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Model-based feedforward for motion systems

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Abstract—This paper considers model-based feedforward for motion systems. The proposed feedforward controller consists of an acceleration feedforward part and an inverse dynamics model of flexible modes. Based on analysis and H_∞ model-based feedforward design, the inverse dynamics part can be restricted to a second order filter in the form of a skew notch even if the motion system has more parasitic modes. The benefit of this is an on-line tuning possibility. Tracking errors and settling times can be reduced significantly compared to acceleration feedforward.

I. INTRODUCTION

IN many today's motion systems, high performance requirements involve short motion times (and hence high accelerations) and small settling times. To fulfill these demands, a combination of feedback and feedforward control is normally used in a two degree of freedom (2 DOF) controller structure, see Figure 1. The feedback controller guards (robust) stability and improves disturbance rejection, while the feedforward controller improves tracking performance.

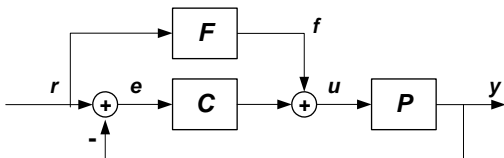


Fig. 1. Standard 2 DOF control architecture.

In Fig. 1, P , C , and F denote plant, feedback controller, and feedforward controller, respectively. Signals are written in lower case: the reference trajectory r , servo error e , plant input u , plant output y , and feedforward function f . In this paper, only feedforward controller design is considered, SISO linear time invariant plants are the focus, in particular motion systems. As objective function we want to minimize the *tracking error function*, defined as:

$$\frac{e}{r} = \frac{1 - PF}{1 + PC} = S(1 - PF) \quad (1)$$

, with $S = (1 + PC)^{-1}$ the sensitivity function. A common strategy is to make the product of plant and feedforward

controller unity, hence to find a feedforward controller that equals the plant inverse

$$F = P^{-1} \quad (2)$$

. This works well as long as the feedforward model approximates the plant inverse sufficiently well. In practice, uncertainties and unmodelled and non-minimum phase dynamics make this difficult, if not impossible. The mismatches are filtered by the sensitivity function which in turn depends on the dynamics of the plant and the feedback controller. Obviously, feedforward control will not affect closed loop stability, provided F itself is stable. Moreover, the size (in some sense) of plant mismatch in the feedforward controller can be up to the size of the plant before reducing the tracking performance, compared to the situation without feedforward [1].

A. Acceleration feedforward

In many industrial motion controllers, *acceleration feedforward* is used. This feedforward action compensates for the low frequent, rigid body behavior of the plant dynamics and therefore compensates the mass line in the frequency response of the plant. In industrial practice, acceleration feedforward is always tuned on-line with a simple gain, correcting for uncertainties in the overall loop gain (mass, amplifier gain, sensor gains etc). With a well tuned feedback [8] and acceleration feedforward controller, servo errors during a motion may not be zero due to flexible dynamics (see Figure 2). These servo errors increase with more severe motions, e.g., shorter motion profiles or steeper trajectories, i.e. higher jerks (derivative of acceleration) and accelerations. The servo errors typically peak during the jerk phases of the reference trajectory. The servo errors are highly reproducible and limit performance in both tracking and point-to-point motion systems.

B. Advanced feedforward

The oscillatory effect in these servo errors can be reduced by using impulse input shapers [2], [7]. Impulse input shapers can be considered as special cases of notch filters, that are uniquely formulated to achieve fast motions with minimum

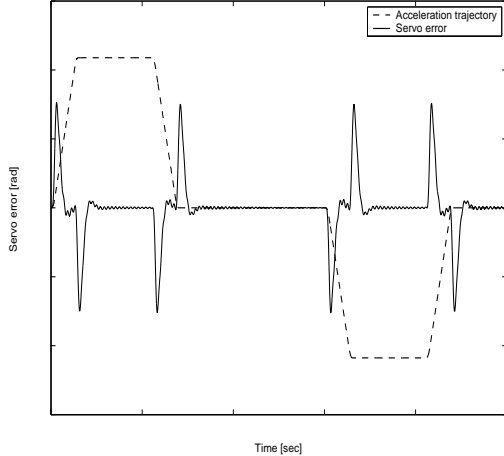


Fig. 2. Servo error using acceleration feedforward and the acceleration of the reference trajectory (scaled).

residual vibrations [4],[5]. Analysis of the inverse dynamics shows that a pure (albeit damped) notch does not match the inverse dynamics exactly. In literature, plant inversion techniques are discussed to make higher mode feedforward controllers [3], [9], [10], [11]. A major drawback is that these advanced feedforward controllers compensate for all parasitic modes taken in the model, whereas these modes often contain high levels of uncertainties. Also, the on-line tuning of these inverse model-based feedforward controllers is very complex in practical environments due to the large number of tuning parameters.

