Mobility Increases the Connectivity of Wireless Networks

Xinbing Wang[1], Xiaojun Lin[2], Qingsi Wang[1], Wentao Luan[1]
1. Department of Electronic Engineering, Shanghai Jiao Tong University, China
2. Department of Electrical and Computer Engineering, Purdue University, USA

***Abstract*—In this paper we investigate the connectivity for large-scale clustered wireless sensor and ad hoc networks. We study the effect of mobility on the critical transmission range for asymptotic connectivity in k-hop clustered networks, and compare to existing results on non-clustered stationary networks. By introducing k-hop clustering, any packet from a cluster member can reach a cluster head within k hops, and thus the transmission delay is bounded as $\Theta(1)$ for any finite k. We first characterize the critical transmission range for connectivity in mobile k-hop clustered networks where all nodes move under either the random walk mobility model with non-trivial velocity or the i.i.d. mobility model. By the term *non-trivial velocity*, we mean that the velocity of a node v is $\omega\big(r(n)\big)$, where $r(n)$ is the transmission range of the node. We then compare with the critical transmission range for stationary k-hop clustered networks. In addition, the critical number of neighbors is studied in a parallel manner for both stationary and mobile networks. We also study the transmission power versus delay trade-off and the average energy consumption per flow among different types of networks. We show that random walk mobility with non-trivial velocities increases connectivity in k-hop clustered networks, and thus significantly decreases the energy consumption and improves the power-delay trade-off. The decrease of energy consumption per flow is shown to be $\Theta\big(\frac{\log n}{n^d}\big)$ in clustered networks. These results provide insights on network design and fundamental guidelines on building a large-scale wireless network.**

I. INTRODUCTION

CONNECTIVITY is a basic concern in designing and implementing wireless networks, and hence is also of paramount significance. Nodes in the networks need to connect to others by adjusting their transmission power and thus carry out the network's functionalities. Therefore, three main schemes of connecting strategies are proposed in literature.

The first type of connecting strategies is distance-based. That is, for a graph (network) $G(V,E)$ and any two nodes $i,j \in V$, $e_{ij} \in E$ if and only if the Euclidean distance between i and j is at most r. The critical value of r for connectivity when the number of nodes grows to infinity has been studied. In [1], Gupta and Kumar proved that with range $r(n) = \sqrt{\frac{\log n + c(n)}{\pi n}}$, overall connectivity can be established with probability one as $n \to \infty$ if and only if $c(n) \to \infty$. And this result was independently determined by Penrose [2] as well. In [3], Wan and Yi determined the precise critical transmission range for k-connectivity.

Of equal importance, the second type of connecting strategies is the number-of-neighbor-based strategy, which means that for $G(V,E)$ and any two nodes $i,j \in V$, $e_{ij} \in E$ if and only if j is among i's ϕ nearest neighbors. Note that this strategy does not ensure that the degree of each node is strictly equal to ϕ. Actually, we have the degree of each node $D_i \geq \phi$, since the ϕ-nearest-neighbor relation is asymmetric. In [4], Xue and Kumar proved that for a network with n nodes to be asymptotically connected, $\Theta\big(\log n\big)$ [1] neighbors are necessary and sufficient. Wan and Yi [3] obtained an improved asymptotic upper bound on the critical neighbor number for k-connectivity.

Another strategy is the sector-based strategy that was proposed as a topology control algorithm by Wattenhofer *et al.* [5] and was further discussed by Li *et al.* in [6]. This strategy is based on the neighbor connection as described above and further concerns with the θ-coverage problem. Given that a node connects bidirectionally to its ϕ_n nearest neighbors in the network, where ϕ_n is a deterministic function of n to be specified, for an angle $\theta \in (0, 2\pi)$, the node is called to be θ-covered by its ϕ_n nearest neighbors if among them, it can find a node in every sector of angle θ. If every node in the graph satisfies this property, the graph is called θ-covered. One then wants to find the relation between θ-coverage and overall connectivity of the network, and to determine the critical value of ϕ_θ which is a deterministic function of θ. In [7], Xue and Kumar determined that the exact threshold function for θ-coverage, including even the pre-constant, is $\log_{\frac{2\pi}{2\pi-\theta}} n$, for any $\theta \in (0, 2\pi)$, and π-coverage with high probability implies overall connectivity with high probability.

The network models studied in these prior works are non-clustered (or *flat*) and stationary networks. Flat networks are found to have poor scalability [8] [9] and energy inefficiency [10] [11]. Clustering and mobility have been found to improve various aspects of network performance. First, clustered networks and clustering algorithms are studied by many researchers [12] [13] [14] [15] and have applications in both sensor networks [10] [16] and ad hoc networks [17] [18]. With random infrastructure support, the throughput capacity of random ad hoc networks can be greatly improved, and the capacity gain is found as $\Theta\left(\sqrt{\frac{n}{\log n}}\right)$ when the number of ad hoc nodes per access point is bounded as $\Theta(1)$ [19]. In [10], Heinzelman *et al.* presented that in sensor networks where nodes have sinks or base stations to gather their data, organizing nodes into clusters and using cluster head electing and rotating can be more energy-efficient than non-clustered multi-hop transmission to base stations which is normally adopted in ad hoc networks. In a separate direction, mobility has been found to increase the capacity [20] and help security [21] in ad hoc networks.

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[1] The following asymptotic notations are used throughout this paper. Given non-negative functions $f(n)$ and $g(n)$:

1) $f(n) = \Theta\big(g(n)\big)$ means for two constants $0 < c_1 < c_2$, $c_1 g(n) \leq f(n) \leq c_2 g(n)$ for sufficiently large n.
2) $f(n) = O\big(g(n)\big)$ means for a constant $c > 0$, $f(n) \leq cg(n)$ for sufficiently large n.
3) $f(n) = \Omega\big(g(n)\big)$ means for a constant $c > 0$, $f(n) \geq cg(n)$ for sufficiently large n.
4) $f(n) \sim g(n)$ means $\lim_{n\to\infty} \frac{f(n)}{g(n)} = 1$.
5) $f(n) = o\big(g(n)\big)$ means $\lim_{n\to\infty} \frac{f(n)}{g(n)} = 0$.
6) $f(n) = \omega\big(g(n)\big)$ means $g(n) = o\big(f(n)\big)$

However, compared to the relatively mature study on the connectivity of flat and stationary networks, studies on the connectivity of mobile and clustered networks are quite limited. (Most previous work on cluster or infrastructure-based mobile network focus on capacity [33] [34]) In a clustered network, a packet only needs to reach one of the cluster heads. We are interested in two cases in this paper. In a stationary k-hop clustered network, a packet must reach a cluster head within k hops. In a mobile k-hop clustered network, a packet must reach a cluster head directly in k time-slots. Clearly, clustering has an inherent advantage compared to flat networks, and it can alter the energy efficiency and delay of the system. First, it can require a different critical transmission range for connectivity, which may depend on the number of cluster heads and whether the network is stationary or mobile. Second, it can lead to different delay. For example, with k-hop clustering, the delay is bounded by k (i.e., $\Theta(1)$). In contrast, in a flat network with the minimum transmission range, the number of hops will increase as $\Theta\left(\sqrt{\frac{n}{\log n}}\right)$, and so does the delay. Finally, both the transmission range and the number of hops can affect the energy consumption of the network. We can then ask the following open question in this paper:

- What is the impact of mobility on connectivity of clustered networks subject to delay constraints?

In this paper, we concentrate on one of the above connecting strategies, namely, the distance-based strategy, and the number-of-neighbor-based strategy is briefly studied in a parallel manner afterwards. We study the critical transmission range for connectivity in mobile k-hop clustered networks where all nodes move under either the random walk mobility model with non-trivial velocity or the i.i.d. mobility model. By the term *non-trivial velocity*, we mean that the velocity of nodes $v = \omega\big(r(n)\big)$. Note that both i.i.d and random walk model can be viewed as the extreme cases of more general classes of mobility models [36], [37]. For example, the i.i.d model may provide useful insights when mobile nodes stay around an area for an extended period of time and then move quickly to another area. Hence, studies under these two models may provide important insights for the performance and inherent tradeoffs in more general system. We then compare with the critical transmission range for connectivity in stationary k-hop clustered networks. We also use these results to study the power-delay trade-off and the energy efficiency of different types of networks, including flat networks. Our results show that random walk mobility with non-trivial velocity does improve connectivity in k-hop clustered networks, and it also significantly decreases the energy consumption and the power-delay trade-off. Hence, these results provide fundamental insights on the design of large-scale wireless networks.

The rest of the paper is organized as follows. In section II, we describe the k-hop clustered network models. We provide the main results and some intuition behind these results in section III. In section IV, V and VI, we give the proofs of the critical transmission range in mobile k-hop clustered networks under the random walk mobility model with non-trivial velocities and the i.i.d. mobility model, and in stationary k-hop clustered networks, respectively. As a parallel discussion, we consider the critical number of neighbors for connectivity in both stationary and mobile clustered network in section VII. We then have a discussion on the impact of mobility on connectivity and network performance in k-hop clustered networks in section VIII. We conclude in section IX.

II. K-HOP CLUSTERED NETWORK MODELS

In this section, we first provide an overview of flat networks and then introduce models of clustered networks. A classification of k-hop clustered networks is given and related issues such as the transmission scheme and the routing strategy are presented, respectively.

A. An overview of flat networks

Before studying clustered networks, we now give an overview of the so-called *flat networks* as depicted in Figure 1. A flat network can be defined as a network in which all nodes have homogeneous roles and functionalities (while they may have different hardware capabilities), and they can reach each other without going through any intermediary service points such as base stations or sinks. In one word, flat networks are self-organized and infrastructure-free, like ad hoc networks in common context.

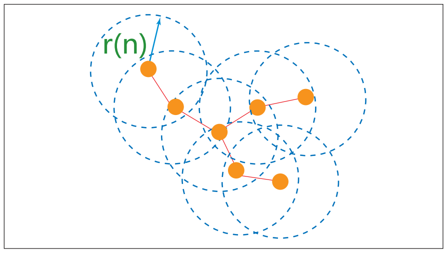

Fig. 1. Flat networks under the distance-based connecting strategies

There are several concepts related to flat networks whose counterparts in clustered networks will be studied in the rest of this paper. The most concerned in this paper is connectivity. Before defining connectivity of flat networks, we formulate flat networks as follows. Let $\mathcal{A}$ denote a unit area in $\Re^2$, and $\mathcal{G}(n)$ be the graph (network) formed when n nodes are placed uniformly and independently in $\mathcal{A}$. An edge e_{ij} exists between two nodes i and j, if the distance between them is less than $r(n)$ under the distance-based strategy. Then, graph $\mathcal{G}(n)$ is connected if and only if there is a path between any pair of nodes in $\mathcal{G}(n)$.

B. Classification of k-hop clustered networks

In contrast to flat networks, in clustered networks nodes are organized into clusters. A cluster head is selected within each cluster to serve the other cluster-member nodes (i.e., clients).

We assume that a clustered network consists of n cluster-member nodes and n^d cluster-head nodes, where d is called *the cluster head exponent* and $0 < d \leq 1$. For ease of presentation, we treat n^d as an integer in this paper, and all nodes are placed uniformly and independently in a unit square $\mathcal{S}$ in $\Re^2$. Moreover, the unit square is assumed to be a torus.

