

Study on Λnn Bound State and Resonance

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We perform the ab initio no-core shell model (NCSM) calculation to investigate the bound state problem of three-body Λnn system in chiral next-to-next-to-leading-order NN and chiral leading-order YN interactions. The calculations show that no Λnn bound state exists, but predict a low-lying Λnn resonant state near the threshold with the energy of $E_r = 0.124$ MeV and the width of about $\Gamma = 1.161$ MeV. In searching for Λnn resonances, we extend the NCSM calculation to the continuum state by employing the J-matrix formalism in the scattering theory with the hyperspherical oscillator basis.

I. INTRODUCTION

Microscopic calculations of few- and many-body systems with strangeness have been a focus in hypernuclear physics to explore the new dynamical features of the structure of hypernuclei and to improve understanding of hyperon-nucleon interactions. Indeed, hyperon-nucleon scattering data is very limited to fully determine the YN interactions. The existing data of few body hypernuclei could provide the important constrain on YN interaction. In hypernuclear physics, hypertriton is used as the first testing ground for YN interaction. It is the simplest and weakly bound hypernuclear system with Λ binding energy about ~ 0.13 MeV [1]. It seems like a lambda bound to deuteron core in the study of the spin-triplet NN interaction [2]. In the Λnn system, two neutrons interact in spin-singlet state and its strength is weaker than that in spin-triplet state. The strength of Λn is also not sufficient to form a bound system. It expects that the existence of a neutral bound state of two neutrons and a hyperon is improbable. But instead, three-body Λnn resonance may exist and that could be used to constrain the YN interaction. If Λnn system were a lightest neutron-rich bound system, it would provide significant information of Λn interaction and a better understanding of the nature of ΛN - ΣN coupling.

There have been a number of theoretical calculations for the Λnn system as a serious doubtful bound state problem. Nonexistence of Λnn bound state was first revealed by Dalitz and Downs [3] using a variational approach. Garcilazo [4] investigated the Λnn system by solving Faddeev equations using YN and NN interactions derived from a chiral constituent quark model and revealed that Λnn bound system was not found. Later the various approach such as hyperspherical harmonics (HH)[5], Faddeev calculations [2, 6–11], variational calculations [12], pionless effective field theory [13–15] with various kinds of baryon-baryon interactions have been

used to analyze the Λnn system and all reported that it is highly unlikely to form a bound system in the theoretical analysis without a significant altering nuclear and hypernuclear forces.

The ${}^3_{\Lambda}n$ hypernucleus could not be produced in the earlier experiments due to no charge of its bound state. However, the HypHI collaboration at GSI [16] reported the first evidence of the existence of the ${}^3_{\Lambda}n$ bound state from analysis of the observed two- and three-body decays mode without describing any statement for the value of binding energy. Their observation was inconsistent with the claim of the above theoretical analysis.

In this paper, we analyze the Λnn bound state problem using the ab initio no-core shell model (NCSM) [17–19] technique. The calculation of the Λnn system ($J^{\pi} = 1/2^+, T = 1$) is performed in Jacobi coordinate HO basis using the NN and YN interactions derived from chiral effective field model. In the extension into the continuum state, we apply the SS-HORSE [20–25] formalism, which is a single state harmonic oscillator representation of scattering equations, to calculate the low-energy phase shifts and scattering amplitudes at the NCSM eigenenergies by employing hyperspherical harmonic oscillator basis. The low-lying Λnn resonance energy and width are extracted from the scattering amplitude parametrization. The NCSM-SS-HORSE method [26] has been successfully applied to study a tetra-neutron unbound system considered as true four-body scatterings. Here we first apply this method to study the three-body system with strangeness.

II. NCSM-SS-HORSE FORMALISM

The hypernuclear Hamiltonian for two nucleon and a hyperon system can be written

$$H = - \sum_{i=1}^3 \frac{\hbar^2}{2m_i} \vec{\nabla}_i^2 + V_{NN}(\vec{r}_1, \vec{r}_2) + \sum_{i=1}^2 V_{YN}(\vec{r}_i, \vec{r}_3) + \Delta M, \quad (1)$$

where the coordinates \vec{r}_i and masses m_i are for the two nucleon with $i = 1, 2$ and the hyperon with $i = 3$. We

