Modal coupling in traveling-wave resonators

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High-Q traveling-wave-resonators can enter a regime in which even minute scattering amplitudes associated with either bulk or surface imperfections can drive the system into the so-called strong modal coupling regime. Resonators that enter this regime have their coupling properties radically altered and can mimic a narrow-band reflector. We experimentally confirm recently predicted deviations from criticality in such strongly coupled systems. Observations of resonators that had $Q > 10^8$ and modal coupling parameters as large as 30 were shown to reflect more than 94% of an incoming optical signal within a narrow bandwidth of 40 MHz.

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The splitting of degenerate levels in the presence of coupling is a general principle in physics; the formation of energy bands in semiconductors (level repulsion) can be attributed to this principle, as can the splitting of atomic levels in the presence of LS coupling. An optical analog of this effect can be encountered in traveling-wave resonators (TWRs). Each mode possesses a natural twofold degeneracy, which results from the two possible directions of propagation, clockwise (CW) and counterclockwise (CCW).[1] Lifting of the degeneracy can occur when a fraction of the mode energy is scattered into the oppositely oriented mode. The consequences of degeneracy lifting by distributed scattering were theoretically investigated recently.[2] Here we experimentally confirm these predictions, using high-Q ($>10^8$) microsphere resonators coupled to fiber-optic tapered waveguides. The long photon lifetimes of high-Q microsphere resonators make possible a counterintuitive effect in which minute scattering gives rise to a regime of strong modal coupling. In this regime scattering into the oppositely oriented mode is the dominant scattering process. Resonances are significantly split, and severe deviations of the critical coupling point occur. We show and observe that in certain regimes the TWR acts as a narrow-bandwidth reflector, causing a strongly reflected signal and vanishing waveguide transmission.

In the case of a slowly varying field amplitude the degenerate modes of a TWR-waveguide system can be described by a simple coupled harmonic oscillator model, equivalent to that presented in Refs. 2 and 3:

$$\frac{\mathrm{d}a_{\mathrm{CW}}}{\mathrm{d}t} = i\Delta\omega a_{\mathrm{CW}} - \frac{1}{2\tau}a_{\mathrm{CW}} + \frac{i}{2\gamma}a_{\mathrm{CCW}} + \kappa s\,,$$

$$\frac{\mathrm{d}a_{\mathrm{CCW}}}{\mathrm{d}t} = i\Delta\omega a_{\mathrm{CCW}} - \frac{1}{2\tau}a_{\mathrm{CCW}} + \frac{i}{2\gamma}a_{\mathrm{CW}}\,, \qquad (1)$$

where a is the amplitude of the CCW and CW modes of the resonator and s denotes the input field ($|s|^2$ is the input pump power). The excitation frequency is detuned by $\Delta\omega$ with respect to resonance frequency ω_0 of the initially degenerate modes and τ is the total lifetime of photons in the resonator, which is related to the quality factor by $Q = \omega\tau$. Coupling coefficient κ denotes the coupling of the input wave to the CW mode of the resonator. The relation $\kappa = \sqrt{1/\tau_{\mathrm{ex}}}$ associates the coupling coefficient with a corresponding lifetime, such that $1/\tau = 1/\tau_{\mathrm{ex}} + 1/\tau_0$. The mutual coupling of the CCW and the CW modes is described by a (scattering) lifetime, γ. The coupling of the resonator modes to the waveguide gives rise to a transmitted (t) and a reflected (r) field:

$$t = -s + \kappa a_{\mathrm{CW}}\,, \qquad r = \kappa a_{\mathrm{CCW}}\,. \qquad (2)$$

In a microsphere TWR, Rayleigh scattering from surface inhomogenities or density fluctuations will transfer power from the initially excited mode to all the confined and radiative modes of the resonator. The scattering to all modes other than the CW and CCW modes is included in the overall effective loss, given through the intrinsic lifetime, τ_0. The eigenmodes are symmetric and antisymmetric superpositions of the original CW and CCW modes centered about new eigenfrequencies $\omega = \omega_0 \pm 1/2\gamma$ (which have a linewidth of $1/\tau$). Modifications to the resonator coupling physics can be described in terms of the normalized mode-coupling parameter $\Gamma \equiv \tau_0/\gamma$. In addition, we introduce the normalized coupling coefficient $K \equiv \tau_0/\tau_{\mathrm{ex}}$ to facilitate the discussion.

