Modal testing using impact excitation and a scanning LDV[1]

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If a laser Doppler vibrometer is used in a continuously-scanning mode to measure the response of a vibrating structure, its output spectrum contains side-bands from which the response mode shape, as defined along the scan line, may be obtained. With impact excitation, the response is the summation of a set of exponentially-decaying sinusoids which, in the frequency domain, has peaks at the natural frequencies and at 'sideband' pseudo-natural frequencies, spaced at multiples of the scan frequency. Techniques are described for deriving natural mode shapes from these, using standard modal analysis procedures. Some limitations as to the types of mode which can be analysed are described. The process is simple and speedy, even when compared with a normal point-by-point impact test survey. Information may also be derived, using a circular scan, on the direction of vibration, and angular vibration, at individual points.

Keywords: Modal testing, hammer, impact tests, spatial-scanning, LDV

1. Introduction

The Laser Doppler Vibrometer (LDV) is extensively used as a vibration measurement transducer, in a wide range of applications [1–4,9], chiefly because the structure is not loaded by attachment of transducers, and because the measurement point is so easily varied. With what is often termed a 'scanning LDV', the measurement is moved successively to a series of points, and the mode-shape derived, often, by a curve-fitting process. An LDV may, however, be used in a continuously-moving scanning mode, and can produce data on a structure's response mode shape [5,8] as defined along the scan line, in one measurement. The direction of vibration [6] and angular vibration at a point [7] can also be measured. These techniques have previously been described [6–8] for sinusoidal excitation, where the output spectrum of a continuously-scanning laser Doppler vibrometer (CSLDV) consists of a component at the excitation frequency, together with sideband components separated by multiples of the scan frequency.

This paper explores the use of a CSLDV in combination with impact testing, which is often preferred to sine testing because of its speed and convenience.

A method of testing, using narrow-band random excitation with a CSLDV has been described [7,8] but, unfortunately, the force input spectrum tended to disappear at and around the natural frequencies, leading to poor coherence and distortion in the frequency response functions (FRFs). With impact excitation the input spectrum does not, in general, suffer from such 'drop-outs' near the natural frequencies so that this problem should be avoided.

If an impact is applied to a structure being scanned by a CSLDV, the scanning produces modulation of the transient response (additional to that produced by the natural decay), and there are corresponding additional sideband response spectrum peaks, again spaced by the CSLDV scan frequency. For meaningful results it is obviously necessary that at least one scan be completed before the transient response ends, which means that there are constraints on the modes which can be dealt with. However, impact tests are widely used: they are quick to set up and easy to carry out, and there is no response distortion due to uncancelled shaker (and push-rod) effects. This extension in technique could therefore prove to be a very useful addition to the vibration engineer's tool-kit.

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2. The continuous-scan laser doppler vibrometer

A laser Doppler vibrometer works, essentially, by detecting the Doppler-shifted wavelengths of light scattered from a point on a vibrating structure, using an interferometer. A coherent laser light beam (usually Class I or II He-Ne) is focused onto a point on a vibrating surface. The Doppler-shifted wavelength of the returned light is a measure of the structure's vibration velocity in the direction of the incident light and, after suitable processing, a voltage signal is produced which is a direct analogue of the vibration. Commercial instruments have been available for some years, embodying this principle. In the applications described here, the point of aim is continuously varied using beam-directing mirrors inside the instrument.

2.1. The CSLDV with sinusoidal excitation

If the point addressed is moved to and fro, sinusoidally at frequency Ω, by virtue of a suitably-oscillating beam-directing mirror, along a straight line over the surface of a vibrating structure, the CSLDV output is modulated because of the variation in vibration amplitude along the scan line – the response mode-shape. We first assume the (linear) structure is responding to sinusoidal forcing at frequency ω. As a consequence, the LDV's output spectrum has sidebands defining the mode shape, as explained below.

It is convenient to assume the scan has a frequency Ω and amplitude ± 1, being zero at the scan mid-point, so that the point addressed is:

$$x(t) = \cos(\Omega t + \beta) \tag{1}$$

Assume a spatial polynomial series for the mode-shape defined along the line, so that the vibration ν_z at the instantaneous scan point is:

$$\nu_z(t) = \sum_{n=0}^{p} \{V_n x^n \cos(\omega t + \alpha_n)\} \tag{2}$$

$V_n \cos\alpha_n$ and $V_n \sin\alpha_N$ being coefficients for power series respectively for in-phase (real) and quadrature (imaginary) components of the so-called Operating Deflection Shape (ODS) – the *response* mode shape – and the series terminates at the x^p term.

