

Cambridge University Press  
978-0-521-68229-9 - Modal Logic for Philosophers  
James W. Garson  
Frontmatter  
[More information](#)

---

## Modal Logic for Philosophers

Designed for use by philosophy students, this book provides an accessible yet technically sound treatment of modal logic and its philosophical applications. Every effort has been made to simplify the presentation by using diagrams in place of more complex mathematical apparatus. These and other innovations provide philosophers with easy access to a rich variety of topics in modal logic, including a full coverage of quantified modal logic, nonrigid designators, definite descriptions, and the *de re–de dicto* distinction. Discussion of philosophical issues concerning the development of modal logic is woven into the text.

The book uses natural deduction systems and includes a diagram technique that extends the method of truth trees to modal logic. This feature provides a foundation for a novel method for showing completeness, one that is easy to extend to systems that include quantifiers.

James W. Garson is professor of philosophy at the University of Houston. He has held grants from the National Endowment for the Humanities, the National Science Foundation, and the Apple Education Foundation. He is also the author of numerous articles in logic, semantics, linguistics, the philosophy of cognitive science, and computerized education.

Cambridge University Press  
978-0-521-68229-9 - Modal Logic for Philosophers  
James W. Garson  
Frontmatter  
[More information](#)

---

# Modal Logic for Philosophers

JAMES W. GARSON

*University of Houston*



Cambridge University Press  
 978-0-521-68229-9 - Modal Logic for Philosophers  
 James W. Garson  
 Frontmatter  
[More information](#)

CAMBRIDGE UNIVERSITY PRESS  
 Cambridge, New York, Melbourne, Madrid, Cape Town, Singapore, São Paulo

Cambridge University Press  
 32 Avenue of the Americas, New York, NY 10013-2473, USA  
[www.cambridge.org](http://www.cambridge.org)  
 Information on this title: [www.cambridge.org/9780521863674](http://www.cambridge.org/9780521863674)

© James W. Garson 2006

This publication is in copyright. Subject to statutory exception and to the provisions of relevant collective licensing agreements, no reproduction of any part may take place without the written permission of Cambridge University Press.

First published 2006

Printed in the United States of America

*A catalog record for this publication is available from the British Library.*

*Library of Congress Cataloging in Publication Data*

Garson, James W., 1943–  
 Modal logic for philosophers / James W. Garson.  
 p. cm.

Includes bibliographical references (p. ) and index.

ISBN-13: 978-0-521-86367-4 (hardback)

ISBN-10: 0-521-86367-8 (hardback)

ISBN-13: 978-0-521-68229-9 (pbk.)

ISBN-10: 0-521-68229-0 (pbk.)

1. Modality (Logic) – Textbooks. I. Title.

BC199.M6G38 2006

160 – dc22 2006001152

ISBN-13 978-0-521-86367-4 hardback

ISBN-10 0-521-86367-8 hardback

ISBN-13 978-0-521-68229-9 paperback

ISBN-10 0-521-68229-0 paperback

Cambridge University Press has no responsibility for the persistence or accuracy of URLs for external or third-party Internet Web sites referred to in this publication and does not guarantee that any content on such Web sites is, or will remain, accurate or appropriate.

Cambridge University Press  
978-0-521-68229-9 - Modal Logic for Philosophers  
James W. Garson  
Frontmatter  
[More information](#)

---

*for Nuel Belnap, who is responsible for anything he likes about  
this book*

## Contents

<i>Preface</i>	<i>page</i> xiii
Introduction: What Is Modal Logic?	1
1 The System K: A Foundation for Modal Logic	3
1.1 The Language of Propositional Modal Logic	3
1.2 Natural Deduction Rules for Propositional Logic: PL	5
1.3 Derivable Rules of PL	9
1.4 Natural Deduction Rules for System K	17
1.5 A Derivable Rule for $\diamond$	20
1.6 Horizontal Notation for Natural Deduction Rules	27
1.7 Necessitation and Distribution	30
1.8 General Necessitation	32
1.9 Summary of the Rules of K	35
2 Extensions of K	38
2.1 Modal or Alethic Logic	38
2.2 Duals	44
2.3 Deontic Logic	45
2.4 The Good Samaritan Paradox	46
2.5 Conflicts of Obligation and the Axiom (D)	48
2.6 Iteration of Obligation	49
2.7 Tense Logic	50
2.8 Locative Logic	52
2.9 Logics of Belief	53
2.10 Provability Logic	54
3 Basic Concepts of Intensional Semantics	57
3.1 Worlds and Intensions	57
3.2 Truth Conditions and Diagrams for $\rightarrow$ and $\perp$	59