We experimentally observed the frequency splitting of a TWR mode in a tapered fiber-coupled[4] microsphere resonator. Because of the high-Q (typically exceeding 10^8), resonances frequently occur as doublets, because only minute scattering is required for easily observable mode splitting.[5] In our experiments, the presence of the waveguide made a negligible contribution to the overall scattering, and distributed scattering centers that are intrinsic to the resonator or to its surface dominated. As evidence of this we found that the splitting frequency increased only slightly (less than 5%) while the taper–sphere gap decreased.

The inset of Fig. 1 is a photograph of a microsphere (diameter, ~70 μm) coupled to a tapered fiber. The taper is attached to a 20-nm-resolution positioning stage to adjust the taper–sphere gap, which permits accurate control of external lifetime τ_{ex}. A 1.55-μm tunable laser source is used to excite a whispering gallery mode of the resonator. The laser is scanned repeatedly through a scan range that contains the resonance doublet. In our experiments, the transmission ($T \equiv |t/s|^2$) and the reflection ($R \equiv |r/s|^2$) through the tapered fiber were recorded as a function of taper–sphere gap. The recorded transmission and

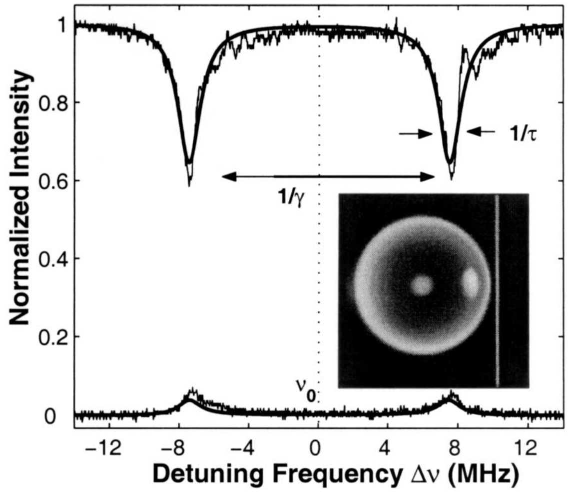

Fig. 1. Spectral transmission ($T \equiv |t/s|^2$) and reflection ($R \equiv |r/s|^2$) properties[6] of a 70-μm sphere with $Q_0 = 1.2 \times 10^8$ and a modal coupling of $\Gamma = 10$. Solid curve, fit from the model from Eqs. (1). Inset, microsphere coupled to a tapered fiber.

reflection spectra were simultaneously fitted to the coupled oscillator model of Eqs. (1) and (2), and the model parameters were inferred. Figure 1 shows a resonance doublet for a 70-μm sphere ($Q_0 = 1.2 \times 10^8$) with a resonance splitting that corresponds to $\Gamma = 10$.

An ideal TWR (characterized by $\Gamma = 0$) allows electromagnetic energy carried by a waveguide to be completely transferred to the resonant mode, a property that in the optical and microwave domains is termed critical coupling. Figure 2 shows power transmission and reflection as a function of external coupling for symmetric excitation ($\Delta\omega = 0$) of a resonator mode. In the absence of scattering ($\Gamma = 0$, corresponding to the dotted–dashed curve in Fig. 2) the critical point, the point where the waveguide transmission vanishes, occurs at $K = 1$. The fact that the critical coupling point coincides with the point of maximum circulating power is due to the unidirectionality of an ideal traveling-wave resonator. The forward input mode of the waveguide is coupled to only one (CCW or CW) mode of the resonator. In the presence of backscattering, unidirectionality is lost, drastically altering the coupling properties. The modifications are particularly interesting in the regime of strong modal coupling ($\Gamma \gg 1$). Even in this regime the condition for critical coupling, vanishing transmission ($T = 0$), can be achieved for $\Delta\omega = 0$, as can be seen from Fig. 2. However, it is not possible to obtain vanishing transmission for the true eigenfrequencies, $\omega = \omega_0 \pm 1/2\gamma$. The point of zero transmission is accompanied by a maximum reflected signal, which occurs for

