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ORIGINAL ARTICLE

MobileMath: exploring mathematics outside the classroom

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Abstract Computer games seem to have a potential for engaging students in meaningful learning, inside as well as outside of school. With the growing availability of mobile handheld technology (HHT), a number of location-based games for handheld mobile phones with GPS have been designed for educational use. The exploitation of this potential for engaging students into meaningful learning, however, so far remains unexplored. In an explorative design research, we investigated whether a location-based game with HHT provides opportunities for engaging in mathematical activities through the design of a geometry game called MobileMath. Its usability and opportunities for learning were tested in a pilot on three different secondary schools with 60 12–14-year-old students. Data were gathered by means of participatory observation, online storage of game data, an online survey and interviews with students and teachers. The results suggest that students were highly motivated, and enjoyed playing the game. Students indicated they learned to use the GPS, to read a map and to construct quadrilaterals. The study suggests learning opportunities that MobileMath provides and that need further investigation.

This article is based on the paper 'MobileMath: the phone, the game and the math' (Wijers, Jonker, & Kerstens, 2008) presented at ECGBL2008, Barcelona, Spain, 16–17 October 2008.

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1 Introduction

So far, the use of handheld technology (HHT) in mathematics education has mainly focused on traditional mathematical content, such as algebra and graphing, and took place in regular teaching settings (Trouche and Drijvers 2010). Nowadays, however, HHT has more to offer: handheld devices offer options for other topics, such as geometry, and can be used outside the classroom, thanks to wireless communication. Such extended mobile devices also provide opportunities for game play in ways that are new for mathematics students and their teachers. These new features are supposed to engage students in meaningful learning, inside as well as outside of school (Gee 2003; Shaffer 2006; DeVane et al. 2008), fit into the digital lifestyle of the new generation and are expected to have great motivational potential. Computer games may enable setting up rich learning environments that appeal for peer collaboration, knowledge constructions, and new roles for both teacher and students (Egenfeldt-Nielsen 2005).

Until today, little is known about the ways in which these assumed potentials can be exploited for the sake of engagement in and quality of learning. Therefore, the overarching aim of the study that we report upon in this paper is to explore the potential of mobile HHT for the engagement in mathematics education, and the learning opportunities of a geometrical out-of-school game in particular.

2 Theoretical background

The backbone of the study's theoretical framework is formed by notions on engagement, and on authentic and realistic mathematics learning.

Mobile gaming is a general term that covers a large range of activities including playing casual games on handhelds as well as playing highly interactive location-based games on mobile devices, where real and virtual worlds are mixed. Digital games seem to be tools for supporting meaningful learning and engagement of students, both inside and out of school (Prensky 2001; Gee 2003; Shaffer 2006; DeVane et al. 2008). Gee (2003) connects engagement in good video games to players being 'enticed to try' (p. 65), put in lots of effort and spend lots of time on task. Prensky (2001) connects engagement to 'total involvement', no off-task behavior and continuous paying attention. He formulates the following elements that need to be carefully designed and combined in order to create engaging games:

- *rules*: provide structure and organize the game play;
- *goals*: motivate players;
- *outcome and feedback*: inform players on the progress, provide opportunities to learn;
- *conflict/challenge/competition*: make games exciting;
- *interaction*: social aspect.

Mobile devices provide the opportunity to actually situate learning outside school and make it possible to integrate characteristics of effective learning such as situated authentic learning, peer collaboration and motivational power. In the definition of O'Malley et al. (2003), mobile learning is any sort of learning that happens when the learner is not at a fixed, predetermined location, or that happens when the learner takes advantage of learning opportunities offered by mobile technologies. A key characteristic of mobile learning is that it enables knowledge building and constructing understanding by learners in different contexts (Winters 2007). Research on game-based learning suggests that the combination of reality and virtual elements leads to a mixed reality experience which can contribute to the students' engagement (Schwabe & Göth 2005). A term used to stress this combination is hybrid reality game (HRG), as this type of games augments the reality by adding virtual elements to it.

> Hybrid reality games (HRGs) employ mobile handheld technologies and GPS devices as tools for transforming physical spaces into interactive game boards (De Souza e Silva & Delacruz 2006, p. 231). HRGs' intrinsic motivational properties transform an otherwise dry curriculum into something entertaining and fun, using affordable and ubiquitous technology. However, these types of games are more than a new snazzy delivery vehicle for the existing content. HRGs force players to look at familiar spaces from unfamiliar perspectives and at content learned in the classroom from a different viewpoint, using learning principles such as elements of social, experiential, and situated learning. (De Souza e Silva & Delacruz 2006, p. 246)

Of course, in serious gaming, we are interested not only in engagement, but also in learning. To enhance the learning effects, games need to be embedded in learning activities (Lave & Wenger 1991). The effectiveness of these learning activities can be stimulated if the tasks are authentic and realistic, and can be worked on in collaboration (O'Donnell et al. 2005). Recent research has shown that the use of mobile location-aware games can contribute to meaningful learning on several school and academic subjects such as history (Admiraal et al. 2007) and science (Squire & Klopfer 2007; Squire 2008). For the case of mathematics, the theory of Realistic Mathematics Education (RME) may help us to design meaningful learning activities.

The theory of RME stresses that problem situations presented in learning activities should be 'experientially' real to students (Gravemeijer 1994) and have meaningful, authentic problem situations as starting points. Mathematics is seen as a constructive human activity that is driven by the act of mathematizing (Freudenthal 1991). Freudenthal (1991) stresses the importance of guided reinvention, which underlies the construction of knowledge as if a student reinvented it. Students' own productions and constructions play an important part in the learning process also because they form input for classroom discussions and interactive reflection. This is in line with a socio-constructivist perspective on learning, in which mathematics is seen as both an individual, constructive activity and as a social practice (Cobb, Yackel, & Wood 1992). Social interaction is seen as a necessary condition for learning mathematics (Freudenthal 1991; Treffers 1987, 1991).

