^+ N_{2,3}$ is promising to test
  - Br$(\pi^+ \longrightarrow e^+ N_{2,3})$ and Br$(K^+ \longrightarrow e^+ N_{2,3})$ may be very small

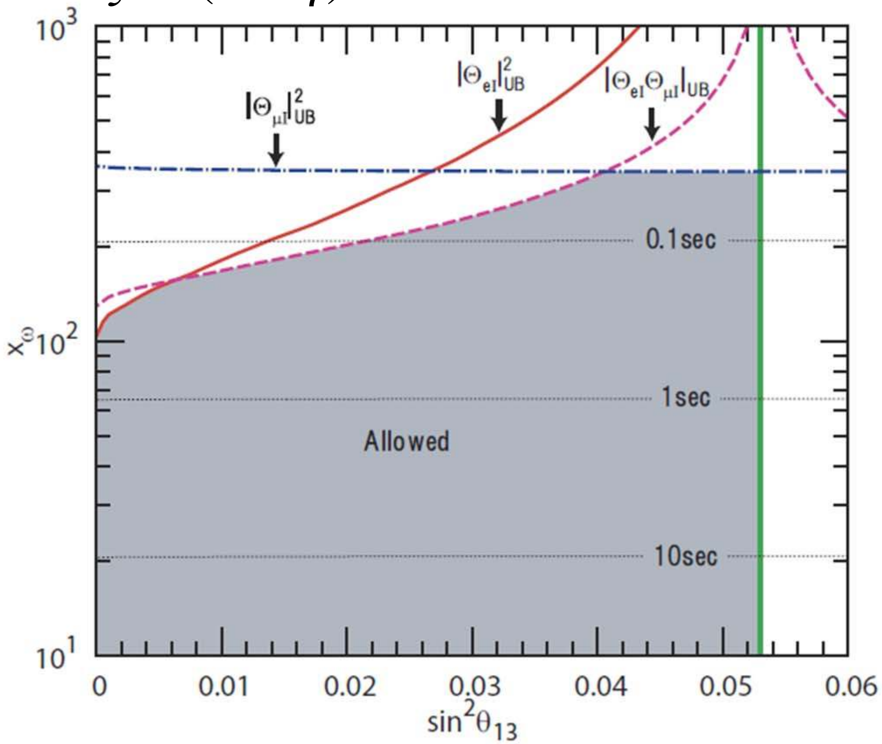
NH case

$M_N = 120 MeV$

$\xi \sin(\delta+\eta) = 1$