1) Mobile k-hop clustered networks:

a. *Mobility pattern*

In a mobile k-hop clustered network, we assume that all cluster members move according to a certain mobility pattern while the clustered heads are fixed with the uniform distribution.

- **Random Walk Mobility Model with Non-Trivial Velocities** [2] [22] [23]: Define a discrete random variable V the speed of a node with the probability mass function $P\big(V = v^{(m)}\big) = p_m$ for all $m \in \mathbb{M}$, where the index set $\mathbb{M}$ is finite and invariant of n. We assume that $v^{(m)} = \omega\left(\sqrt{\frac{\log n}{n^{d'}}}\right)$ $(d' < \frac{d}{2})$ and $v^{(m)} = O(1)$, for all $m \in \mathbb{M}$. This assumption, combined with the k-time-slots deadline that we will introduce next, implies that we are interested in the case when the speed is fast enough so that nodes can move multiple transmission ranges before the deadline (please refer to Remark 4.1), for which we expect the scaling laws to differ substantially with that in stationary networks. In addition, we assume that p_m for all $m \in \mathbb{M}$ does not change with n, and $p_m > 0$ for all m. Further, we assume that there exists an index $m_\star$ such that for all sufficiently large n, $\frac{v^{(m)}}{\log_n np_m} \geq \frac{v^{(m_\star)}}{\log_n np_{m_\star}}$ for all m. Let v_{min} represent the minimum value of V, and we assume that $v_{min} \leq \frac{1}{k}$. In other words, not all nodes can traverse a side of the torus in k slots. Note that we do allow $v^{(m)}$ to scale with n. We will see later that the probability of full connectivity will depend heavily on the dynamics of the nodes belonging to class $m_\star$. We then partition the data transmission process into time-slots with unit length. At the beginning of each period (i.e., every k slots; see *Transmission scheme* for the definition of a *period*) each member node randomly and independently select a speed $V = v$ according to the distribution of V, and uniformly and independently choose a random direction $\theta \in [0, 2\pi)$. The node then moves along this direction θ with the constant speed v for the entire period. Note that the mobility pattern of nodes in our model is slightly different from that defined in [22] and they do not bounce off the border since we have assumed the unit square to be a torus.
- **I.I.D. Mobility Model**: The transmission process is also divided into slots as we did above and at the beginning of each time-slot each member node will randomly and uniformly choose a position within the unit torus and remain static during the rest of the time-slot. In addition, we assume that $d > \frac{1}{k}$. (Please refer to the proof of Proposition 5.1 where we need the condition that $d > \frac{1}{k}$.)

b. *Transmission scheme*

We divide the channel into $W (W \geq 2)$ sub-channels, and thus the network can accommodate at least W flows initiated in a certain time-slot. Moreover, we assume that for each flow, the packet is forwarded for one hop in each time-slot. Therefore, the maximum delay for the transmission of a packet in our network model is k time-slots, or the delay constraint is $D = k$. In section VIII, we will mainly use the notation D as the delay constraint in our discussion on the power-delay trade-off in k-hop clustered networks.

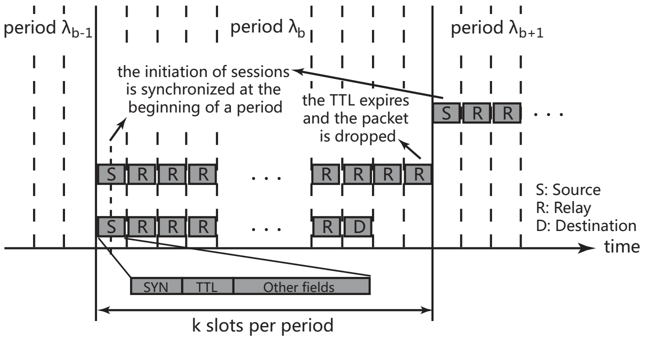

Fig. 2. Transmission scheme in mobile k-hop clustered networks.

We use the term *session* to refer to the process that a packet is forwarded from its source cluster member to a cluster head. In every packet, we assume that there is a **TTL** (time to live) field to record the number of hops that the packet has been forwarded. The initial value of TTL is set to 1 and each relay increases the counter by one when it receives the packet. When the hop counter is greater than k, the packet is discarded and we say that the session is *failed*. Every k time-slots constitute a period. We assume that there is a **SYN** (synchronize) field for all nodes to be synchronized and data-flows are initiated only at the beginning of each period. This assumption accords with the design of some novel energy-efficient duty-circle MAC protocols (RMAC [24], DW-MAC [25]). Our proposed transmission scheme is illustrated in Figure 2.

c. *Routing strategy*

As to the routing strategy, we simply assume that a cluster member holds the packet (acting as the relay of itself), if it does not have a cluster head in its transmission range during its course of movement, or sends the packet to the cluster head once they meet.

Note that this assumption requires that a cluster member can know the existence of a cluster head within the transmission range. Such an assumption would be valid when (1) the cluster heads are static and the cluster member has knowledge of its own position and the positions of cluster heads; or (2) the cluster heads broadcast a pilot signal that covers nearby cluster members. Our routing strategy under the random walk mobility assumption is illustrated in Figure 3.

Note that in the above model we choose not to use multi-hop transmissions in mobile k-hop clustered networks. Although

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[2] In the random walk mobility model defined in [22], each movement either corresponds to a constant time interval t, or corresponds to a constant distance traveled. The model we use conforms with the former case.

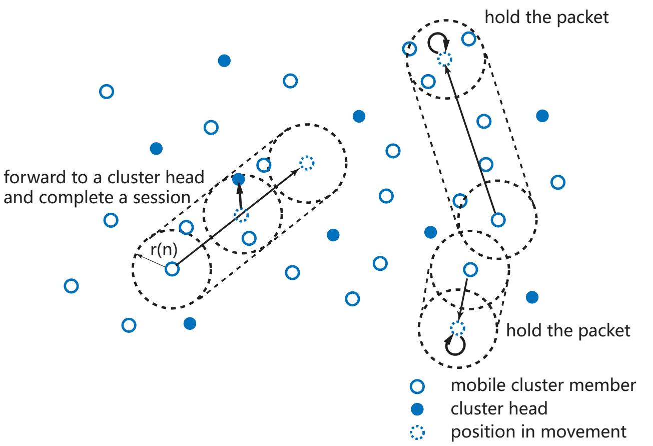

Fig. 3. Routing strategy in mobile k-hop clustered networks, random walk mobility.

multi-hop transmissions may further improve system performance, establishing multi-hop paths to cluster-heads would have required the mobile nodes to dynamically discover the cluster-heads that are k times its transmission range away.This would require either a significantly larger pilot signal transmitted by the cluster-heads, or location information of both the mobile and the cluster-heads. In contrast, our study in this paper does not requite these mechanisms. Further, as we can see, even without multi-hop transmissions, the analysis is already quite complicated due to various difficulties in the proofs. Hence, we decided to leave multihop transmissions to the cluster head as future work.

d. *Memoryless assumption*

For both mobility models, we further make the following memoryless assumption. That is, all cluster-member nodes are memoryless about their past experience of the success or failure of sessions. Furthermore, all cluster-member nodes do not record the positions of any cluster-head nodes with which they may have communicated. Thus, under this memoryless assumption, in each period, the distribution of head nodes is still uniform in the area of network, as seen by the member nodes.

2) Stationary k-hop clustered networks: In a stationary k-hop clustered network, all nodes remain static after uniformly distributed in the unit area. As in its mobile counterpart, we also assume that the packet is forwarded for one hop in each time-slot.

3) Redefining connectivity in clustered networks: Due to clustering and mobility, the definition of connectivity in clustered networks is different from that in flat networks. For stationary k-hop clustered networks, we say that a cluster member is *connected* if it can reach a cluster head within k hops. For mobile clustered networks, a cluster member is *connected* if it can reach a cluster head within k slots. If all the cluster members in a network are connected, we define that the network has *full connectivity*.

III. MAIN RESULTS AND INTUITIONS

A. *Defintions*

Before we state our main results, we first formally define the critical transmission range and the critical number of neighbors in both mobile and stationary k-hop clustered networks.

Recall that for mobile networks, in every period of k time-slots, each node may attempt to connect to the cluster head. For mobile k-hop clustered networks, let E denote the event that all cluster members are connected in a given period Λ, and let $P^{\Lambda}(E)$ denote the the corresponding probability. We then are ready to define the critical transmission range for clustered networks.

Definition 3.1: For mobile k-hop clustered networks, $r(n)$ is the critical transmission range if

$$\lim_{n\to\infty} P^{\Lambda}(E) = 1, \text{if } r \geq cr(n) \text{ for any } c > 1;$$
$$\lim_{n\to\infty} P^{\Lambda}(E) < 1, \text{if } r \leq c'r(n) \text{ for some } c' < 1,$$

For stationary networks, we define E to be the event that all cluster members are connected to a cluster head in k hops.

Definition 3.2: For stationary k-hop clustered networks, $r(n)$ is the critical transmission range if

$$\lim_{n\to\infty} P(E) = 1, \text{if } r \geq cr(n) \text{ and } c > 1;$$
$$\lim_{n\to\infty} P(E) < 1, \text{if } r \leq c'r(n) \text{ and } c' < 1.$$

In parallel, we have the following definition for the critical number of neighbors. Note that critical number of neighbors (CNoN) in cluster and mobile networks is different from that in flat and stationary network. Due to mobility, the CNoN is the number of neighbors a node needs to maintain contact within a time period. And due to clustering, each cluster member only needs to maintain contact with the cluster heads. Hence, the CNoN is the number of neighbors a cluster member needs to check to see whether there is a cluster head.

Definition 3.3: For mobile k-hop clustered networks, given that the state of network is observed in the period Λ, $\phi(n)$ is the critical number of neighbors if

$$\lim_{n\to\infty} P^{\Lambda}(E) = 1, \text{if } \phi \geq c\phi(n) \text{ and } c > 1;$$
$$\lim_{n\to\infty} P^{\Lambda}(E) < 1, \text{if } \phi \leq c'\phi(n) \text{ and } c' < 1.$$

Definition 3.4: For stationary k-hop clustered networks, $\phi(n)$ is the critical number of neighbors if

$$\lim_{n\to\infty} P(E) = 1, \text{if } \phi \geq c\phi(n) \text{ and } c > 1;$$
$$\lim_{n\to\infty} P(E) < 1, \text{if } \phi \leq c'\phi(n) \text{ and } c' < 1.$$

B. *Main results and intuitions*

We summarize our main results in this paper as follows:

- Under the random walk mobility assumption, the critical transmission range is $r(n) = \frac{\log n}{2kv_{\star}n^d}$, where d is *the cluster head exponent*, $0 < d \leq 1$, $v_{\star} = \min\{\frac{v^{(m)}}{\log_n np_m}, \forall m \in \mathbb{M}\}$.Note that $v_{\star}$ is a function of n. (See Section II.B)

- Under the i.i.d. mobility assumption, the critical transmission range is $\sqrt{\frac{\log n}{k\pi n^d}}$, where $\frac{1}{k} < d \leq 1$.
- For stationary k-hop clustered networks, the critical transmission range is $r(n) = \frac{1}{k}\sqrt{\frac{d \log n}{\pi n^d}}$, where $0 < d < 1$.
- For both mobile and stationary k-hop clustered networks, $\Theta(n^{1-d}\log n)$ neighbors are necessary and sufficient.