C. Problem formulation and outline

In this paper, a feedforward controller is proposed to compensate for higher mode dynamic effects, such that on-line tuning is feasible. The feedforward controller is derived from a simplified model of the dynamics of the plant (Section II). The proposed controller consists of an acceleration part as well as a skew notch part, see Section (III). In Section (IV) the solution is compared to an advanced feedforward controller designed with H_∞ techniques. The skew notch has better performance, since it can be constructed in a non proper way and thereby approximates the inverse plant more accurately. Also, the skew notch controller can be tuned on-line, which is an advantage especially for higher order systems. Finally, conclusions are drawn in Section V.

II. PLANT DYNAMICS

Electro-mechanical motion systems are studied in this paper. Typical high performance motion systems are direct drive and behave like rigid body (open-loop unstable) dynamics in low frequency regions. In higher frequency regions, effects of

limited mechanical stiffness show up as resonance behavior in the frequency response. A physical interpretation of the dynamics can be made using a lumped parameter model, i.e., a series connection of masses, springs, and dampers (Figure 3). The position is measured on one of these masses (x_n),

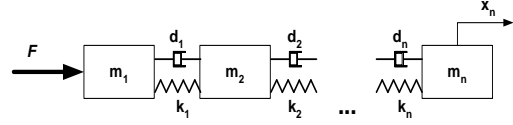


Fig. 3. Lumped parameter model of plant dynamics

while the actuator force F is applied on the most left mass m_1 only. In this paper, we will use as a practically relevant and representative example a sixth order non co-located plant, i.e., with three masses, and with measurement on the load side (third mass). A Bode diagram is given in Figure 4.

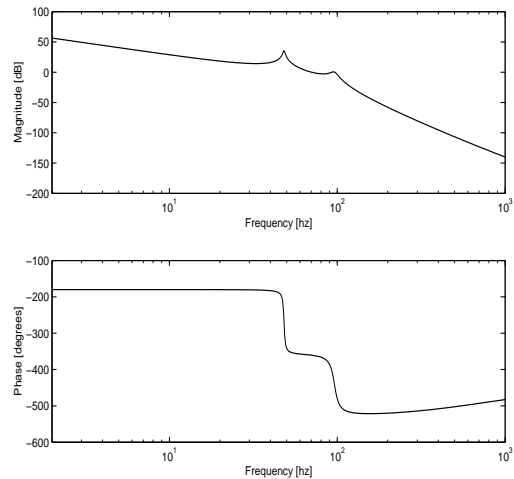


Fig. 4. Bode diagram of the example system as in Fig. 3 with $n=3$.

III. MODEL-BASED FEEDFORWARD DESIGN

As shown in (2), the inverse model of the plant is the best model based feedforward controller. Complete model inversion techniques, even for non-minimum phase systems, are discussed in [3], [9], and [11]. In the spirit of multi-body feedforward, one could derive 6th, 8th, or higher even-order feedforward controllers for the compensation of multiple plant modes [11]. The complexity of the total inverse plant feedforward will increase which makes on-line tuning more difficult. In practice, higher modes come with high levels of uncertainty and the increase of tracking performance is not spectacular as practical reference trajectories contain less energy at high frequencies [10].

The strategy used in this paper is to design a feedforward controller for higher dynamic modes as an addition to

the conventional rigid body (acceleration) feedforward. An important desirable feature is that the designed feedforward controller will not disrupt the rigid body feedforward compensation.

A. Skew notch in feedforward

From feedback design considerations, compensating for a resonance mode would involve the use of a notch, located in the feedback loop as close to the plant as possible (Figure 5).