Substituting for $x(t)$ from equation (1) gives:

$$\nu_z(t) = \sum_{n=0}^{p} \{V_n \cos(\omega t + \alpha_n) \times \cos^n(\Omega t + \beta)\} \tag{3}$$

When this is expanded out, multiple terms arise at frequencies ($\omega \pm n\Omega$), upper and lower side-bands magnitudes for each n being identical, i.e. $S{-n} = S_n$ and $\alpha_{-n} = \alpha_n$.

$$\nu_z(t) = \sum_{n=-p}^{+p} \{S_n \cos[(\omega + n\Omega)t + \alpha_n + n\beta]\} \tag{4}$$

The phase angles associated with each sideband pair are $(\alpha_n + n\beta)$ and $(\alpha_N + N\beta)$, their average giving α_n directly. If the mode-shape is real, the α_n values are identical (or differ by 180°); if, additionally, the phase reference is the response at a point on the structure, they are all zero (or 180°).

After some manipulation, the S (sideband) coefficients in equation (4) can be shown [8] to be related to the polynomial coefficients in equation (2) by a simple matrix transform

$$\{V\} = [T][S] \tag{5}$$

Equation (5) can be applied to a vector $\{S\}$ of (signed) side-band magnitudes to give a vector $\{V\}$ of polynomial coefficients for a real mode. Otherwise it can be applied separately to the in-phase and quadrature side-band components, to give complex mode polynomial coefficients.

For up to 10 sideband pairs,

$$[\mathrm{T}] = \begin{bmatrix} 1 & 0 & -2 & 0 & 2 & 0 & -2 & 0 & 2 & 0 & -2 \\ 0 & 2 & 0 & -6 & 0 & 10 & 0 & -14 & 0 & 18 & 0 \\ 0 & 0 & 4 & 0 & -16 & 0 & 36 & 0 & -64 & 0 & 100 \\ 0 & 0 & 0 & 8 & 0 & -40 & 0 & 112 & 0 & -240 & 0 \\ 0 & 0 & 0 & 0 & 16 & 0 & -96 & 0 & 320 & 0 & 800 \\ 0 & 0 & 0 & 0 & 0 & 32 & 0 & -224 & 0 & 864 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 64 & 0 & -512 & 0 & 2240 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 128 & 0 & -1152 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 256 & 0 & -2560 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 512 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1024 \end{bmatrix} \tag{6}$$

2.2. The CSLDV with impact excitation

With impact excitation, the desired input is a short, quasi-half-sine impulse, the Fourier transform of which is, in general, a force spectrum whose magnitude and phase characteristics are smooth and continuous, over the frequency range used. For a non-scanned measurement, the response is the summation of a set of exponentially-decaying sinusoids, one for each natural mode. For the example in Fig. 1(a), the response is predominantly in one mode (detail is not accurately represented here because of a low sample rate).

If a structure is excited at a point k by a forcing function with a continuous spectrum, the response at a point j will have a response spectrum, $\nu_j(\omega)$, such that, assuming a linear structure with modal behaviour and

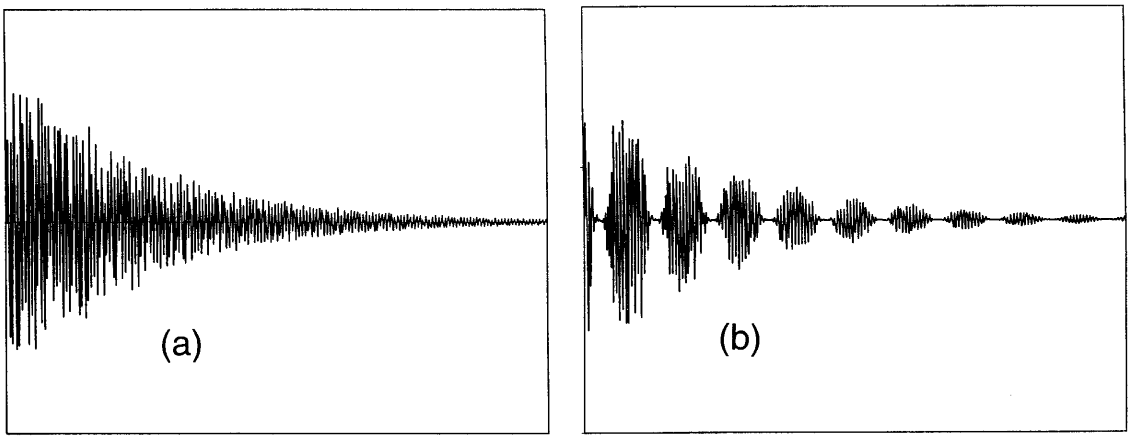

Fig. 1. Vibration response of a structure (in one mode of vibration) to an impact, Measured (a) at a fixed point, (b) using a scanning laser Doppler vibrometer.

hysteretic damping.

$$\nu_j(\omega) = f_k(\omega) \sum_{r=1}^{m} \frac{{}_rA_{jk}\omega}{\omega_r^2 - \omega^2 + i\eta_r\omega_r^2} \quad (7)$$

ν_j being the response component at frequency ω, f_k the forcing at this frequency, ω_r and η_r the natural frequency and (hysteretic) damping for mode r, and ${}_rA_{jk}$ a modal constant.