$\xi \sin(\delta+\eta) = -1$



Cancellation in $|\Theta_{eI}|$ occurs if

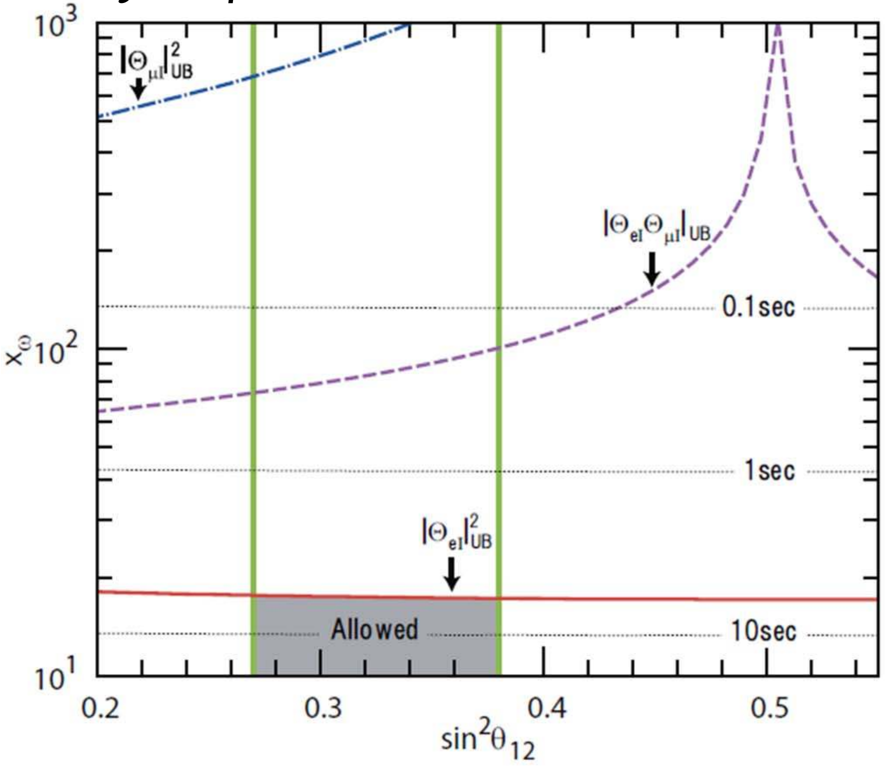
$$\xi \sin(\delta+\eta) = -1$$

$$\tan\theta_{13} = \sqrt{r_m}\,\sin\theta_{12}$$

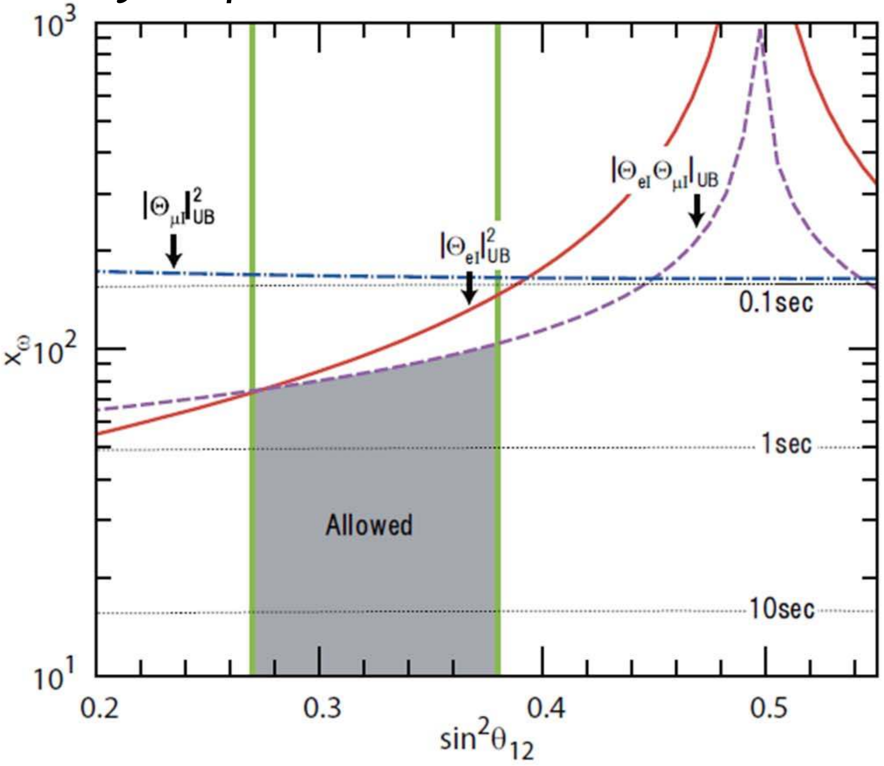
IH case

$M_N = 120 MeV$