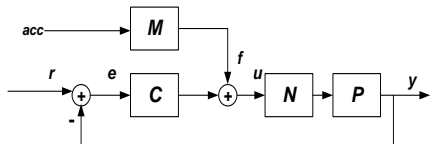


Fig. 5. Notch (N) before the plant to cancel the effect of a single mode

Cancellation of a resonance, is obtained if the poles and zeros of the notch are both placed on the exact frequency of the mode frequency. For motion systems as we consider here, i.e., non co-located, an additional 180 degree phase lag after the resonance frequency occurs, which is not compensated by a pure notch. In fact, an approximate solution is to make a *skew notch*, i.e., the pole pair will be placed at a higher frequency, see Fig. 6 for a Bode plot. Because the skew notch has a high gain (square of the pole/zero frequency ratio) at high frequencies, measurement noise will be amplified as the complementary sensitivity $PC(1 + PC)^{-1}$ will increase. Also, higher frequency resonances will be amplified. A

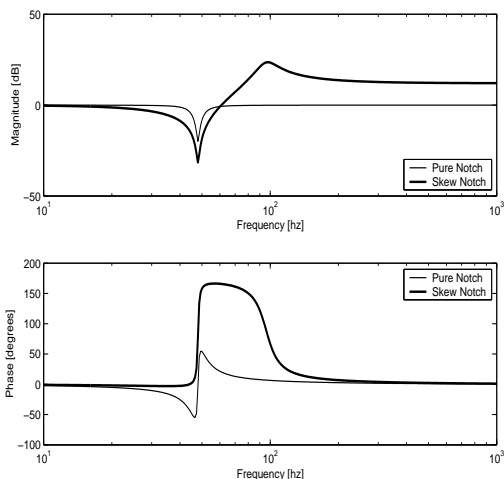


Fig. 6. Bode diagram of a pure and skew notch filter

possible solution is to rearrange the control structure and to place a notch in both the feedback and feedforward path before the injection point of the feedforward signal (Figure

7). Notice that we make use of the acceleration signal acc (second derivative of the position reference profile), which is a given function in most motion control firmware. The feedforward filter N_{FF} operates on the acceleration profile. It should therefore be equal to one at low frequencies (rigid body dynamics) and unequal to one at those frequencies where compensation of parasitic dynamics is desired. In this configuration, the notch N_{FB} in the feedback path can be tuned in a commonly used strategy for increasing bandwidth in motion systems [8]. Herein, the pole pair of the feedback notch is placed just high enough to obtain phase for increasing bandwidth and low enough to constrain the complementary sensitivity $PC(1 + PC)^{-1}$. In our example, we take a pure notch in the feedback controller. In the feedforward path, the freedom exists to place the pair of poles as high as preferred. Ultimately, the skew notch can be approximated only by its

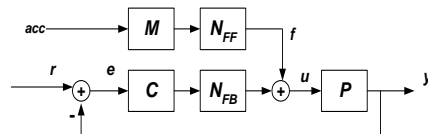


Fig. 7. Notch in feedback and feedforward path

zeros:

$$N_{FF} = \frac{s^2 + 2\zeta\omega_m s + \omega_m^2}{\omega_m^2} \quad (3)$$

The interpretation is that the feedforward filter times the plant, i.e., $N_{FF}P$, gives a highly accurate approximation of a double integrator plant, in the case P is fourth order. For higher order plants, it only compensates for a single resonance mode, which can be chosen to be the most critical one. Because the notch is filtering the acceleration feedforward signal, it is clear that jerk and the derivative of jerk should be finite. The fourth order feedforward compensator therefore requires at least a fourth order reference trajectory.

The benefit of the formulation of a skew notch filter as above, is that on-line tuning becomes practically feasible: frequency and damping are the only knobs and can easily be tuned for the best result. It should be emphasized that this is an important requirement to have new design approaches accepted in industrial practice.

B. H_∞ feedforward design

The design of the feedforward filter N_{FF} can also be done using a model-based technique like, for instance, H_∞ optimization. Again, the design will only focus on the compensation of non-rigid body plant modes. To achieve this, the plant is split up in the (un)stable rigid body part (P_r : mass-line) and the stable high modal part (P_h : resonance dynamics). The H_∞ controller $N_{FF} = K$ will then try to invert P_h under the constraint of the chosen weighting filters.

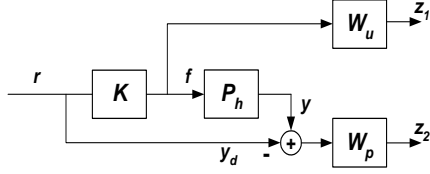


Fig. 8. Interconnection structure for H_∞ feedforward controller design

For the H_∞ design, the interconnection (Figure 8) is used. The H_∞ problem is then formulated as:

$$\min_K \left\| \begin{array}{c} M_{11} \\ M_{21} \end{array} \right\|_\infty = \left\| \begin{array}{c} W_p(P_h K - I) \\ W_u K \end{array} \right\|_\infty \quad (4)$$

As the interconnection is stable, controller K is stable as well. This means that in case of non-minimum phase dynamics in P_h no unstable zeros are cancelled. The performance weight W_p is chosen as a 2nd order low-pass filter. The high frequency amplification is then reduced and properness is guaranteed. The input weighting filter W_u is chosen as a constant that is small enough not to affect the M_{11} problem, but still non-zero in order to fulfill Ricatti solver conditions. In practical situations, W_u can be used to avoid actuator saturation.