With a CSLDV, the response amplitude is further modulated by the deflection shape as the scan passes to and fro over the surface, introducing multiple exponentially-decaying components which beat together to give a modulated transient, as illustrated by Fig. 1(b). As described above for sinusoidal excitation, each component input forcing frequency, ω, produces additional responses at frequencies $(\omega + n\Omega)$. With an input having a continuous spectrum, and a continuous repetitive scan, therefore, the side-band responses repeat the curve given by one term of equation (7) at different scales, and shifted by $n\Omega$, as illustrated by Fig. 2.

The sidebands are labelled: $S_1, S_2 \ldots, S_{-1}, S_{-2} \ldots$, and S_0 for the central, natural frequency. The frequency response function (FRF), $H(\omega)$, expressed as the (complex) response divided by the (complex) force, of a CSLDV response to an impact excitation, $\nu(\omega,\Omega)/f(\omega)$, from a scanning LDV will therefore have additional terms corresponding to side-bands around each mode, so that

$$H(\omega) = \sum_{r=1}^{m} \sum_{n=-p}^{+p} \frac{{}_rA_{jkn}(\omega - n\Omega)}{\omega_r^2 - (\omega - n\Omega)^2 + i\eta_r\omega_r^2} \quad (8)$$

The modal constants ${}_rA_{jkn}$ in equation (7) can refer to a particular response point j, or to any weighted combination of such points. In equation (8), the constant ${}_rA_{jkn}$ for sideband N is a summation over all points on the scan line. Since, (see equation (5)), at each frequency, the polynomial coefficients are linearly related to the CSLDV sideband responses, modal analysis can legitimately be applied to the sidebands, and the modal constants thus-derived used, with equation (5), to arrive at polynomial modal coefficients.

2.2.1. *FRF correction*

The zero-order modal constant may be obtained by modal analysis of the scanned FRF without error. The FRFs for the sideband 'resonances' are, however, 'wrong', because they are based on force datum signals at the wrong frequencies, i.e. the response frequencies are shifted, but the force input is not. Derived modal constants can be corrected for this, using the known force input spectrum, but it is probably easier in practice to apply the modal analysis process to the complex *response* spectrum, to establish a relative mode shape. This can then be related to the forcing to give $H(\omega)$, if necessary, scaling from the FRF at one point, separately-measured, or from the zero-order FRF modal constant.

Problems may be experienced with structures with high modal density because of the proliferation of closely-spaced sidebands. With heavily-damped modes the response decays more quickly, requiring a faster scan rate, which inevitably spreads the sidebands over a wider frequency range, so that other modes' sidebands may cause confusion at lower modal densities.

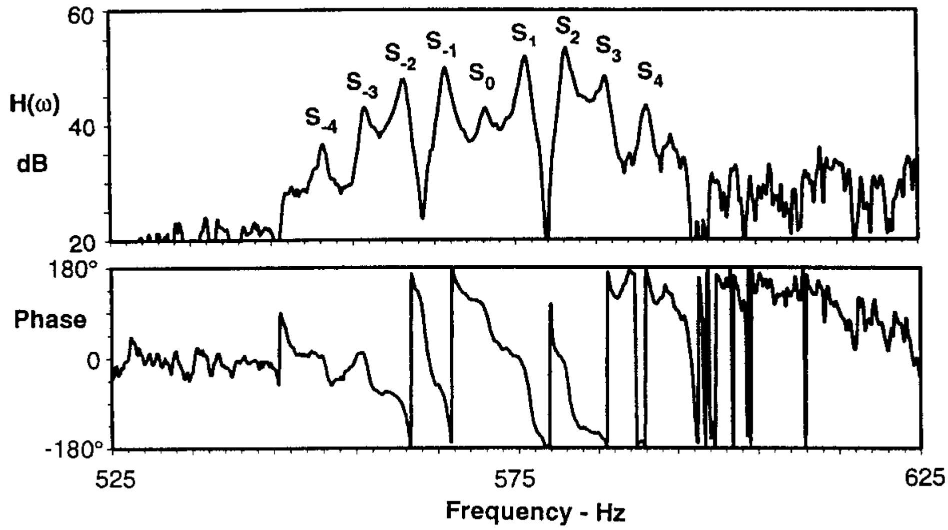

Fig. 2. Apparent Frequency Response Function of a structure with impact excitation and a CSLDV for response measurement.

