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Mobility Costs, Frictional Unemployment, and Efficiency

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Moving and training costs play a significant role in job-taking decisions.[1] Even if workers were equally productive in all jobs, these costs would make it sometimes worthwhile to refuse a job offer while waiting for a more attractive offer. The rate at which workers are offered jobs with different moving costs depends on the decisions of other workers as to which jobs to refuse. This externality implies that equilibrium will not generally be efficient. With the plausible assumption that job offerings become more attractive on average when the number of available jobs increases, efficiency increases when workers are induced to pass up jobs with relatively high moving costs (by unemployment compensation, for example).

These results are derived in a model of steady state search equilibrium similar to that which has been used elsewhere.[2] Earlier analyses have focused on decisions which affect the rate at which workers receive job offers, taking as given the distribution of the quality of job offers. Here the arrival rate of job offers is taken as given, with the quality distribution endogenous. It is assumed that there are fixed and equal numbers of jobs and workers. Production is governed by a fixed coefficients technology. Workers and firms are taken to be risk neutral

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[1] For a discussion of the role of employment in mobility decisions see Bartel (1979).

[2] Diamond (1980), Diamond and Maskin (1979, forthcoming), Mortensen (1979).

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and to face a common exogenous interest rate. Both job termination and the arrival of new job offers are modeled as Poisson processes with fixed parameters. It is assumed that the wage and sharing of moving costs are negotiated, with workers and firms equally good bargainers. The steady state unemployment level is then determined by the job taking decisions of workers. More stringent standards for job taking raise the vacancy rate which, in turn, improves the distribution of job offers. Thus there can be multiple steady state equilibria. From any steady state equilibrium, inducing a permanent further decrease in the moving costs which workers are willing to bear raises the present discounted value of output in the economy. The optimal unemployment compensation benefit is derived in Section 6. Presentation of these results takes as given the relationship between the unemployment rate and the quality of job offers. In Section 7, a simple model of job information flows is presented to derive an example of the way higher job availability might improve average job offer quality. An example with an exponential distribution of moving costs is examined in detail in Sections 8 and 9. This example results in optimal benefits in the neighborhood of 60% of the wage. Another example is presented in Section 10.

1. Employment

Assume that all jobs are the same, with a fixed coefficients technology that results in a flow of output, y. All jobs are subject to a risk of termination at the constant breakup rate b. This represents exogenous factors such as a transfer of the worker or the job opportunity to a different location. We ignore the possibility of the worker and the job moving together. We also ignore endogenous reasons for job

termination such as quits for better jobs or layoffs to hire different workers.

At initial employment, the wage is negotiated, as is the sharing of set up costs. For simplicity we shall assume complete symmetry between workers and jobs implying that the net gain from commencing production is shared equally between worker and employer. In the absence of labor disutility capital user cost, and unemployment compensation, this implies equal sharing of the flow of output and so a wage equal to y/2. Workers are assumed to be risk neutral and to face a constant interest rate r. With these assumptions, it is appropriate to focus attention on the expected present discounted value of earnings less moving costs. Denote by W_E and W_U the expected discounted value of earnings less moving costs for employed and unemployed workers respectively. With complete symmetry assumptions about both finding and losing jobs, an infinite expected life, and with analysis restricted to the steady state, W_E and W_U will not vary over time or workers.

For an employed worker, the rate of interest times expected earnings equals the wage less the expected capital loss from job termination:

$$rW_E = y/2 - b(W_E - W_U). \tag{1}$$

The next step in the analysis is to consider job taking, and so the determination of W_U.

2. Job-taking

Assume that unemployed workers learn about job opportunities with an exogenous[1] arrival rate, a. We assume no costs and no decisions of an individual which affect this arrival rate. While the productivity of all jobs is taken to be the same for all workers, jobs differ across workers in the setup costs before production begins.[2] These costs reflect moving costs when workers must relocate to take new jobs and training costs before production can begin. The pattern of a fixed cost followed by constant output can be viewed as an approximation to the increased output that comes with on-the-job learning. We ignore the variation among jobs in commuting costs since these represent a variation in flow benefits rather than set up costs.