Before we prove these results rigorously, we now give an intuitive approach to estimate the order of critical transmission range in each scenario here:

Suppose there are n cluster members and n^d cluster heads uniformly distributed in a unit square. Thus, roughly speaking, there is one cluster head within an area of $\frac{1}{n^d}$.

- Under the random walk mobility assumption, the area covered by movement during k time-slots constitutes the dominating part of a cluster member's coverage area. Assume that the velocities of nodes are uniformly a constant v. Thus, in order to reach a cluster head, on average we need

  $$2kvr(n) = \frac{1}{n^d}, \text{ or } r(n) = \frac{1}{2kvn^d};$$

  With certain transmission range, consider the number of other nodes within the coverage of an arbitrary cluster member during its movement in one period, we have

  $$\phi(n) = (n + n^d) \cdot 2kvr(n) = n^{1-d} + 1.$$

- Under the i.i.d. mobility assumption, considering that cluster members actually remain static during any time-slot, the coverage consists of the overall area of k disks. Thus, on average we need

  $$k\pi r^2(n) = \frac{1}{n^d}, \text{ or } r(n) = \sqrt{\frac{1}{k\pi n^d}};$$

  Similarly, we have

  $$\phi(n) = (n + n^d) \cdot k\pi r^2(n) = n^{1-d} + 1.$$

- In the stationary networks, a reachable cluster head is roughly within a disk with a radius $kr(n)$ of the cluster member, and thus we need

  $$\pi\big(kr(n)\big)^2 = \frac{1}{n^d}, \text{ or } r(n) = \frac{1}{k}\sqrt{\frac{1}{\pi n^d}};$$

  Similarly, we obtain

  $$\phi(n) = (n + n^d) \cdot \pi\big(kr(n)\big)^2 = n^{1-d} + 1.$$

In the following, we will prove the necessary and sufficient conditions for the critical transmission range $r(n)$ under random walk, i.i.d. and stationary k-hop models. The main idea for the proofs of necessary condition is to show that the probability of disconnection would be lower bounded from zero if the critical transmission range is no greater than $r(n)$. Similarly, for proofs of sufficient conditions, we will prove that the probability of session failure (i.e. network disconnected) would approach 0 asymptotically.

IV. THE CRITICAL TRANSMISSION RANGE FOR MOBILE K-HOP CLUSTERED NETWORKS, RANDOM WALK MOBILITY

We first define several key notations that will be used throughout this section. Let $v_\star = \frac{v^{(m_\star)}}{\log_n np_{m_\star}} = \min\{\frac{v^{(m)}}{\log_n np_m}, \forall m \in \mathbb{M}\}$, $m_\star = \arg\min\{\frac{v^{(m)}}{\log_n np_m}, \forall m \in \mathbb{M}\}$ and $p_{m_\star} = p_\star$, where $v^{(m)}$ is the value of the discrete random variable V — the speed of cluster member. Recall that by our assumption, $m_\star$ is independent of n when n is large while $v_\star$ is a function of n for all n. (See Section II.B) Also $v_\star$ is not equal to $v^{(m_\star)}$, which can be easily seen from the definition of $v_\star$.

In this section, we have the following main result.

Theorem 4.1: Under the random walk mobility assumption, the critical transmission range is $r(n) = \frac{\log n}{2kv_\star n^d}$, where $0 < d \leq 1$.

Remark 4.1: Recall the assumption that $v^{(m)} = \omega(\sqrt{\frac{\log n}{n^{d'}}})$ with $d' < \frac{d}{2}$. Combined with $r(n) = \frac{\log n}{2kv_\star n^d}$, it implies that $\frac{r(n)}{kv_\star} = o(n^{d'-d})$. In other words, the speed is fast enough so that nodes can move multiple transmission ranges before the deadline. Further, it is easy to verify that $r(n) = o\left(\frac{\sqrt{\log n}}{n^{d-\frac{d'}{2}}}\right)$.

A. Necessary condition on $r(n)$ of Theorem 4.1

We start with the following lemma.

Lemma 4.1: If $r(n) = \frac{\frac{d_0}{2}\log n + \kappa}{2kv_\star n^d}$, where $d' < d_0 < d$, then for any fixed $\theta < 1$ and $\epsilon(n) = \frac{1}{\log n}$, there exists N_0 such that for all $n \geq N_0$, the following holds

$$n^{\frac{d_o}{2}}\Big(1 - \big(1 + \epsilon(n)\big)2kv^{(m_\star)}r(n)\Big)^{n^d} \geq \theta e^{-\kappa - \frac{d_0}{2}\log p_\star - \frac{d_0}{2}}, \tag{1}$$

where $0 < d \leq 1$.

Proof: Taking the logarithm of the left hand side of (1), we get

$$\log\big(\text{L.H.S. of } (1)\big) = \frac{d_0}{2}\log n + n^d \log\Big(1 - \big(1 + \epsilon(n)\big)2kv^{(m_\star)}r(n)\Big). \tag{2}$$

Using the power series expansion for $\log(1 - x)$,

$$\begin{aligned} &\log(\text{L.H.S. of } (1)) \\ &= \frac{d_0}{2}\log n - n^d \sum_{i=1}^{\infty} \frac{\Big(\big(1 + \epsilon(n)\big)2kv^{(m_\star)}r(n)\Big)^i}{i} \\ &= \frac{d_0}{2}\log n - n^d \bigg(\sum_{i=1}^{2} \frac{1}{i}\Big(G\big(n, \kappa, \epsilon(n)\big)\Big)^i + \delta(n)\bigg) \end{aligned} \tag{3}$$

where

$$G\big(n, \kappa, \epsilon(n)\big) = \big(1 + \epsilon(n)\big)2kv^{(m_\star)}r(n)$$

and

$$\begin{aligned}
\delta(n) &= \sum_{i=3}^{\infty} \frac{1}{i}\Big(G\big(n,\kappa,\epsilon(n)\big)\Big)^{i} \\
&\leq \sum_{i=3}^{\infty} \frac{1}{3}\Big(G\big(n,\kappa,\epsilon(n)\big)\Big)^{i} \\
&= \frac{1}{3}\frac{\Big(G\big(n,\kappa,\epsilon(n)\big)\Big)^{3}}{1-G\big(n,\kappa,\epsilon(n)\big)} \\
&\leq \frac{1}{3}\frac{\Big(G\big(n,\kappa,\epsilon(n)\big)\Big)^{3}}{G\big(n,\kappa,\epsilon(n)\big)} \\
&= \frac{1}{3}\Big(G\big(n,\kappa,\epsilon(n)\big)\Big)^{2}, \qquad (4)
\end{aligned}$$

for all large n. Substituting (4) in (3), we get

$$\begin{aligned}
&\log\big(\text{L.H.S. of } (1)\big) \\
&\geq \frac{d_0}{2}\log n - n^d\bigg(G\big(n,\kappa,\epsilon(n)\big) + \frac{5}{6}\Big(G\big(n,\kappa,\epsilon(n)\big)\Big)^2\bigg) \\
&= \frac{d_0}{2}\log n - n^d\bigg(\big(1+\epsilon(n)\big)\frac{\frac{d_0}{2}\log n + \kappa}{n^d}\log_n\left(np_\star\right) + \\
&\qquad \frac{5}{6}\Big(\big(1+\epsilon(n)\big)\frac{\frac{d_0}{2}\log n + \kappa}{n^d}\log_n\left(np_\star\right)\Big)^2\bigg) \\
&= -\kappa - \frac{d_0}{2}\log p_\star - \epsilon(n)(\frac{d_0}{2}\log n + \frac{d_0}{2}\log p_\star + \kappa) \\
&\quad -\big(1+\epsilon(n)\big)\kappa\log_n p_\star \\
&\quad -\frac{5}{6}\frac{\Big(\big(1+\epsilon(n)\big)(\frac{d_0}{2}\log n + \kappa)\log_n\left(np_\star\right)\Big)^2}{n^d}.
\end{aligned}$$

Since $\epsilon(n) = \frac{1}{\log n}$, the right hand side converges to $-\kappa - \frac{d_0}{2}\log p_\star - \frac{d_0}{2}$ as $n \to \infty$. Hence, for any $\tilde{\epsilon} > 0$ we can choose $N_{\tilde{\epsilon}}$ such that

$$\log\big(\text{L.H.S. of } (1)\big) \geq -\kappa - \frac{d_0}{2}\log p_\star - \frac{d_0}{2} - \tilde{\epsilon},$$

for all $n > N_{\tilde{\epsilon}}$. Taking the exponent of both sides and using $\theta = e^{-\tilde{\epsilon}}$, the result follows. ■

Let $\mathcal{G}_{rw}(n, r(n))$ denote the network where two nodes can communicate if their Euclidean distance is at most $r(n)$ and $P^{\Lambda}_{f\_rw}(n, r(n))$ be the probability that $\mathcal{G}_{rw}(n, r(n))$ has some node that is not connected in the period Λ. Then we have the following proposition.

Proposition 4.1: If $r(n) = \frac{\frac{d_0}{2}\log n + \kappa(n)}{2kv_\star n^d}$, then

$$\begin{aligned}
&\liminf_{n\to\infty} P^{\Lambda}_{f\_rw}(n, r(n)) \\
&\quad \geq \; e^{-(\kappa + \frac{d_0}{2}\log p_\star)}\left(e^{-\frac{d_0}{2}} - e^{-(\kappa + \frac{d_0}{2}\log p_\star)}\right)
\end{aligned}$$

where $\kappa = \lim_{n\to\infty}\kappa(n), \kappa > \frac{d_0}{2} - \frac{d_0}{2}\log p_\star$.

Proof: Let $u(n) = O\Big(\sqrt{\frac{\log n}{n}}\Big)$. Then we divide the unit square into $\frac{1}{u(n)} \times \frac{1}{u(n)}$ cells and each cell is of size $u^2(n)$.

Now, among these cells, pick $n^{\frac{d_0}{2}}$ of them such that each of them is at least $\sqrt{\frac{1}{n^{d_0}}}$ away from others. For example, we can choose a subset of the highlighted cells in Figure 4.