2.2.2. Modal analysis correction

Response FRFs may be distorted if the force input is not uniform over the small frequency range associated with a single, lightly-damped resonance peak. However, if the force spectrum magnitude and phase variations are nearly linear over this range, the result is, in general, that the response FRF, plotted in the Nyquist plane, is rotated but not seriously distorted: a single mode, for example, retaining its circular form. FRFs are usually processed by an analysis that, for a particular mode, requires data only in this narrow frequency range. As a consequence, modal analysis of CSLDV hammer test data is usually achieved quite satisfactorily.

Sideband modal analysis is based on frequency series that are shifted by $n\Omega$ where Ω is the scan frequency and n the sideband order (noting that n may be a positive or negative integer). The resulting 'natural' frequencies will therefore be shifted by the same amount. Taking a simple view of modal analysis, the modal damping is given by $\Delta\omega/\omega_r$ where Δ_ω is the frequency difference corresponding to response levels 3dB below the maximum. When applied to sidebands, the derived modal damping will be in error by $(\omega_r + n\Omega)/\omega_r$ because of the frequency shift of ω_r (see Fig. 2). In a similarly simplistic way, the modal constant is merely the diameter of the resonance 'circle' in the Nyquist plane, multiplied by the modal damping. Hence, again, the modal constants derived from sidebands have to be multiplied by

$$(\omega_r + n\Omega)/\omega_r \tag{9}$$

For the example given in Table 1, this correction is quite insignificant compared with experimental uncertainty.

2.2.3. Setting scan rates

The above analysis assumes implicitly that the transient response is noise-free and linear, with linear damping, so that the modal contributions diminish exponentially. If the response measurement had infinite resolution, it could be measured accurately no matter how much it was attenuated – but this can never be true in practice. The CSLDV, in particular, is subject to 'speckle' noise which gives irregular short-term noise spikes, which equate, in the frequency domain, to a random noise floor typically at 1% of full scale. It is necessary that the vibration response transient persists, above the noise, at a measurable level, for at least one complete scan cycle before it is excessively affected by noise. The scan rate which can be used therefore depends on the amount of damping in the mode being studied. Several scan cycles can be counted in the example in Fig. 1(b).

In impact testing there another criterion which has to be applied: namely that the response transient must decay to an insignificant level before the end of the (time) sample acquired for FFT analysis. Two stratagems are commonly employed to achieve this: either an exponential window is applied to the response data so as to increase its attenuation rate (and artificially increase the apparent damping in the system), or a zoom FFT transform is used, reducing the analysed frequency range but increasing the sample time length.

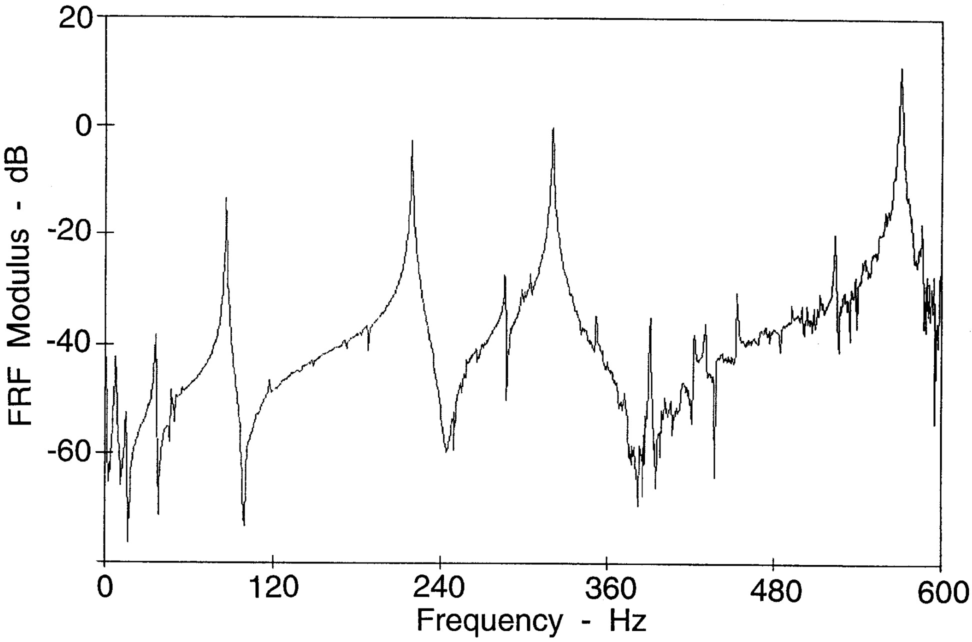

Fig. 3. Fixed-point FRF of the plate shown in Fig. 4.