Denote by $G(c\ ,u)$ the distribution of set up costs associated with jobs an individual learns about when the unemployment rate is u. A higher unemployment rate (and higher vacancy rate) is assumed to improve the distribution of moving costs in the sense that $G_u(c,u) > 0$ where $G(c,u)$ is positive and less than one.[3] In section 7 we consider this assumption in more detail. We take the distribution to be constant over time for a given worker. Maintaining the assumed symmetry between workers and jobs, set up costs are assumed to be equally divided between worker and job. If an unem-

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[1]Below, we will allow a to vary with the unemployment rate.

[2]For a detailed analysis of individual choice with setup costs see H. Loikkanen and U. Pursiheimo (1979).

[3]To interpret this assumption one needs to compare alternative economies with different steady state unemployment and vacancy rates. It is not appropriate to consider a single economy over a business cycle since, in that case, a rise in unemployment is accompanied by a decline in vacancies rather than a move in the same direction.

ployed worker accepts any job with a setup cost less than c*, then, in the absence of unemployment benefits, we can write the expected discounted value of net earnings implicitly as

$$rW_U = a\int_0^{c^*} [W_E - W_U - c/2]dG(c,u) \tag{2}$$

That is, the interest rate times the expected value of earnings equals the expected gain from job taking, less set up costs.

The choice problem for the individual worker is the selection of c* to maximize W_U. Naturally, this involves accepting any job for which the set-up cost is less than the expected gain from job taking:

$$c^* = 2(W_E - W_U) = [y + a\int_0^{c^*} c\,dG]/[r + b + aG(c^*, u)] \tag{3}$$

where (1) and (2) have been solved to give the implicit equation for c*. Workers are more willing to bear set-up costs when output is greater, job finding is more difficult, the interest rate is lower, expected job duration (b^{-1}) is longer or the unemployment rate is lower. That is, from implicit differentiation of (3) we have $\partial c^*/\partial y > 0$, $\partial c^*/\partial a < 0$, $\partial c^*/\partial r < 0$, $\partial c^*/\partial b < 0$, $\partial c^*/\partial u < 0$.

3. Equilibrium

In steady state equilibrium, the aggregate rate of job finding must equal the rate of job losing. Denoting the unemployment rate by u, this gives us

$$b(1 - u) = au\,G(c^*,u) \tag{4}$$

That is, the job break-up rate times the proportion employed equals the job acceptance rate times the proportion unemployed. Since G_u is assumed positive, we have c* decreasing with u. One would not generally expect to find that the rate, a, at which workers learn of potential jobs to be independent of the unemployment and vacancy rates (which are equal by assumption). We would expect a to increase with u across steady states. Since the implications of this relationship have been explored elsewhere, we take a to be exogenous here but note the implied differences in footnotes. Similar considerations hold for b.

Since c* decreases with u in both (3) and (4) we have the possibility of multiple steady state equilibria; that is, multiple solutions to (3) and (4). When more jobs are available (higher u) anticipated mobility costs are lower (higher G) and individuals are more selective in the jobs they take (lower c*). Greater selectivity by workers, in turn, raises the unemployment rate. This is shown in Figure 1, where equations (3) and (4) have been drawn.

We note that an individual will not accept a job with an expected return below set up costs. Thus the chosen cutoff c* is less than $y/(r+b)$. Even if expected future setup costs are close to zero, the potential loss in foregone wages implies that the chosen cutoff c* will never be less than $y/(r+b+a)$. If all offered jobs are accepted, the equilibrium unemployment rate is $b/(a+b)$. If none are accepted, the unemployment rate goes to one.