With notions of engagement and authentic, RME in mind, we can now phrase the central question of this pilot study:

> Can a social mobile game, based on the geographical reality, its virtual map representation, and location-sensitive handheld technology engage 12-14 year old students in meaningful mathematical activities?

The words 'engage' and 'meaningful' evoke the following sub-questions:

- *Game play*: does the game engage the students? Do they appreciate the mixed reality experience?
- *Learning opportunities*: Do the students recognize the mathematics embedded in the mobile game? Do students feel they learned something?

3 Methods

To address the research questions, a game called MobileMath was designed, with corresponding learning activities, and a sequence of pilots was set up. A first pilot was held with the teachers of the three pilot schools, followed by three pilots with students at different schools. In all pilots, researchers observed the team activities during the game and debriefed the teams afterward. Observations were made either by accompanying a team outside as a participating researcher or by watching all teams in the game in real time on the website. Notes were made and the game data were stored online. Players completed an online questionnaire immediately after playing.

3.1 Design of the MobileMath game

With the design of the game MobileMath (Demeyer et al. 2008), the design team consisting of researchers on mathematics education[1] and developers of creative technology for social innovation[2] has tried to combine principles of RME with principles of mobile game-based learning in order to design a mobile mathematical game, i.e. engaging and provides opportunities for mathematics learning.

As a first design heuristic, the elements described by Prensky (2001) guided the design of the game structure. The players' position is to be involved into the game play and thus should support situative, authentic and collaborative learning of mathematics. Collaboration and interaction, including both social interaction and interaction with the tools, were realized by having students play in eight competing teams of two students. The game rules were designed to promote both types of interaction.

As a second design principle, the RME theory led us to search for an authentic and realistic problem situation—where we take the word 'realistic' as meaningful rather than as 'from everyday life'. By incorporating students' own mathematical constructions in the heart of the game play of MobileMath, we wished to integrate an important tenet of RME with principles of situated and game-based learning.

MobileMath is designed to be a HRG. In MobileMath, we wanted the players to be immersed in a mixed reality game environment, in which they create virtual elements, in this case mathematical shapes, by interacting with the real world.

We wanted to design a geometric game. By playing the game, students were expected to deepen their experiential knowledge of geometrical concepts related to shapes and orientation/navigation. This includes: properties of angles and edges of parallelograms (including rectangles and squares); geometrical concepts such as parallel and perpendicular; and on the boundary of geography and math: orientation and navigation in 2D (map) and 3D (real world). These are all topics addressed in the math curriculum in lower secondary education in grade 7 or 8. The objectives are to learn the names of the shapes and gain (at least implicit) knowledge of some fundamental properties of parallelograms regarding sides being in pairs parallel and of equal length; and angles being straight or not. In the game, they create parallelograms and thus must use and make explicit the knowledge of these properties.

We identified the starting point for the design of MobileMath to be the game play. The mathematical content to be learnt would be integrated into the game in an intrinsic way. Previous research on game-based learning has shown that the content of a game can be integrated into the game in an intrinsic or an extrinsic way (Kafai, 1998, 2001; Malone and Lepper, 1987). Intrinsic integration of content (mathematics) in a game provides students with greater opportunities to construct new relationships with knowledge in the process of playing. In this type of games, the educational and game components are inseparable (Klopfer, 2005). A large number of the available games for mathematics do not reflect this design principle. By realizing this intrinsic integration of game and mathematics, MobileMath should give students the feeling of playing a game, and at the same time have them experiencing another way of doing mathematics.

A second design issue concerns the location-based character of MobileMath. It was seen as important to make sure that every school could use the game in its own direct environment. Players can thus use their knowledge of the environment as well as expand that knowledge by playing the game. Although the game is location-based in the sense that it makes use of elements in the real world, MobileMath is not based on one specific location, but can be played anywhere.

3.2 Pilot with the teachers

The pilot was carried out with one geography teacher, three mathematics teachers and one computer science teacher. This pilot had three objectives: to test the game play with a group of 'new' players, to prepare for the pilots with students and to discuss and co-design the educational setting. The game test was performed with five teams, each consisting of one teacher and one researcher as participating observer. Afterward, the teachers filled in a questionnaire and the game was discussed with all participants. The closing discussion, in which the tracks were viewed and the strategies of the teams were discussed, provided opportunities for reflecting and making connections between game

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[2] Waag Society, Amsterdam.

strategies and aspects of mathematics (geometry of navigation and orientation and shapes) and geography (map skills).

3.3 Pilots with students

MobileMath was piloted in grades 7 and 8. By the end of grade 7, all students have worked on mathematical content related to MobileMath. This includes: navigation and orientation; measurements such as length (distance) and area; 2D shapes and their properties, especially those of specific quadrilaterals. The construction of 2D shapes is not a core topic in the mathematics curriculum for grade 7.

Three secondary schools (from the researchers' networks) participated in the pilot with a total of 60 students, 54 of whom filled in the questionnaire afterward (see Table 1). All pilots were observed by at least three researchers, notes were taken and in two of the three pilots video recordings were made.

