Model derivations for nutrient diversion on lakes

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Abstract

Lakes are frequently subjected to wastewater discharges. If nutrients like nitrogen, phosphorus and carbon are present in sufficiently large amounts, the water body will be polluted and eutrophication can take place. Eutrophication of a lake induced by municipal wastes can be retarded by removing the source of plant nutrients, notably phosphorus. This is accomplished by diversion of the effluent around the lake, or by treatment of the wastewater employing advanced treatment processes.

In this study, first a mathematical model with different assumptions about nutrient exchange processes is introduced, then the mass balance considering input, output, net loss to the sediments was used to predict the lake nutrient concentration as a function of nutrient residence time. Lake Iznik located in the southern part of the Marmara region of Turkey is subjected to three different levels of nutrient loadings, with a certain amount of diversions each time to study the response of the lake.

1 Introduction

Eutrophication is the enrichment of water bodies with nutrients such as nitrogen and phosphorus. One of the methods for controlling or minimizing lake eutrophication is to reduce the rate of nutrient flux into lakes. Regulatory agencies require predictions of the impact of present and future nutrient inputs on lake quality. Those who are concerned with lake management need to know the rate of improvement which might be expected as a result of reduced nutrient discharge into a lake. It is important to know how fast and to what extent will a lake respond to a change in the nutrient influx.[1] Diversion of wastewaters away from a lake has various degrees of success in decelerating or reversing the process of eutrophication.[2]

A simple model is presented in this paper to predict the response of lakes to changes in nutrient influx. The model give a good solution of the problem for the purpose of watershed planning and management.

2 Lake Iznik

Lake Iznik is located in the southern part of Marmara region within the province of Bursa (see Figure 1). It has an average surface area of 304.30 km^2, and a mean volume of 12.12 billion m^3.[3] Drainage area of the lake is 626 km^2. Lake Iznik was formed by a tectonic depression which took place during geological ages.

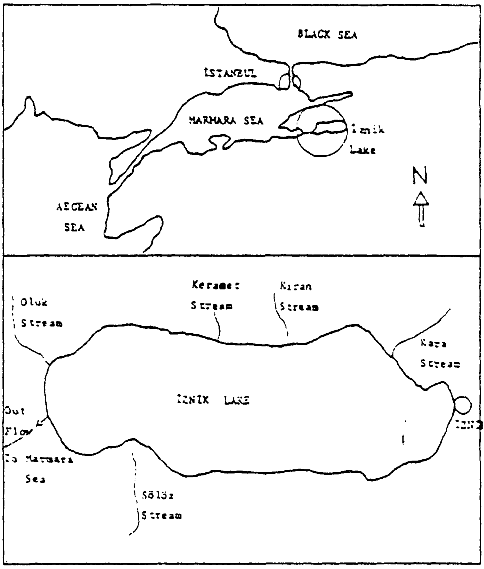

Figure 1: Drainage area of lake Iznik

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3 A Simple Model for nutrient exchange processes in Lakes

In this paper a model is derived taking into consideration different assumptions about nutrient exchange processes.[4]

Assuming that long term effects can be approximated by average annual values in a well-mixed lake, a mass balance can be written on the nutrient of interest. A diagrammatic representation of this case is shown below.

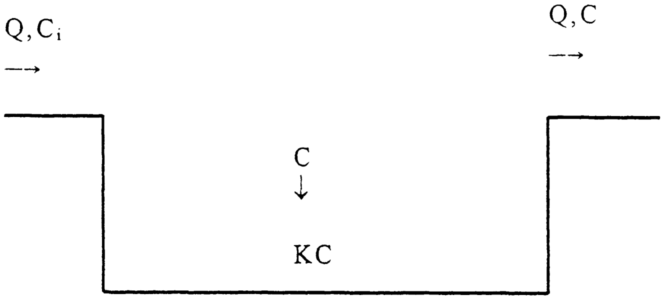

Figure 2: Diagrammatic representation of the model

The rate of change in concentration with time is given by

$$\frac{dc}{dt} = \frac{M}{V} - \frac{CQ}{V} - \frac{KCA}{V} \tag{1}$$

where

M = mass flow in from all sources, g/yr

C = average annual nutrient concentration, g/m^3

k = net specific rate of loss to sediments, m/yr

Q = average annual outflow, m^3/yr

V = lake volume, m^3

A = surface area

C_0 = nutrient concentration at time zero, g/m^3

t = time, yrs

Integration of Equation 1 from C_0 at time zero to C at time t yields

$$C = \frac{M}{Q + KA}\left(1 - e^{-\frac{Q+KA}{V}t}\right) + C_0 e^{-\frac{Q+KA}{V}t} \tag{2}$$