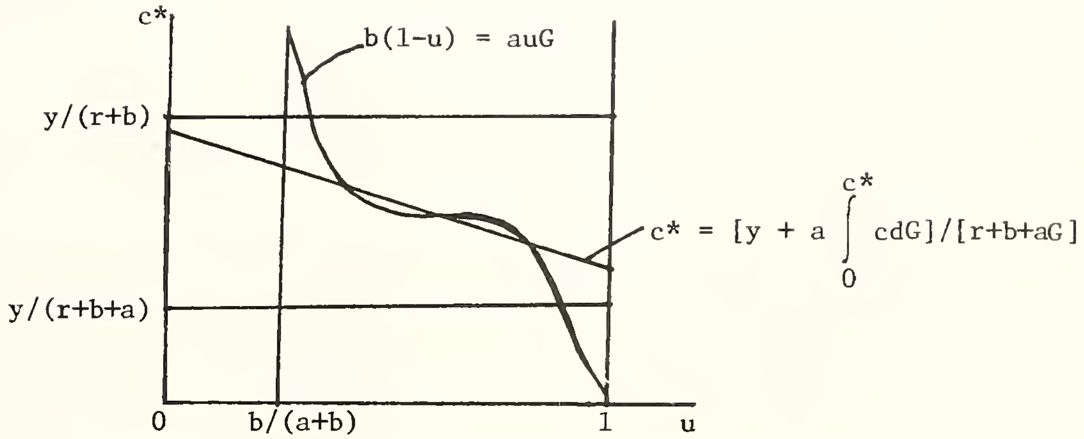

Figure 1

4. Steady-State Output

All steady state equilibria lie along the curve given in (4). At any point on this curve we can calculate the steady state output level per person. There are 1-u employed per person giving a gross output flow per person of (1-u)y. Moving costs per person equal the rate of new job taking per person of au G times the average cost of moving, $\int_0^{c^*} cdG/G$. Thus, net output, Q, satisfies

$$Q = (1-u)y - au \int_0^{c^*} cdG. \tag{5}$$

Differentiating with respect to u (with c* given as an implicit function of u by (4)), evaluating at an equilibrium (where (3) holds), and using integration by parts, we have

$$\frac{dQ}{du} = -y - a\int_0^{c^*} c\,dG - auc^*g\,\frac{dc^*}{du} - au\int_0^{c^*} cg_u\,dc$$

$$= -rc^* + au\int_0^{c^*} G_u\,dc. \qquad (6)$$

where $g = G_c$. The second term reflects the externality in lower moving costs of a higher unemployment rate. The first factor reflects the absence of discounting in steady state comparisons.

5. Dynamics

The economy analyzed here cannot move directly from one steady state to another. While it is interesting to compare alternative steady states which might have occurred, proper policy analysis requires consideration of the comparative statics of the actual path of the economy. We shall analyze the effects of a policy which controls c* directly. That is, we consider an economy where job acceptance behavior is sufficiently closely monitered to make it a government control variable. Below we will examine how unemployment compensation can be used to induce the same

steady state equilibrium without monitoring job acceptance behavior. These two modes of control correspond to stylised versions of the German and American economies.

Unemployment grows by job terminations and declines by job takings

$$\dot{u} = b(1-u) - auG(c^*,u) \tag{7}$$

Starting at a steady state equilibrium given by (3) and (4) we shall calculate the change in the present discounted value of aggregate net output, W, from a permanent differential change in c*, with unemployment given by (7).

That is, we want to calculate the derivative with respect to c* of

$$W \equiv \int_0^\infty e^{-rt}\{(1-u(t))y - au(t)\int_0^{c^*} c\,dG(c;u(t))\}dt \tag{8}$$

where u(t) satisfies (7) with an initial condition satisfying (3) and (4). Calculating this derivative[1] we have

$$\frac{\partial W}{\partial c^*} = \frac{-auc^*g}{r} + \left(\frac{y+a\int_0^{c^*} c\,dG + au\int_0^{c^*} cg_u\,dc}{r}\right)\left(\frac{aug}{r+b+aG+auG_u}\right) \tag{9}$$

Using (3) we can write this as[2]

$$\frac{\partial W}{\partial c^*} = \left(\frac{aug}{r}\right)\left(-c^* + \frac{(r+b+aG)c^* + au\int_0^{c^*} cg_u\,dc}{r+b+aG+auG_u}\right)$$

$$= \frac{-(a^2u^2g)(c^*G_u - \int_0^{c^*} cg_u\,dc)}{r(r+b+aG+auG_u)} = \frac{-a^2u^2g}{r}\left(\frac{\int_0^{c^*} G_u\,dc}{r+b+aG+auG_u}\right) < 0. \tag{10}$$

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[1]See Diamond (forthcoming).

[2]If a is function of u, (10) becomes $\frac{\partial W}{\partial c^*} = \frac{-au^2g}{r}\left(\frac{\int_0^{c^*}(aG_u+a'G)dc}{r+b+aG+auG_u+a'uG}\right).$

Thus a government policy to move the economy out of a steady state equilibrium in the direction of higher unemployment raises efficiency by improving the average quality of job offers.