$\xi \sin\eta = -1$



$\xi \sin\eta = 1$



Cancellation in $|\Theta_{\mu I}|$ and $|\Theta_{\tau I}|$ occurs if

$$\xi \sin\eta = -1$$

$$\tan\theta_{12} = (1 + r_m^2)^{+1/4}$$

Cancellation in $|\Theta_{eI}|$ occurs if

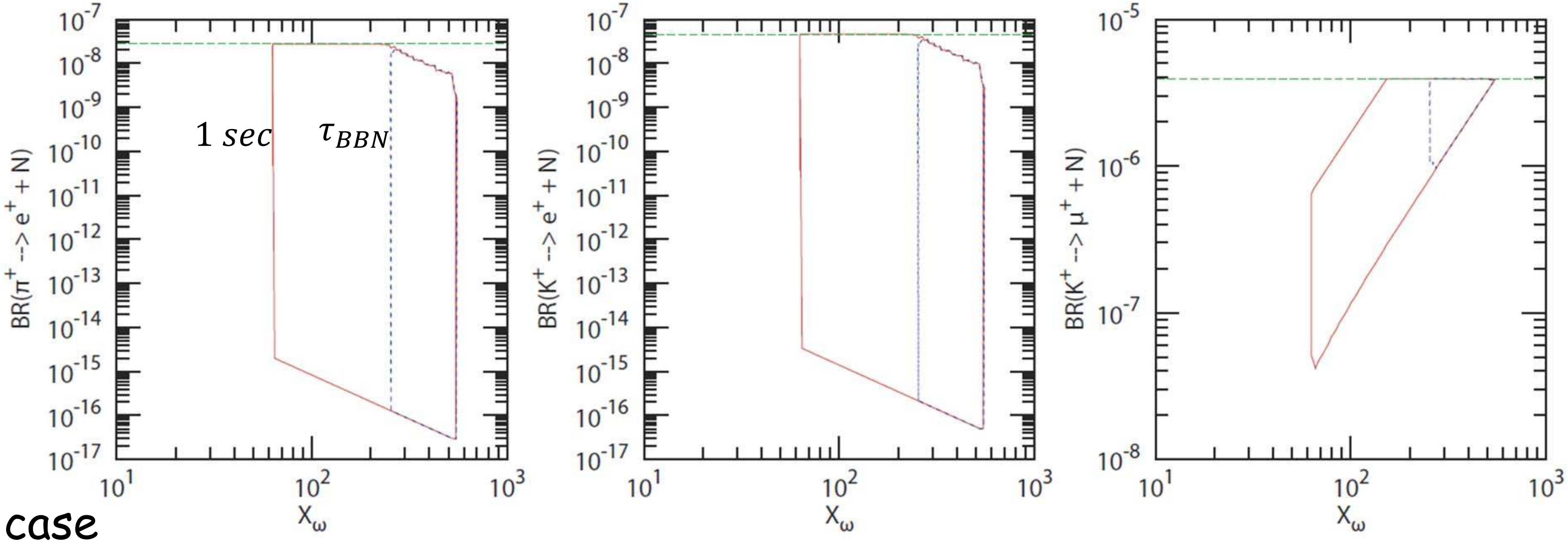
