Applications of Mathematics

Jaroslav Mlýnek; Roman Knobloch

Model of shell metal mould heating in the automotive industry

Applications of Mathematics, Vol. 63 (2018), No. 2, 111–124

Persistent URL: http://dml.cz/dmlcz/147185

63 (2018) APPLICATIONS OF MATHEMATICS No. 2, 111–124

MODEL OF SHELL METAL MOULD HEATING IN THE AUTOMOTIVE INDUSTRY

Jaroslav Mlýnek, Roman Knobloch, Liberec

Received March 30, 2017. First published March 30, 2018.

Abstract. This article focuses on heat radiation intensity optimization on the surface of a shell metal mould. Such moulds are used in the automotive industry in the artificial leather production (the artificial leather is used, e.g., on car dashboards). The mould is heated by infrared heaters. After the required temperature is attained, the inner mould surface is sprinkled with special PVC powder. The powder melts and after cooling down it forms the artificial leather. A homogeneous temperature field of the mould is a necessary prerequisite for obtaining a uniform colour shade and material structure of the artificial leather. The article includes a description of a mathematical model that allows to calculate the heat radiation intensity on the outer mould surface for each fixed positioning of the infrared heaters. Next, we use this mathematical model to optimize the locations of the heaters to provide approximately the same heat radiation intensity on the whole outer mould surface during the heating process. The heat radiation intensity optimization is a complex task, because the cost function may have many local minima. Therefore, using gradient methods to solve this problem is not suitable. A differential evolution algorithm is applied during the optimization process. Asymptotic convergence of the algorithm is shown. The article contains a practical example including graphical outputs. The calculations were performed by means of Matlab code written by the authors.

Keywords: heat radiation; heat conduction; optimization; differential evolution algorithm; mathematical model; parallel programming

MSC 2010: 65C20, 80M50, 93A30

1. Introduction

Procedures and calculations described in this article form part of technology of artificial leather production. The leather is used for surfacing of car interiors (dashboards, inside parts of car doors). The producer utilizes the production process

This work was supported by SGS project of FP, Technical University of Liberec.

DOI: 10.21136/AM.2018.0086-17

based on heating of relatively thin shell metal moulds by means of infrared heaters. Infrared heaters are used, since this way of heating is characterized by approximately 30% lower energy consumption compared to oil heating, hot air heating or hot sand heating. When infrared heating is used, it is necessary to ensure a uniform heat radiation intensity (within a given tolerance) on the whole outer mould surface to attain the same colour shade and material structure of the artificial leather. The uniform heat radiation can be achieved by finding suitable locations of the heaters.

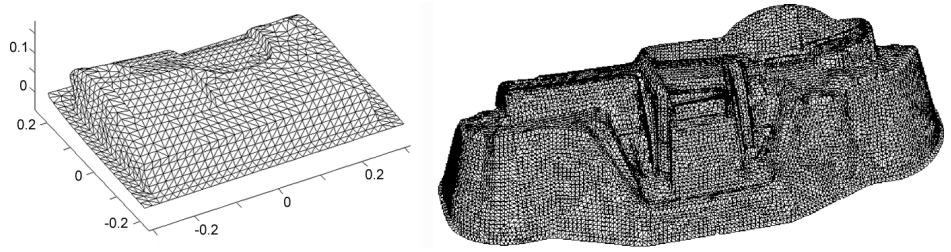
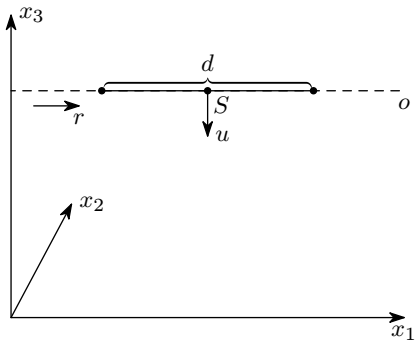


Figure 1. Shell metal moulds.

Shell moulds (of aluminium or nickel alloys) of constant thickness 6–8 mm which usually have complicated shapes and weigh from 100 to 300 kg are used in the production (see Fig. 1). The infrared heaters have a tubular form and their length is approximately 20 cm. Each heater is equipped with a mirror located above the radiation tube that reflects the heat radiation in the set direction (see the right part of Fig. 2).



S—centre of the heater
d—length of the heater
o—heater axis
r—unit vector of the heater axis direction
u—unit vector of the radiation direction

Figure 2. Schematic representation of the infrared heater (on the left) and Philips infrared heater with 1000 W capacity (on the right).