Table 1
Derivation of (unscaled) mode shape

Side band	Frequency Hz	Modal constant	Corrected Eq.(9)	Average $\frac{(A_n+A_{-n})}{2}$	V_n Eq.(5)
−4	551.4	0.753	0.726		
−3	556.4	1.93	1.88		
−2	561.4	−4.31	−4.24		
−1	566.4	−4.99	−4.95		
0	571.3	2.38	2.38	2.38	13.32
1	576.3	−4.72	−4.76	−4.85	−21.8
2	581.3	−4.89	−4.98	−4.61	−32.2
3	586.3	2.12	2.17	2.02	16.2
4	591.3	0.957	0.991	0.86	13.8

Another factor to be considered is the fact that high scan speeds increase the response sideband spacing (and eight sideband pairs may typically have to be accommodated in the frequency transform range). They also put high mechanical loads on the CSLDV mirror drive system.

A preliminary non-scanned FRF may be useful, so that the damping can be estimated before selecting a scan rate. It is not safe merely to view the response in the time domain, as in Fig. 1 (b), as the attenuation of one mode may be masked by the slower attenuation of another, if there is more than one mode in the transform range.

It should be realised that all the other modes also produce a set of sideband response peaks, and there may be considerable difficulties in separating their responses when they overlap. Some alleviation of this problem may be given by careful adjustment of the scan frequency, but it may well be impossible to avoid sideband coincidence. Difficulties are also caused when sideband frequencies become negative.

2.2.4. Demonstration of hammer-test SLDV mode-shape determination

Figure 3 shows a conventional impact-excited drive-point FRF, measured on a $175 \times 175 \times 1$ mm cantilevered plate, using a fixed-point LDV measuring at the excitation point. The plate and the drive point are illustrated in Fig. 4. A number of lightly-damped modes are clearly evident.

Figure 2 shows a CSLDV FRF measured on this plate, using a zoom-FFT and an exponential window.

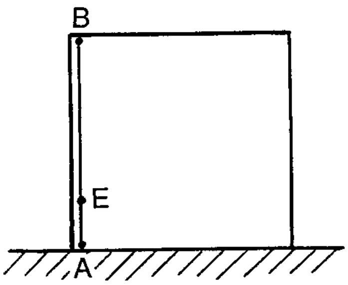

Fig. 4. Cantilevered plate, showing the impact excitation point, E, (behind) and the CSLDV scan line A-B.

A zoom range of 100 Hz, centred on the mode at 571 Hz was used, with a sine scan at 5 Hz along line A-B, in Fig. 4. Time-differentiation was applied to the CSLDV signal to produce a frequency response function in accelerance terms, rather than mobility, to correspond with modal analysis data input requirement. There is a fairly symmetrical group of sideband peaks in Fig. 2, centred on S_0, spaced 5 Hz apart – the scan frequency.

Figure 5 is the corresponding CSLDV output response spectrum, magnitude and phase, plotted directly, without referencing it to the forcing input. As mentioned above, Figs 2 and 5 are generally similar, but the FRF magnitude spectrum is less symmetrical than that of the response because the force spectrum was not completely flat over this 100 Hz frequency range. The phase spectrum is offset, but otherwise not much altered. Obviously-unusual characteristics of Figs 2 and 5 are the sheared phase spectra. These are a consequence of non-coordination between impact timing and laser scan position which, in effect, leads to an unknown and uncontrolled value of β in equation (1), so that the phase spectrum is, in general, different for each new impact.

A better technique is to trigger response data acquisition when the CSLDV scan point passes through the scan centre, so that β is zero. In practice this implies arming the trigger from the force impulse signal, and triggering, using the CSLDV scan signal, when it next passes through zero, if necessary with an allowance for time delay in the CSLDV mirror-drive system. The force impact event may well have ended before data acquisition, but this is of little consequence if modal analysis is applied only to the response spectrum, rather than to the FRF, as recommended above. An example is included, as Fig. 6. The test subject in this case was a cantilevered beam, tested in a similar way as the plate in Fig. 4, but triggering data acquisition from the scan signal, as described above. Zero, first, second, third and fifth-order sideband peaks were generated around the third flexural mode, at 1303 Hz – with 180° phase transitions at each sideband, just like a standard FRF phase spectrum.

After the corrections described, modal analysis should yield modal constants with identical magnitudes for each pair of sidebands. Application of equation (5) in principle gives a set of polynomial coefficients for the *natural* mode-shape, as opposed to the *response* mode-shape obtained from single-frequency sine testing. For the examples illustrated here, the modes were so nearly real that these would be indistinguishable.

