Modal decomposition of partially coherent flat-topped beams produced by multimode lasers

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Received October 7, 1997

We present a simple mathematical model giving a possible description of a partially coherent light beam exhibiting a flat-topped transverse intensity profile. Such a model allows us to deduce the modal distribution inside a multimode stable optical cavity, assuming that the modes are of the Hermite–Gauss type. The analytical expression used to represent flat-topped profiles is of the flattened Gaussian type and leads to an exact, closed-form expression for the M^2 factor of the output beam. An analogous procedure could be used to treat the general problem of deducing the modal distribution inside a laser cavity starting from intensity measurements of the output beam.

OCIS codes: 140.3460, 030.4070.

Inside a multimode optical cavity a light field can oscillate simultaneously on a multitude of different transverse modes that, for most practical applications, can be considered to be oscillating independently of one another.[1] As far as the mutual intensity is concerned,[2] this incoherent mode superposition reduces the spatial coherence of the beam.[3] The coherence features of the beam depend on the functional structure of the modes and on their strengths, i.e., on the relative power carried by each of them. The determination of the modes of partially coherent sources and their relative weights is a subject of considerable relevance and interest.[4–7]

In this Letter we consider beams produced by stable resonators with spherical mirrors whose modes can be considered to be of the Hermite-Gauss (HG) type.[1] We assume that the parameters of such modes, i.e., the positions and sizes of their waists, can be deduced from the geometrical characteristics of the empty resonator.[3] On the other hand, the power distribution of the modes depends on the presence of intracavity elements and on the working conditions of the laser.

As an example of partially coherent sources obtainable by incoherently superimposing HG beams we quote the so-called Collett–Wolf (or Gaussian Schell-model) sources, for which both the intensity profile and the degree of spatial coherence are Gaussian shaped.[8] In this case the weights of the modes follow a decreasing exponential law as functions of the order of the mode.[9,10]

In the general case one can obtain information about expansion coefficients by measurement of the degree of coherence.[11] If the mode structure is known, in principle the intensity profile of the beam should be enough. Following these lines, Siegman and Townsend proposed a numerical algorithm, based on a least-squares optimization procedure, to determine the HG mode distribution inside a laser cavity, producing a flat-topped intensity profile.[12] Here we show that such a problem can be solved analytically by use of a flattened Gaussian (FG) profile[13–15] to describe the transverse intensity distribution of the beam.

The present approach could be extended to other intensity profiles. In addition, for the present case it leads to an analytical description of the weights of the component modes and provides the exact, closed-form expression for the M^2 quality factor of the output beam, thus giving a complete characterization of its paraxial propagation features.

We start from the following intensity distribution, which belongs to the class of FG profiles[13,14]:

$$I_N(x) = \exp\left[-\frac{(N+1)}{{w_0}^2}x^2\right]\sum_{n=0}^{N}\frac{1}{n!}\left[\frac{(N+1)}{{w_0}^2}x^2\right]^n, \quad (1)$$

where N is a positive integer and w_0 is a real, positive parameter. The intensity behavior, as a function of the normalized variable x/w_0, is shown in Fig. 1 for several values of the parameter N. In that figure only the range $[0, +\infty)$ is visualized, the function I_N being even. The shape of $I_N(x)$ is controlled by two parameters, w_0 and N, related to the width of the transverse region on which I_N is reasonably different from zero and to the rapidity of the transition from the maximum value to zero, respectively.

If we consider the distribution in Eq. (1) as arising from an incoherent superposition of HG beams, we can

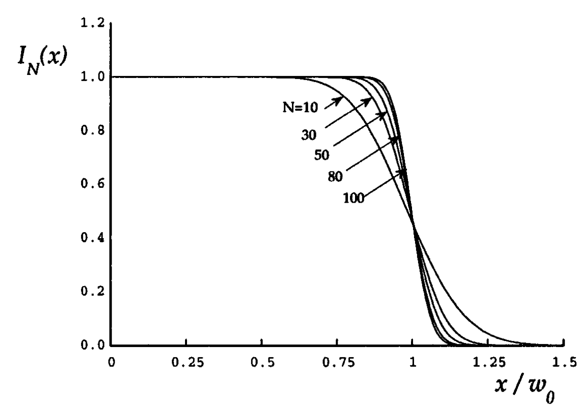

Fig. 1. Flattened Gaussian intensity profiles for several values of N.

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$$I_N(x) = \sum_{n=0}^{N} \lambda_n^{(N)} |\Phi_n(x)|^2, \quad (2)$$

where $\lambda_n^{(N)}$ are suitable (positive) coefficients and $\Phi_n(x)$ are the normalized HG functions,[1] i.e.,

$$\Phi_n(x) = \left(\frac{\sqrt{2}}{v_0}\right)^{1/2} \frac{1}{\sqrt{2^n n!\sqrt{\pi}}} H_n\left(\frac{x\sqrt{2}}{v_0}\right) \exp\left(-\frac{x^2}{v_0^2}\right), \quad (3)$$

where H_n is the nth-order Hermite polynomial[16] and v_0 is the spot size. To ensure that Eq. (2) coincides with Eq. (1), we first have to require that the arguments of the exponentials be equal to each other, i.e., that

$$v_0 = w_0\left(\frac{2}{N+1}\right)^{1/2}. \quad (4)$$

In the present case the value of spot size v_0 is fixed by the geometry of the resonator[1] and the width of flat-topped profile w_0 is determined by the transverse size of the gain medium. As a consequence of Eq. (4) the value of the order N is fixed and turns out to be

$$N = \left[2\left(\frac{w_0}{v_0}\right)^2\right] - 1, \quad (5)$$

where square brackets denote the nearest integer. With Eq. (4) taken into account, the equality between Eqs. (1) and (2) requires that

$$\sum_{n=0}^{N} \alpha_n^{(N)} H_n^2(\xi) = \sum_{n=0}^{N} \frac{1}{n!}\xi^{2n}, \quad (6)$$

where, for simplicity, the following quantities have been introduced:

$$\xi = \frac{x\sqrt{2}}{v_0}, \quad (7)$$

$$\alpha_n^{(N)} = \left(\frac{2}{\pi}\right)^{1/2} \frac{\lambda_n^{(N)}}{2^n n! v_0} \qquad (n = 0, 1, \ldots N). \quad (8)$$