$$K \equiv (\tau_0/\tau_{\mathrm{ex}}) = \sqrt{1 + \Gamma^2}. \qquad (3)$$

This expression shows that a significant shift of the critical point ($K = 1$ without modal coupling) is possible in the strong-coupling regime.

The experimental data presented in Fig. 2 were obtained for a mode that exhibited a modal coupling of $\Gamma = 10$ and $Q_0 = 1.2 \times 10^8$. The observed maximum backreflection was 84%, which agrees very well with the theory predicted maximum (see Fig. 3). In addition, the points of maximum backreflection and zero forward transmission were correctly predicted by theory. For reference, the dashed–dotted curve in Fig. 2 gives the case of an ideal TWR characterized by no modal coupling ($\Gamma = 0$). The solid curve in Fig. 2 is a fit obtained by use of a constant splitting frequency and a constant intrinsic lifetime. The theoretical fit shows excellent agreement with the experimental data, despite the fact that the splitting frequency was found to vary slightly as a function of resonator loading.

The maximum reflection is observed at the critical point and is given by

$$R_{\mathrm{crit}} = \left(\frac{\Gamma}{1 + \sqrt{1 + \Gamma^2}}\right)^2. \qquad (4)$$

The reflection asymptotically approaches unity in the limit of large Γ, with all incoming power coupled back into the direction of the source. We experimentally verified this functional dependence, as is shown in Fig. 3. The inset of Fig. 3 illustrates that in the strong-modal-coupling regime the TWR behaves as a frequency-selective reflector. The highest intermode coupling of $\Gamma = 31$ was observed in a sphere with a diameter of 30 μm. This large modal coupling implies that the probability for scattering a photon into the oppositely oriented mode was 31 times higher than the probability of a photon's being lost (absorbed or scattered into nonresonant modes). This surprisingly efficient coupling process can be understood qualitatively if one considers the spatial distribution of the mode in the microsphere. Only light that is scattered into an angular segment that exceeds the mode's cutoff angle is lost[2]; all the remaining light is channeled back into the CW and CCW propagating

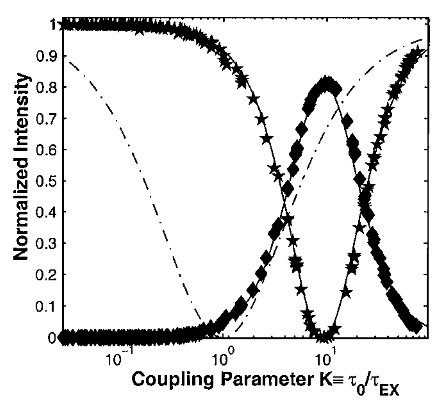

Fig. 2. Transmission (stars) and reflection (diamonds) behavior for symmetric $\Delta\omega = 0$ excitation relative to K (Ref. 7) for a mode with $Q_0 = 1.2 \times 10^8$ and a modal coupling of $\Gamma = 10$. Solid curve, a theoretical fit with the model from Eqs. (1) and (2). The minimum $T = 0$ occurs at $K \approx \Gamma$ and is accompanied by a maximum backreflection of 84%. Dotted–dashed curve, transmission for an ideal TWR in the absence of backscattering, where critical coupling occurs for $K = 1$.