The H_∞ designed feedforward controller shows a good approximation of the inverse plant up to the chosen cut-off frequency of W_p . The effects of this cut-off frequency are similar to the frequency of the pole pair in the skew notch configuration. An H_∞ controller for a fourth order plant will therefore be a worse approximation to the inverted plant than the proposed skew notch controller in equation (3). Also notice that it in general, the DC-gain of the calculated filter is not exactly equal to one. In our case, it appeared to be 0.9998. Even such a small deviation has a significant effect on the rigid body compensation, i.e., the acceleration feedforward. Hence, the DC-gain is forced to equal unity with a negligible loss of high frequent optimality, by adjusting the gain of the filter afterwards. For multi mode systems ($n > 2$ for Figure 3), the same H_∞ design setup can be used. Again, the inverse of the plant will be approximated up to the specified cut-off frequency which is limited by implementation.

IV. SIMULATIONS AND ANALYSIS

Feedforward design for the sixth order plant example is illustrated. The system shows two resonance peaks in the frequency response (48 and 100 Hz, respectively), see Fig. 4. For this system, a feedback controller is used, consisting of a lead/lag filter as well as a pure notch to suppress the first resonance. The cross-over frequency (where the open-loop gain PC equals 0 dB) is at 12 Hz with appropriate robustness margins [8]. Conventional acceleration feedforward is used to compensate for the rigid body behavior. The servo errors

with this setup are shown in Figure 11. The skew notch (3), and the H_∞ feedforward controller will be designed on a model containing only the first plant mode.

The frequency responses of the skew notch and H_∞ controller are very similar, see Fig. 9. Note that the skew notch and the H_∞ controller approximate the plant inverse up to a certain frequency. The skew notch (3) has the advantage of a non-proper construction possibility, i.e., to translate it in terms of a feedforward signal. The H_∞ controller has to rely on the filtering of the acceleration signal and only approximates the first mode plant inverse up to the cut-off frequency specified in the design weight filters (W_p).

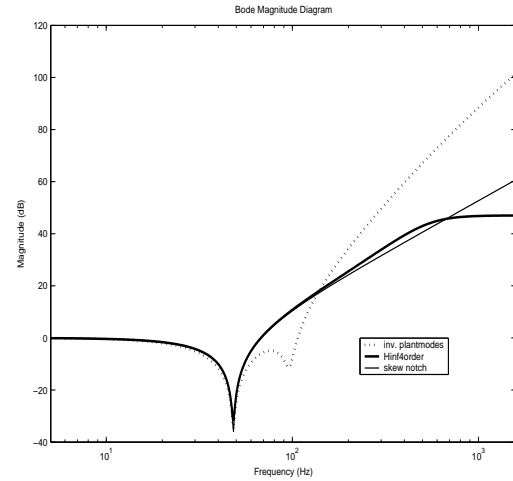


Fig. 9. Magnitude frequency responses of the feedforward controllers compared to the inverse plant modes.

A. Tracking error function

The tracking error function in equation (1) relates $r \rightarrow e$, the tracking error to the reference trajectory. The frequency response function of this transfer function shows the performance of the feedforward controllers over the whole frequency range, see Figure 10. From the figure it is clear that all newly proposed feedforward controllers reduce the function very well around the first mode. The skew notch and the H_∞ feedforward controller almost give the same result. As both controllers only compensate the -4 slope, the second plant mode will be amplified. Tracking performance will become worse in this frequency region, compared to rigid body feedforward.

B. Tracking performance

A direct performance indicator is the servo error during motion. In a finite order polynomial reference trajectory motion, servo errors in different motion phases are directly related to compensated plant modes. In the simulation, a reference trajectory is used where the fourth derivative of position is finite. In

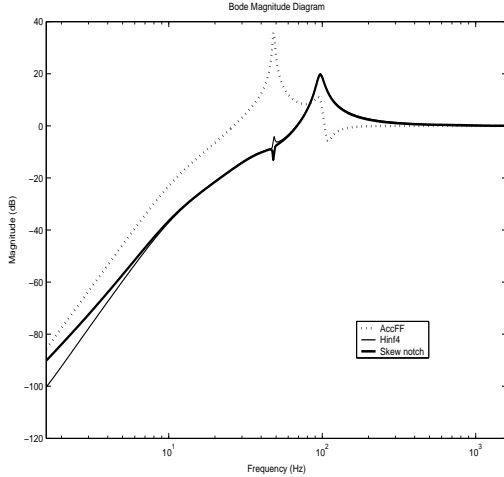


Fig. 10. Magnitude frequency responses of the tracking error function for several feedforward controllers.