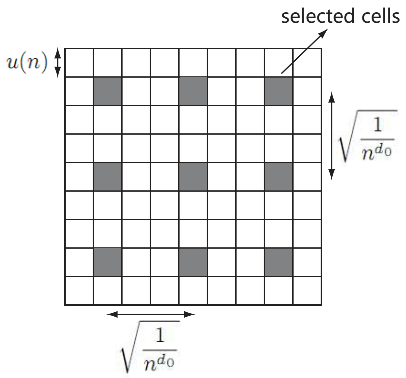

Fig. 4. Cell Selection

Note that, by appropriately choosing $u(n)$ (e.g. choosing $u(n) = \sqrt{\frac{C\log n}{n}}$, where the factor C can be set according to the value of d_0 and $p_{m_\star}$. For a rigorous proof, see [35, lemma 11]), with high probability, there are at least one cluster member in each of these selected cells taking the speed $v^{(m_\star)}$. Pick such a cluster member node from each of these selected cells. There are a total of $n^{\frac{d_0}{2}}$ of these nodes. Let $\mathbb{Y}$ denote the set of such cluster member nodes. Note that any two nodes in $\mathbb{Y}$ are at a distance of at least $\sqrt{\frac{1}{n^{d_0}}}$ away. Let s_i be the session initiated by node i and we say that session s_i fails if i is not connected (i.e., it cannot reach a cluster head in k time-slots). Then, consider an arbitrary period Λ, we have[3]

$$\begin{aligned}
&P^{\Lambda}_{f\_rw}(n, r(n)) \\
&\geq P^{\Lambda}(\{\text{some session } s_i \text{ fails in } \mathcal{G}_{rw}(n, r(n))\}) \\
&\geq \sum_{i\in\mathbb{Y}} P^{\Lambda}(\{s_i \text{ is the only failed session in } \mathcal{G}_{rw}(n, r(n))\}) \\
&\geq \sum_{i\in\mathbb{Y}} P^{\Lambda}(\{s_i \text{ is a failed session in } \mathcal{G}_{rw}(n, r(n))\}) \\
&\quad -\sum_{i\in\mathbb{Y}}\sum_{j\neq i} P^{\Lambda}(\{s_i \text{ and } s_j \text{ are failed sessions} \\
&\qquad\qquad \text{in } \mathcal{G}_{rw}(n, r(n))\}). \qquad (5)
\end{aligned}$$

Next, we will evaluate the two terms on the right hand side of (5), respectively. We will find a lower bound for the first term and an upper bound for the second term. Then, $P^{\Lambda}_{f\_rw}(n, r(n))$ will be proved to be bounded away from zero. Proposition 4.1 will then follows.

---

[3] Equation (2) is similar to the corresponding equation in the proof of Theorem 2.1 in [1]. However, the summation in [1] is over all nodes, while here the summation is only over nodes i and j in the set $\mathbb{Y}$. The reason that we have to consider a smaller set $\mathbb{Y}$ is because otherwise the second summation may diverge. In [1], in order to ensure the convergence of the corresponding second summation, it is essential that, when nodes i and j are both disconnected, they must be at least some distance apart. In our case, the corresponding property would require that, if s_i and s_j are both failed sessions, nodes i and j must be some distance apart. Unfortunately, this property does not hold for all nodes. On the other hand, the restriction to the set $\mathbb{Y}$ helps to enforce this property.

Specifically, for the first term, using Lemma 4.1 with $\epsilon(n) = \frac{1}{\log n}$, we know that it is bounded by

$$\begin{aligned} & n^{\frac{d_0}{2}}\Big(1-\big(\pi r^2(n)+2kv^{(m_\star)}r(n)\big)\Big)^{n^d} \\ = \;& n^{\frac{d_0}{2}}\Big(1-\big(1+\epsilon_1(n)\big)2kv^{(m_\star)}r(n)\Big)^{n^d} \\ > \;& n^{\frac{d_0}{2}}\Big(1-\big(1+\epsilon(n)\big)2kv^{(m_\star)}r(n)\Big)^{n^d} \\ \geq \;& \theta e^{-\kappa-\frac{d_0}{2}\log p_\star-\frac{d_0}{2}}, \end{aligned}\tag{6}$$

where $\epsilon_1(n) = \frac{\pi r(n)}{2kv^{(m_\star)}} < \epsilon(n)$ according to the Remark 4.1 and $0 < \theta < 1$.

To bound the second term, note that the main difficulty here is that the trajectories of s_i and s_j may overlap. Let $\phi (0 \leq \phi < \pi)$ be the angle of intersection. Then the overlapping area can be as large as $4r^2(n)/\sin\phi$. In comparison, the trajectories of s_i and s_j cover an area of $2r(n)kv^{(m_\star)}$ each. Hence, in order to show that the overlapping area does not significantly affected the probability of connectivity, the key is to show that with high probability the angel ϕ cannot be too close to either 0 or π.

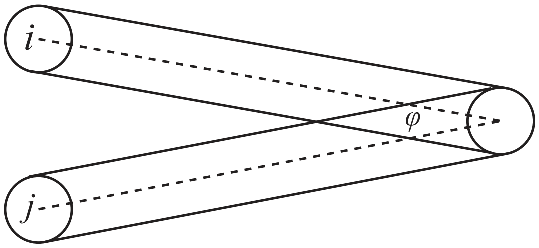

Fig. 5. Overlapped Area–1

There are two cases. First, if $\phi < \pi/2$, then as illustrated in Figure 5, the angle of intersection must be of order

$$\Theta\left(\frac{\sqrt{1/n^{d_0}}}{kv^{(m_\star)}}\right)=o\left(\sqrt{\frac{1}{n^{d_0-d'}\log n}}\right)=\Omega\left(\sqrt{\frac{1}{n^{d_0}}}\right).\tag{7}$$

Hence, in this case, the intersection of the trajectories of i and j, with probability one, is of area

$$\begin{aligned} \big(2r(n)\big)^2\frac{1}{\sin\frac{\sqrt{1/n^{d_0}}}{kv^{(m_\star)}}} &\sim 4r(n)kv^{(m_\star)}\cdot r(n)n^{\frac{d_0}{2}} \\ &= o\left(\frac{4r(n)kv^{(m_\star)}}{\log^2 n}\right), \end{aligned}\tag{8}$$

where we have used Remark 4.1.

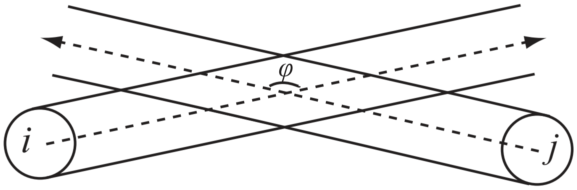

Fig. 6. Overlapped Area–2

Next, consider the case when $\phi > \pi/2$ as shown in Figure 6. The angle of intersection can be close to π if the two tracks are along a straight line. But we can show that this event happens with very low probability. Let ϕ be the angle of intersection and $\varepsilon = \varepsilon(n) = \frac{\log^2 n}{n^{d-d'}}$, where $d' < \frac{d}{2}$ and $d' < d_0 < d$. Note that for the angle ϕ to be greater than $\pi - \varepsilon$, the trajectories from both i and j must be no more than an angle $\frac{\varepsilon}{2}$ away from the line connecting i and j. Then we have

$$P(\phi > \pi-\varepsilon) < \left(\frac{\varepsilon}{2\pi}\right)^2 = \frac{\log^4 n}{4\pi^2 n^{2(d-d')}} = \frac{\log^4 n}{4\pi^2 n^{d_\star}},\tag{9}$$

where $d_\star = 2(d-d') > d > d_0$. Then, for such $\pi/2 \leq \phi \leq \pi - \varepsilon$, the intersection is of area

$$\begin{aligned} \frac{2r(n)}{\sin\phi}\cdot 2r(n) &\leq 4r(n)kv^{(m_\star)}\cdot\frac{r(n)}{kv^{(m_\star)}}\cdot\frac{1}{\varepsilon(n)} \\ &= o\left(\frac{4r(n)kv^{(m_\star)}}{\log^2 n}\right), \end{aligned}\tag{10}$$

where we have used Remark 4.1.

Let S_{i+j}^{Λ} denote the total area covered by i and j during the period Λ. Then for some $\epsilon' = o\left(\frac{1}{\log^2 n}\right)$ we can obtain that, whenever $\phi \leq \pi - \varepsilon$ the following holds.

$$\begin{aligned} S_{i+j}^{\Lambda} &\geq 4r(n)kv^{(m_\star)} - o\left(\frac{4r(n)kv^{(m_\star)}}{\log^2 n}\right) \\ &= (1-\epsilon')4r(n)kv^{(m_\star)}, \end{aligned}\tag{11}$$

for all $n > N_{\epsilon'}$. Let T_{ij} be the event that $S_{i+j}^{\Lambda} \geq (1-\epsilon')4r(n)kv^{(m_\star)}$, taking into account both cases we have

$$P(T_{ij}) > 1 - \frac{\log^4 n}{4\pi^2 n^{d_\star}}.$$

Therefore, each term in the second summation is bounded by

$$\begin{aligned} & P^{\Lambda}(\{s_i \text{ and } s_j \text{ are failed sessions in } \mathcal{G}_{rw}\}) \\ & = P^{\Lambda}(\{s_i \text{ and } s_j \text{ are failed sessions in } \mathcal{G}_{rw}\} \mid T_{ij})P(T_{ij}) \\ & \quad + P^{\Lambda}(\{s_i \text{ and } s_j \text{ are failed sessions in } \mathcal{G}_{rw}\} \mid \overline{T_{ij}})P(\overline{T_{ij}}) \\ & \leq P^{\Lambda}(\{s_i \text{ and } s_j \text{ are failed sessions in } \mathcal{G}_{rw}\} \mid T_{ij}) \\ & \quad + P(\overline{T_{ij}}) \\ & \leq \Big(1-\big(1-\epsilon'\big)4r(n)kv^{(m_\star)}\Big)^{n^d} + \frac{\log^4 n}{4\pi^2 n^{d_\star}} \\ & \leq e^{-4n^d(1-\epsilon')r(n)kv^{(m_\star)}} + \frac{\log^4 n}{4\pi^2 n^{d_\star}} \\ & \leq e^{-2(1-\epsilon')\log_n(np_\star)(\frac{d_0}{2}\log n+\kappa)} + \frac{\log^4 n}{n^{d_\star}} \end{aligned}\tag{12}$$

where the forth step is due to the well-known bound

$$1 - x \leq e^{-x} \text{ for } x \in [0,1].\tag{13}$$

Therefore, combined with (6) and (12) in (5), we obtain

$$\begin{aligned} & P_{f\_rw}^{\Lambda}(n, r(n)) \\ & \quad \geq \theta e^{-\kappa-\frac{d_0}{2}\log p_\star-\frac{d_0}{2}} \\ & \qquad - n^{d_0}\left(e^{-2(1-\epsilon')\log_n(np_\star)(\frac{d_0}{2}\log n+\kappa)} + \frac{\log^4 n}{n^{d_\star}}\right), \end{aligned}$$

for all $n > N_{\theta,\epsilon'} = \max\{N_\theta, N_{\epsilon'}\}$.