3.3	Derived Truth Conditions and Diagrams for PL	61
3.4	Truth Conditions for $\Box$	63
3.5	Truth Conditions for $\Diamond$	66
3.6	Satisfiability, Counterexamples, and Validity	67
3.7	The Concepts of Soundness and Completeness	69
3.8	A Note on Intensions	70
4	Trees for K	72
4.1	Checking for K-Validity with Trees	72
4.2	Showing K-Invalidity with Trees	81
4.3	Summary of Tree Rules for K	91
5	The Accessibility Relation	93
5.1	Conditions Appropriate for Tense Logic	93
5.2	Semantics for Tense Logics	99
5.3	Semantics for Modal (Alethic) Logics	104
5.4	Semantics for Deontic Logics	108
5.5	Semantics for Locative Logics	111
5.6	Relevance Logics and Conditional Logics	112
5.7	Summary of Axioms and Their Conditions on Frames	115
6	Trees for Extensions of K	116
6.1	Trees for Reflexive Frames: M-Trees	116
6.2	Trees for Transitive Frames: 4-Trees	121
6.3	Trees for Symmetrical Frames: B-Trees	123
6.4	Trees for Euclidean Frames: 5-Trees	129
6.5	Trees for Serial Frames: D-Trees	133
6.6	Trees for Unique Frames: CD-Trees	135
7	Converting Trees to Proofs	136
7.1	Converting Trees to Proofs in K	136
7.2	Converting Trees that Contain Defined Notation into Proofs	147
7.3	Converting M-Trees into Proofs	149
7.4	Converting D-Trees into Proofs	151
7.5	Converting 4-Trees into Proofs	152
7.6	Converting B-Trees into Proofs	154
7.7	Converting 5-Trees into Proofs	159
7.8	Using Conversion Strategies to Find Difficult Proofs	163
7.9	Converting CD-Trees into Proofs in CD and DCD	164
7.10	A Formal Proof that Trees Can Be Converted into Proofs	165
8	Adequacy of Propositional Modal Logics	172
8.1	Soundness of K	172
8.2	Soundness of Systems Stronger than K	180
8.3	The Tree Model Theorem	182

<i>Contents</i>		ix
8.4	Completeness of Many Modal Logics	188
8.5	Decision Procedures	189
8.6	Automatic Proofs	191
8.7	Adequacy of Trees	191
8.8	Properties of Frames that Correspond to No Axioms	192
9	Completeness Using Canonical Models	195
9.1	The Lindenbaum Lemma	195
9.2	The Canonical Model	198
9.3	The Completeness of Modal Logics Based on K	201
9.4	The Equivalence of PL+(GN) and K	210
10	Axioms and Their Corresponding Conditions on R	211
10.1	The General Axiom (G)	211
10.2	Adequacy of Systems Based on (G)	215
11	Relations between the Modal Logics	221
11.1	Showing Systems Are Equivalent	222
11.2	Showing One System Is Weaker than Another	224
12	Systems for Quantified Modal Logic	228
12.1	Languages for Quantified Modal Logic	228
12.2	A Classical System for Quantifiers	231
12.3	Identity in Modal Logic	234
12.4	The Problem of Nondenoting Terms in Classical Logic	239
12.5	FL: A System of Free Logic	242
12.6	fS: A Basic Quantified Modal Logic	245
12.7	The Barcan Formulas	248
12.8	Constant and Varying Domains of Quantification	250
12.9	A Classicist's Defense of Constant Domains	254
12.10	The Prospects for Classical Systems with Varying Domains	256
12.11	Rigid and Nonrigid Terms	260
12.12	Eliminating the Existence Predicate	262
12.13	Summary of Systems, Axioms, and Rules	263
13	Semantics for Quantified Modal Logics	265
13.1	Truth Value Semantics with the Substitution Interpretation	265
13.2	Semantics for Terms, Predicates, and Identity	268
13.3	Strong Versus Contingent Identity	270
13.4	Rigid and Nonrigid Terms	276
13.5	The Objectual Interpretation	278
13.6	Universal Instantiation on the Objectual Interpretation	281
13.7	The Conceptual Interpretation	286