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work with nonrelativistic two-body NN and YN potentials, employing the leading-order chiral hyperon-nucleon interactions with regulator cutoff $\Lambda_{YN} = 600$ MeV [27] and a family of 42 different nuclear interactions at next-to-next-to-leading order (also called chiral NNLO_{sim} family of NN interactions) [28]. These nuclear interactions were constructed by varying the chiral regulator cutoff Λ_{NN} between 450 and 600 MeV in steps of 25 MeV and the truncation of the input NN scattering $T_{\text{lab}} \leq T_{\text{Lab}}^{\text{max}}$ between 125 and 290 MeV in six steps, which were obtained from a simultaneous optimization of all 26 low-energy constants (LECs) to different sets of NN and πN scattering plus bound state observables [28]. In this work, we mainly use the NN interactions with $\Lambda_{NN}=500$ and $T_{\text{Lab}}^{\text{max}}=290$ MeV. The effect of ΛN - ΣN coupling is taken into account [19].

In NCSM, three active particles are considered in the three-dimensional harmonic oscillator (HO) basis. In the construction of HO basis states for such a few-body Λnn system, it is more effective to use the relative Jacobi coordinates where the center of mass (c.m.) coordinate $\vec{\xi}_0$ is separated, which allows us to perform NCSM calculations in large model space. The relative Jacobi coordinates in terms of the rescaled version of the single-particle coordinates $\vec{x}_i = \sqrt{m_i} \vec{r}_i$ are defined as

$$\begin{aligned}\vec{\xi}_1 &= \sqrt{\frac{1}{2}}(\vec{x}_1 - \vec{x}_2), \\ \vec{\xi}_2 &= \sqrt{\frac{2m_N m_Y}{2m_N + m_Y}} \left[\frac{1}{2\sqrt{m_N}}(\vec{x}_1 + \vec{x}_2) - \frac{1}{\sqrt{m_Y}}\vec{x}_3 \right],\end{aligned}\quad (2)$$

where m_N and m_Y are the masses of nucleon and hyperon. $\vec{\xi}_1$ is the relative coordinate of the two-nucleon pair and $\vec{\xi}_2$ is the relative coordinate of the hyperon with respect to the c.m. of the two-nucleon pair. Following the general Jacobi coordinate formulation in Ref. [19], we construct the JT-coupled HO basis states for the system of a two-nucleon pair and a hyperon,

$$|(n_{NN}(l_{NN}s_{NN})j_{NN}t_{NN}, \mathcal{N}_Y \mathcal{L}_Y \mathcal{J}_Y \mathcal{T}_Y)JT\rangle, \quad (3)$$

depending on the coordinates $\vec{\xi}_1$ and $\vec{\xi}_2$ respectively. n_{NN} , l_{NN} , s_{NN} , j_{NN} , t_{NN} (\mathcal{N}_Y , \mathcal{L}_Y , \mathcal{J}_Y , \mathcal{T}_Y) are the HO radial quantum number, orbital angular momentum, spin, angular momentum and isospin of the relative two-nucleon (hyperon) state. J and T are the total angular momentum and total isospin of the system. The basis (3) is antisymmetrized with respect to the exchange of two nucleon by restricting the two nucleon relative quantum numbers with the condition $(-1)^{l_{NN}+s_{NN}+t_{NN}} = -1$. The basis (3) is suitable for evaluating two-body NN interaction matrix elements but not for evaluating two-body YN interaction matrix elements.

For a subsystem including YN pair and a nucleon, another set of Jacobi coordinate is correspondingly intro-

duced,

$$\begin{aligned}\vec{\eta}_1 &= \sqrt{\frac{(m_N + m_Y)m_N}{2m_N + m_Y}} \left[\frac{1}{\sqrt{m_N}}\vec{x}_1 \right. \\ &\quad \left. - \frac{1}{(m_N + m_Y)}(\sqrt{m_N}\vec{x}_2 + \sqrt{m_Y}\vec{x}_3) \right], \\ \vec{\eta}_2 &= \sqrt{\frac{m_N m_Y}{m_N + m_Y}} \left(\frac{1}{\sqrt{m_N}}\vec{x}_2 - \frac{1}{\sqrt{m_Y}}\vec{x}_3 \right),\end{aligned}\quad (4)$$

where $\vec{\eta}_1$ is the relative coordinate of a nucleon with respect to the c.m. of the YN pair and $\vec{\eta}_2$ is the relative coordinate of the YN pair. By using orthogonal transformation, the antisymmetrized HO basis (3) can be expanded as