Because $H_n^2(\xi)$ is a polynomial of order n in ξ^2, we set

$$H_n^2(\xi) = \sum_{k=0}^{n} p_{n,k}\xi^{2k}, \quad (9)$$

where the matrix $p_{n,k}$ ($n = 0, \ldots, N$; $k = 0, \ldots, n$) can be deduced starting from the recurrence properties of the Hermite polynomials. After some algebra, the following linear system for the unknown quantities $\alpha_n^{(N)}$ ($n = 0, 1, \ldots, N$) is obtained:

$$\sum_{n=k}^{N} \alpha_n^{(N)} p_{n,k} = \frac{1}{k!} \qquad (k = 0, 1, \ldots, N). \quad (10)$$

System (10) is upper triangular and can be solved by means of the following recurrence rule:

$$\alpha_N^{(N)} = \frac{1}{p_{N,N} N!},$$

$$\alpha_k^{(N)} = \frac{1}{p_{k,k}}\left[\frac{1}{k!} - \sum_{n=k+1}^{N} \alpha_n^{(N)} p_{n,k}\right]$$

$$(k = N-1, N-2, \ldots, 0). \quad (11)$$

In summary, once the parameters of the FG intensity profile are fixed, the present algorithm uniquely leads to the modal distribution $\lambda_n^{(N)}$ ($n = 0, \ldots, N$) through the inversion of a simple linear system.

In Fig. 2 the coefficients $\lambda_n^{(N)}$, whose values have been normalized according to the condition $\sum_n \lambda_n^{(N)} = 1$, are shown for some values of N. Note that a peak in the distribution is present at a value of n, say, $\bar{n}$, that depends on N. More precisely, such a peak occurs approximately at $N/2$, that is, $\bar{n} \simeq (w_0/v_0)^2$. This confirms the relationship between the flatness of the beam profile and the presence of a saturable-gain medium.[12] The curve for $N = 88$ is obtained with $v_0 = 0.15$ and $w_0 = 1$ and can be compared with the one in Fig. 4(b) of Ref. 12, where such values of v_0 and w_0 were used. Such a comparison shows that the agreement between our results and those of Ref. 12 is very good.

To provide a more comprehensive characterization of the output beam, its M^2 factor,[17] which represents one of the most used quality parameters, can be evaluated. In particular, by starting from Eq. (1) and exploiting the shape-invariance property of the beam[18] one can give M^2 a closed form. The determination of such an analytical expression is not trivial, but for the sake of brevity we omit it and report only the final result, which turns out to be

$$M^2 = 1 + 2N/3. \quad (12)$$

This results shows how the quality of the beam decreases when N increases. This means that, as expected, the flatter the intensity profile of the source, the wider the spreading angle of the radiated beam.

Furthermore, Eq. (12) could suggest a way to deduce the modal content of a FG source: By measuring the spreading angle of the beam radiated by it, one

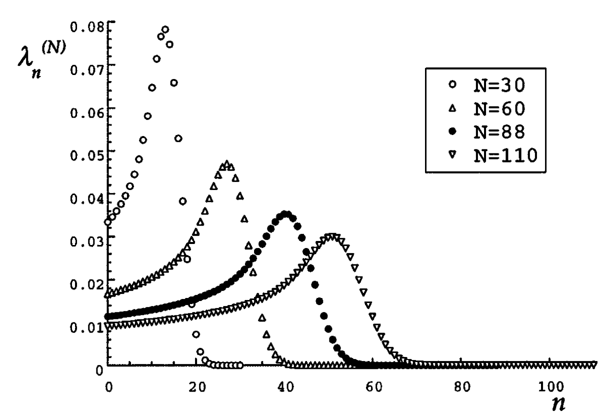

Fig. 2. Normalized coefficients $\lambda_n^{(N)}$ for several values of N.

can easily evaluate the value of N and completely determine the modal structure of the source.

In conclusion, we have presented a procedure for retrieving the modal power decomposition of a partially coherent source showing a flat-topped intensity profile and whose modes are assumed to be of the HG type. Such a procedure, for which the sole intensity distribution is needed, can provide analytical expressions for significant quantities pertinent to the beam's features, such as its M^2. Results seem to be consistent and agree well with those obtained by numerical optimization methods.

The present approach is based on the assumption that the spot size of the modes is known, and this is true whenever the physical characteristics of the laser resonator are given. If such an assumption is not verified, more-cumbersome coherence measurements are needed, as shown, for instance, in Ref. 6.

It should be stressed that the proposed algorithm could be extended to the case of more general intensity distributions to take account of experimental situations in which different beam shapes are encountered. In such cases an analytical description of the modal distribution and of other relevant quantities could be useful in laser design or in beam characterization.

The authors are very grateful to Franco Gori and Anthony Siegman for many stimulating discussions during the preparation of this work.

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