The pilots consisted of a whole-class introduction, one round of game play of 1 h and a debriefing session. The whole-class introduction focused on map reading skills, characteristics of quadrilaterals, the game rules and the technology of the phone. In schools 1 and 2, both a math teacher and a geography teacher were involved. Introduction in school 1 took about 50 min, whereas in school 2 only 30 min was used. In school 3, the game was introduced by a mathematics teacher the day before the game was played. On all schools, a designer–researcher demonstrated how to use the phone. After the introduction students played a 1-h game of MobileMath in a playing field located around school, with a radius of 1 km. Because of the large number of students who participated in school 1, two rounds of game play were needed: half of the students played the game outside, while the other half watched the game play online in the computer room at school and kept a log of what (they thought) was happening.

At school 1, for safety reasons, all outside teams were accompanied by a teacher or researcher acting as a participating observer. At the other schools, the teams went outside on their own and the teachers and researchers observed the game play online on the game website. In a debriefing session, the stored game data and the observation notes were used to have the teams reflect on their game play. Two forms of debriefing were used: a debriefing session with each team immediately upon its arrival back at school or a whole-class debriefing session. At schools 1 and 2, both forms were used, at school 3 only individual debriefings were held. Notes were taken during the debriefing sessions, and in two schools debriefing was videotaped.

4 The MobileMath game

The MobileMath game, as it resulted from the design process, is a location-based game that can be played on a mobile phone with a GPS receiver and Windows Mobile as Operating System. Geometry was chosen by the team to provide the mathematical content to be addressed in the game.

The game can be played by two to eight teams. Each team creates geometrical shapes on a previously defined playing field (in the real world), using a mobile phone with GPS functionality and an on-screen map. The size of the playing field and the duration of the game can be set by the first player starting the game from a mobile phone. Other teams then subscribe to this game by choosing a team name. Players are connected with the game space and each other through the map on their screens. During the game, all teams can see themselves and the others as colored dots moving in real time in the playing field on the underlying map. The goal is for a team to gain points by covering as much area as possible with virtually constructed parallelograms ((including squares and rectangles). Players do this outside by walking to locations where they place, and virtually connect, vertices. To support the process of constructing and deconstructing quadrilaterals, auxiliary lines are visible on screen. If the fourth vertex finishes a shape correctly (which is evaluated by the game engine, within a margin of about 10 m), it appears on the playing field in the color of the team and is visible for all teams. If the fourth vertex does not finish a correct shape, it will disappear (see Fig. 1).

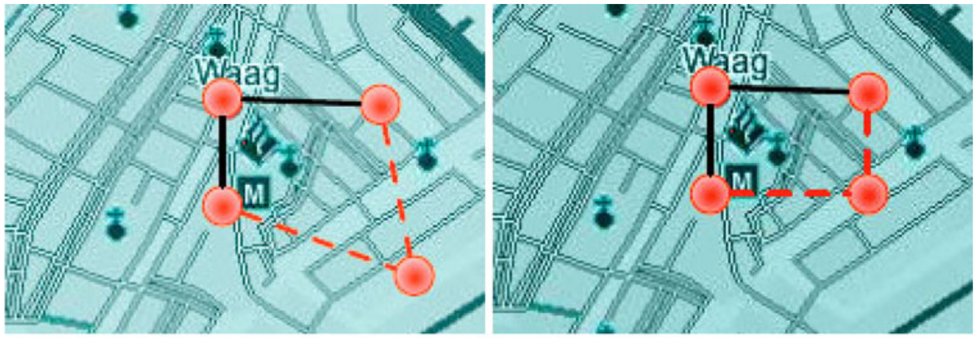
Since the shapes cannot overlap, it is possible to 'hinder' a team by trying to create a shape within a shape under construction.

Based on a conversion rule, each shape is awarded with a score equivalent to its area; this score is multiplied by a factor based on the difficulty of the construction of the shape. The difficulty of the construction is defined by the number of constraints. Squares having the most constraints

Table 1 Overview of the pilots

	# of students	Grade	Male	Female	Voluntary?	Accompanied?
School 1	28	7	17	11	No	Yes
School 2	12	8	4	8	No	No
School 3	16	8	11	5	Yes	No

Fig. 1 Finding the proper location of the fourth vertex



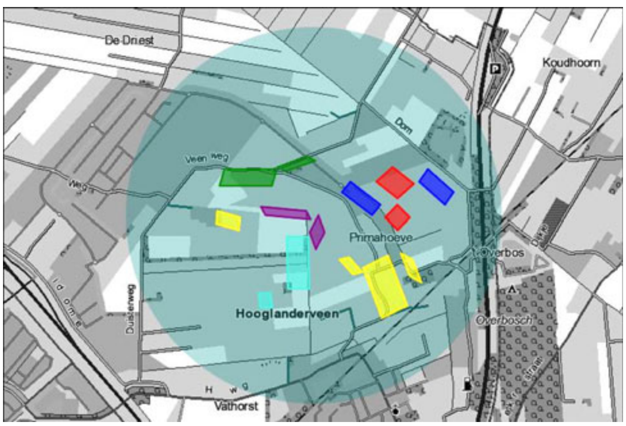


Fig. 2 View of a finished MobileMath game showing the created quadrilaterals and the playing field

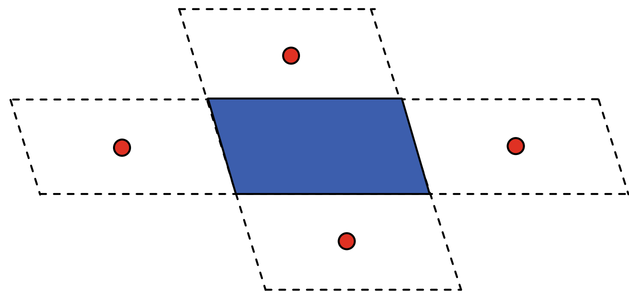


Fig. 3 The four deconstruction points of the parallelogram in the center

(exact right angles and equal sides) are most difficult to construct: the factors are 2× for a square and 1.5× for a rectangle and 1 for any other parallelogram. The created shapes are virtual elements added to the real world as a kind of 'overlay' (see Fig. 2): the physical world has thus become an interactive game board.