$$\begin{aligned}& |(n_{NN}(l_{NN}s_{NN})j_{NN}t_{NN}, \mathcal{N}_Y \mathcal{L}_Y \mathcal{J}_Y \mathcal{T}_Y)JT\rangle \\ &= \sum_{LS} \hat{L}^2 \hat{S}^2 \hat{J}_{NY} \hat{J}_N \hat{J}_{NN} \hat{J}_Y (-1)^{s_{NY} + \frac{1}{2} + s_{NN} + \frac{1}{2} + \mathcal{L}_N + \mathcal{L}_Y} \\ &\times \left\{ \begin{matrix} l_{NY} & s_{NY} & j_{NY} \\ \mathcal{L}_N & \frac{1}{2} & \mathcal{J}_N \\ L & S & J \end{matrix} \right\} \left\{ \begin{matrix} l_{NN} & s_{NN} & j_{NN} \\ \mathcal{L}_Y & \frac{1}{2} & \mathcal{J}_Y \\ L & S & J \end{matrix} \right\} \left\{ \begin{matrix} \frac{1}{2} & \frac{1}{2} & s_{NN} \\ \frac{1}{2} & S & s_{NY} \end{matrix} \right\} \\ &\times (-1)^{t_{NY} + \mathcal{T}_N + t_{NN} + \mathcal{T}_Y} \hat{t}_{NY} \hat{t}_{NN} \left\{ \begin{matrix} \frac{1}{2} & \frac{1}{2} & t_{NN} \\ \mathcal{J}_Y & T & t_{NY} \end{matrix} \right\} \\ &\times \langle n_{NY} l_{NY} \mathcal{N}_N \mathcal{L}_N | n_{NN} l_{NN} \mathcal{N}_Y \mathcal{L}_Y \rangle_{d=\frac{2m_N+m_Y}{m_Y}} \\ &\times |(n_{NY}(l_{NY}s_{NY})j_{NY}t_{NY}, \mathcal{N}_N \mathcal{L}_N \mathcal{J}_N)JT\rangle,\end{aligned}\quad (5)$$

in terms of HO basis states

$$|(n_{NY}(l_{NY}s_{NY})j_{NY}t_{NY}, \mathcal{N}_N \mathcal{L}_N \mathcal{J}_N)JT\rangle, \quad (6)$$

depending on the coordinates $\vec{\eta}_2$ and $\vec{\eta}_1$ respectively. The general HO bracket $\langle n_{NY} l_{NY} \mathcal{N}_N \mathcal{L}_N | n_{NN} l_{NN} \mathcal{N}_Y \mathcal{L}_Y \rangle_d$ follows the agreement of Ref. [29]. YN interaction matrix elements involving Λ and Σ hyperons are evaluated in the antisymmetrized basis (3) through its expansion in the basis (6) as

$$\left\langle \sum_{i=1}^2 V_{YN}(\vec{r}_i, \vec{r}_3) \right\rangle = 2 \langle V_{YN}(\vec{\eta}_2) \rangle, \quad (7)$$

where the matrix elements on the right-hand side are diagonal in all quantum numbers of the basis states (6), except for n_{NY} and l_{NY} . The lowest eigenstates of the Λnn system are calculated by the diagonalization of the truncated Hamiltonian matrix.

To look for resonances, we extend our study to the continuum state by employing J-Matrix formalism, also known as Harmonic oscillator representation of scattering equation (HORSE) formalism, which arms one to study continuum spectrum using only positive energies obtained from bound state approach like NCSM applying HO basis. The HORSE method can be used to describe the open channels in the external subspace while the internal subspace is associated with the NCSM approach.

For details of the HORSE formalism, we may refer to Refs. [22, 30].

In the extension into continuum, the three-body extension of the J-matrix formalism for all three-body decay channels is very complicated. We apply the democratic decay approximation (also known as true three-body scattering or $3 \rightarrow 3$ scattering) [31] which employs the hyperspherical harmonic (HH) basis to describe the Λnn system decaying through only three-body break-up channel and it does not allow for other possible two-body channels associated with two-body sub-bound states.