We have been cooperating with the producer of artificial leather approximately for 10 years and conceived a mathematical model of heating of the mould. The model

has been improved gradually during this period. The aim of the model is to optimize the heat radiation incident onto the mould surface. We presented in the meantime partial results and developed procedures used in the production technology. In this article we provide a comprehensive survey of the procedures and calculations that proved most successful and that make part of the current production technology.

The heating optimization is relatively complicated. The moulds have usually complex shapes—see Fig. 1 and the cost function may have many local minima. In addition, possible collisions between two heaters as well as collisions between a heater and the mould have to be avoided during the optimization process.

Therefore, the application of gradient methods is unsuitable. Instead, we have used evolution optimization algorithms (see [12], [1]). First, genetic algorithms were tested. The obtained results were further improved by a Hill Climbing procedure (see [9]). However, better optimized positions of the heaters were found by differential evolution algorithms. Finally, we modified the differential evolution algorithm known as `DE/rand/1/bin` (for details see [11]) and obtained the best results. The modified algorithm is hereafter denoted by MDEA. When using MDEA, the asymptotic convergence to the global minimum of the cost function can be proved.

2. Mathematical model of the heat radiation

In this chapter the mathematical model of the heat radiation is described. The heaters and the heated mould are considered in the Euclidean space E_3 with the Cartesian coordinate system (O, x_1, x_2, x_3) with basis vectors $e_1 = (1, 0, 0)$, $e_2 = (0, 1, 0)$ and $e_3 = (0, 0, 1)$.

2.1. Heater representation. Each heater is represented by a straight line segment of length d (see the left part of Fig. 2). The position of the heater can be described by the following parameters:

(i) The coordinates of the heater centre $S = [s_1, s_2, s_3]$.

(ii) The unit vector $u = (u_1, u_2, u_3)$ of the heat radiation direction. We can suppose that the component $u_3 < 0$, which means that the heater always radiates (at least partially) "downward".

(iii) The unit vector $r = (r_1, r_2, r_3)$ of the heater longitudinal axis o.

In this case we would have nine quantities describing the heater position. But some of these quatities are dependent. For instance the vectors u and r are orthogonal. In mechanics it is a well known fact that six parameters are sufficient to fully determine a body position in a space. So, we have to reduce the number of quantities characterizing the heater position. It is suitable to use just the first two coordinates u_1, u_2 of the vector u. The vector r can then be described just by one quantity, for instance

by the angle φ between the vertical projection of the vector r onto the x_1x_2-plane and the positive part of axis x_1 ($0 \leqslant \varphi < \pi$). The position of each heater Z can then be defined by 6 parameters

$$Z = (s_1, s_2, s_3, u_1, u_2, \varphi). \tag{2.1}$$

Since we consider heating by M heaters, we have $6M$ parameters that describe the positions of all heaters.

2.2. Mould representation. The outer mould surface P is described by elementary surfaces p_j, where $1 \leqslant j \leqslant N$, such that $P = \bigcup p_j$ and $\operatorname{int} p_i \cap \operatorname{int} p_j = \emptyset$ for $i \neq j$, $1 \leqslant i, j \leqslant N$. Each elementary surface p_j can be determined by the following parameters:

(i) Its centroid $T_j = [t_1^j, t_2^j, t_3^j]$.

(ii) The unit vector of the outer normal $v_j = (v_1^j, v_2^j, v_3^j)$ at the point T_j (we can suppose v_j faces "upwards" and therefore it is defined through the first two components v_1^j and v_2^j).

(iii) The area c_j of the elementary surface.

Each elementary surface p_j can then be defined by 6 parameters

$$p_j = (t_1^j, t_2^j, t_3^j, v_1^j, v_2^j, c_j). \tag{2.2}$$

2.3. Calculation of the heat radiation intensity on an elementary surface. To be able to calculate the heat radiation intensity incident on an elementary surface we need to know how the heat radiation intensity is distributed in the heater neighbourhood. The heaters manufacturer did not provide this distribution of the heat radiation in space and we had to measure it experimentally. First, we describe the experimental measuring of the heat radiation intensity in the heater neighbourhood for the heater in the "basic position". Next, we show a suitable transformation of the measured values to determine the heat radiation intensity for the heater in a general position.