Deconstructing quadrilaterals of other teams is also part of the game play and teams are rewarded points for it. Deconstruction brings extra challenge and competition in the game. Both construction and deconstruction require that players use their knowledge of characteristics of specific types of parallelograms and their constructions. Figure 3 shows the locations of the four points of deconstruction for the parallelogram in the center. Determining the location of one of these deconstruction points on the map as well as in the real world requires a mathematical 'construction'.

As the game proceeds, teams occupy territory with their shapes and thus the free playing space gets smaller, which provokes interaction between competing teams. Territory occupied cannot be used anymore, unless the shape covering it is destroyed. Deconstruction of shapes clears space and the deconstructing team 'steals' half the points connected to that shape from the team that created it. The game ends after a set duration, the team with the highest score being the winner.

The game is supported by a website on which each game can be observed in real time online and on which the game data are stored and can be viewed back later. These data include the tracks of all teams, which are not visible on the phones during game play, as well as the quadrilaterals that remain at the end of the game.

What does playing MobileMath come down to in terms of student actions? Students look at the map and imagine where they want to make a shape. They walk to the location for the first vertex and then enter this location using the phone. On the map, they see the location marked with a dot in the color of the team. When they start walking again, to the location of the second vertex of their imagined shape, a line occurs on the screen connecting the first vertex with the current (moving) team location. Once the team is at the location they see fit as the second vertex of their shape, they enter it and a second colored dot appears; the line connecting the dots is now solid and is an edge of the shape. The team now plans the location for the third vertex; they use the map on their screen in combination with properties of the real world. There are several constraints: the type of shape they want to make determines whether they will go for a straight angle or not; properties of the terrain pose constraints as well: a vertex cannot be in water or within a building because the students must enter the vertex on the location itself. As soon as they start walking, two dotted lines connecting the two 'set' vertices with the current location appear on the screen (see Fig. 1).

Note that it is not necessary to walk along an imaginary edge; the connecting line will become the edge once the location of the vertex is 'entered'; this edge can run through buildings or over water. After entering the location for the third vertex, the location of the last one is 'determined': students now must go to exactly that position that finishes the shape in a correct way. Until now, there was always the opportunity to change the strategy. For the last vertex, accuracy is very important. It is in this phase that we see students using the zoom options of the game: zooming out to see their whole intended shape, zooming in to accurately place the last vertex and, e.g., make sure the angle is indeed a straight one. Students walk back and forth to fine-tune the location of the last vertex.

5 Results

In this section, we present the results based on the analysis of the observation data, the stored game data and the data from the online questionnaire.

The first pilot with the teachers showed that the game play proved to be engaging, that the technology worked well and that the rules and goals were clear. Based on the observations during game play, the group discussion and the data of the questionnaire, it was decided that some elements needed extra attention when introducing the game to the students. These included: the mathematics involved in finding the deconstruction points, the inaccuracy of the GPS, the 'mechanics' of the grid and the use of the map, which is not a conventional street map. As a result, the educational embedding was designed in the form of teaching suggestions for a whole-class introduction focusing on the mathematics, the geography, the actual handheld phone (HTC)—provided by the researchers—and the game play. Also, the online evaluation questionnaire for the students was designed based on these pilot experiences.

We present the results of the pilots with students in five sections: (1) engagement, (2) game play, (3) technical issues (all related to sub-question 1), (4) mathematics: the construction process, and (5) learning opportunities (the latter two related to sub-question 2).

5.1 Engagement

Students easily engaged in MobileMath as was observed during all of the four games that were played: no large technical difficulties were observed and all teams played for the full hour.

On the questionnaire, the game was rated as being 'fun' (see Table 2). This is supported by data from the observations made by the accompanying researchers. Typically, students reacted very exuberant when they completed or destroyed a shape. This was visible in their moves as well as in their verbal outings: "yes, we did it".

Table 2 Results on the question: 'Did you enjoy playing the game?'

	1 (not at all)	2 (no)	3 (neutral)	4 (yes)	5 (yes a lot)
School 1	–	–	2	10	16
School 2	–	–	3	6	3
School 3	–	–	–	9	5
Total	–	–	5	25	24

In an open-ended follow-up question, students were asked to motivate their 'fun score'. The reasons most frequently given are summarized in Table 3.

The most frequently mentioned positive reason refers to the mathematical content of the game. A sample student's response for this is: "The game is fun because, you learn to use GPS and you learn to construct different shapes: quadrilaterals, rectangles, etcetera, and at the same time you play a game".

5.2 Game play

Most students easily understood the goal and the rules of MobileMath. Most of the 30 teams constructed or deconstructed at least one correct quadrilateral and scored points. There are large differences in scores, both between the four games as well as between the teams within one game, as can be seen in Table 4.

Table 4 shows that the students in school 1 who played the game in the second round did better than the students who played in the first round. The students who played game 2 had watched and discussed the game being played

Table 3 Results on the question 'Why or why not, did you enjoy playing the game?'