The hyperspherical oscillator basis can be labeled as $|\kappa K \gamma\rangle$, where κ is the principal quantum number and K is the hypermomentum, $\gamma \equiv \{l_1, l_2, L, s_1, s_2, S, t_1, t_2, T\}$ collects all possible quantum numbers corresponding to the Jacobi coordinates for a three-body system. The external subspace is spanned by hyperspherical oscillator functions with $N \equiv 2\kappa + K > N_{\max}$ where the Hamiltonian $H = T$ is used. Here N_{\max} is the maximum number of excitation quanta defining the many-body NCSM basis space. Because of high centrifugal barrier $\mathcal{L}(\mathcal{L} + 1)/\rho^2$, the HH states with larger K can be neglected in the case of no sub-bound Λnn system [ρ is hyper radius with the mass scaled Jacobi coordinates and $\mathcal{L} = K + 3/2$ is the effective momentum]. It is adequate to consider a single hyperspherical channel with minimum hypermomentum $K_{\min} = 0$ to describe democratic three-body decays.

We follow the SS-HORSE approach [21, 22, 26] to compute the $3 \rightarrow 3$ scattering phase shifts at the eigenenergies $E_\nu > 0$ obtained directly from NCSM calculation,

$$\tan \delta(E_\nu) = -\frac{S_{N_{\max}+2, \mathcal{L}}(E_\nu)}{C_{N_{\max}+2, \mathcal{L}}(E_\nu)}, \quad (8)$$

where $S_{N\mathcal{L}}$ and $C_{N\mathcal{L}}$ are regular and irregular solutions of free Schrödinger equation in the hyperspherical oscillator representation, which can be applied in the case of arbitrary \mathcal{L} (both integer and half integer), taking simple analytical expressions [21, 23, 31]

$$S_{N\mathcal{L}}(E) = \sqrt{\frac{(N - \mathcal{L} + \frac{3}{2})!}{\lambda \Gamma(\frac{N}{2} + \frac{\mathcal{L}}{2} + \frac{9}{4})}} q^{\mathcal{L}+1} e^{-\frac{q^2}{2}} L_{(N-\mathcal{L}+\frac{3}{2})/2}^{\mathcal{L}+\frac{1}{2}}(q^2), \quad (9)$$

$$C_{N\mathcal{L}}^{(\pm)}(E) = \frac{1}{\pi \sqrt{\lambda}} \sqrt{(N - \mathcal{L} + \frac{3}{2})! \Gamma(\frac{N}{2} + \frac{\mathcal{L}}{2} + \frac{9}{4})} \Psi(\frac{N}{2} + \frac{\mathcal{L}}{2} + \frac{9}{4}, \mathcal{L} + \frac{3}{2}; e^{\mp i\pi} q^2) q^{\mathcal{L}+1} e^{\frac{q^2}{2}} e^{\mp i\pi(\mathcal{L}+\frac{1}{2})}, \quad (10)$$

$$C_{N\mathcal{L}}(E) = \frac{1}{2} (C_{N\mathcal{L}}^{(+)}(E) + C_{N\mathcal{L}}^{(-)}(E)), \quad (11)$$

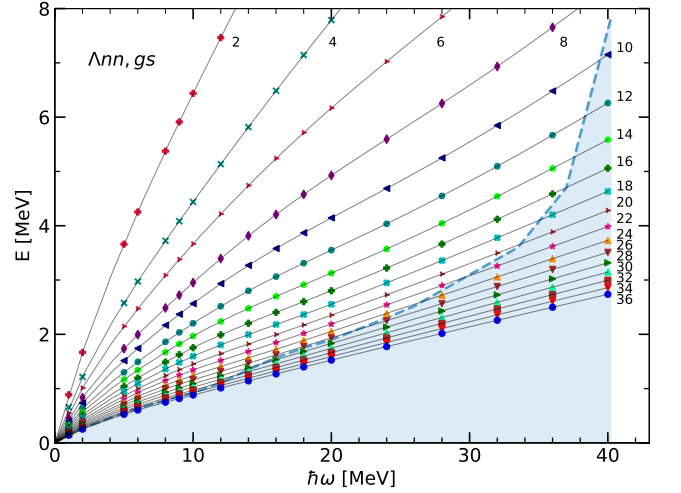


FIG. 1. The eigenenergies of the NCSM Hamiltonian with various model space sizes N_{\max} as a function of oscillator frequency $\hbar\omega$. The numbers at the end of each line represent N_{\max} . The blue shaded area shows the selected energies for parametrization of the scattering amplitude.