2.3.1. Experimental measurement of the heat radiation intensity. We set up the experimental measurement of the heat radiation intensity for the location of the heater $Z = (0, 0, 0, 0, 0, 0)$ in accordance with relation (2.1). It means that the centre S of the heater is at the origin of the Cartesian coordinate system (O, x_1, x_2, x_3), the unit radiation vector has coordinates $u = (0, 0, -1)$ and the vector of the heater axis has coordinates $r = (1, 0, 0)$. Only points under the heater are relevant. The heat radiation is symmetric with respect to the plane going through axis o and vector u located in the centre of the heater S (see the left part of Fig. 2)

and the plane going through the centre S and perpendicular to the heater axis o. Therefore, it is sufficient to carry out the experimental measurement of the heat radiation in one octant (determined by the two planes and the ground plane). The experimental measurements were carried out at grid points of a regular rectangular mesh in several horizontal planes parallel to the ground plane by means of the robot KUKA 8.2 equipped with a sensor (see Fig. 3). The specific arrangement of the experimental measurements of the heat radiation intensity around the infrared heater is described in detail in [8]. The subsequent processing utilizes the linear interpolation of a function of five variables to interpolate the measured values (for details see [2]).

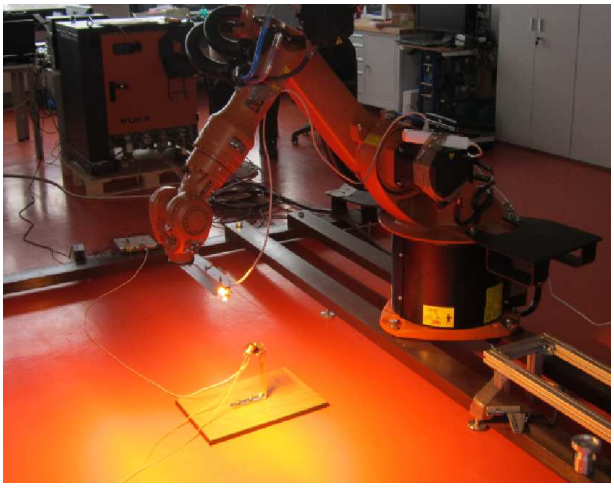


Figure 3. Experimental measurement of the heat radiation intensity using the robot KUKA 8.2.

2.3.2. General case of a heater and an elementary surface locations. Now, we focus on a case when a heater and an elementary mould surface p_j are in general positions. We can assume that the heat radiation intensity on the whole elementary surface p_j is the same as at its centroid T_j since the elementary surface is small. The heat radiation intensity at T_j depends on the position of this point (determined by the first three parameters of the elementary surface p_j given by (2.2)), on the direction of the outer normal vector v_j at the point T_j (determined by the fourth and fifth parameters of the elementary surface p_j given by (2.2)) and on the location of the heater. For a heater in a general position, we briefly describe the transformation of the previous Cartesian coordinate system (O, e_1, e_2, e_3) into a positively oriented Cartesian system $(S, r, n, -u)$, where S is the centre of the heater, r is the vector of the heater axis, and u is the direction vector of the heat radiation. The vector n

is determined by the vector product of the vectors $-u$ and r (i.e. $n = (-u) \times r$, for more details see [3], [13]). The vectors r, u, and n are normalized to have the unit length. Then we can define an orthogonal transformation matrix $\mathbf{A}$. If S is the triple of parameters representing (in (O, e_1, e_2, e_3)) the centre of the heater that determines the coordinate system $(S, r, n, -u)$, then T_j and v_j are transformed as

$$(T_j')^{\mathrm{T}} = \mathbf{A}^{\mathrm{T}}(T_j - S)^{\mathrm{T}} \quad \text{and} \quad (v_j')^{\mathrm{T}} = \mathbf{A}^{\mathrm{T}} v_j^{\mathrm{T}};\ \mathbf{A} = \begin{pmatrix} r_1 & n_1 & -u_1 \\ r_2 & n_2 & -u_2 \\ r_3 & n_3 & -u_3 \end{pmatrix},$$

where T_j' and v_j' are the coordinates in $(S, r, n, -u)$. In this way, we transform the general case of the heater location to the experimentally measured case described at the beginning of this subsection.