Reasons	Frequency
Positive	
Constructing or deconstructing (digital) shapes	13
Just fun	10
Game play exciting (e.g. destroying shapes)	8
Scoring points	3
Use GPS	3
Learn something	3
Walking and being outside	3
Collaborate with peers	3
Other	3
Negative	
Fun but: tiring/bad weather/guided/no points	13
No reason	5

Table 4 Range and average scores per game

Game	Score range	Average score	Remarks
School 1, game 1	0–140	36	140 is an outlier
School 1, game 2	20–201	93	
School 2, game 3	0–155	25	4 of 7 teams scored 0
School 3, game 4	11–111	51	

by their classmates on the game website. Observations revealed that the teams in this second round were more aware of each others locations and actions.

Six out of the 30 teams that played MobileMath scored no points at all, which means they did not succeed in constructing or deconstructing any shape. In game 3, more than half of the teams (4 out of 7) did not score any points. During this game, the weather conditions were very bad: storm and rain. An analysis of the tracks of these teams reveals that two of them did not seem to attempt to create a correct shape but instead wandered of to a dry and warm place (the nearby shopping center), while the other two teams tried but were not accurate enough.

5.3 Technical issues

None of the students had problems using the phone. Observations showed that most students easily interacted within the mixed reality environment, and were able to combine the information from the map with the information from the physical world. One team, e.g., placed the edge they just made on screen, parallel to the street to decide whether to go left or right.

A small number of students, however, were seen to have problems reading and interpreting the map on screen. They had a hard time figuring out how the map matched with their environment. This may be partly due to the fact that the map on screen was not a conventional street map (see Figs. 1, 2).

The technical features 'zooming' and 'checking the scores' were used by most students (see Table 5).

Almost all students checked the scores frequently, which helped them to make tactical and strategic decisions, like destroying a large shape of the leading team to steal half of their points and take over the lead. Students who used the zoom function mentioned several reasons for doing so. They indicated to use zoom in during the (de)construction process to check the accuracy of their construction. Using the zoom-in and walking back and forth in the physical world is required to find the proper location for a vertex or for a deconstruction point.

Table 5 Results on the questions 'did you zoom?' and 'did you check the scores?'

	Yes	No
Did you zoom in or out?	44	10
Did you check the scores when playing?	50	4

Zooming out was used to get an overview of the playing field where the locations of all teams and the shapes that had been created so far could be seen. It was also used by teams to check their own location, and plan their course and actions. The inaccuracy of the GPS readings, combined with the margins and the grid, proved to be frustrating for some of the teams.

5.4 Mathematics: the construction process

The stored game data, especially the tracks of all teams, show that for the creation of shapes different strategies were employed. A first strategy is based on the (virtual) reality of the map: students deploying this strategy used elements from the map—like rectangular street patterns and buildings—to identify possible quadrilaterals. A second observed strategy can be characterized as 'using knowledge of the environment around school'—students who used this strategy knew, e.g., where a rectangular playground was located and used this knowledge of the physical world, to identify and construct a shape. This strategy in which the virtual and physical reality merges can also cause conflicts. This happened in a team of students that wanted to make a rectangle around the sports field near school. The students walked around it (on the sidewalk) and 'entered' the vertices on their handheld. But three times over when they entered the last vertex, the game gave as feedback that their shape was wrong. Only back at school when reviewing their tracks on the computer, they noticed that the streets around the sports field did not form a proper rectangle; two of the angles were not 90°. During the game, they did not zoom out in the virtual layer to check their assumptions that the streets in the physical world around the field formed a proper rectangle.

The third observed strategy is a mathematical one. Students using this strategy made their quadrilaterals as 'free standing' shapes. Two girls even leaped over a ditch to overcome the physical barriers of the location to finish their shape. The tracks of teams deploying this strategy are often not related to the edges of their shapes. They indentify locations for their vertices and use their maps only to plan a route to walk from one to the next. For these students, the characteristics of the physical environment are hardly important for their constructions, they mainly focus on the mathematical properties of the shapes under construction.

5.5 Learning opportunities

The questionnaire presented the following three open questions related to learning and school subjects:

- Did you learn something? Illustrate.
- Is this game related to geography? If so, in what way?
- Is this game related to mathematics? If so, in what way?

Most students (78%) confirmed that they had learned something. In the open follow-up question, the majority of them (36 out of 42) reported what it was they had learned. Students could list more than one example. The results are summarized in Table 6.

An example of learning occurring during reflection-in-action is the following: Two students (a team) were creating a rectangle: after they fixated the third vertex, they noticed that they had not created a right angle. One of the students wanted to delete this shape because 'in this way we cannot finish the rectangle', the other student while looking at the screen suddenly realized that another option existed "we can make it into a parallelogram if we put the fourth vertex here instead of here (pointing at the map on the screen)."

Learning opportunities could also be identified in the introduction and debriefing sessions as is exemplified in the following examples:

The geography teacher introduces the map used in MobileMath by projecting it on screen. He tells the class that this is a very simple map on which not everything can be recognized.

T Can you see the railway-tracks?
S1 Yes, black and white
T And the water?
S2 You see two small ferry boats on the map
T Can you name something you recognize on this map?
S3 The police station (by the icon of a flame)
T Where can I walk?
S4 On this type of lines ['roads']

Table 6 Examples of topics learned, for the question 'Did you learn something? Clarify.' ($N = 42$)

Yes	# mentioned
Using the GPS	14
Constructing shapes	13
Collaborating	5
Using a map	4
Strategic thinking	4
Being accurate	2
Playing the game	2

The mathematics teacher introduces the shapes that can be made in MobileMath. He uses a flexible device made of four strips to demonstrate the shapes.