Table 1 lists modal constants obtained using a conventional modal analysis package, ICATS MODENT (Inverse) Line-Fit, on the response spectrum in Fig. 5. Positive or negative signs were assigned to the modal constants, based on the vector directions of the modal constants, which were consistent with a 'real' mode shape. The set of (five) derived polynomial mode-shape coefficients is also included in the Table. Mode-shape curves determined in this way are shown in Fig. 7, for two of the natural modes of the plate, at 321 and 571.3 Hz, the latter using the data in Table 1.

3. Circular scanning

It is sometimes appropriate to scan over a vibrating surface in a circle at a uniform speed Ω_R, rather than in a straight line. This is easily arranged in practice, by supplying sine- and cosine-inputs to the x and y SLDV beam-deflecting mirrors simultaneously.

As with straight-line scans, there are then sideband pairs in the spectrum, spaced at $\pm n\Omega_R$ about the vibration frequency, ω. The sums and differences of the sideband amplitudes give the amplitudes of 'real' and 'imaginary' spatial Fourier mode-shape components directly [8]. The sums and differences of the phase angles give, respectively, $2\alpha_n$ and $2n(\gamma_n - \beta)\alpha_N$, being the temporal phase angle. γ_n defines the spatial orientation of the n^{th} Fourier component.

In an impact test, the mode shape is given correctly, relative to a datum position which depends on the position of the scan point, round the circle, at the start of data acquisition. This position may be identified, as for a straight-line scan, by triggering data acquisition from one of the LDV mirror drive signals.

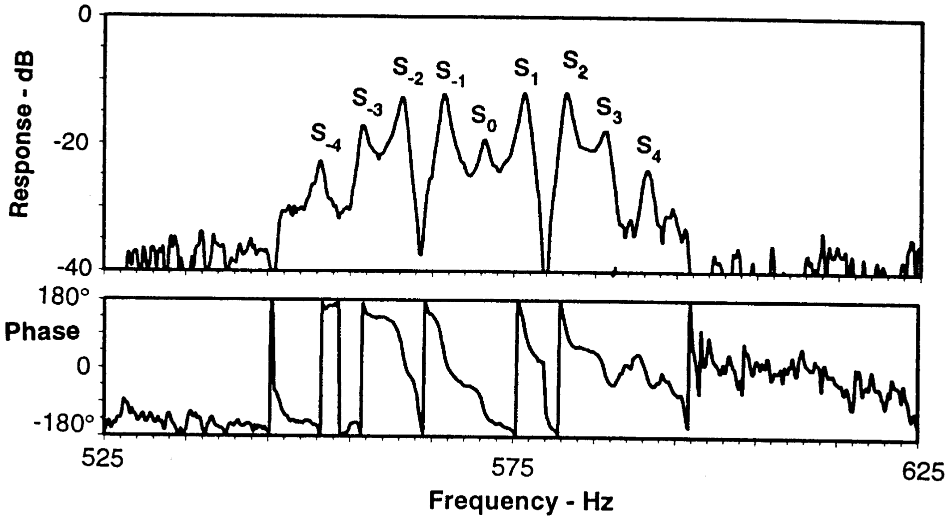

Fig. 5. CSLDV response spectrum, corresponding to Fig. 2.