2.4. Total heat radiation intensity and uniformity of its distribution. Now, we describe the numerical computation of the total heat radiation intensity on the mould surface. Let us suppose that all the heaters are located in fixed positions. We denote by L_j the set of all heaters radiating onto the jth elementary surface p_j $(1 \leqslant j \leqslant N)$. By I_{jl} we denote the heat radiation intensity from the lth heater incident on the p_j elementary surface. Then the total radiation intensity I_j on the elementary surface p_j is given by (see [4])

$$I_j = \sum_{l \in L_j} I_{jl}. \tag{2.3}$$

The producer of artificial leather recommends a constant value of the heat radiation intensity on the mould. Let us denote this constant value by I_{rec}. This specific value is hardly attainable with a limited number of heaters. The goal is to achieve approximately uniform intensity close to the recommended value on the whole outer mould surface. We can define functions F and $\widetilde{F}$ that quantify the deviation of the intensity from the recommended value I_{rec} by

$$F = \frac{1}{W} \sum_{j=1}^{N} |I_j - I_{\mathrm{rec}}| c_j, \quad \widetilde{F} = \sqrt{\frac{1}{W} \sum_{j=1}^{N} (I_j - I_{\mathrm{rec}})^2 c_j}, \tag{2.4}$$

where $W = \sum\limits_{j=1}^{N} c_j$. Let us recall that c_j denotes the area of the elementary surface p_j. The arguments of the deviation functions are the $6M$ parameters describing the positions of all heaters in compliance with (2.1). We need to find such locations of the heaters that minimize the value of the deviation function F (alternatively the value of the deviation function $\widetilde{F}$).

3. Optimization of the locations of the heaters

When we use optimization terminology, the functions F and $\widetilde{F}$ defined in (2.4) can be called cost functions and we need to minimize them. These functions may have many local minima. This means that gradient methods are in this case not appropriate. If we applied them, we would find with high probability only a local minimum. Instead, we use MDEA to identify the global minimum of the corresponding cost function. We use the notation of parameters usual in the area of differential evolution algorithms (see e.g. [11], [14]). The position of each heater is defined in accordance with the relation (2.1) by 6 parameters. Therefore, $6M$ parameters are necessary to define the positions of all M heaters. One individual in MDEA represents one possible configuration of all M heaters. In the algorithm we successively construct generations of many individuals. Each generation includes N_P individuals, where each individual is a potential solution of the problem. We seek an individual $y_{\min} \in C$ satisfying the condition

$$F(y_{\min}) = \min\{F(y);\ y \in C\}, \tag{3.1}$$

where $C \subset E_{6M}$ is a search space of all possible heaters positions. Each element of C is a set of $6M$ admissible parameters and defines one configuration of the heaters above the mould. In practice, we may not be able to find the individual $y_{\min}$ defined by (3.1). Nonetheless, we are able to determine an optimized solution y_{opt}.

3.1. Modified differential evolution algorithm (MDEA). In this part we briefly describe the operation of MDEA.

First, we define a specimen SPEC which determines a type and value ranges of each parameter of the individual $y \in C$. Then SPEC can be expressed in the form

$$\text{SPEC} = \{\{\text{type}_1, Lo_1, Hg_1\}; \{\text{type}_2, Lo_2, Hg_2\}; \ldots; \{\text{type}_6, Lo_6, Hg_6\}\}. \tag{3.2}$$

Here type_m specifies the type of the mth parameter in relation (2.1) (in this case type_m is a real parameter), the values Lo_m and Hg_m determine its lower and upper limit, where $1 \leqslant m \leqslant 6$. The specimen determines the admissible parameter values for one heater. It is used for position limitations of all M heaters in each individual. In this way the relation (3.2) determines the domain of the cost function F or $\widetilde{F}$ defined by relation (2.4). One suitable way of forming the initial generation of individuals is

$$y_{i,j} := Lo_j + \text{rand}(0,1) \cdot (Hg_j - Lo_j), \tag{3.3}$$

where index j determines the jth component of the ith individual ($1 \leqslant i \leqslant N_P$, $1 \leqslant j \leqslant 6M$). The function rand$(0,1)$ randomly generates a value from a closed interval $\langle 0, 1\rangle$.

We create a series of generations $G(k)$, where k denotes the generation number. Each generation consists of individuals y and we look for an individual with the smallest value $F(y)$. Four individuals of the current generation participate in the creation of an individual of the next generation. The generated individuals are saved in a matrix $\mathbf{B} \in \mathbb{R}^{N_P \times (6M+1)}$. Each row of this matrix represents one individual y and its evaluation $F(y)$.