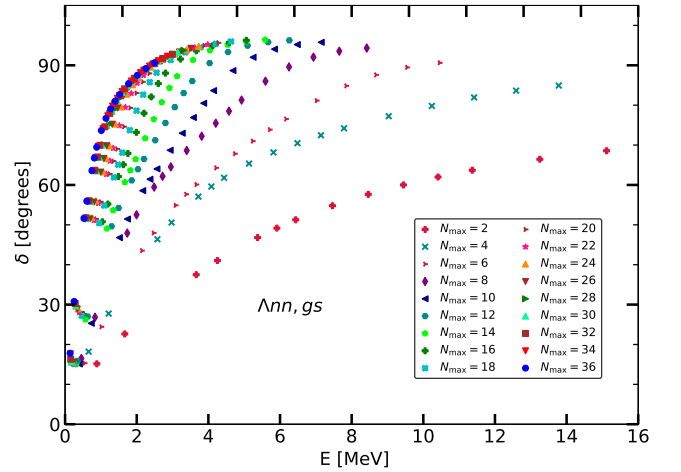


FIG. 2. $3 \rightarrow 3$ scattering phase shifts obtained directly from the NCSM eigenstates using Eq. (8).

where $q = \sqrt{\frac{2E}{\hbar\omega}}$ is dimensionless momentum, $L_{\kappa}^{\mathcal{L}+\frac{1}{2}}(x)$ is the associated Laguerre polynomial, $\lambda = \sqrt{\frac{m\omega}{\hbar}}$ is the oscillator radius at $\Psi(a, c; x)$ which is the Tricomi function.

The SS-HORSE scattering amplitude for neutral particles may be calculated in the standard way,

$$f(E_\nu)q = \frac{1}{(\cot \delta_{\mathcal{L}}(E_\nu) - i)}. \quad (12)$$

We parameterize the scattering amplitude in the method proposed in [32] for the case that a resonance is not sharp, but both the potential scattering (non-resonant background) and resonance contribution are not negligi-

ble. The scattering amplitude may be parametrized as

$$F(E)q = e^{i\delta_0(E)} \sin \delta_0(E) + \frac{-\Gamma/2}{E - E_r + i\Gamma/2} e^{2i\delta_0(E)}, \quad (13)$$

where $\delta_0(E)$ is the potential scattering phase shift, depending on the energy E . We will fit the SS-HORSE scattering amplitude by the complex-valued function $F(E)q$ in the next section to determine the form of the $\delta_0(E)$ and derive the resonance energy E_r and width Γ .

III. RESULTS AND DISCUSSION

The Λnn system is analyzed using the NCSM approach with chiral NNLO_{sim} NN and LO YN interactions. The NCSM computational model space is characterized by a chosen maximal total HO quanta N_{\max}^{tot} , that is,

$$2n_{NN} + l_{NN} + 2N_Y + \mathcal{L}_Y \leq N_{\max}^{\text{tot}} \equiv N_{\max} + N_0, \quad (14)$$

where the minimal possible number of HO quanta is $N_0 = 0$. In Λnn case, $N_{\max}^{\text{tot}} = N_{\max}$. We have computed the total energy of Λnn system in the oscillator basis with model space truncations $N_{\max} \leq 36$, and in the range of the HO frequencies $1 \text{ MeV} \leq \hbar\omega \leq 40 \text{ MeV}$. It is found that there is no Λnn bound system. The Λnn ground-state energy as a function of the model space truncation N_{\max} and HO frequency $\hbar\omega$ is presented in Figure 1. The NCSM energies decrease with increasing N_{\max} and with decreasing $\hbar\omega$. Our model used here can reproduce well the binding energy of hypertriton [33] and also for s-shell hypernuclei, ${}^4_{\Lambda}H$ and ${}^4_{\Lambda}He$, which will be a future publication.

The SS-HORSE phase shifts covering all computed NCSM energies calculated by using Eq. (8) are shown in Figure 2. The phase shifts obtained with smaller N_{\max} lie in a wide energy region as the obtained Λnn ground-state energies spread widely. With N_{\max} increasing, however, the obtained Λnn ground-state energies converge to lower values, as shown in Figure 1, and hence the corresponding phase shifts shift to the lower energy region. The first convergence of phase shifts is achieved at smaller energies with larger N_{\max} , almost the same results at $N_{\max} = 34$ and 36 MeV. We follow the selection procedure of Ref. [21, 26, 34] and select a set of eigenvalues E_ν from the $N_{\max} = 10$ -36 model spaces, which is illustrated by the shaded area in Figure 1, to produce a single smooth curve of phase shifts for parametrization. The SS-HORSE phase shifts corresponding to these selected smaller eigenvalues are plotted in Figure 3.