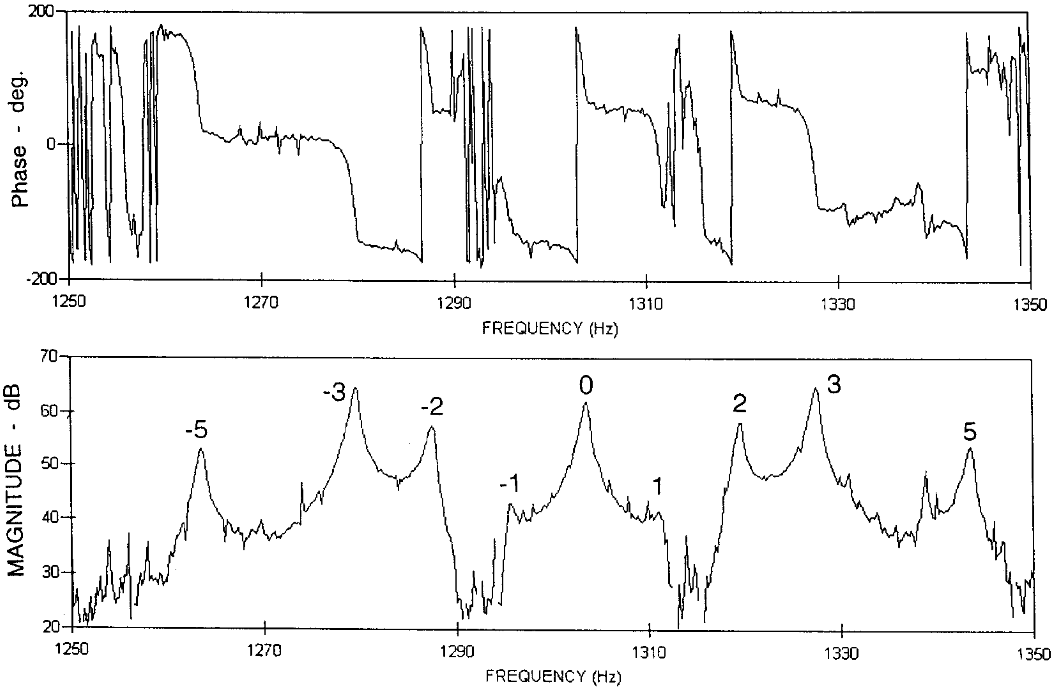

Fig. 6. CSLDV response spectrum for a cantilever beam – triggered from the scan signal.

3.1. Angular vibration measurement

If the scale of a straight-line sinusoidal scan is sufficiently reduced [7], the surface scanned will exhibit only translational and angular vibration, so that the CSLDV response has only a single pair of sidebands. This makes quantification of the translation and angular rotation modal constants particularly simple, using an impact test with short straight-line scans in x and y directions. The fact that only one pair of sidebands is produced gives considerably more scope in selecting the measurement time and reducing the confusion from close natural frequencies.

With a complex vibration mode it is possible for an-

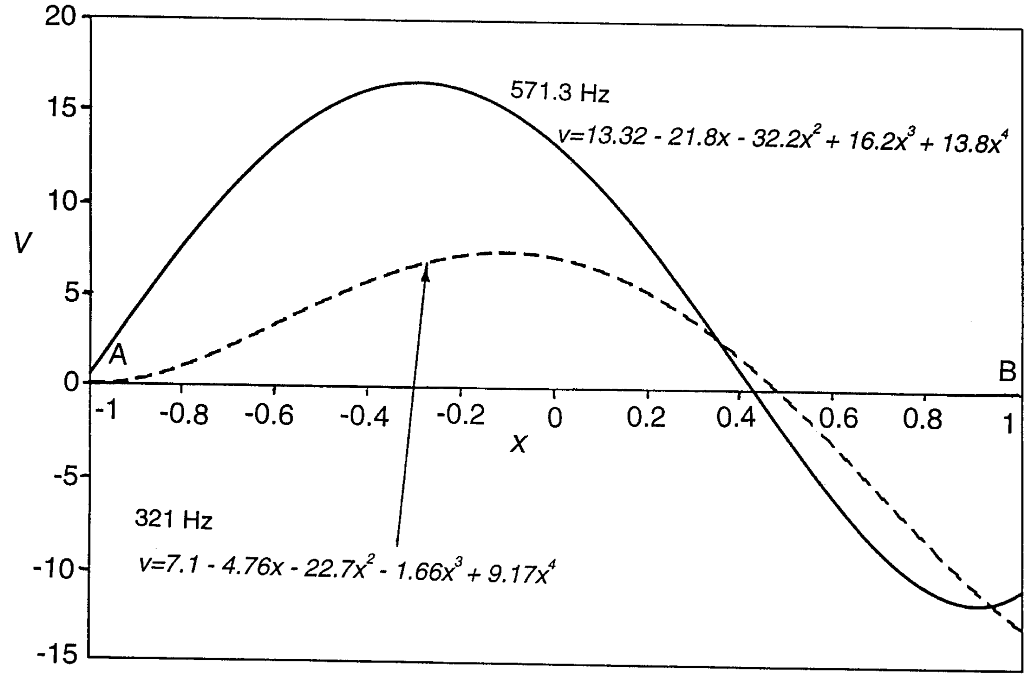

Fig. 7. Plate mode shapes, measured on scan line A-B.

gular motion to include a swash component. Spatially orthogonal straight-line sine scan responses will then generally not be in phase, and further calculation is necessary to derive principal rotation amplitudes and directions from the x, y scans. With a small-diameter circular scan, swash leads to sidebands that are of unequal magnitude, whence the principal rotation amplitudes are easily derived, from one measurement [7]. The *directions* of the principal rotation vectors require phase data and, again, data acquisition must be triggered from the scan signal.

3.2. Measurement of vibration direction

Circular scans, in conjunction with a short focal-length lens to focus the conical scan onto a single point on a sinusoidally vibrating surface, can determine the direction of vibration [6]. The CSLDV output spectrum comprises a component at the vibration frequency, plus one pair of sidebands, and an impact test procedure can be derived, corresponding to that for sinusoidal excitation. When the vibration is sinusoidal, along a straight-line path, the sidebands are of equal magnitude and the vibration direction is easily determined. When, as may arise with complex modes, the vibration follows an elliptical path, its determination may be a little more difficult.