Recall that $\lim_{n\to\infty}\kappa(n)=\kappa$. Then for any $\hat{\epsilon}>0$, there exists $N_{\hat{\epsilon}}$ such that for all $n\geq N_{\hat{\epsilon}}$, $\kappa(n)\leq\kappa+\hat{\epsilon}$. Considering that the probability of disconnectedness is monotonously decreasing in κ, we obtain,

$$\begin{aligned} &P_{f\_rw}(n,r(n)) \\ &\quad\geq\quad \theta e^{-(\kappa+\frac{d_0}{2}\log p_\star+\frac{d_0}{2}+\hat{\epsilon})} \\ &\qquad -n^{d_0}\left(e^{-2(1-\epsilon')\log_n(np_\star)(\frac{d_0}{2}\log n+\kappa+\hat{\epsilon})}+\frac{\log^4 n}{n^{d_\star}}\right), \end{aligned}$$

for all $n>\max\{N_{\theta,\epsilon'},N_{\hat{\epsilon}}\}$. Note that $\epsilon'=o\left(\frac{1}{\log^2 n}\right)$, then

$$\begin{aligned} &n^{d_0}e^{-2(1-\epsilon')\log_n(np_\star)(\frac{d_0}{2}\log n)} \\ &\quad=\quad \frac{n^{d_0}}{n^{d_0(1-\epsilon')}}e^{-(1-\epsilon')d_0\log p_\star} \\ &\quad=\quad e^{-d_0\log p_\star},\ \text{as } n\to\infty. \end{aligned}$$

Taking limits, we then have

$$\begin{aligned} &\liminf_{n\to\infty}P_{f\_rw}(n,r(n)) \\ &\quad\geq\quad \theta e^{-(\kappa+\frac{d_0}{2}\log p_\star+\frac{d_0}{2}+\hat{\epsilon})}-e^{-2(\kappa+\frac{d_0}{2}\log p_\star+\hat{\epsilon})}. \end{aligned}$$

Since this holds for all $\hat{\epsilon}>0$ and any fixed $\theta<1$, Proposition 4.1 gets proved. Note that κ has a lower bound $\frac{d_0}{2}-\frac{d_0}{2}\log p_\star$, thus the right hand side of Proposition 4.1 will be above zero. ■

Remark 4.2: Proposition 4.1 provides the necessary condition on $r(n)$ in terms of both $\kappa(n)$ and d_0, i.e., in addition to the requirement for $\kappa(n)$ to approach infinity and a lower bound for κ, this condition also says that any $d_0<\frac{d}{2}$ will result in a positive probability of disconnection. To explain this implication, suppose two range $r_0(n)$ and $r_1(n)$, such that $\pi r_0^2(n)=\frac{\frac{d_0}{2}\log n+\kappa(n)}{2kv_\star n^d}$ with $\kappa(n)=o(\log n)$, and $\pi r_1^2(n)=\frac{\frac{d_1}{2}\log n+\kappa}{2kv_\star n^d}$, where $d_0<d_1<\frac{d}{2}$, $\kappa(n)\to\infty$ and κ is a constant. By Proposition 4.1, we know that $r_1(n)$ will result in a disconnected network. Meanwhile, since $\kappa(n)=o(\log n)$ we have $r_0(n)<r_1(n)$ for all sufficiently large n and the probability of disconnection is monotonously decreasing in r. Therefore, $r_0(n)$ will also result in a disconnected network.

As an consequence of the Proposition 4.1 and Remark 4.2, we have

Corollary 4.1: Under the random walk mobility assumption, $r(n)\geq\frac{\frac{d}{2}\log n}{2kv_\star n^d}$ is necessary for the connectivity of the mobile k–hop clustered networks with *random walk* mobility model.

Hence we have proved the necessity part of Theorem 4.1.

B. Sufficient condition on $r(n)$ of Theorem 4.1

Suppose there are at most n sessions in a period Λ, and let E_i denote the event that s_i is a failed session, where $i=1,2,\ldots,n$. Let the transmission range of each node be $r=cr(n)$, where $c>1$. Then, it suffices to show that

$$\lim_{n\to\infty}P^\Lambda\left(\bigcup_{i=1}^n E_i\right)=0.$$

Using the union bound we have

$$P^\Lambda\left(\bigcup_{i=1}^n E_i\right)\leq\sum_{i=1}^n P^\Lambda(E_i). \tag{14}$$

Next we divide the mobile nodes into two sets. The first set $\mathbb{A}$ contains nodes satisfying $v_i\leq\frac{1}{k}$, while the second set $\mathbb{B}$ contains nodes with $v_i>\frac{1}{k}$. For each node in $\mathbb{A}$, because we assume that the space is a torus, no path of trajectory overlaps. In contrast, for each in $\mathbb{B}$, if it moves at a small angle along one of the four sides, paths of trajectory may overlap and loop around the torus. Hence, these two sets need to be treated differently. Specifically

For node in $\mathbb{A}$, since every node's speed satisfies $v_i\geq v_{min}$,

$$P^\Lambda(E_i)\leq\Big(1-\big(\pi r^2+2kv_{min}cr(n)\big)\Big)^{n^d}<\big(1-2kv_{min}cr(n)\big)^{n^d}.$$

For node in $\mathbb{B}$, because v_i is larger than $\frac{1}{k}$, the minimum area swept by the trajectory is $2cr(n)$. This circumstance occurs, e.g. when the node moves parallel to one of the four sides of the space and loops around the torus. Hence, for node in $\mathbb{B}$,

$$P^\Lambda(E_i)\leq\Big(1-2cr(n)\Big)^{n^d}\leq\big(1-2kv_{min}cr(n)\big)^{n^d},$$

where the second inequality holds on because we have assumed that $v_{min}\leq\frac{1}{k}$ in the definition of the random walk model in Section II. Then, we have

$$\begin{aligned} \sum_{i=1}^n P^\Lambda(E_i) &= \sum_{i\in\mathbb{A}}P^\Lambda(E_i)+\sum_{i\in\mathbb{B}}P^\Lambda(E_i) \\ &\leq \sum_{i\in\mathbb{A}}\big(1-2kv_{min}cr(n)\big)^{n^d}+ \\ &\quad \sum_{i\in\mathbb{B}}\big(1-2kv_{min}cr(n)\big)^{n^d} \\ &= n\big(1-2kv_{min}cr(n)\big)^{n^d} \\ &\leq ne^{-2kv_{min}cr(n)n^d} \quad \text{(by (13))} \\ &= n^{-cv_{min}/v_\star+1} \\ &\leq n^{-c\log_n(np_{min})+1} \\ &= \frac{n}{\big(np_{min}\big)^c} \end{aligned}$$

Consequently, for any $c>1$, we have

$$\sum_{i=1}^n P^\Lambda(E_i)\leq\frac{n}{\big(np_{min}\big)^c}\quad\to\quad 0, as\ n\to\infty. \tag{15}$$

Thus, using (15) in (14), we have

$$\lim_{n\to\infty}P^\Lambda\left(\bigcup_{i=1}^n E_i\right)=0,$$

and the result follows.

Remark 4.3: Note that there is a small gap between the necessary and sufficient conditions. The necessary condition roughly requires $r(n)=\frac{\frac{d}{2}\log n}{2kv_\star n^d}$, while the sufficient condition roughly requires $r(n) = \frac{\log n}{2kv_\star n^d}$. Nonetheless, the order in terms of n is the same.

Also note that by assuming the network area is a torus, we have eliminated the possibility that nodes get reflected at the boundary. If reflection is allowed, it will lead to minimal change to our conclusions. Specifically, the critical transmission range under this case is $r(n) = \frac{\log n}{kv_\star n^d}$, where $0 < d \leq 1$. Note that it is two times larger than the critical transmission range in a torus. An intuitive explanation for this result is that, if nodes get reflected and follow their impinging path, the sweep area might be $kv^{(m)}r(n)$ under the worst situation, $m \in \mathbb{M}$. Thereby $r(n)$ should be two times larger than that in a torus. We omit the proof due to space constraints since it is very similar to the earlier proofs in this section.

V. THE CRITICAL TRANSMISSION RANGE FOR MOBILE K-HOP CLUSTERED NETWORKS, I.I.D. MOBILITY

The main result of this section is as follows.

Theorem 5.1: Under the i.i.d. mobility assumption, the critical transmission range is $r(n) = \sqrt{\frac{\log n}{k\pi n^d}}$, where $\frac{1}{k} < d \leq 1$.

A. Necessary condition on $r(n)$ of Theorem 5.1

We start with the following technical lemma.

Lemma 5.1: If $\pi r^2(n) = \frac{\log n + \kappa}{kn^d}$, for any fixed $\theta < 1$ and $\mu \leq 1$, and for all sufficiently large n

$$n\big(1 - \mu\pi r^2(n)\big)^{kn^d} \geq \theta e^{-\kappa}, \tag{16}$$

where $0 < d \leq 1$.

Proof: The proof of this lemma follows the same argument as that of Lemma 4.1, and we thus omit this proof. ■

Let $\mathcal{G}_{iid}(n, r(n))$ denote the network where two nodes can communicate if their Euclidean distance is at most $r(n)$ and $P_{f\_iid}(n, r(n))$ be the probability that $\mathcal{G}_{iid}(n, r(n))$ has failed sessions (i.e., it has a node that is not connected). Then we have the following proposition.

Proposition 5.1: If $\pi r^2(n) = \frac{\log n + \kappa(n)}{kn^d}$, where $\frac{1}{k} < d \leq 1$, then

$$\liminf_{n\to\infty} P_{f\_iid}(n, r(n)) \geq e^{-\kappa}(1 - e^{-\kappa}),$$

where $\kappa = \lim_{n\to\infty} \kappa(n), \kappa > 0$.

Proof: Unlike the techniques employed in the proof of Proposition 4.1, we will consider all pairs of node i and j that are disconnected cluster members.

To evaluate $P_{f\_iid}(n, r(n)$, we have

$$\begin{aligned} &P_{f\_iid}(n, r(n)) \\ &\geq \sum_{i=1}^{n} P(\{s_i \text{ is a failed session in } \mathcal{G}_{iid}(n, r(n))\}) \\ &\quad - \sum_{i=1}^{n} \sum_{j\neq i} P(\{s_i \text{ and } s_j \text{ are failed sessions} \\ &\qquad\qquad \text{in } \mathcal{G}_{iid}(n, r(n))\}). \end{aligned} \tag{17}$$

Then we evaluate the two terms on the right hand side of (17), respectively, and we have

$$P(\{s_i \text{ is a failed session}\}) \geq \Big(\big(1 - \pi r^2(n)\big)^{n^d}\Big)^k \tag{18}$$

and

$$\begin{aligned} &P(\{s_i \text{ and } s_j \text{ are failed sessions}\}) \\ &\leq \Big(4\pi r^2(n)\big(1 - \pi r^2(n)\big)^{n^d} \\ &\qquad + \big(1 - 4\pi r^2(n)\big)\big(1 - 2\pi r^2(n)\big)^{n^d}\Big)^k. \end{aligned} \tag{19}$$

The two terms in the parenthesis on the right hand side of (19) take into account the cases where the two nodes carrying the packet of s_i and s_j are at a distance less than $2r(n)$ and greater than $2r(n)$ when initially distributed, respectively. Using (18) and (19) in (17), we obtain

$$\begin{aligned} &P_{f\_iid}(n, r(n)) \\ &\geq n\big(1 - \pi r^2(n)\big)^{kn^d} - n^2\Big(4\pi r^2(n)\big(1 - \pi r^2(n)\big)^{n^d} \\ &\qquad\qquad + \big(1 - 2\pi r^2(n)\big)^{n^d}\Big)^k. \end{aligned} \tag{20}$$

Using Lemma 5.1 and (13), for any fixed $\theta < 1$, we have

$$\begin{aligned} &P_{f\_iid}(n, r(n)) \\ &\geq \theta e^{-\kappa} - \Big(n^{\frac{2}{k}}\Big(4\pi r^2(n)e^{-n^d\pi r^2(n)} + e^{-2n^d\pi r^2(n)}\Big)\Big)^k \\ &= \theta e^{-\kappa} - \Big(\frac{4(\log n + \kappa)}{kn^{d-\frac{1}{k}}}e^{-\frac{1}{k}\kappa} + e^{-\frac{2}{k}\kappa}\Big)^k \\ &\geq \theta e^{-\kappa} - (1+\epsilon)e^{-2\kappa}, \end{aligned}$$

for any $\epsilon > 0$ and for all $n > N_{\epsilon,\theta,\kappa}$. Note that we need $d > \frac{1}{k}$ in the last step.