Figure 11, the servo error is shown for the various feedforward filters. Compared to the conventional acceleration feedforward,

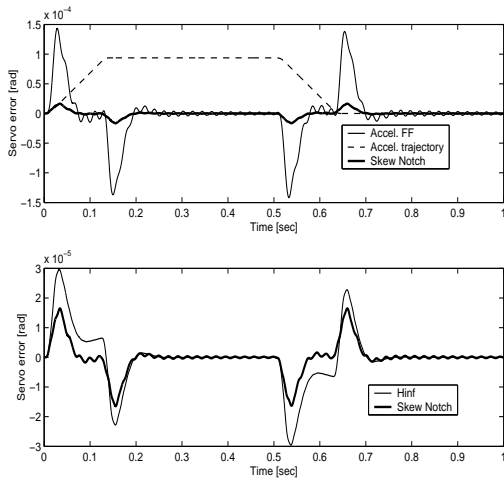


Fig. 11. Servo error during acceleration.

tracking performance increases significantly using any of the proposed feedforward controllers. The H_∞ controller performs worse than the skew notch controller. The amplification of the second mode, as noticed in section IV-A, is negligible due to the limited frequency content of the reference trajectory.

C. Feedforward signal

A good indicator to analyse the effect of the feedforward controller is the feedforward signal (Fig. 12) during a reference motion. As sixth order reference is very smooth, differences between the proposed feedforward controllers are more difficult to visualize. For off-line analysis of feedforward signals, effects are mostly visible using a reference trajectory

with the same order of the feedforward controller. In this case, a fourth order reference trajectory is used.

The major part of the signal consists of rigid body compensation (acceleration feedforward). The effect of higher modes is visible during the slopes of acceleration. Also a small offset during jerk phase is noticeable (not shown here) as damping in the first mode is compensated. More importantly, small parts are taken from the feedforward function during derivative of jerk motion phases. These parts compensate for the first mode of the plant. The H_∞ controller has to construct these parts by filtering the acceleration feedforward signal. The effects of the limited frequency approximation (properness) are visible as smoother edges in the forcing function. The skew notch has the property of constructing the feedforward signal in a non-proper way. The edges in the forcing function can be sharper and result in smaller tracking errors. For both controllers, the parts taken from the forcing function are larger with more aggressive motions. These parts will be large peaks (∞ for the non-proper skew notch) if the reference profile is of third order or lower.

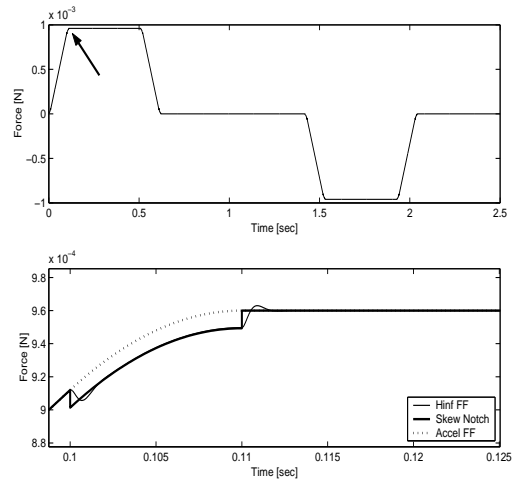


Fig. 12. Feedforward signal during motion, top figure. Bottom figure; zoomed in at the corner of the feedforward function.

V. CONCLUSIONS

Tracking performance can improve significantly with compensation of the first resonance plant mode. Compensation of higher modes can reduce the tracking error even further. However, uncertainties in modelling high frequency modes are often large. A skew notch in feedforward approximates the inverse of a single mode of the plant.

As frequency content of reference profiles is high in low frequency regions, compensation for lower plant modes is more important. In feedforward design, it is therefore not always necessary to make inversions for all plant modes. From the simulations results, it is clear that a skew notch put in

the acceleration feedforward path compensates for tracking errors of a multi mode system. The performance improvement depends on the maximum cut-off frequency of this notch. The skew notch can be formulated as a non-proper inverse of the plant. The mode compensation is then not limited by the cut-off frequency.

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