6. Unemployment Compensation

There are several modes of intervention which will raise the equilibrium unemployment rate and so the efficiency of the economy. By increasing the importance of adjustment costs relative to the financial gains of employment, the government can induce greater selectivity in job-taking. Thus taxing output without allowing a deduction for adjustment costs or subsidizing unemployment or vacancies will have the desired end. Of these we shall analyze unemployment compensation. In addition to making workers more selective in job-taking, unemployment compensation raises the wage by one-half the unemployment compensation benefit, given the negotiation assumptions we have made here.[1]

From the perspective of the worker and employer, the net gain from production is y-B, where B is the unemployment benefit. Sharing equally both this gain above the value of being unemployed as well as adjustment costs results in a wage equal to B + .5(y-B). Rewriting the two value equations, (1), and (2) we have

[1] For a more detailed presentation, see Diamond (1980).

$$rW_E = .5\ (y+B) - b(W_E - W_u)$$

$$rW_U = B + a\int_0^{c*}(W_E - W_U - c/2)dG(c,u) \tag{11}$$

Then, the chosen cutoff cost satisfies

$$c* = 2(W_E - W_U) = (y - B + a\int_0^{c*} cdG)/(r+b+aG). \tag{12}$$

Increasing the unemployment benefit by one dollar decreases the cutoff level of costs selected at a constant unemployment rate by $(r+b+aG)^{-1}$ dollars. The induced increase in unemployment magnifies this effect.

Setting equation (9) equal to zero we can derive the cutoff cost level which would be optimal in a steady state. That is, an optimal trajectory will, asymptotically, have the cutoff cost level, c*, satisfying[1]

$$c* = \frac{y + a\int_0^{c*} cdG + au\int_0^{c*} cg_u dc}{r + b + aG + auG_u} . \tag{13}$$

Solving (12) and (13), we can derive the unemployment compensation benefit which holds asymptotically on the efficient trajectory (i.e., which supports the asymptotic optimum).

$$B* = \frac{(y + a\int_0^{c*} cdG)auG_u - au\int_0^{c*} cg_u dc(r + b + aG)}{r + b + aG + auG_u} . \tag{14}$$

Substituting from the first order condition for c*, B* can be written, alternatively as[2]

$$B* = au[c*\ G_u - \int_0^{c*} cg_u dc] = au\int_0^{c*} G_u dc > 0 . \tag{15}$$

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[1]From (13) and the first line of (6), we note that $\frac{dQ}{du} < 0$ at u*.

[2]If a varies with u, (15) becomes $B* = u\int_0^{c*}(aG_u + a'G)dc$.

Passing up a job offer alters the trajectory of the economy and so generates a pattern of externalities that varies over time. The present discounted value of the social gain from accepting a job with set up cost c can be derived by differentiating W with respect to u. Evaluated at a steady state this gives

$$-\frac{\partial W}{\partial u} - c = \frac{y}{r+b+aG+auG_u} - c. \qquad (16)$$

In an efficient equilibrium this social gain must equal the private gain $2(W_E - W_U) - c$. Equating the social and private gains (using (12)) we have an alternative derivation of the asymptotically optimal unemployment compensation.

7. Discrete example

Having examined the implications of the dependence on the unemployment rate of the distribution of adjustment costs of job offers, we now consider an example of how word of mouth communication of job availability can generate such dependence.[1] In this section we consider an example with two locations. Training costs are c_1 if the job and worker are in the same location. In addition, there are moving costs c_2 if the job and worker are in different locations. For the next section we consider an example where c is distributed exponentially with a parameter that depends linearly on the unemployment rate.

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[1] For previous analyses of word of mouth communication see S. Boorman (1975) and M. Satterthwaite (1979).

Assume that employed workers learn of a job opportunity in the same location and inform their unemployed friends of its existence. Assume that the process of attempted job communication is such that communication about any vacancy is a Poisson process with constant parameter a. (In practice it is likely that there are more attempted communications of new job vacancies than of old ones.) Each potential communicator knows n workers who could fill this job. Assume that each of the n has a probability u of being unemployed and an independent probability p of being in the same location. With probability $(1-u)^n$ all n friends are employed and there is no one to whom to tell of the job. If the communicator does have unemployed friends, he only tells a friend in the other location if none of his unemployed friends are in the same location. The probability of his telling someone in the other location is $(1-up)^n - (1-u)^n$ - the probability of no unemployed friends with the same location less the probability of no unemployed friends. Then $1 - (1-up)^n$ is the probability that he tells a friend in the same location. The greater the unemployment rate, the greater the probability of telling a friend in the same location.