T How can you be sure if a shape is a rectangle?

Students name several properties:

- the upper side is long
- the length and width are not equal
- it has four corners

T Who can tell more about the corners?
S1 The corners are the same … same
S2 … 90 degrees
S3 … straight
T How can we make a square?
S By making the length and width equal
T What shape do you think will be easier to make in the game: a rectangle or a square?
S1 Square: you do not have to walk that far
S2 A rectangle, the length and width don't need to be equal
T What is easier to make: a rectangle or a parallelogram?
S Rectangle

The large majority of the students recognized the relation to geography in the use of the map or the GPS. Eight of the 54 students saw no relation to geography. Three students saw no relation to mathematics. Most students specify how they observed the relation between mathematics and MobileMath: 32 students used terminology related to constructing shapes, 4 students connected this with the calculating of the score; 6 students (also) used other geometrical terms such as area, angles and measurements; calculating (in general) was mentioned five times.

Other learning opportunities were identified in the debriefing sessions. Students explained their strategies and constructions, referring to their tracks on the map projected in the classroom. For example, one girl (age 13) was asked to describe and explain her team's subsequent activities during the game in front of the classroom. The tracks of the team are projected as a curly line on the map (see Fig. 4).

T Where did you place your vertices to make your shapes? [T. points to the location of the school on the lower left of the map on the interactive whiteboard]
S [Pointing to the tracks on the map] First we made this small rectangle here. Then we made a block around the church. Then we made a third rectangle and a fourth one (see Fig. 5). At the end of the game three shapes were destroyed by other teams

Reports such as the one above led to lively discussions between students centered around the game strategies

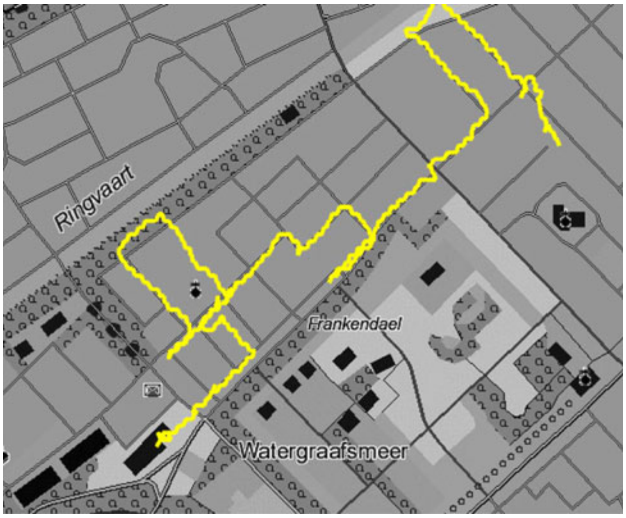


Fig. 4 Tracks of one team after they finished the game saved on the computer

employed, the mathematical 'constructions' and shapes each team made, and around the geographical aspects of the game. Students were able to reflect on their actions: they indicated, e.g., why in some cases their shapes under construction proved to be incorrect, how they tried to 'hinder' other teams and how they determined the location of a deconstruction point.

6 Conclusion

The research questions phrased in Sect. 2 focus on the engagement provoked by a social mobile game and its opportunities for learning mathematics. The questions are investigated through design and through field tests with teachers and students. We now address the two foci of engagement and learning opportunities, respectively.

6.1 The game play engagement

6.1.1 MobileMath characteristics

What characteristics of the MobileMath game were important for the students' engagement? We first conclude that MobileMath indeed fulfills the characteristics of a HRG with its intrinsic motivational properties as defined by De Souza e Silva & Delacruz (2006). It allows players to create an imaginary layer on top of the physical reality by drawing shapes. MobileMath adds a geometrical dimension to the world which is transformed into a game board. Observations show that for the players the physical and digital space merge, which contributes to students' engagement. Interaction between the teams mainly took place in the virtual reality: they spotted each other more easily on screen than in the physical reality. This matches with similar results found by others Squire and Klopfer (2007), who conclude that students 'within minutes were diving into this mixed-reality environment' (p. 403).

A prerequisite for engagement is appropriate technical functioning. We conclude that the technical environment of MobileMath is user-friendly, robust and flexible enough to be used in an actual (educational) playing setting. Students easily used the functionality of the phones. However, the inaccuracy of the GPS readings sometimes interfered with and frustrated the game play. For example, what looked like a parallelogram on screen was identified by the game as being a rectangle, or a seemingly correctly placed vertex was evaluated to be wrong. This is in line with Schwabe & Göth (2005) who discussed this type of effects of (in)accuracy and concluded that for tasks that require very specific locations the inaccuracy can cause frustration and loss of engagement. Other, more accurate location technologies may be preferred in that case. For MobileMath, the margins and the grid size built into the software need to be reconsidered.