Equation 2 describes the concentration of nutrient in the water as a function of time as a result of changing the nutrient input rate, M.

The model can be incorporating the nutrient fluxes into and out of the sediments independently as shown in Figure 3.

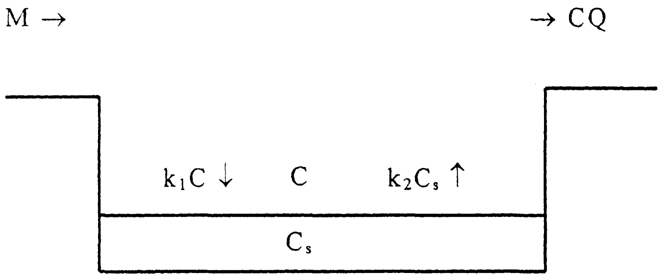

Figure 3: Diagrammatic representation of the Model

For cases in which the nutrient concentration in the sediments does not change significantly over time, mass balance yields

$$\frac{dc}{dt} = \frac{M}{V} + \frac{k_2C_sA}{V} - \frac{k_1CA}{V} - \frac{CQ}{V} \tag{3}$$

in which

M, C, Q, V, A, C_0 are as previously defined
k_1 = specific rate of nutrient transfer to sediment, m/yr
k_2 = specific rate of nutrient transfer from sediment, m/yr
C_S = nutrient concentration in sediment, g/m^3

The solution to Equation 3 is

$$C = \frac{M + k_2C_sA}{Q + k_1A}\left(1 - e^{-\frac{Q+k_1A}{V}t}\right) + C_0 e^{-\frac{Q+k_1A}{V}t} \tag{4}$$

In this case the specific rates k_1 and k_2 must be determined independently.

Thirdly we can take into account the possibility that nutrients may be depleted from the sediments when nutrient input to the water is decreased. In this case the value of C_s is variable.

The diagrammatic representation of this model is shown in Figure 4. Notice that here we have a set of coupled differential equations representing the nutrient concentration in water (C) and sediment (C_s).

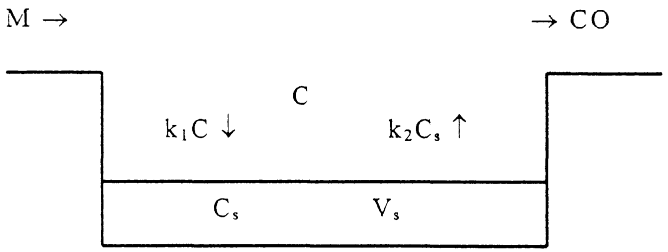

Figure 4: Diagrammatic representation of the Model

$$\frac{dc}{dt} = \frac{M}{V} + \frac{k_2 C_s A}{V} - \frac{k_1 CA}{V} - \frac{CQ}{V} \tag{5}$$

$$\frac{dC_s}{dt} = \frac{k_1 CA}{V_s} - \frac{k_2 C_s A}{V_s} \tag{6}$$

in which M, C, Q, V, A, C_0, k_1, k_2, C_s are as previously defined and V_s volume of sediment. The solution to the coupled set of Equations 5 and 6 is somewhat more complicated and the solution procedure is outlined in detailed in the reference.[4]

3.1 First form of the model presented in this study

The last form of the model presented above require information about the k_1, k_2 rates, C_s nutrient concentration in sediment and V_s volume of sediment. Therefore at first step the simple model represented in Equation 1 was studied.

The model considers a completely mixed system with phosphorus input, out flow and internal loss to sediment (See Figure 2). As stated earlier Equation 2 describes the concentration of nutrient in the water as a function of time as a result of changing the nutrient input rate, M.