Since $\lim_{n\to\infty} \kappa(n) = \kappa$, for any ϵ there exits $N'(\epsilon)$ such that for all $n \geq N'(\epsilon)$, $\kappa(n) \leq \kappa + \epsilon$. Considering that the probability of disconnectedness is monotonously decreasing in κ, then we have

$$P_{f\_iid}(n, r(n)) \geq \theta e^{-(\kappa+\epsilon)} - (1+\epsilon)e^{-2(\kappa+\epsilon)},$$

for $n \geq \max\{N_{\epsilon,\theta,\kappa+\epsilon}, N'_\epsilon\}$. Taking limits

$$\liminf_{n\to\infty} P_{f\_iid}(n, r(n)) \geq \theta e^{-(\kappa+\epsilon)} - (1+\epsilon)e^{-2(\kappa+\epsilon)}.$$

Since this holds for all $\epsilon > 0$ and $\theta < 1$, the result follows. ■

Then we have the following corollary to prove the necessity part of Theorem 5.1.

Corollary 5.1: Under the i.i.d. mobility assumption, the mobile k-hop clustered network is to have failed sessions with positive probability bounded away from zero if $\pi r^2(n) = \frac{\log n + \kappa(n)}{kn^d}$ and $\lim_{n\to\infty} \kappa(n) < +\infty$, which means $\pi r^2(n) \geq \frac{\log n}{kn^d}$ is necessary for the connectivity of mobile k–hop clustered networks with *i.i.d.* mobility model.

B. Sufficient condition on $r(n)$ of Theorem 5.1

Suppose there are at most n sessions in a period λ_b, and let E_i denote the event that s_i is a failed session, where $i = 1, 2, \ldots, n$. Let each node have the transmission range $r = cr(n)$, where $c > 1$. Using the existing approach in Section IV-B, we have

$$\begin{aligned} P\left(\bigcup_{i=1}^{n} E_i\right) &\leq \sum_{i=1}^{n} P(E_i) \\ &\leq n\left(\left(1-\pi r^2\right)^{n^d}\right)^k \\ &\leq n e^{-kn^d \pi r^2} \\ &= \frac{1}{4n^{c^2-1}}. \end{aligned}$$

For any $c > 1$, the result follows.

VI. THE CRITICAL TRANSMISSION RANGE FOR CONNECTIVITY OF STATIONARY K-HOP CLUSTERED NETWORKS

In this section, our main result is the following theorem.

Theorem 6.1: For the stationary k-hop clustered networks, the critical transmission range is $r(n) = \frac{1}{k}\sqrt{\frac{d \log n}{\pi n^d}}$, where $0 < d < 1$.

A. Necessary condition on $r(n)$ of Theorem 6.1

Let $\mathcal{G}_{stat}(n, r(n))$ denote the network where two nodes are connected if their Euclidean distance is at most $r(n)$ and we use the term *disconnected* to describe a cluster member whose packets cannot reach a cluster head within k hops. Then, if there is at least one disconnected member node in the network, we define that $\mathcal{G}_{stat}$ is disconnected. Let $P_{d\_stat}(n, r(n))$ be the probability that $\mathcal{G}_{stat}$ is disconnected and we have the following proposition.

Proposition 6.1: If $\pi r^2(n) = \frac{d_0 \log n + \kappa}{k^2 n^d}$, where $d_0 < d$, then

$$\liminf_{n\to\infty} P_{d\_stat}(n, r(n)) \geq e^{-\kappa}(1 - e^{-\kappa}),$$

where $\kappa = \lim_{n\to\infty} \kappa(n), \kappa > 0$.

Proof: The proof applies similar techniques as that of Proposition 4.1, we will not considered all pairs of nodes i and j that are disconnected cluster members. The problem for this is that there are too many of these pairs. The intuitive explanation is depicted in the following figure.

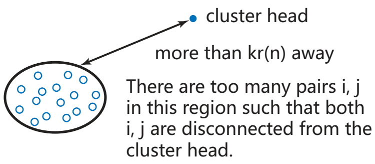
cluster head
more than kr(n) away
There are too many pairs i, j in this region such that both i, j are disconnected from the cluster head.

Fig. 7. An intuitive explanation of the problem in the above approach

Further, such i and j may be very close to each other, making it difficult to bound the probability.

The specific set of is and js are selected as follow. Let $u(n) = O\left(\sqrt{\frac{\log n}{n}}\right)$. Then we divide the unit square into $\frac{1}{u(n)} \times \frac{1}{u(n)}$ cells such that each cell is of size $u^2(n)$.

Now, among these cells, pick n^{d_0} of them such that each of them is at least $\sqrt{\frac{1}{n^{d_0}}}$ away from others, as shown in Figure 8.

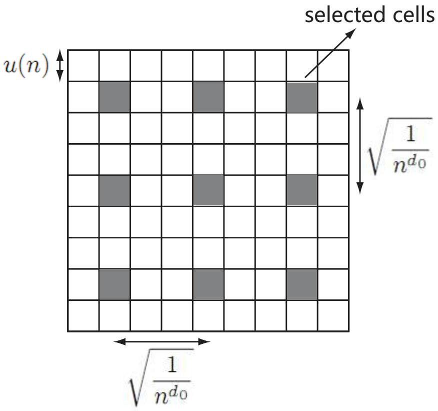

Fig. 8. Cell selection

Pick one cluster member node from each of these selected cells to form a set consisted of n^{d_0} nodes. Let $\mathbb{Y}$ denote the set of such cluster member nodes. Note that any two nodes in $\mathbb{Y}$ are at a distance of at least $\sqrt{\frac{1}{n^{d_0}}}$ away. Further, this distance is larger than $kr(n)$ when n is sufficiently large.

Then, using the existing approach of computing the probability that the network is disconnected, we have

$$\begin{aligned} &P_{d\_stat}(n, r(n)) \\ &\quad\geq \sum_{i\in\mathbb{Y}} P(\{i \text{ is an disconnected cluster member} \\ &\qquad\qquad \text{in } \mathcal{G}_{stat}(n, r(n))\}) \\ &\qquad - \sum_{\substack{i,j\in\mathbb{Y},\\ j\neq i}} P(\{i \text{ and } j \text{ are disconnected cluster} \\ &\qquad\qquad \text{members in } \mathcal{G}_{stat}(n, r(n))\}). \end{aligned} \tag{21}$$

As to the first term in the right hand side of (21), it is bounded by

$$n^{d_0}\left(1 - \pi\left(kr(n)\right)^2\right)^{n^d} \geq n^{d_0} \cdot \theta \cdot \frac{1}{n^{d_0}} e^{-\kappa} = \theta e^{-\kappa},$$

where $0 < \theta < 1$. To bound the second term, note that $i, j \in Y$ are at least $\sqrt{\frac{1}{n^{d_0}}}$ away, which will be larger than $2kr(n)$ when n is sufficiently large. Thus, each term in the second summation is bounded by

$$P(\{i \text{ and } j \text{ are disconnected}\}) \leq \left(1 - 2\pi\left(kr(n)\right)^2\right)^{n^d}.$$

Therefore,

$$\begin{aligned} P_{d\_stat}(n, r(n)) &\geq \theta e^{-\kappa} - n^{2d_0} e^{-2n^d \pi (kr(n))^2} \\ &\geq \theta e^{-\kappa} - e^{-2\kappa}, \end{aligned}$$

for all $n > N_{\theta,\kappa}$.

The rest of steps are omitted and follows these in the earlier proofs. ■

Like Proposition 4.1, Proposition 6.1 provides the necessary condition on $r(n)$ in terms of both $\kappa(n)$ and d_0. The implication behind this is the same as explained in Remark 4.2. We can have the following corollary.

Corollary 6.1: $r(n) \geq \frac{1}{k}\sqrt{\frac{d\log n}{\pi n^d}}$ is necessary for the connectivity of stationary k-hop clustered networks.

B. Sufficient condition on $r(n)$ of Theorem 6.1

Heuristically, we can use the tessellation-based approach to prove the sufficiency part of Theorem 6.1. However, it can also be shown that we cannot develop a sufficient condition on $r(n)$ by directly applying the similar technique as in the previous proofs. The problem is that when $\frac{1}{k} < d \leq 1$, we cannot bound the probability that at least one cell has no cluster heads to asymptotically approach zero, if the size of cell is roughly smaller than $\pi\big(kr(n)\big)^2$. On the other side, when the unit square is tessellated such that the size of cell is roughly equal to $\pi\big(kr(n)\big)^2$, we are also unable to show that any clustered head within the distance of $kr(n)$ is reachable for a cluster member. However, we can overcome this technical problem by the trick of considering a distance that is reachable for a cluster member and approaches $kr(n)$ as well.

First, we divide the unit square into cells with side length $\frac{\sqrt{2}}{2}\frac{r(n)}{\log n}$. Thus, there are $\log n$ cells along the transmission range $r(n)$. This tessellation is shown in Figure 9.

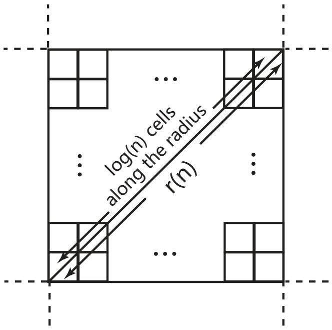

Fig. 9. There are $\log n$ cells along the transmission range $r(n)$.

Then we evaluate the probability that at least one cell is empty, and we have

$$\begin{aligned}
&P(\{\text{at least one cell is empty}\})\\
&\leq \quad \frac{2}{d}\pi k^2 n^d \log n\Big(1-\frac{d}{2\log n\pi k^2 n^d}\Big)^n\\
&\leq \quad \frac{2}{d}\pi k^2 \log n\Big/ n^{\frac{d}{2\pi k^2}\frac{n^{1-d}}{\log^2 n}-d}\\
&\to \quad 0,\ \text{as } n\to\infty.
\end{aligned}$$

Consequently, we know that with high probability there is at least one node in each cell. Then, with the transmission range $r(n)$, each hop can jump over at least $(\log n - 1)$ cells. Thus, any cluster head within the distance of $\frac{\log n-1}{\log n}kr(n)$ is reachable in k hops with high probability. Now we have a proper cell size to construct the main tessellation.