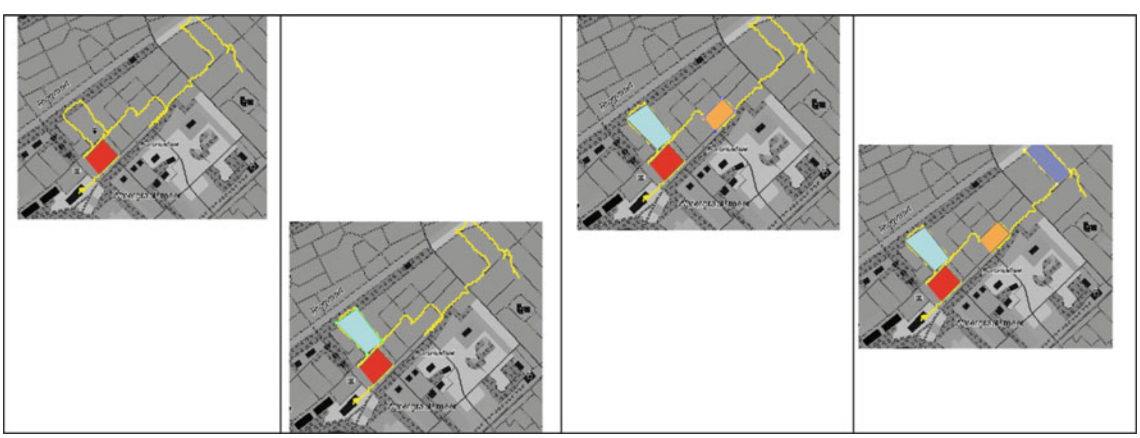


Fig. 5 Shapes created by one team, pasted manually in the tracks to clarify the protocol

6.1.2 *Fun and engagement*

Main factors that showed to be essential in enhancing students' engagement are clear goals, objectives, and rules, competition and interaction, which is in agreement with the elements identified by Prensky (2001). In MobileMath, the *goal and objectives* proved to be clear. Furthermore, all teams kept setting new goals for themselves, during the full hour. Although they were moving freely outside of school, most students were seen to be highly motivated and engaged during game play, almost no off-task behavior was observed. We can also identify the other elements: MobileMath has clear *rules* that structure the game, and students had no problems understanding and following these rules. The feedback in this game is immediate: every action, its outcome and its consequences are immediately visible on the map on the phone. Without this immediate feedback, the game would be unplayable.

Competition provided an important contribution to the engagement with MobileMath. There are two elements in the game that can be described as competitive. First, the game in itself as a whole is competitive: students are all competing to cover as much area as possible on a restricted playing field, and gain the highest number of points. Second, the option of destroying shapes of others brings in an extra competitive element. This feature proved to be very engaging: eight students explicitly mentioned it in the questionnaire as the main reason for the game being fun. Also during debriefing sessions team specifically and in much detail reported their deconstruction actions and successes. Some teams during the game even specialized in destroying instead of creating shapes. MobileMath offers ample opportunities for *interaction*. Because there was only one phone for two players in one team, the players had to work together. In most teams this turned out to be positive: often the two students were seen bent over the screen together, discussing and planning their actions. In a small number of teams, the student carrying the phone did all the work and the other students just walked along. Because all students were eager to play, most students did not let this happen.

Altogether, we conclude that with the design of MobileMath we succeeded in creating an engaging game activity. MobileMath was showed to be an accessible HRG with engaging characteristics such as clear goals, objectives and rules, as well as competitive and interactive elements.

6.2 Learning opportunities for mathematics

The design of MobileMath shows that mathematics can successfully be integrated into an engaging mobile location-aware game. Almost all students recognized the mathematics in the game and described it in mathematical terms. The mathematical concepts build into MobileMath (orientation and navigation, measurement, properties and especially 'constructions' of specific quadrilaterals) were already partly known by the students (see Sect. 5.5). However, in the regular curriculum, creating shapes does not receive much attention and the bodily experience of making shapes—as is done in MobileMath—is fully new.

MobileMath invites several types of mathematical activity, such as the (re)discovery and use of characteristics of squares, rectangles and parallelograms, the notice of geometrical aspects of the world, and to combine mathematical skills.

Students (re)*discovered and used characteristics of squares, rectangles and parallelograms* when they (de)constructed these shapes. In doing so, they went back and forth between the realities of the map and the real world. Students needed to imagine a shape before actually creating it. They had to mentally 'see' it on the map, and then walk to the locations of the vertices in the real world. Students' perception of the city changed like in HRGs and other augmented reality games. They *noticed and discussed geometrical aspects of the world*, such as if streets make right angles or not and whether they are running parallel; they identified blocks, buildings or other structures as having a rectangular 'top view' on the map. *Combining math skills* was needed when a team studied the map and decided on a strategy related to their position. They had to consider aspects such as time and distance: which distances can be covered walking? How much time is left before the game ends? Where will a correct shape fit, can each vertex be reached? How long will the construction process take? Students were also observed using mathematical skills and reasoning when reflecting on the scores, which also involves reasoning about area.

Observations also show that during game play students went through micro-cycles of learning and reflection in a natural way. For each action, they needed to reflect on the game status, which involved taking note of the playing field and of the characteristics of the shape they were creating in relation to the physical aspects of the terrain. They discussed, e.g., where to place a vertex to make sure that the angles and the length of the edge were suited to create the intended shape. Apart from the names of the quadrilaterals, they frequently used mathematical terms such as parallel, (straight) angle, straight line, perpendicular, of equal length. Because they were 'acting out' these concepts in their movements and constructions, it is likely that their understanding has deepened and has become experientially real.

For the further exploitation of the mathematical aspects of the game play, debriefing sessions are important. Although in the pilot these sessions were very short, we could glimpse the opportunities these sessions have for reflecting on the game play as well as on the math involved and thus for promoting learning. Some work is still needed to fully exploit this potential of MobileMath for learning.

Although, as stated before, students reported they learned something, were able to name aspects of the mathematics involved in the game and were seen to engage in mathematical activities and talk, we cannot yet support the claim that MobileMath has had a learning effect. The fact that in MobileMath students experience mathematics to be engaging may in itself be seen as a positive result. In future research on MobileMath, we hope to be able to measure learning effects, and thus show that MobileMath can contribute to students' learning of mathematics, just as some other digital games have proven to do (Kebritchi et al. 2008; Rosas et al. 2003; Shin et al. 2006).