With everyone following the same behavior rule, there are two candidates for equilibrium - accept only jobs at the same location or accept any job offer. Let u_1 and u_2 be the unemployment rates under these two behavior rules. Then the equilibria satisfy

$$b(1-u_1) = au_1(1 - (1-u_1p)^n)$$
$$b(1-u_2) = au_2(1 - (1-u_2)^n) \quad . \qquad (17)$$

For some parameter values both of these equilibria will exist. Rather than pursue this example in more detail, we turn to a similar example with a continuum of locations, which can be considered to be around a circle. It is then assumed that the likelihood of knowing an individual is exponentially distributed with the distance to his location and communication goes to the unemployed person for whom the setup costs are smallest.

8. Exponential example - steady state properties

We assume that c has an exponential distribution with coefficient nu. This example is chosen to fit the discussion above since the minimum of a random sample of size nu from the exponential distribution with coefficient 1 is exponential with coefficient nu. Thus we assume that

$$G(c,u) = 1 - e^{-nuc} \tag{18}$$

With this distribution, the two equilibrium equations (3) and (4) satisfy

$$c^*(r + b + a) = y + \frac{a}{nu}(1 - e^{-nuc^*}) \tag{19}$$

$$b(1-u) = au(1 - e^{-nuc^*}) \tag{20}$$

There exists a unique solution to this pair of equations with u between b/(a+b) and 1 and c* between y/(r + b + a) and y/(r + b).

In a steady state (whether equilibrium or not), the net output flow satisfies (from (5))

$$Q = (1-u)y - an^{-1}[1 - e^{-nuc*}(1 + nuc*)]$$

$$= (1-u)y - \frac{b(1-u)}{n\,u} - (\frac{a}{n})(1 - \frac{b(1-u)}{a\,u})\ \ln\ (1 - \frac{b(1-u)}{a\,u}) . \quad (21)$$

The asymptotically optimal unemployment compensation satisfies (from (15))

$$B* = (\frac{a}{nu})\ (1 - e^{-nuc*}(nuc* + 1))$$

$$= \frac{b(1-u)}{nu^2} + (\frac{a}{nu})\ (1 - \frac{b(1-u)}{a\,u})\ \ln\ (1 - \frac{b(1-u)}{a\,u}) . \quad (22)$$

From (21) and (22) we note that

$$Q = (1-u)y - uB* , \quad (23)$$

Alternatively this follows from (15) since, with the exponential distribution, $\int uG_u = \int cg$. From (13) we note that at the steady state optimum the maximal acceptable moving costs satisfy

$$\frac{y}{c*} = \frac{b}{u*} + r. \quad (24)$$

To find the optimal unemployment rate, we solve (20) and (24) simultaneously.

Some examples are shown in Table 1. For these calculations, parameters were chosen for a, b, and r. Next was chosen an equilibrium unemployment rate in the absence of government intervention, u. This implied a particular value for the product ny, and the remaining calculations were done for this value. In addition to steady state comparisons,

the last column of Table 1 reports the percentage change in the present discounted value of net output along the trajectory from the equilibrium steady state to the optimal one where the fraction of accepted jobs, G, is held constant at its asymptocally optimal value. As detailed in the next section, the increase in net output along the trajectory exceeds the difference between steady states.