$$\xi \sin\eta = 1$$

$$\tan\theta_{12} = (1 + r_m^2)^{-1/4}$$

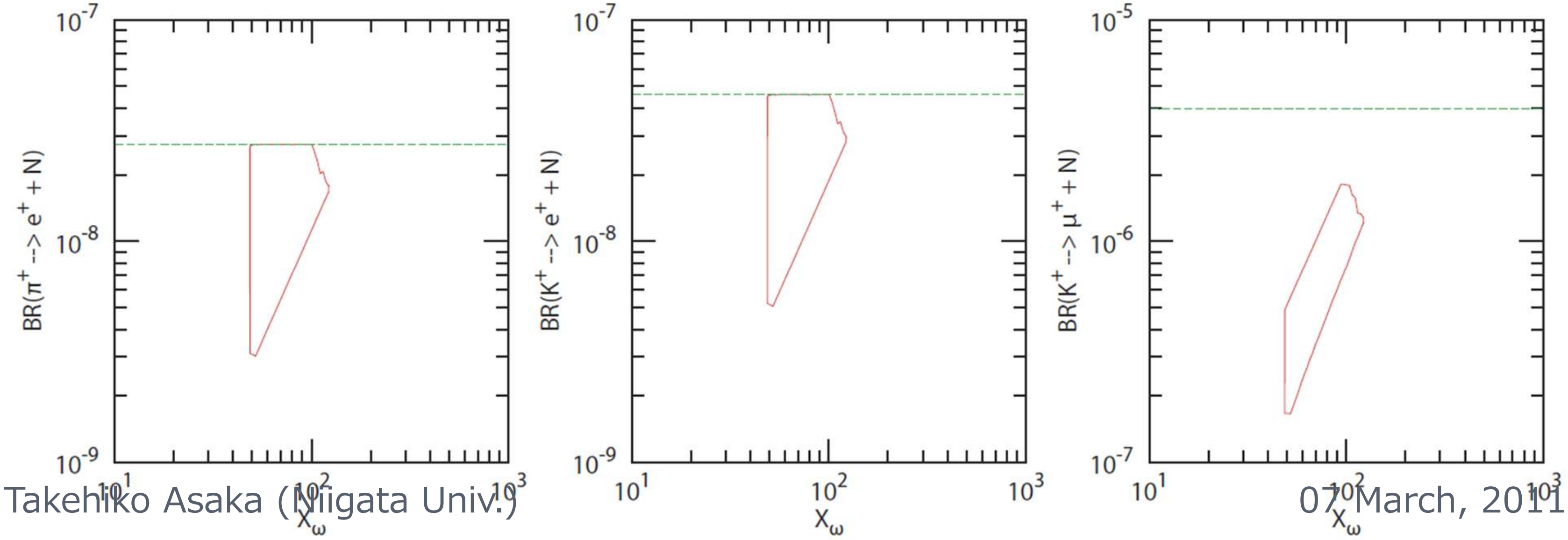
Branching ratios

$M_N = 120 MeV$

NH case

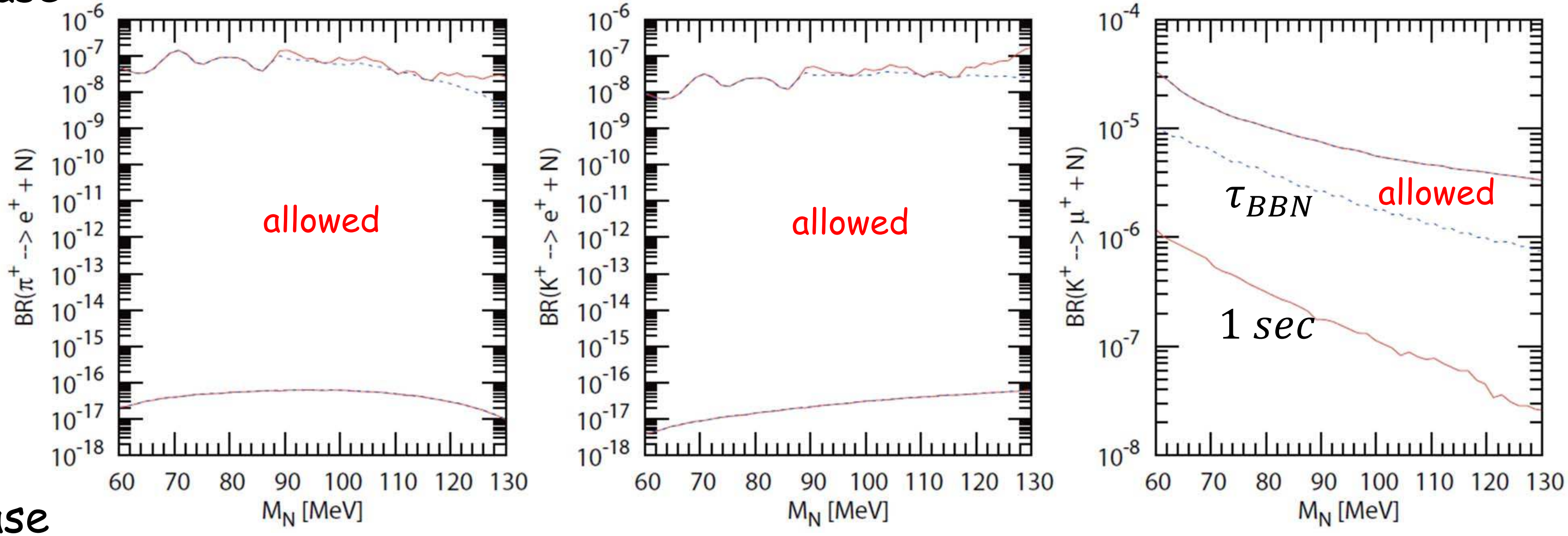


IH case



Branching ratios

NH case



IH case