During the creation of individuals it is necessary to ensure that the components of each generated individual are in conformity with relation (3.2), and to eliminate possible collisions between heaters and between heaters and the mould. Only such individuals are formed when no two heaters are in a collision. Specifically, each two heaters have a defined minimal distance and each heater has a minimal distance from the upper part of the mould. Now, we briefly describe the MDEA algorithm.

INPUT: Generation size N_P, crossover probability C_R, mutation factor f, the number of calculated generations N_G, lower limits Lo_m and upper limits Hg_m in relation (3.2), $1 \leqslant m \leqslant 6$. The dimension of individuals is $6M$ (where M is the number of heaters).

COMPUTATION:

1. Create the initial generation ($k = 0$) of N_P individuals y_i^k, $1 \leqslant i \leqslant N_P$ (e.g. by use of relation (3.3)).
2. a) Evaluate all the individuals y_i^k of the kth generation (calculate $F(y_i^k)$ for each individual y_i^k).
   b) Store the individuals y_i^k and their evaluations $F(y_i^k)$ in the matrix $\mathbf{B}$.
3. *while* $k \leqslant N_G$
   a) *for* $i := 1$ *step* 1 *to* N_P *do*
   collision := *true*
   repeat
   (i) randomly select index $m_i \in \{1, 2, \ldots, 6M\}$,
   (ii) randomly select indices $\varrho_1, \varrho_2, \varrho_3 \in \{1, 2, \ldots, N_P\}$, where $\varrho_t \neq i$ for $1 \leqslant t \leqslant 3$ and $\varrho_1 \neq \varrho_2$, $\varrho_1 \neq \varrho_3$, $\varrho_2 \neq \varrho_3$;
   (iii) *for* $j := 1$ *step* 1 *to* $6M$ *do*
   if (rand$(0,1) \leqslant C_R$ *or* $j = m_i$) *then* $y_{i,j}^{\text{trial}} := y_{\varrho_3,j}^k + f(y_{\varrho_1,j}^k - y_{\varrho_2,j}^k)$
   else $y_{i,j}^{\text{trial}} := y_{i,j}^k$
   end if
   end for (j)

(iv) Testing of possible collisions between the heaters determined by y_i^{trial}.
if y_i^{trial} does not include collisions *and* y_i^{trial} satisfies (3.2) *then*
collision := *false*
until collision = *false*
end repeat
(v) *if* $F(y_i^{\text{trial}}) \leqslant F(y_i^k)$ *then* $y_i^{k+1} := y_i^{\text{trial}}$
else $y_i^{k+1} := y_i^k$
end if
end for (i),

b) Store individuals y_i^{k+1} and their evaluations $F(y_i^{k+1})$ $(1 \leqslant i \leqslant N_P)$ of the new $(k+1)$st generation in the matrix $\mathbf{B}$, $k := k+1$.

c) Find index n which satisfies the condition $F(y_n^k) \geqslant F(y_i^k)$, $1 \leqslant i \leqslant N_P$, $y_n^k := y^{\text{rand}}$, where y^{rand} satisfies (3.2)

end while (k).

OUTPUT: The row of matrix $\mathbf{B}$ that contains the corresponding value $\min\{F(y_i^k); y_i^k \in \mathbf{B}\}$ represents the best found individual y_{opt}.

Note: Unlike the *while condition cycle*, the *repeat until condition cycle* is always executed at least once, since the controlling condition is checked at the end of the cycle.

The notation $y_{i,j}^k$ means the jth component of the individual y_i^k in the kth generation. The notation y^{rand} means a randomly generated individual from the search space C. The individual y_{opt} is the final optimized solution and includes information about the location of each heater in the form (2.1).

The MDEA differs from the original algorithm standardly named `DE/rand/1/bin` in the supplemented part 3.c) of the algorithm. After creating a new generation of individuals, the individual with the highest value of the cost function is replaced by a randomly generated individual. The asymptotic convergence of the algorithm MDEA is ensured by this adjustment. The asymptotic convergence is proved in the next subsection.

In general, differential evolution algorithms are characterized by higher computational demands and slower convergence. The parallel programming tools can be applied to accelerate the calculation process (see [10]).