Table 1 Exponential Example

a	b	r	u	$\frac{Q}{(1-u)y}$	c*/y	G(c*,u)	u*	B*/w	G*	Q*/Q	W'/W
10	.2	.05	.01965	.996	.12	.998	.01971	.31	.994	1.0000	1.0000
			.02	.99	.13	.98	.0205	.42	.96	1.0001	1.0001
			.03	.98	.25	.65	.0344	.59	.56	1.0013	1.0014
			.04	.97	.34	.48	.0470	.60	.41	1.0023	1.0024
			.05	.96	.43	.38	.0593	.61	.32	1.0032	1.0034
			.06	.95	.52	.31	.0715	.61	.26	1.0040	1.0043
			.07	.94	.61	.27	.0835	.60	.22	1.0048	1.0051
			.08	.93	.69	.23	.0954	.60	.19	1.0054	1.0059
			.09	.93	.77	.20	.1071	.59	.17	1.0061	1.0066
			.10	.92	.85	.18	.1188	.59	.15	1.0066	1.0073
5	.2	.05	.04	.99	.27	.96	.0415	.45	.92	1.0002	1.0002
10				.97	.34	.48	.0470	.60	.41	1.0023	1.0024
20				.97	.36	.24	.0484	.62	.20	1.0031	1.0032
50				.96	.37	.10	.0491	.63	.08	1.0036	1.0037
100				.96	.37	.05	.0493	.64	.04	1.0037	1.0038
1000				.96	.38	.005	.0494	.64	.004	1.0038	1.0039
10	.05	.05	.04	.97	1.40	.12	.0487	.61	.10	1.0028	1.0032
	.1			.97	.71	.24	.0483	.62	.20	1.0029	1.0031
	.2			.97	.34	.48	.0470	.60	.41	1.0023	1.0024
10	.2	.02	.04	.97	.35	.48	.0471	.61	.40	1.0024	1.0025

As a guide to interpreting the table, consider the fourth row. In an economy where the unemployed receive 10 offers a year, the expected duration of a job is 5 years, the interest rate is 5% and the equilibrium unemployment rate is 4%, 3% of gross output is spent on moving costs, workers are just willing to spend 34% of a years output on moving, workers accept 48% of jobs they hear about. The asymptotically optimal unemployment rate is 4.7% which can be induced by unemployment compensation equal to 60% of the wage. In this equilibrium workers accept 41% of job offers representing an 18% increase in the expected duration of unemployment, steady state output is higher than in the no compensation equilibrium by .23%, with a gain of .24% along the constant G trajectory from initial equilibrium to the asymptotic optimum.

The examples show a surprisingly consistent pattern. When the equilibrium unemployment rate is very close to b/(a+b), the minimum achievable,[*] the optimal unemployment compensation is small. As u rises, B*/w rises very rapidly, reaching the neighborhood of 60% when u is about one percentage point above the achievable minimum. B*/w stays in the neighborhood of 60% for all calculated values, which included values of u as integer percentages up to 10%. The same pattern arose for all calculated values of a, b, and r.

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[*] When a = 10 and b = .2, b/(a+b) = .019608

9. Continuous example-dynamics

The transition from the equilibrium steady state to the optimal one shows a larger change in net output than does the steady state comparison. To see this let us consider a policy of changing the proportion of jobs accepted by an unemployed worker. (We shall also consider the optimal policy below.) If workers are accepting all jobs with costs below $c^*(t)$, then the proportion of jobs accepted equals $1 - e^{-nu(t)c^*(t)}$. Let us consider the policy which holds this proportion constant over time at the level that occurs in the asymptotic steady state which we write as G^*. Then the economy follows the differential equations

$$\dot{u}(t) = b(1 - u(t)) - au(t)G^*$$
$$\frac{\dot{c}^*(t)}{c^*(t)} = \frac{-\dot{u}(t)}{u(t)} \quad . \tag{25}$$

We assume an initial condition at time t_0 of the steady state equilibrium. The immediate effect of the change in policy is to decrease aggregate setup costs without changing gross output. Over time the unemployment rate rises, decreasing gross output but keeping aggregate setup costs constant. This pattern is shown in Figure 2.

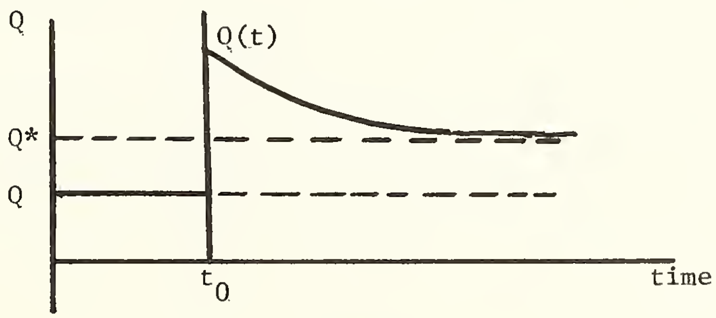

Figure 2

To calculate W along this trajectory, begin by considering aggregate setup costs. These equal