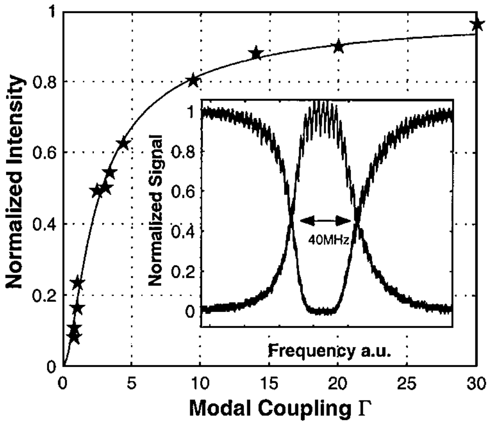

Fig. 3. Experimentally observed and theoretically determined reflection at the critical point as a function of modal coupling Γ. Inset, transmission and reflection at the $T = 0$ point for a mode with $\Gamma = 31$. In this case 94% of the optical power is reflected. Note that the doublet structure is masked, because the lifetime of the mode is of the same order as the lifetime of the modal coupling process. However, the doublet structure is still evident in the spectrum, causing a flattened frequency response at resonance.

modes, thereby inducing modal coupling. Because the ratio of modal volume (approximately quadratic in radius) to sphere volume increases for smaller spheres, the modal coupling is expected to increase as well. This result is in agreement with our experimental findings, as the largest intermode coupling of $\Gamma \approx 30$ was generally observed only in spheres with diameters of less than approximately 40 μm. We studied the origin of the scattering amplitudes that cause strong modal coupling by analyzing the sphere's surface about the equatorial plane, using scanning-electron imaging. We found that, for spheres with large intermode coupling, small, localized, and randomly distributed surface defects, which had subwavelength dimensions (typically hundreds of nanometers), were present, whereas in spheres that exhibited negligible mode splitting these defects were absent.

It is important to note that the critical coupling point in the presence of scattering does not correspond to the point of maximum circulating power. In fact, maximum circulating power occurs with finite transmission. We theoretically investigated the shift of the point of maximum power transfer when one of the new eigenmodes, $\omega = \omega_0 \pm 1/2\gamma$, was excited; and we determined that the largest shift occurs for a modal coupling of $\Gamma = 1.5$, where it shifts to $K = 1.52$. Interestingly, in the case of large modal coupling ($\Gamma \gg 1$) the maximum power transfer condition approaches the condition $K = 1$, as was the case for weak or no modal coupling ($\Gamma = 0$). The condition $K = 1$ has the special property of corresponding to transmission and reflection of equal amplitude. In addition, the circulating power is reduced in the presence of modal coupling. Defining a power reduction factor C for the total circulating power, and assuming that one of the new eigenfrequencies $\omega = \omega_0 \pm 1/2\gamma$ is excited, we can write C as

$$C \equiv \frac{|a_{\mathrm{CW}}|_\gamma{}^2 + |a_{\mathrm{CCW}}|_\gamma{}^2}{|a_{\mathrm{CW}}|_{\mathrm{ideal}}{}^2} = \frac{[2\Gamma^2 + (1+K)^2]^2 + (1+K)^2\Gamma^2}{[4\Gamma^2 + (1+K)^2]^2} + \frac{\Gamma^2}{4\Gamma^2 + (1+K)^2}. \tag{5}$$

The first term on the right-hand side of Eq. (5) is the power contributed by the CW mode, and the second term is the power in the CCW direction. In the limit of no scattering ($\Gamma = 0$), one obtains the ideal circulating power, whereas for large scattering the amount is reduced by a factor of 2.

In conclusion, we have experimentally confirmed modified coupling properties in the presence of large modal coupling. We observed a shift in the critical coupling point, and a strong reflected signal. The extent to which the resonator properties are altered depend on normalized modal coupling parameter Γ. Backreflection as large as 94% was observed, suggesting the use of TWRs as narrowband reflectors. The resultant modifications of criticality and circulating power are important in cavity QED and nonlinear optical experiments.[8,9] In addition, the results are potentially important in design considerations for resonator-based optical telecom devices. Finally, it was recently demonstrated that it is possible to couple a sharp tip to a whispering-gallery mode without degrading the Q factor of the mode.[10] This result combined with the observation of large intermode coupling, as shown in this Letter, suggests that a tip can in principle leave intact the Q factor while inducing a significant amount of intermode coupling.

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