13.8	The Intensional Interpretation	288
13.9	Strengthening Intensional Interpretation Models	293
13.10	Relationships with Systems in the Literature	294
13.11	Summary of Systems and Truth Conditions	300
14	Trees for Quantified Modal Logic	303
14.1	Tree Rules for Quantifiers	303
14.2	Tree Rules for Identity	307
14.3	Infinite Trees	309
14.4	Trees for Quantified Modal Logic	310
14.5	Converting Trees into Proofs	314
14.6	Trees for Systems that Include Domain Rules	319
14.7	Converting Trees into Proofs in Stronger Systems	320
14.8	Summary of the Tree Rules	321
15	The Adequacy of Quantified Modal Logics	323
15.1	Preliminaries: Some Replacement Theorems	324
15.2	Soundness for the Intensional Interpretation	326
15.3	Soundness for Systems with Domain Rules	329
15.4	Expanding Truth Value (tS) to Substitution (sS) Models	332
15.5	Expanding Substitution (sS) to Intensional (iS) Models	337
15.6	An Intensional Treatment of the Objectual Interpretation	339
15.7	Transfer Theorems for Intensional and Substitution Models	342
15.8	A Transfer Theorem for the Objectual Interpretation	347
15.9	Soundness for the Substitution Interpretation	348
15.10	Soundness for the Objectual Interpretation	349
15.11	Systems with Nonrigid Terms	350
15.12	Appendix: Proof of the Replacement Theorems	351
16	Completeness of Quantified Modal Logics Using Trees	356
16.1	The Quantified Tree Model Theorem	356
16.2	Completeness for Truth Value Models	361
16.3	Completeness for Intensional and Substitution Models	361
16.4	Completeness for Objectual Models	362
16.5	The Adequacy of Trees	364
17	Completeness Using Canonical Models	365
17.1	How Quantifiers Complicate Completeness Proofs	365
17.2	Limitations on the Completeness Results	368
17.3	The Saturated Set Lemma	370
17.4	Completeness for Truth Value Models	373



<i>Contents</i>		xi
17.5	Completeness for Systems with Rigid Constants	377
17.6	Completeness for Systems with Nonrigid Terms	379
17.7	Completeness for Intensional and Substitution Models	382
17.8	Completeness for the Objectual Interpretation	383
18	Descriptions	385
18.1	Russell's Theory of Descriptions	385
18.2	Applying Russell's Method to Philosophical Puzzles	388
18.3	Scope in Russell's Theory of Descriptions	390
18.4	Motives for an Alternative Treatment of Descriptions	392
18.5	Syntax for Modal Description Theory	394
18.6	Rules for Modal Description Theory: The System !S	396
18.7	Semantics for !S	400
18.8	Trees for !S	402
18.9	Adequacy of !S	403
18.10	How !S Resolves the Philosophical Puzzles	407
19	Lambda Abstraction	409
19.1	<i>De Re</i> and <i>De Dicto</i>	409
19.2	Identity and the <i>De Re–De Dicto</i> Distinction	413
19.3	Principles for Abstraction: The System $\lambda S$	415
19.4	Syntax and Semantics for $\lambda S$	416
19.5	The Adequacy of $\lambda S$	422
19.6	Quantifying In	424
	Answers to Selected Exercises	432
	<i>Bibliography of Works Cited</i>	445
	<i>Index</i>	449

## Preface

The main purpose of this book is to help bridge a gap in the landscape of modal logic. A great deal is known about modal systems based on propositional logic. However, these logics do not have the expressive resources to handle the structure of most philosophical argumentation. If modal logics are to be useful to philosophy, it is crucial that they include quantifiers and identity. The problem is that quantified modal logic is not as well developed, and it is difficult for the student of philosophy who may lack mathematical training to develop mastery of what is known. Philosophical worries about whether quantification is coherent or advisable in certain modal settings partly explains this lack of attention. If one takes such objections seriously, they exert pressure on the logician to either eliminate modality altogether or eliminate the allegedly undesirable forms of quantification.