Next, we introduce the disk tessellation (with a minor abuse of the term *tessellation*) of the unit torus, as depicted in Figure 10. Let the radius of each disk $R(n)$ be such that $R(n) = \frac{\log n-1}{\log n}kr(n)$. Then we evaluate the probability that at least one disk does not have cluster heads, and we have

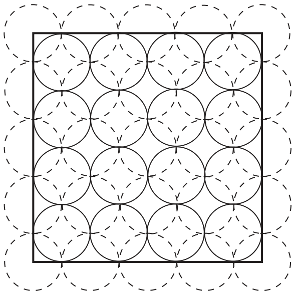

Fig. 10. Disk tessellation of the unit torus

$$\begin{aligned}
&P(\{\text{at least one disk does not have cluster heads}\})\\
&\leq 2\bigg(\frac{1}{2\big(\frac{\log n-1}{\log n}kr(n)\big)}\bigg)^2\bigg(1-\pi\Big(\frac{\log n-1}{\log n}kr(n)\Big)^2\bigg)^{n^d}\\
&\leq \frac{\pi n^d\log n}{2d(\log n-1)^2}\exp\{-\Big(\frac{\log n-1}{\log n}\Big)^2 n^d\pi\big(kr(n)\big)^2\}\\
&\leq \frac{\pi\log n}{2d(\log n-1)^2}n^{d-d\left(\frac{\log n-1}{\log n}\right)^2}\\
&\to 0,\ \text{as } n\to\infty.
\end{aligned}$$

Therefore, $r(n) = \frac{1}{k}\sqrt{\frac{d\log n}{\pi n^d}}$ is sufficient to guarantee the connectivity of network.

VII. THE CRITICAL NUMBER OF NEIGHBORS FOR CONNECTIVITY OF K-HOP CLUSTERED NETWORKS

In this section, we briefly have a parallel discussion on connectivity under the number-of-neighbor-based connecting strategy. We show the critical number of neighbors (CNoN) in k-hop clustered networks for both stationary and mobile cases, respectively.

A. The CNoN in stationary k-hop clustered networks

As we assumed before, n cluster members and n^d cluster heads are uniformly and independently placed in the unit square network. We denote by $\mathcal{G}_{stat}(n,\phi_n)$ the network formed when each node is connected to its ϕ_n nearest neighbors and we have the following result.

Theorem 7.1: For $\mathcal{G}_{stat}(n,\phi_n)$ to be asymptotically connected, $\Theta\big(n^{1-d}\log n\big)$ neighbors are necessary and sufficient.

Proof:

1) the necessity part: We consider a disk $\mathcal{D}_i$ which is centered at a node n_i with radius $r = \frac{1}{k}\sqrt{\frac{d\log n}{\pi n^d}}$. Hence, the probability of the event that any other node falls into $\mathcal{D}_i$ is

$$p = \pi r^2 = \frac{d\log n}{k^2 n^d},$$

Note that r is the critical transmission range for overall connectivity of a stationary k-hop clustered network, which is argued in Theorem 6.1. We then want to determine the number of nodes within $\mathcal{D}_i$, denoted by N_i, which would help us to derive the lower bound of the number of neighbors for connectivity. To evaluate this, we make use of Chernoff's bounds [30]: Let X be a binomially distributed random variable, and m and p are the number of Bernoulli experiments and the probability of each Bernoulli experiments to be successful, respectively. The expectation of X is mp. Then, for any $0 < \delta \le 1$,

$$P\big(X < (1-\delta)mp\big) < e^{-mp\frac{\delta^2}{2}} \tag{22}$$

and for any $\delta > 0$,

$$P\big(X > (1+\delta)mp\big) < e^{-mpf(\delta)}, \tag{23}$$

where $f(\delta) = (1+\delta)\log(1+\delta) - \delta$.

Using (22) with $\delta = \frac{1}{2}$ and $m = n + n^d - 1$, we have

$$P\Big(N_i < \frac{d}{2k^2}(1 + n^{1-d} - \frac{1}{n^d})\log n\Big) < e^{-\frac{d}{8k^2}(1+n^{1-d}-\frac{1}{n^d})\log n}$$

Using the union bound, we obtain

$$\begin{aligned} & P\Big[N_i \ge \frac{d}{2k^2}(1 + n^{1-d} - \frac{1}{n^d})\log n,\ \forall i\Big] \\ = &\ 1 - P\Big[N_i < \frac{d}{2k^2}(1 + n^{1-d} - \frac{1}{n^d})\log n,\ \exists i\Big] \\ \ge &\ 1 - \sum_{i=1}^{n+n^d} P\Big(N_i < \frac{d}{2k^2}(1 + n^{1-d} - \frac{1}{n^d})\log n\Big) \\ \ge &\ 1 - \big(n + n^d\big)e^{-\frac{d}{8k^2}(1+n^{1-d}-\frac{1}{n^d})\log n} \\ \to &\ 1, \text{as } n \to \infty. \end{aligned} \tag{24}$$

Thus, each disk is to have more than $\frac{d}{2k^2}(1+n^{1-d}-\frac{1}{n^d})\log n$ nodes with high probability. We then know that if $\phi_n < \frac{d}{2k^2}n^{1-d}\log n$, the equivalent transmission range r_{eq} for all node is no greater than $\frac{1}{k}\sqrt{\frac{d\log n}{\pi n^d}}$ with high probability. Therefore, referring to the proof of the necessity part of Theorem 6.1, the overall network is disconnected then. Hence we have proved the necessity part of Theorem 7.1.

2) the sufficiency part: We still consider a disk $\mathcal{D}_i$ with radius $r = \frac{1}{k}\sqrt{\frac{d\log n}{\pi n^d}}$. Then applying (23) with $\delta = 1$ and $m = n + n^d - 1$, we have

$$\begin{aligned} & P\Big(N_i > \frac{2d}{k^2}(1 + n^{1-d} - \frac{1}{n^d})\log n\Big) \\ < &\ e^{-\frac{d}{k^2}f(1)(1+n^{1-d}-\frac{1}{n^d})\log n},\ f(1) = 2\log 2 - 1 > 0. \end{aligned}$$

In a similar manner as (24), it can be shown that

$$\begin{aligned} & P\Big[N_i \le \frac{2d}{k^2}(1 + n^{1-d} - \frac{1}{n^d})\log n,\ \forall i\Big] \\ \ge &\ 1 - \big(n + n^d\big)e^{-\frac{d}{k^2}f(1)(1+n^{1-d}-\frac{1}{n^d})\log n} \\ \to &\ 1, \text{as } n \to \infty. \end{aligned}$$

Thus, each disk is to have less than $\frac{2d}{k^2}(1 + n^{1-d} - \frac{1}{n^d})\log n$ nodes almost surely. With this result, we then know that if $\phi_n > \frac{2d}{k^2}(1 + n^{1-d} - \frac{1}{n^d})\log n$, the equivalent transmission range r'_{eq} for all node is greater than $\frac{1}{k}\sqrt{\frac{d\log n}{\pi n^d}}$ with high probability. As a result of the sufficiency part of Theorem 6.1, the overall network is connected then. The final result then follows. ■

B. The CNoN in mobile k-hop clustered networks

Let $\mathcal{G}_m(n, \phi_n)$ be the mobile clustered network formed when each node is connected to its ϕ_n nearest neighbors. Applying similar procedure of the proof in stationary clustered networks, we have the following results under mobile networks.

Theorem 7.2: In k-hop clustered networks with i.i.d. mobility or random walk mobility, for $\mathcal{G}_m(n, \phi_n)$ to be asymptotically connected, $\Theta\big(n^{1-d}\log n\big)$ neighbors are necessary and sufficient.

Proof: Under i.i.d. mobility model, we consider a disk $\mathcal{D}'_i$ which is centered at a node n_i with radius $r = \sqrt{\frac{\log n}{k\pi n^d}}$. Hence the probability of the event that any other node falls into $\mathcal{D}'_i$ is

$$p'_i = 1 - (1 - \pi r^2)^k.$$

Using the equation

$$1 - x^k = (1-x)(1 + x + x^2 + ... + x^{k-1})$$

we can have

$$\begin{aligned} & p'_i = \pi r^2\big(1 + (1-\pi r^2) + (1-\pi r^2)^2 + ... + (1-\pi r^2)^{k-1}\big) \\ & \quad > \pi r^2 = \frac{\log n}{kn^d} \end{aligned}$$

Let N'_i be he number of nodes within D'_i. Then using the existing approach, we can show that

$$\begin{aligned} & P\Big(N'_i < \frac{1}{2k}(1 + n^{1-d} - \frac{1}{n^d})\log n\Big) < P\Big(N'_i < \frac{1}{k}(n + n^d - 1)p'_i\Big) \\ & \qquad < e^{-\frac{1}{8}(n+n^d-1)p'_i} < e^{-\frac{1}{8k}(1+n^{1-d}-\frac{1}{n^d})\log n}. \end{aligned}$$

and hence

$$\begin{aligned} & P\Big[N'_i \ge \frac{1}{2k}(1 + n^{1-d} - \frac{1}{n^d})\log n,\ \forall i\Big] \\ = &\ 1 - P\Big[N_i < \frac{1}{2k}(1 + n^{1-d} - \frac{1}{n^d})\log n,\ \exists i\Big] \\ \ge &\ 1 - \sum_{i=1}^{n+n^d} P\Big(N_i < \frac{1}{2k}(1 + n^{1-d} - \frac{1}{n^d})\log n\Big) \\ \ge &\ 1 - \big(n + n^d\big)e^{-\frac{1}{8k}(1+n^{1-d}-\frac{1}{n^d})\log n} \\ \to &\ 1, \text{as } n \to \infty. \end{aligned}$$

Similarly, we also have

$$P\Big[N'_i \le \frac{2d}{k^2}(1 + n^{1-d} - \frac{1}{n^d})\log n,\ \forall i\Big] \to 1, \text{as } n \to \infty.$$

With the existing argument, we can conclude that $\Theta\big(n^{1-d}\log n\big)$ neighbors are necessary and sufficient.