$$au(t) \int_0^{c^*(t)} cnu(t)e^{-nu(t)c}\, dc \tag{26}$$

$$= \frac{a}{n}\left(1 - e^{-nu(t)c^*(t)}(nu(t)c^*(t) + 1)\right).$$

Since $u(t)c^*(t)$ is constant we have

$$Q(t) = Q^* + y(u^* - u(t)). \tag{27}$$

Solving (25) for u(t) we have

$$u(t) = u^* + (u - u^*)e^{-\frac{b}{u^*}t} . \tag{28}$$

Thus the present discounted value of output satisfies

$$W' = \int_0^{\infty} e^{-rt} Q(t)dt = \frac{Q^*}{r} + \frac{y(u^* - u)}{(r + \frac{b}{u^*})} . \tag{29}$$

Since the optimal unemployment rate exceeds the equilibrium rate, the value of output along this path exceeds its value at the optimal steady state. The excess of W' over equilibrium output, W, is shown in Table 1. The addition to the value of unemployment compensation from analysing the dynamic path is small but noticeable for the examples calculated.

In addition to considering this path, which was chosen for its ease of analysis, it is interesting to analyze the optimal path

assuming the government could control $c^*(t)$. The choice problem is

$$\underset{c^*(t)}{\text{Max}} \int_0^\infty e^{-rt}\{(1 - u(t))y - \frac{a}{n}(1 - e^{-nu(t)c^*(t)}(1 + nu(t)c^*(t))\}dt$$

$$\text{s.t.} \quad \dot{u}(t) = (1 - u(t))b - au(t)(1 - e^{-nu(t)c^*(t)}) \ . \qquad (30)$$

The first order condition for $\dot{c}^*$ satisfies

$$\dot{c}^*(t) = (r+b)c^*(t) - y + ac^*(t)(1 - e^{-nu(t)c^*(t)}) \ . \qquad (31)$$

The phase diagram for the optimal path is shown in Figure 3.

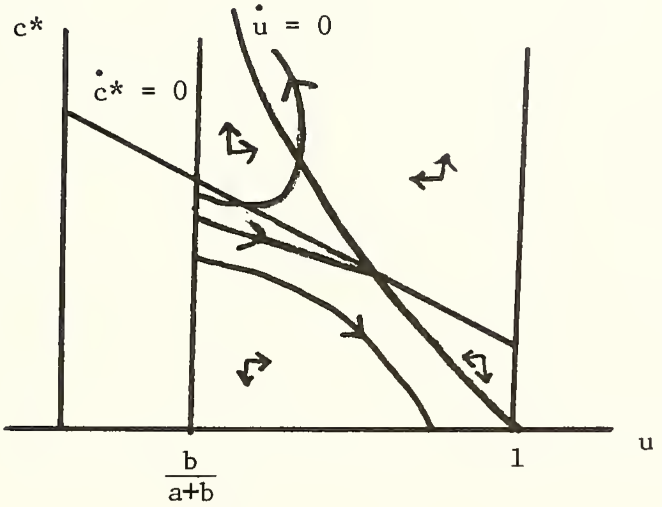

Figure 3

Along the optimal path $c^*(t)$ is falling while $u(t)$ is rising. Their product, and so the proportion of job offers accepted, is rising. In terms of Figure 2 the optimal path of net output converges to Q^* more rapidly than the path with uc^* constant, starting from a higher initial net output level.

10. Another Exponential Example.

The example above has a simple relationship between setup costs and the externalities generated by job refusals (equation (23)). With more complicated descriptions of information flows, we would not expect this relationship to necessarily hold. To allow variation in the relationship between costs and externalities, we turn now to an example with two Poisson processes. Job communication is simultaneously proceding on both networks. On the first network the structure is the same as above, with a rate of communication a_1 and a distribution of costs

$$G_1 = 1 - e^{-n_1 uc}. \tag{32}$$

On the second network, the communication process is independent of the unemployment rate and we have a parameter a_2 and distribution