In Equation 2 (Q+KA)/V is equal to $1/R_n$.R_n is the nutrient residence time for a lake. For time infinite it yields an equilibrium concentration of

$$C_{eq} = \frac{M}{Q + KA} \qquad (7)$$

The above stated equilibrium concentration is approached exponentially as a function of phosphorus residence time (Equation 2), and is represented in Figure 5.

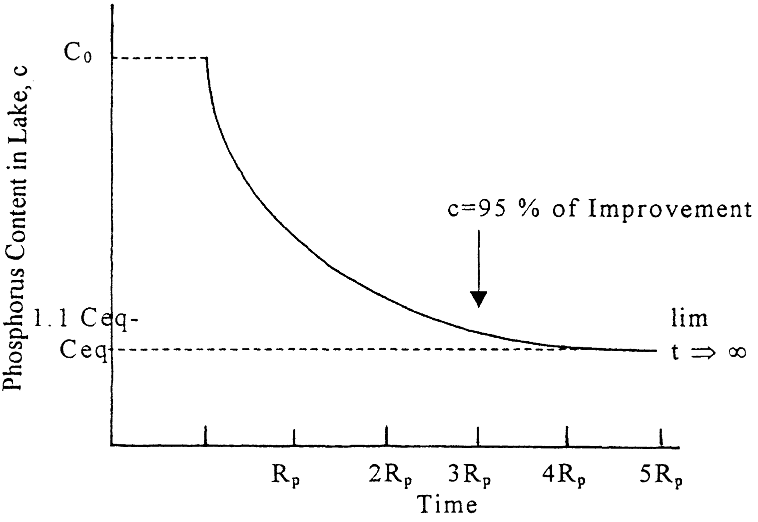

Figure 5: Rate of recovery of a lake

The time required to change from one equilibrium concentration C_0 to within 10 percent of a new equilibrium value of C_{eq} can be obtained from Equation (1) with the use of expression 7. Integration of Equation 1 yields

$$\ln\frac{(C - C_{eq})}{(C_0 - C_{eq})} = -\frac{(Q + KA)}{V}t \tag{8}$$

If 1.1 C_{eq} is substituted for C in above equation, the corresponding time t_{10} will result as

$$t_{10} = \frac{V}{Q + KA}\left\{2.3 + \ln\left[\frac{(Q + KA)C_0}{M} - 1\right]\right\} \tag{9}$$

From Figure 5, information can be seen that for practical purposes this t_{10} gives a satisfactory result in order to calculate the time elaps to reach the new equilibrium concentration.

4 Predicting the effects of nutrient diversion on lake Iznik

The above model can be adapted to nutrients such as nitrogen and carbon, but the relationship would necessarily be more complex since a gas phase must be considered in the aqueous chemistry of nitrogen and carbon. Therefore in this study the model is applied only to phosphorus. In an earlier study supervised by the author of this paper[5] phosphorus was found to be the most limiting element for lake Iznik.

The major inputs to a water body generally are wastewater discharges, land runoff, the atmosphere and groundwater. Calculated results of the phosphorus loadings in the drainage area of lake Iznik from different sources are given in Table 1.

Table 1. Estimated annual phosphorus loads to lake Iznik.

Source	Phosphorus (P)x10^3 kg P/year
Agricultural area	0.624
Farm animal	30.85
Sewage	36.24
Forests	2.4
Aerial	7.61
Total	77.71

The data presented in Table 1 was obtained from the Directory of Agriculture in Bursa city. Census results and the nutrient transfer coefficients were gathered from Census of Population records published by the State Statistical Institute and from a paper written by Rast and Lee.[6]

The other data used in the model are given in Table 2 below

Table 2. Some characteristics of lake Iznik

Out flow rate Q (m^3/yr)	37.17×10^6
Net specific rate of loss to sediments, K(m/yr)	8.60
Surface area, A (km^2)	304.30
Volume, V(m^3)	12.12×10^9

Q was calculated from the water balance of Iznik lake, and the other parameters in the table were obtained from the 1st Region Directory of the State Hydraulic Works (DSI).[3]

The Model can now be used to analyse the response of Iznik lake when the phosphorus loading rate, given in Table 1, is subjected to changes. For this reason the discharge is diverted from the lake or the wastewater is subjected to treatment.