In the example shown in Fig. 8, a gas turbine rotor blade was subjected to a single impact and a conical-scanned LDV directed at it, near the blade tip, to measure the response [6]. Modal constants for the zero-order and first-order sidebands give respectively $V \cos\theta \cos\gamma$ and $V \sin\theta \sin\gamma$, where V is the amplitude of vibration, θ is the semi-angle of the conical scan and γ defines the direction of vibration. From these, as illustrated in the Figure, the blade tip vibration direction was found to be at 39.8° to the scan axis. In this particular case, since the vibration was known to be in a plane perpendicular to the blade longitudinal axis, the second angle defining the vibration direction was not required and phase information was therefore unnecessary.

4. Test technique summarised

When conducting a CSLDV impact test measurement, the scan rate must be such that at least one scan cycle is completed before the transient response becomes masked by noise, a requirement which may exclude measurements on some highly-damped structures. The response transient must attenuate to an insignificant level by the end of the sample in order to avoid errors in the FRF. A zoom FFT and/or exponen-

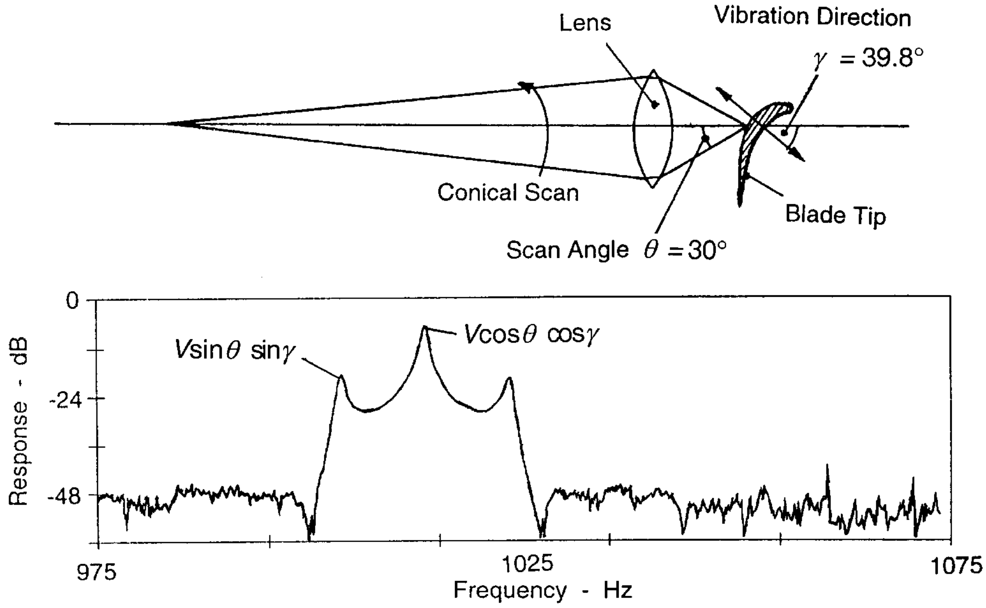

Fig. 8. Measurement of vibration direction using a conical scan.

tial response weighting may be used, as usual, for this purpose. The proliferation of sideband response peaks can cause great confusion if the modal density is too high.

Standard modal analysis techniques can be used on the resulting response spectrum. Equipment requirements, other than that needed for standard impact modal testing, are a laser Doppler vibrometer and a means of producing the scan, e.g. steerable beam-directing mirrors with a sine-wave drive source. Data acquisition should be triggered from the LDV scan signal, prearmed by the force input signal. An FFT may be used to transform the response into the frequency domain, no special arrangements are necessary, as they are with sine testing, for deriving a phase spectrum.

There are, of course, some practical limitations in impact testing with a CSLDV. Fairly obviously, the impact has to be applied in such a way as not to obstruct the scanning laser beam. Also, the structure must be attached fairly firmly to earth, so that rigid body responses do not affect the scan path on the structure. Also in normal fixed-point impact testing, particularly where a structure is flexible, it may be impossible to produce a clear, single, short input pulse. This should not be a problem with a CSLDV as the impact position can be chosen to be away from such regions.

5. Conclusion

Impact tests have an important place in vibration modal testing. Sinusoidal or pseudo-random input techniques have some advantages but they are more complicated, and sometimes impossible, to set up, and may cause inadvertent distortion of the test structure's characteristics.

On the other hand, where repeated impacts are used to measure mode shapes, or to measure rotational degrees of freedom, or x, y, z components of vibration at a point, there may be considerable difficulty in achieving an input at exactly the point, and in exactly the direction intended. Another important disadvantage of discrete-point modal testing is that it is difficult to predefine the test point spacing necessary adequately to describe a mode shape. These problems are eliminated if a CSLDV is used, although it is not of universal application, and note should be taken of the restrictions and inaccuracies mentioned above.

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