$$G_2 = 1 - e^{-n_2 c}. \tag{33}$$

For such an economy we have the equilibrium condition

$$b(1 - u) = u(a_1(1 - e^{-n_1 uc^*}) + a_2(1 - e^{-n_2 c^*})). \tag{34}$$

To find the asymptotic optimum, we can solve (34) simultaneously with (13), the general equation for the optimal cutoff. Having found u* and c*, we can substitute into (15) to find the unemployment compensation which supports the optimum. Instead of taking this route, we will compare this economy to an alternative, satisfying the conditions of the previous example, with the parameters a_o, b_o, n_1. By suitable choice of a_o and b_o, we can find an economy with the same optimum, u*, c*, and r. From (15), we then know

that unemployment compensation should equal a_1/a_o times that in the constructed economy. For equal values of u*, c*, and r, a_o and b_o must satisfy

$$\frac{a_o(1 - e^{-n_1 u^* c^*})}{b_o} = \frac{a_1(1 - e^{-n_1 u^* c^*}) + a_2(1 - e^{-n_2 c^*})}{b} = \frac{1}{u^*} - 1. \tag{35}$$

$$\frac{y + a_o u^* c^{*2} n_1 e^{-n_1 u^* c^*}}{r + b_o + a_o(1 - e^{-n_1 u^* c^*}(1 - u^* n_1 c^*))}$$

$$= \frac{y + a_1 u^* c^{*2} n_1 e^{-n_1 u^* c^*} + \frac{a_2}{n_2}(1 - e^{-n_2 c^*}(1 + n_2 c^*))}{r + b + a_1(1 - e^{-n_1 u^* c^*}(1 - n_1 u^* c^*)) + a_2(1 - e^{-n_2 c^*})} = c^*. \tag{36}$$

Reversing the logic above, we can find many economies which have the same u*, c*, and r by selecting a_o, b_o, and n_1 and using (35) and (36) to restrict the choice among a_1, a_2, b, and n_2. Rather than working with these parameters directly, we consider the shares of the unemployment sensitive process in total job takings, s_1, and in total setup costs, s_2 (measured at the optimum):

$$s_1 = \frac{a_1 G_1^*}{a_1 G_1^* + a_2 G_2^*} \tag{37}$$

$$s_2 = \frac{a_1 \int_0^{c^*} c \, dG_1}{a_1 \int_0^{c^*} c \, dG_1 + a_2 \int_0^{c^*} c \, dG_2}. \tag{38}$$

Using (35) - (38), and the properties of the exponential distribution we can write the critical ratio as

$$\frac{a_1}{a_o} = [y(b_o + a_o G)a_o^{-1} + u^* G_u (y - r c^*)] \Big/ [y(b_o + a_o G)a_o^{-1} s_1^{-1} \tag{39}$$

$$+ u^* G_u (y - r c^*) + (s_2^{-1} - 1)(\int_0^{c^*} c\, dG)(r + b_o + a_o G + a_o u^* G_u)$$

$$+ (s_1^{-1} - 1)\, u^* c^* G_u (b_o + a_o G)].$$

where G is (equivalently) G_1 or G_o. In Table 2 we show some examples.

Table 2 B^*/w

$a_o = 10$, $b_o = .2$, $r = .05$, $n_1 = 1$, $u^* = .0470$, $c^*/y = .34$

s_1 = \ s_2	.2	.4	.6	.8	1.0
.2	.12	.14	.15	.16	.16
.4	.18	.24	.27	.28	.29
.6	.22	.31	.36	.39	.41
.8	.25	.37	.44	.48	.51
1.0	.27	.41	.50	.56	.60

To interpret this table, consider an array of economies, all of which are at their asymptotic optima and all of which have the same u*, c*, and r in this steady state. The economy which has only the unemployment sensitive communications network supports this equilibrium with an unemployment benefit equal to 60% of the wage. The economy which has only the

insensitive network has no externalities and achieves the optimum without unemployment compensation. The economy which fills 60% of its jobs through the sensitive network and which has 40% of its setup costs from these job fillings, supports the optimum with an unemployment compensation equal to 31% of the wage.

M. Baily (1977) and J.S. Flemming (1978) have analysed optimal unemployment compensation assuming risk averse workers and no externalities in the labor allocation process. This paper takes the opposite tack of assuming risk neutral workers and externalities. It seems likely that workers are risk averse and externalities are present, making a much stronger case for unemployment compensation.

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