We compute the SS-HORSE low-energy scattering amplitude for the purpose of extracting the resonance parameters from scattering amplitude parametrization. The function $|f(E_\nu)q|^2$ of the scattering amplitude given in Eq. (12) is shown by symbols in Figure 4. The fitting to the SS-HORSE result $|f(E_\nu)q|^2$ by the function $|F(E)q|^2$ leads the $\delta_0(E)$ to the form

$$\delta_0(E) = a_0 + a_2(\sqrt{E})^2 + a_4(\sqrt{E})^4, \quad (15)$$

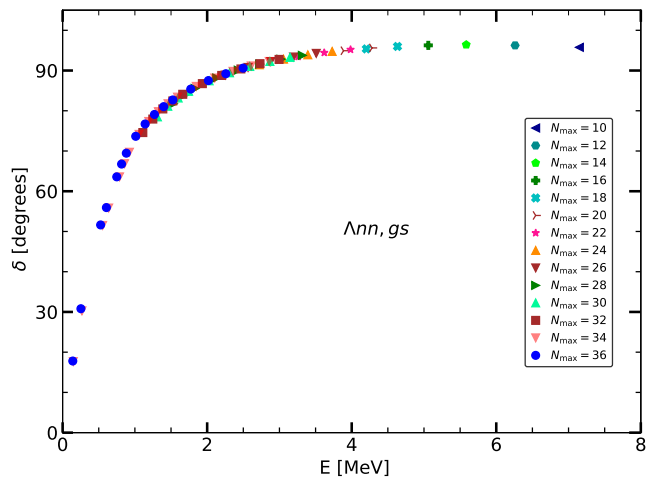


FIG. 3. $3 \rightarrow 3$ scattering phase shifts obtained from selected NCSM eigenstates with $N_{\max} \in [10, 36]$ for scattering amplitude parametrization.

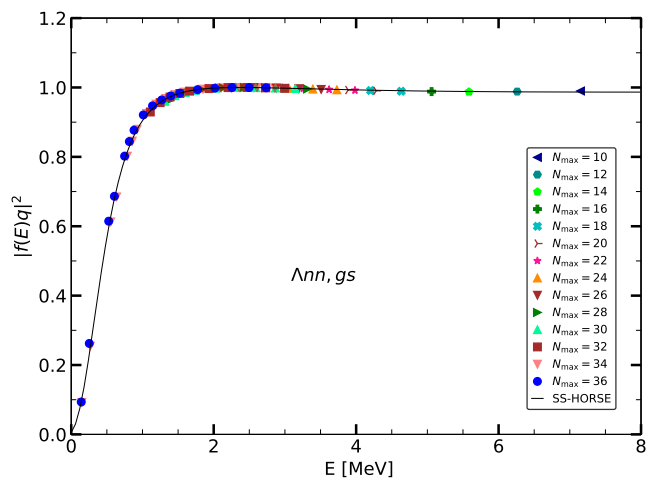


FIG. 4. The scattering amplitude $|f(E)q|^2$ using Eq. (12) obtained from NCSM eigenstates (symbol). The solid line shows the parametrization of scattering amplitude in Eq. (13).

with the adjustable parameters $a_0 = 1.856$, $a_2 = -0.014 \text{ MeV}^{-1}$, $a_4 = 2.959 \times 10^{-4} \text{ MeV}^{-2}$. The resonance energy and width are derived, $E_r = 0.124 \text{ MeV}$ and $\Gamma = 1.161 \text{ MeV}$. The result is in good agreement with those in Ref. [7, 11] and lies within the estimated range of the location and width of a Λnn pole [35]. We look forward to the results of Λnn bound and resonance states from the ongoing experiment (E12-17-003) at Jefferson Lab (JLab) [36]. Such Λnn bound and resonance states, if any, are expected to provide new perspective on Λn interactions.

SUMMARY

We have performed ab initio no-core shell model calculations for the Λnn system ($J^\pi = 1/2^+$, $T = 1$) without tuning the strength of realistic NN and YN potentials at

various N_{\max} and $\hbar\omega$ values with full inclusion of ΛN - ΣN coupling, and found that no bound state exists. To look for resonance states of the Λnn , we have applied the NCSM-SS-HORSE technique to calculate the Λnn scattering phase shifts which suggest a Λnn resonant state at energy $E_r = 0.124$ MeV and $\Gamma = 1.161$ MeV. Further theoretical studies and experimental searches for Λnn resonances would be of great benefit of constraining Λn interactions.

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