Recent available measured data for the lake is for the year 1990. Therefore the initial lake nutrient concentration corresponds to this year, and its magnitude is 0.029 mg/l of total phosphorus.[7]

In Figure 6 three different discharge rates are shown. If a reduction of 50 % of the total loading is assumed, a phosphorus equilibrium concentration of

$$C_{eq} = \frac{M}{Q+KA} = \frac{0.50 \times 77.71 \times 10^6}{37.17 \times 10^6 + 8.60 \times 304.3 \times 10^6}$$

$$C_{eq} = 1.47 \times 10^{-2} \text{ mg/l}$$

is expected, and t_{10} is

$$t_{10} = 11.21 \text{ yr}$$

As was stated earlier with the use of Equation 9 t_{10} is calculated and above magnitude gives us a close approximation about the time at which new equilibrium concentration of 1.47 x 10^{-2} mg/l is reached.

The second reduction is 2/3 of the total discharge. In this case, a new equilibrium concentration of 0.98 x 10^{-2} mg/l is attained and t_{10} is 14.64 yr. The third reduction is 75% and the resulting C_{eq} and t_{10} are 7.33 x 10^{-3} mg/l and t_{10}=16.6 yr respectively (see Figure 6).

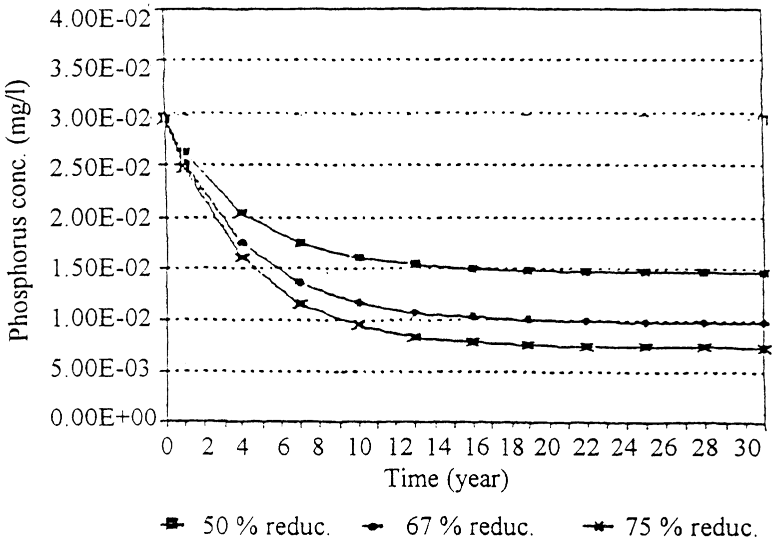

Figure 6: Expected recovery rates of lake Iznik.

5 Conclusion

It can be seen from Equation 7 that, the mean phosphorus content of the lake will be reduced in direct proportion to the change in the influx. As seen in Figure 6 the initial concentration in the lake is 0.0293 mg/l for total phosphorus for the year 1990, and its reduction is 50%, 67% and 75% with the resulting C_{eq} values of 0.0147 mg/l, 0.0098 mg/l and 0.00733 mg/l.

The recovery of the lake can be given in terms of the phosphorus residence time. For this purpose Equation 2 shows that the new steady state concentration will be approached exponentially as a function of the phosphorus residence time. In terms of the phosphorus residence time, the time required to reach 50% of the expected change is 0.69 R_p, 95% of the expected change will be reached in a period of 3 R_p.

Those equilibrium concentrations are theoretically reached at time infinity. As described earlier, the time t_{10} gives the approximate time of new equilibrium concentrations. Magnitudes of t_{10} are 11.21 yr, 14.64 yr, 16.6 yr for the 3 cases studied.

C_{eq} and t_{10} represent the new equilibrium concentrations and the time when they will be reached for the three different scenarios considered for lake Iznik.

The model presented here represents a simple yet useful tool for regulatory agencies responsible for watershed planning and management.

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