References

Admiraal, W., Raessens, J., & Van Zeijts, H. (2007). Technology enhanced learning through mobile technology in secondary education. In P. Cunningham & M. Cunningham (Eds.), *Expanding the knowledge economy. Issues, applications, case studies (Part 2)* (pp. 1241–1248). Amsterdam: IOS Press.

Cobb, P., Yackel, E., & Wood, T. (1992). A constructivist alternative to the representational view of mind in mathematics education. *Journal for Research in Mathematics Education, 23*, 2–33.

De Souza e Silva, A., & Delacruz, G. C. (2006). Hybrid reality games reframed: Potential uses in educational contexts. *Games and Culture, 2006*(1), 231–253.

Demeyer, T., Hopman, A., Jonker, V., Kerstens, K., & Wijers, M. (2008). *MobileMath*. Retrieved May 15, 2008, from http://www.mobilemath.nl/.

DeVane, B., Squire, K., & Durga, S. (2008). *Games, learning & society: Centers of expertise for academic learning through open-ended video games*. Paper presented at the International Conference of the Learning Sciences, Utrecht, The Netherlands.

Egenfeldt-Nielsen, S. (2005). *Beyond edutainment: Exploring the educational potential of computer games*. Copenhagen: IT-university.

Freudenthal, H. (1991). *Revisiting mathematics education. China Lectures*. Dordrecht: Kluwer Academic Publishers.

Gee, J. (2003). *What video games have to teach us about learning and literacy*. New York: Palgrave Macmillan.

Gravemeijer, K. P. E. (1994). *Developing realistic mathematics education*. Utrecht: Cd-β Press.

Kafai, Y. (1998). Game design as an interactive learning environment for fostering students' and teachers' mathematical inquiry. *International Journal of Computers for Mathematical Learning, 3*, 149–198.

Kafai, Y. (2001). *The educational potential of electronic games: From games-to-teach to games-to-learn*. Retrieved December 20, 2009, from http://www.savie.ca/SAGE/Articles/1182_1232-KAFAi-2001.pdf.

Kebritchi, M., Hirumi, A., & Bai, H. (2008). *The Effects of modern math computer games on learners' math achievement and math course motivation in a public high school setting*. Retrieved December 20, 2009, from http://dimensionm.com/docs/UCF Research_Brief_June_202008.pdf.

Klopfer, E. (2005). *Playing to learn: State-of-the-art computer games go to school*. Retrieved December 20, 2009, from http://educationarcade.org/files/articles/Cable_in_the_classroom.pdf.

Lave, J., & Wenger, E. (1991). *Situated learning: Legitimate peripheral participation*. Cambridge: Cambridge University Press.

Malone, T. W., & Lepper, M. R. (1987). Making learning fun: A taxonomy of intrinsic motivations for learning. In R. E. Snow & M. J. Farr (Eds.), *Aptitude, learning and instruction, Vol. 3: Cognitive and affective process analyses* (pp. 223–253). Hillsdale, NJ: Erlbaum.

O'Donnell, A., Hmelo-Silver, C., & Erkens, G. (Eds.). (2005). *Collaborative learning, reasoning, and technology*. Mahwah, NJ: Erlbaum.

O'Malley, C., Vavoula, G., Glew, J. P., Taylor, J., Sharples, M., & Lefrere, P. (2003). *Guidelines for learning/teaching/tutoring in a mobile environment*. Ricerca, Italy: MOBIlearn/Giunti.

Prensky, M. (2001). *Digital game-based learning*. New York: McGraw-Hill.

Rosas, R., Nussbaum, M., Cumsille, P., Marianov, V., Correa, M., Flores, P., et al. (2003). Beyond Nintendo: Design and assessment of educational video games for first and second grade students. *Computers & Education, 40*, 71–94.

Schwabe, G., & Göth, C. (2005). Mobile learning with a mobile game: Design and motivational effects. *Journal of Computer Assisted Learning, 21*, 204–216.

Shaffer, D. W. (2006). *How computer games help children learn*. New York: Palgrave Macmillan.

Shin, N., Norris, C., & Soloway, E. (2006). *Effects of handheld games on students learning in mathematics*. Paper presented at the ICLS, Bloomington, IN.

Squire, K. (2008). *Designing place-based augmented reality games for literacy*. Paper presented at the International Conference of the Learning Sciences, 24–28 June, Utrecht, The Netherlands.

Squire, K., & Klopfer, E. (2007). Augmented reality simulations on handheld computers. *Journal of the Learning Sciences, 16*(3), 371–413.

Treffers, A. (1987). *Three dimensions, a model of goal and theory description in mathematics instruction—The Wiskobas Project*. Dordrecht, The Netherlands: D. Reidel.

Treffers, A. (1991). Didactical background of a mathematics program for primary education. In L. Streefland (Ed.), *Realistic mathematics education in primary school* (pp. 21–57). Utrecht: Cd-β Press.

Trouche, L., & Drijvers, P. (2010). Handheld technology: Flashback into the future. *ZDM, The International Journal on Mathematics Education* (online first).

Wijers, M., Jonker, V., & Kersten, K. (2008). MobileMath: The phone, the game and the math. In T. Connoly & M. Stansfield (Eds.), *Proceedings of the European Conference on Game Based Learning, Barcelona 2008* (pp. 507–516). Reading: Academic Publishing Limited.

Winters, N. (2007). What is mobile learning? In M. Sharples (Ed.), *Big issues in mobile learning* (pp. 7–12). Nottingham: LSRI.