3.2. Convergence of the modified differential evolution algorithm (MDEA). Generally, the convergence of a differential evolution algorithm `DE/rand/1/bin` is not guaranteed. In this subsection we focus on the asymptotic convergence of MDEA. The function F (and similarly the function $\widetilde{F}$) defined by relations (2.4)

is a continuous function of $6M$ variables. These functions may have many local minima in the search space C. We introduce the *optimal solution set*

$$C^* = \{y_{\min} \in C;\ F(y_{\min}) = \min\{F(y);\ y \in C\}\}.$$

That is, $y_{\min}$ represents the global minimum of the cost function F. Further, we consider an *expanded optimal solution set*

$$C^*_\varepsilon = \{y \in C;\ |F(y_{\min}) - F(y)| < \varepsilon\},$$

where ε is a small positive number. We suppose that $\mu(C^*_\varepsilon) > 0$ for each ε, where μ denotes the Lebesgue measure. In the following definition we introduce the term *convergence in probability* (see [5]).

Definition. Let $\{G(k),\ k = 0, 1, 2, \ldots\}$ be the generation sequence created by MDEA to solve the optimization problem (3.1). We say that the MDEA converges to the *expanded optimal solution set* C^*_ε in probability if

$$\lim_{k\to\infty} p\{G(k) \cap C^*_\varepsilon \neq \emptyset\} = 1, \tag{3.4}$$

where p denotes the probability of an event.

Now let us consider using MDEA to solve the optimization problem (3.1). Then the following theorem holds.

Theorem 3.1. *MDEA converges in probability to the expanded optimal solution set C^*_ε, i.e., the relation* (3.4) *holds.*

P r o o f. Recall that the individual $y^{\text{rand}} \in C$ is randomly generated in each generation $G(k)$ of MDEA. We understand the probability in geometric sense and put

$$p\{y^{\text{rand}} \in C^*_\varepsilon\} = \frac{\mu(C^*_\varepsilon)}{\mu(C)} = \alpha,$$

where $0 < \alpha < 1$. The relation $p\{y^{\text{rand}} \notin C^*_\varepsilon\} = 1 - \alpha$ holds for each generation $G(k)$. Thus the inequality $p\{G(k) \cap C^*_\varepsilon = \emptyset\} \leqslant 1 - \alpha$ is true for each generation $G(k)$. Therefore, the upper estimate of the probability that the first k generations do not include an individual $y \in C^*_\varepsilon$ is given by the relation

$$\prod_{i=1}^{k} p\{G(i) \cap C^*_\varepsilon = \emptyset\} \leqslant (1 - \alpha)^k.$$

Based on the construction of individuals in generation $G(k)$ in the algorithm MDEA described in Subsection 3.1 (the best individual in generation $G(k)$ has the same or better evaluation than the best individual from all previous generations) it is true that

$$\lim_{k\to\infty} p\{G(k)\cap C_\varepsilon^* \neq \emptyset\} = 1 - \lim_{k\to\infty} p\{G(k)\cap C_\varepsilon^* = \emptyset\} \geqslant 1 - \lim_{k\to\infty}(1-\alpha)^k = 1,$$

which completes the proof. □

Convergence in probability of the generalized problem of the modified differential algorithm `DE/rand/1/bin` with random choice of more individuals in each generation is proved in article [6].

4. Practical example

Now, we describe a practical example of the heating of an aluminium mould. The dimensions of the mould are $0.8\,\mathrm{m} \times 0.4\,\mathrm{m} \times 0.15\,\mathrm{m}$ (see Fig. 4), the number of elementary surfaces $N = 2064$. The heat radiation intensity recommended by the producer of artificial leather is $I_{\mathrm{rec}} = 47\,\mathrm{kW/m^2}$. We use 16 infrared heaters ($M = 16$) of the same type (producer Philips, heating capacity 1600 W, length 15 cm, width 4 cm). For the specification of values defined in relation (3.2), we need to determine the position of the mould in the Cartesian coordinate system. The front left mould corner lies at the origin of the coordinate system, the longer front lower edge of the mould lies in the positive half-axis x and the shorter lower edge lies in the positive half-axis y. Then the upper and lower limits of individual components in relation (3.2) are: $Lo_1 = -0.3\,\mathrm{m}$, $Hg_1 = 1.1\,\mathrm{m}$; $Lo_2 = -0.3\,\mathrm{m}$, $Hg_2 = 0.7\,\mathrm{m}$; $Lo_3 = 0.05\,\mathrm{m}$, $Hg_3 = 0.65\,\mathrm{m}$; $Lo_4 = -0.9\,\mathrm{m}$, $Hg_4 = 0.9\,\mathrm{m}$; $Lo_5 = -0.9\,\mathrm{m}$, $Hg_5 = 0.9\,\mathrm{m}$; $Lo_6 = 0.0$, $Hg_6 = \pi$. In the first step we calculate the value $F(y_1)$, where the initial individual y_1 corresponds to the locations of the heaters as in the left part of Fig. 4. The centres of the heaters lie in the plane parallel to the x_1x_2-plane at a distance of 10 cm from the centroid T_j of the elementary surface p_j with the highest value t_3^j ($1 \leqslant j \leqslant N$). In the initial position, all the heaters have $r = (1, 0, 0)$ and $u = (0, 0, -1)$. It means that all the heaters radiate downwards and are parallel to the axis x_1. Then the cost function value is $F(y_1) = 20.74$.