Even if one lays those philosophical worries aside, serious technical problems must still be faced. There is a rich menu of choices for formulating the semantics of quantified modal languages, and the completeness problem for some of these systems is difficult or unresolved. The philosophy of this book is that this variety is to be explored rather than shunned. We hope to demonstrate that modal logic with quantifiers can be simplified so that it is manageable, even teachable. Some of the simplifications depend on the foundations – in the way the systems for propositional modal logic are developed. Some ideas that were designed to make life easier when quantifiers are introduced are also genuinely helpful even for those who will study only the propositional systems. So this book can serve a dual purpose. It is, I hope, a simple and accessible introduction to propositional modal logic for students who have had a first course

in formal logic (preferably one that covers natural deduction rules and truth trees). I hope, however, that students who had planned to use this book to learn only propositional modal logic will be inspired to move on to study quantification as well.

A principle that guided the creation of this book is the conviction that visualization is one of the most powerful tools for organizing one's thoughts. So the book depends heavily on diagrams of various kinds. One of the central innovations is to combine the method of Haus diagrams (to represent Kripke's accessibility relation) with the truth tree method. This provides an easy and revealing method for checking validity in a wide variety of modal logics. My students have found the diagrams both easy to learn and fun to use. I urge readers of this book to take advantage of them.

The tree diagrams are also the centerpiece for a novel technique for proving completeness – one that is more concrete and easier to learn than the method of maximally consistent sets, and one that is extremely easy to extend to the quantifiers. On the other hand, the standard method of maximally consistent sets has its own advantages. It applies to more systems, and many will consider it an indispensable part of anyone's education in modal logic. So this book covers both methods, and it is organized so that one may easily choose to study one, the other, or both.

Three different ways of providing semantics for the quantifiers are introduced in this book: the substitution interpretation, the intensional interpretation, and the objectual interpretation. Though some have faulted the substitution interpretation on philosophical grounds, its simplicity prompts its use as a centerpiece for technical results. Those who would like a quick and painless entry to the completeness problem may read the sections on the substitution interpretation alone. The intensional interpretation, where one quantifies over individual concepts, is included because it is the most general approach for dealing with the quantifiers. Furthermore, its strong kinships with the substitution interpretation provide a relatively easy transition to its formal results. The objectual interpretation is treated here as a special case of the intensional interpretation. This helps provide new insights into how best to formalize systems for the objectual interpretation.

The student should treat this book more as a collection of things to do than as something to read. Exercises in this book are found embedded throughout the text rather than at the end of each chapter, as is the more common practice. This signals the importance of doing exercises as soon

as possible after the relevant material has been introduced. Think of the text between the exercises as a preparation for activities that are the foundation for true understanding. Answers to exercises marked with a star (\*) are found at the end of the book. Many of the exercises also include hints. The best way to master this material is to struggle through the exercises on your own as far as humanly possible. Turn to the hints or answers only when you are desperate.

Many people should be acknowledged for their contributions to this book. First of all, I would like to thank my wife, Connie Garson, who has unfailingly and lovingly supported all of my odd enthusiasms. Second, I would like to thank my students, who have struggled through the many drafts of this book over the years. I have learned a great deal more from them than any of them has learned from me. Unfortunately, I have lost track of the names of many who helped me make numerous important improvements, so I apologize to them. But I do remember by name the contributions of Brandy Burfield, Carl Feierabend, Curtis Haaga, James Hulgán, Alistair Isaac, JoBeth Jordon, Raymond Kim, Kris Rhodes, Jay Schroeder, Steve Todd, Andy Tristan, Mako Voelkel, and especially Julian Zinn. Third, I am grateful to Johnathan Raymon, who helped me with the diagrams. Finally, I would like to thank Cambridge University Press for taking an interest in this project and for the excellent comments of the anonymous readers, some of whom headed off embarrassing errors.