Under random walk mobility model, we consider the coverage area $\mathcal{D}''_i$ by a disc centered at a node n_i with radius $r = \frac{\log n}{2kvn^d}$ as node n_i moves within a given period. Then due to mobility, the probability of the event that any other node falls into $\mathcal{D}''_i$ is

$$p'' = 2rkv = \frac{\log n}{n^d}.$$

Using the same technique, the result follows. ■

<table>
<tr><th colspan="3">Network Type</th><th>Transmission Range $(r(n))$</th><th>Number of Neighbors</th><th>Transmission Power (P_t) vs. Delay (D) Trade-off</th><th>Average Hops $(\bar{H})$</th><th>Average Energy Cons. per Flow $(\bar{E})$</th><th>Cluster Head Exponent (d)</th></tr>
<tr><td colspan="3">Flat Network</td><td>$\sqrt{\frac{\log n}{\pi n}}$</td><td>$\Theta(\log n)$</td><td>$P_t = \Theta\Big(\frac{\log n}{n}\Big)$
(non-trade-off, to denote P_t's order)</td><td>$\Theta(\sqrt{\frac{n}{\log n}})$</td><td>$\Theta\left(\frac{\sqrt{\log n}}{n^{1/2}}\right)$</td><td>——</td></tr>
<tr><td rowspan="3">Clustered Network</td><td colspan="2">Stationary</td><td>$\frac{1}{k}\sqrt{\frac{d\log n}{\pi n^d}}$</td><td rowspan="3">$\Theta\big(n^{1-d}\log n\big)$</td><td>$P_t = \Theta\left(\frac{d}{D^2}\frac{\log n}{n^d}\right)$</td><td rowspan="3">$\Theta(1)$</td><td>$\Theta\left(\frac{\log n}{n^d}\right)$</td><td>$0 < d < 1$</td></tr>
<tr><td rowspan="2">Mobile</td><td>I.I.D. Mobility</td><td>$\sqrt{\frac{\log n}{k\pi n^d}}$</td><td>$P_t = \Theta\left(\frac{1}{D}\frac{\log n}{n^d}\right)$</td><td>$\Theta\left(\frac{\log n}{n^d}\right)$</td><td>$\frac{1}{k} < d \leq 1$</td></tr>
<tr><td>R.W. Mobility (Non-trivial)</td><td>$\frac{\log n}{2kv_\star n^d}$</td><td>$P_t = \Theta\left(\frac{1}{D^2}\Big(\frac{\log n}{n^d}\Big)^2\right)$</td><td>$\Theta\left(\Big(\frac{\log n}{n^d}\Big)^2\right)$</td><td>$0 < d \leq 1$</td></tr>
</table>

TABLE I

The transmission power vs. delay trade-off and energy consumption in k-hop clustered networks

VIII. The Impact of Mobility on Connectivity and Network Performance in K-hop Clustered Networks

In this section, we use the results that we obtained from the previous sections to study the impact of mobility on connectivity and network performance. We first characterize the power-delay trade-off and energy consumption per flow in clustered networks, based on the critical transmission ranges that we have obtained. We then summarize these results and explore the insights and implications that they may provide us.

We assume the free space propagation model[4], and thus

$$P_r = P_t G_t G_r \Big(\frac{\lambda}{4\pi d}\Big)^2,$$

where

$$\begin{aligned} P_t &= \text{transmission power of an isotropic source}, \\ G_t &= \text{transmitting antenna gain}, \\ G_r &= \text{receiving antenna gain}, \\ d &= \text{propagation distance between antennas}, \\ \lambda &= \text{carrier wavelength}. \end{aligned}$$

Let G_t, G_r and λ be constants. We then have

$$P_t = \frac{1}{G_t G_r}\Big(\frac{4\pi d}{\lambda}\Big)^2 P_r = cP_r d^2, \qquad (25)$$

where c is a constant. To ensure sufficient signal strength for receiving the packet, we require that $P_r \geq P_{r\_th}$, where P_{r_th} is a threshold of the receiving power at the receiver. Replace P_r with P_{r_th} in (25) and replace the propagation distance d by the transmission range r. We then have

$$P_t \propto r^2.$$

Consequently, let $\bar{E}$ denote the energy consumption per flow. We have

$$\bar{E} = \bar{H}P_t \propto \bar{H}r^2,$$

where $\bar{H}$ is the average number of hops per flow.

P_t and $\bar{E}$ are both of great engineering significance, while they have different influence on a network. P_t is correlated to the node-level operation and it has a dominating impact on the total number of transmissions that a single node can undertake in energy-constrained networks like wireless sensor networks [27]. On the other hand, $\bar{E}$ is a flow-level description of energy consumption and thus it provides a picture of the life-time expectation both of each single node and of the entire network.

Using the results in the critical transmission range $r(n)$ from the earlier sections, we can compute the order of P_t and $\bar{E}$. All the results for the case where $v^{(m)} = \Theta(1)$ are reported in Table I.

Before we discuss these results, we make a cautious note regarding the energy consumption. Note that in these calculations, we have ignored the energy consumption due to mobility. Hence, these results should not be interpreted as a reason to introduce mobility to an otherwise static network, but rather represent an inherent advantage of having mobility in the system. Similarly, the comparison with the flat network is not entirely fair, since in a clustered network, a packet only needs to reach a cluster head. Hence, our following results should be viewed as an inherent advantage of clustered network due to the availability of infrastructure support.

We now discuss the insights on the impact of mobility on connectivity and network performance based on these results. By the implication from [28], we know that when $d < \frac{1}{2}$, bottleneck of capacity may appear, and thus we assume $d > \frac{1}{2}$ in our following discussion. Note that we do allow the speed of nodes $v^{(m)}$ to scale with n and $v^{(m)} = \omega\left(\sqrt{\frac{\log n}{n^{d'}}}\right)$ $(d' < \frac{d}{2}) = O(1)$. If $2d - d' > 1$, we can then demonstrate that the speed $v^{(m)}$ is large enough to guarantee the improvement of connectivity in mobile clustered networks. For simplicity, we discuss the case where $v^{(m)} = \Theta(1)$ which can bring the best improvement of connectivity under our model.

- $P_t\{\text{r.w.}\} = o(P_t\{\text{flat}\})$ and $\bar{E}\{\text{r.w.}\} = o(\bar{E}\{\text{flat}\})$, which means that random walk mobility with non-trivial velocity plus k-hop clustering can greatly decrease both the transmission power and the average energy consumption per flow. Thus, random walk mobility with non-trivial velocity plus k-hop clustering can increase the number of transmission that a node can undertake and extend the life-time both of each single node and of the entire network.

---

[4] Other propagation model (e.g., the two-ray model) can be studied in a similar way.

- To identify the contribution of mobility and k-hop clustering on the improvement of network performance, we have

  $$\bar{E}\{\text{r.w.}\} = \frac{\log n}{n^d}\bar{E}\{\text{stat}\}; P_t\{\text{r.w.}\} = \frac{\log n}{n^d}P_t\{\text{stat}\},$$

  and

  $$\bar{E}\{\text{stat}\} = \frac{\sqrt{\log n}}{n^{d-1/2}}\bar{E}\{\text{flat}\}; P_t\{\text{stat}\} = n^{1-d}P_t\{\text{flat}\}.$$

  Thus, combining the results of above equations and using $\Uparrow$ and $\Downarrow$ to denote the positive and negative impacts, respectively, we provide the following formulations to identify the effects of mobility and k-hop clustering on network performance.

  $$\bar{E}\{\text{r.w.}\} = \underset{\substack{\Uparrow\textbf{ due to}\\ \textbf{r.w. mobility}\\ \textbf{non-trivial}}}{\frac{\log n}{n^d}} \cdot \underset{\substack{\Uparrow\textbf{ due to}\\ k\textbf{-hop}\\ \textbf{clustering}}}{\frac{\sqrt{\log n}}{n^{d-1/2}}} \cdot \bar{E}\{\text{flat}\};$$

  $$P_t\{\text{r.w.}\} = \underset{\substack{\Uparrow\textbf{ due to}\\ \textbf{r.w. mobility}\\ \textbf{non-trivial}}}{\frac{\log n}{n^d}} \cdot \underset{\substack{\Downarrow\textbf{ due to}\\ k\textbf{-hop}\\ \textbf{clustering}}}{n^{1-d}} \cdot P_t\{\text{flat}\}.$$

- From the perspective of energy consumption per flow, clustered networks have an inherent advantage in terms of energy-efficiency due to the availability of infrastructure support.
- Mobile k-hop clustered networks under the i.i.d mobility model and stationary clustered networks may have comparable performance and this can be understood intuitively since nodes under the i.i.d. mobility model actually remain static during the time-slot.

In conclusion, **random walk mobility with non-trivial velocity increases connectivity in k-hop clustered networks, and thus significantly improves the energy efficiency and the power-delay trade-off of the network.**

IX. CONCLUDING REMARKS

In this paper, we have studied the effects of mobility on the critical transmission range for asymptotic connectivity in k-hop clustered networks. Our results could be applied to the large scale wireless sensor networks like [29],. Our contributions are twofold. We have developed the critical transmission range for the mobile k-hop clustered network under the random walk mobility model with non-trivial velocities and the i.i.d. mobility model, and for the stationary k-hop clustered network, respectively. In addition, results of the critical number of neighbors are consequently derived for both stationary and mobile clustered networks. These formulations do not only provide an asymptotic description of the critical power needed to maintain the connectivity of the network, but also help to identify the contribution of mobility in the improvement of network performance. Thus, based on these results that we have developed in this paper, our second contribution is to present that random walk mobility with non-trivial velocity increases connectivity in k-hop clustered networks, and thus significantly improves the energy efficiency and the power-delay trade-off of the network.

There are several interesting direction for future work. First, we can improve our random walk mobility model to make it more realistic and general. For example, in a “lazy” random walk, cluster members might remain static with certain probability. Note that this assumption is also reasonable since in most circumstance, mobile terminals will not keep moving all the time. Second, we plan to extend the results to account for multi-hop transmissions. See the selected work in [31] [32], which deal with both heterogeneous and homogeneous settings. Third, the capacity of such network is also a non–ignorable issue. Finally, in this paper we assume that the cluster heads are stationary, even though the cluster members may move. It would be interesting to study the case where cluster-heads may move as well. Further, it is also interesting to study k-hop connectivity in flat networks, where no difference exists among all the nodes.

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Xinbing Wang (SM'12) received the B.S. degree (with hons.) in automation from Shanghai Jiao Tong University, Shanghai, China, in 1998, the M.S. degree in computer science and technology from Tsinghua University, Beijing, China, in 2001, and the Ph.D. degree with a major in electrical and computer engineering and minor in mathematics from North Carolina State University, Raleigh, in 2006. Currently, he is a professor with the Department of Electronic Engineering, Shanghai Jiao Tong University. His research interests include resource allocation and management in mobile and wireless networks, TCP asymptotics analysis, wireless capacity, cross-layer call admission control, asymptotics analysis of hybrid systems, and congestion control over wireless ad hoc and sensor networks. Dr. Wang has been a member of the Technical Program Committees of several conferences including ACM MobiCom 2012, ACM MobiHoc 2012, IEEE INFOCOM 2009-2013.

Xiaojun Lin (S'02 / M'05) received his B.S. from Zhongshan University,Guangzhou, China, in 1994, and his M.S. and Ph.D. degrees from Purdue University, West Lafayette, Indiana, in 2000 and 2005, respectively. He is currently an Associate Professor of Electrical and Computer Engineering at Purdue University.

Dr. Lin's research interests are in the analysis, control and optimization of wireless and wireline communication networks. He received the IEEE INFOCOM 2008 best paper award and 2005 best paper of the year award from Journal of Communications and Networks. His paper was also one of two runner-up papers for the best-paper award at IEEE INFOCOM 2005. He received the NSF CAREER award in 2007. He was the Workshop co-chair for IEEE GLOBECOM 2007, the Panel co-chair for WICON 2008, the TPC co-chair for ACM MobiHoc 2009, and the Mini-Conference co-chair for IEEE INFOCOM 2012. He is currently serving as an Area Editor for (Elsevier) Computer Networks journal, and has served as a Guest Editor for (Elsevier) Ad Hoc Networks journal.

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Qingsi Wang received the BE degree in electronic engineering in Shanghai Jiao Tong University, China. His research interests are in the area of asymptotic analysis of coverage and connectivity in wireless ad hoc network. He is a student member of the IEEE.

Wentao Luan received the BE degree in electronic engineering in Shanghai Jiao Tong University, China, in 2012. His research interests are in the area of asymptotic analysis of coverage and connectivity in wireless ad hoc network. He is a student member of the IEEE.