Subsequently, we use MDEA described in Subsection 3.1 to optimize the locations of the heaters. The parameters of the algorithm are: generation size $N_P = 192$, crossover probability $C_R = 0.60$, mutation factor $f = 0.98$ and the number of generations $N_G = 4000$. Fig. 4 shows a graphical representation of the heat radiation on the mould surface (where the levels of radiation intensity in $\mathrm{kW/m^2}$ correspond

to different colour shades) and the locations of the heaters. The right part shows the optimized locations of the heaters (individual y_{opt}, $F(y_{\text{opt}}) = 2.02$) after 4000 generations of MDEA.

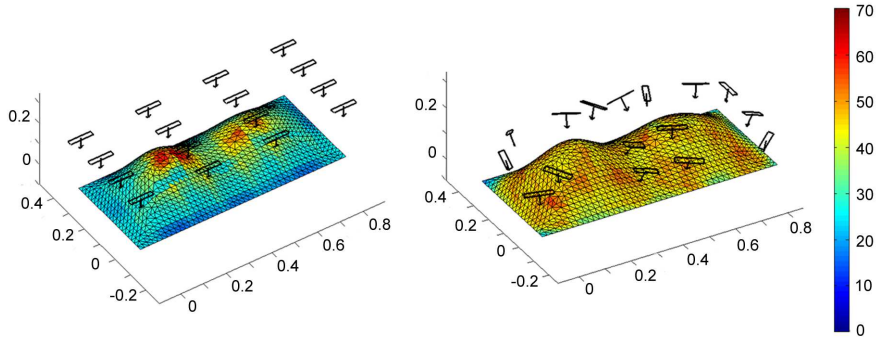


Figure 4. Heat radiation intensity in kW/m^2 on the mould surface and the locations of the heaters, $F(y_1) = 20.74$ and $F(y_{\text{opt}}) = 2.02$.

Furthermore, to run the optimization from a different starting point, we took $\tilde{y}_1$, an initial individual corresponding to the locations of the heaters recommended as optimal by manufacturing technicians. The corresponding cost function value is $F(\tilde{y}_1) = 11.2204$. We obtained again the same optimized individual $\tilde{y}_{\text{opt}}$ with the value $F(\tilde{y}_{\text{opt}}) = 2.02$ after 4000 generations of MDEA. The dependence of the deviation $F(\tilde{y}_{\text{opt}})$ on the number of generations is shown in Fig. 5.

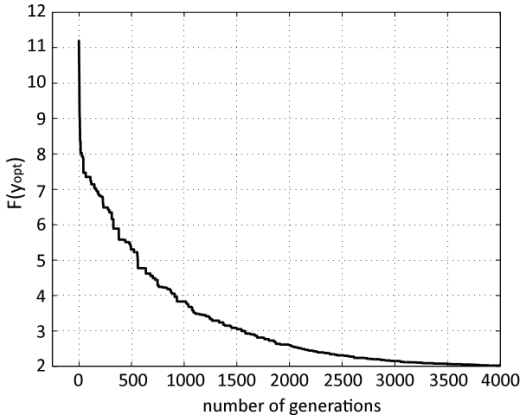


Figure 5. Dependence of $F(\tilde{y}_{\text{opt}})$ on the number of generations.

Another possibility how to decrease the value $F(\tilde{y}_{\text{opt}})$ is to increase the number of heaters. The heaters would be in general farther from the mould and we would obtain a more uniform heat radiation intensity on the mould surface. But this would lead to higher energy consumption which is not feasible for the manufacturer.

It is possible to calculate the time dependent temperature field (during the heating of the mould) on the inner surface of the mould (where artificial leather is produced) for the found optimized locations of the heaters. The partial differential parabolic equation of the heat conduction in the mould with an initial condition and boundary conditions including the own heat radiation of the mould determined by the Stefan-Boltzmann law by the finite element method using the ANSYS software package is presented in [7].

5. Conclusions

Based on numerical calculations, we get an optimized solution for the locations of the heaters over the mould. The results obtained by MDEA are better than the results produced by classic differential or genetic algorithms. The quality of the solution can be measured by evaluating the function F for the given optimized solution.

The locations of the heaters based on the experience of qualified technicians produce significantly worse results compared to the numerical computation. In addition, the manual approach is much more time consuming (approximately two to three weeks depending on the mould size and the number of heaters). Furthermore, the calculated optimization of the locations of the heaters is more accurate than the optimization based on experience. Besides, the described optimization process is economically feasible for the producer and induces virtually no additional costs.

References

[1] *M. Affenzeller, S. Winkler, S. Wagner, A. Beham*: Genetic Algorithms and Genetic Programming. Modern Concepts and Practical Applications. Numerical Insights 6, CRC Press, Boca Raton, 2009. zbl MR doi

[2] *H. M. Antia*: Numerical Methods for Scientists and Engineers. Birkhäuser, Basel, 2002. zbl MR

[3] *B. Budinský*: Analytic and Differential Geometry. Matematika pro vysoké školy technické, SNTL, Praha, 1983. (In Czech.) zbl

[4] *Y. A. Cengel, A. J. Ghajar*: Heat and Mass Transfer: Fundamentals and Applications. McGraw-Hill Education, New York, 2015.

[5] *Z. Hu, S. Xiong, Q. Su, X. Zhang*: Sufficient conditions for global convergence of differential evolution algorithm. J. Appl. Math. *2013* (2013), Article ID 193196, 14 pages. MR doi

[6] *R. Knobloch, J. Mlýnek, R. Srb*: The classic differential evolution algorithm and its convergence properties. Appl. Math., Praha *62* (2017), 197–208. zbl MR doi

[7] *J. Mlýnek, R. Knobloch, R. Srb*: Temperature field optimization on the mould surface. Advanced Mechatronics Solutions (R. Jablonski, T. Brezina, eds.). Advances in Intelligent Systems and Computing 393, Springer, Cham, 2015, pp. 225–230. doi

[8] *J. Mlýnek, T. Martinec, R. Srb*: Heating of mould in manufacture of artificial leathers in automotive industry. Mechatronics 2013. Recent Technological and Scientific Advances (T. Březina, R. Jabloński, eds.). Springer, Cham, 2014, pp. 119–126. doi

[9] *J. Mlýnek, R. Srb*: The process of an optimized heat radiation intensity calculation on a mould surface. Proceedings 26th European Conference on Modelling and Simulation, ECMS 2012 (K. G. Troitzsch, M. Möhring, U. Lotzmann, eds.). European Council for Modelling and Simulation, Dudweiler, 2012, pp. 461–467. doi

[10] *J. Mlýnek, R. Srb, R. Knobloch*: The use of graphics card and nVidia CUDA architecture in the optimization of the heat radiation intensity. Programs and Algorithms of Numerical Mathematics 17 (J. Chleboun et al., eds.). Proceedings of Seminar, Dolní Maxov, 2014, Institute of Mathematics, Academy of Sciences of the Czech Republic, Praha, 2014, pp. 150–155. zbl MR

[11] *K. V. Price, R. M. Storn, J. A. Lampien*: Differential Evolution. A Practical Approach to Global Optimization. Natural Computing Series, Springer, Berlin, 2005. zbl MR doi

[12] *D. Simon*: Evolutionary Optimization Algorithms. Biologically Inspired and Population-Based Approaches to Computer Intelligence. John Wiley & Sons, Hoboken, 2013. zbl MR

[13] *J. J. Stoker*: Differential Geometry. Wiley Classics Library, John Wiley & Sons, New York, 1989. zbl MR

[14] *J. Zhang, A. C. Sanderson*: Adaptive Differential Evolution. A Robust Approach to Multimodal Problem Optimization. Adaptation, Learning, and Optimization 1, Springer, Berlin, 2009. MR doi

Authors' address: *Jaroslav Mlýnek*, *Roman Knobloch*, Department of Mathematics, Technical University of Liberec, Studentská 2, 461 17 Liberec, Czech Republic, e-mail: `jaroslav.mlynek@tul.cz`, `roman.knobloch@tul.cz`.