Model Predictive Control for Energy-saving and Comfortable Temperature Control in Buildings

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Abstract— **Model predictive control has been recognized as one of the essential solutions to achieve considerable energy savings in buildings. However, its performance on a building zone level can be inferior to a well-tuned conventional controller, especially in situations with constant energy prices and conservative comfort constraints. Optimization problem in the background has to be chosen to guarantee recursive feasibility and considerable energy savings without compromising the users comfort at the same time. This paper gives a novel formulation of the model predictive temperature control problem in buildings and its fair comparison with conventional controllers with the same level of flexibility allowed in zone temperature control. All controllers are tested for a system with seasonal heating and cooling, which is the most common case in real applications. It is shown that the introduced formulation leads to the model predictive controller that significantly outperforms conventional controllers both in energy consumption and users comfort.**

INTRODUCTION

In the last few decades, the increased awareness of the limitations of fossil fuels combined with increasing energy demand worldwide and noticeable effects of irrational energy consumption has resulted in energy-efficiency policies for advocating and encouraging rational energy consumption. The improvement of the buildings sector energy efficiency becomes critical to attain a balance in many sectors. This is most notably the case with the power sector, as almost half of all the energy consumed today is used in buildings [1]. Given the large share of energy consumed in buildings, improvement of buildings energy efficiency is crucial to ensure long-term energy security. Model Predictive Control (MPC) framework, due to its distinct advantages, significantly outstands among other conventional methods applicable for the building control design. Conventional control algorithms mostly rely on the calibration of algorithms designed for a typical building according to the approximate rule of thumb or trial and error method. Two most common conventional controllers used within Building Energy Management Systems (BEMSs) on a zone level are standard proportional-integral (PI) controller and hysteresis-type (on/off) controller.

The MPC is an optimization-based control approach where control actions are calculated by solving finite-horizon optimal control problem and applied in a receding horizon fashion [2]. To achieve energy savings and outperform conventional controllers, this optimization problem needs to be chosen very carefully. Potential energy savings are up to 40% [3]–[7], but they must be evaluated in a fair set-up. In most of the reported studies, energy savings are gained by setting users comfort zone very wide, mostly within the interval 20-25 ^{o}C or even wider [8]–[11]. The most commonly recommended temperature that ensures comfort is 24 ^{o}C and the majority of users are not so flexible to allow temperature deviations of $\pm$ 2.5 ^{o}C or even larger [12], [13]. An additional problem of MPC formulations reported in [8] is a lack of recursive feasibility, e.g. if the initial state violates the control problem constraints, small enough sampling time under control input constraints will make the control problem infeasible. One way to deal with feasibility issues of this type is to replace hard constraints with so-called soft constraints. Another solution is to replace classic hard constraints by chance constraints [9], [10], but for south oriented building zones with a large glazing area and only cooling or heating available at the certain time, this formulation often results in infeasibility as well. Furthermore, for MPC formulations with wide comfort zones, it is very hard to determine obtained gains since conventional PI controller has a task to follow the reference and on-off type hysteresis controller usually has much narrower hysteresis bounds. In addition to extensive literature with claimed gains of MPC over conventional controllers, there are several reports showing that for standard applications performance of the MPC controller on a zone level is approximately the same, or even worse than the performance of a well-tuned conventional controller [14], [15]. Standard application implies constant energy prices and disabled advanced options such as peak shaving, uncertainty handling, etc.

This paper is focused on a deterministic MPC application in building zones temperature control, and its comparison with standard hysteresis and PI controllers, where MPC constraints are matched with hysteresis bounds. The MPC and hysteresis approach are both tested for three standard cases with allowed temperature deviation from set-point set to $\pm$ [0.2 0.5 0.7] ^{o}C, which corresponds to limits of cyclic temperature variations of A, B and C classes of the thermal environment defined by ISO 7730 standard [12]. The MPC problem formulation presented in this paper enables considerable energy savings without compromising the users comfort. Simulations are performed for the test-site comprised of the 9th floor of University of Zagreb, Faculty of Electrical Engineering and Computing (FER) skyscraper building with 23 zones equipped with fan-coils. The gain is obtained by optimally manipulating the temperature in the given comfort interval, with respect to current and predicted outdoor conditions.

This work is organized as follows. In Section I, conventional controllers used in this study are presented. Section II gives the formulation of MPC with a description of building model and detailed description of the optimization criterion. In Section III simulation set-up with used comfort metrics is described and results are given. Section IV concludes the paper.

I. Conventional controllers

PI controller

Conventional PI controller represents a typical decentralized control approach which can be found in many building applications. Comparison with PI controller will give the baseline for energy consumption since PI controller ensures tracking of user reference all the time when it is possible. The synthesis of PI controller is performed automatically within the MATLAB environment to ensure the best performance regarding reference tracking [16].

Hysteresis control

The test-site zones are currently controlled by an industrial RXC controller [17] based on hysteresis control of fan-coils. To assess the possible energy savings on FER building, achievable by improvement of its BEMS with MPC, hysteresis control is also included into comparison. The fan-coils operate at 3 different fan speeds (FS). The amount of power at certain speed depends on the temperature and mass flow of the heating/cooling medium and is considered as constant. To be as close as possible to the real system, power amounts are estimated from calorimeter readings available on each major supply duct of the heating/cooling system on the considered floor of the Faculty building. The RXC controller switches between available power outputs based on the temperature difference between current j^{th} zone temperature T_j and set-point value SP_j set by the end-users (Fig. 1). Hysteresis width 2Δ is predefined and equal for all zones.

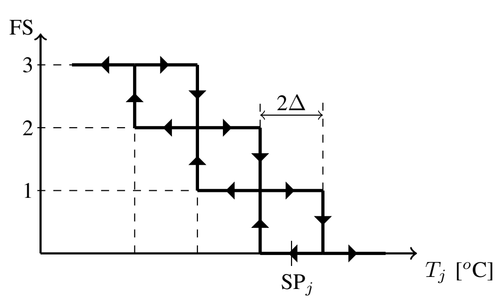

Fig. 1. Hysteresis control law for the j^{th} zone in the heating season.

II. Model Predictive Controller

Possible energy savings by using an MPC have been widely reported in the literature. The aim of this paper is to identify the best formulation of MPC problem such that it outperforms conventional controllers even under the same conditions. Distinct advantages of MPCs lie in (i) using the relevant future information in making control decisions; (ii) inherent handling of multi-input multi-output (MIMO) systems; (iii) routine respecting of system constraints (e.g. finite amount of heating/cooling power, comfort intervals) and (iv) explicit orientation of the control actions towards the specified goal such as economic, environmental or their combination. All the mentioned advantages make the MPC a favorable choice for the BEMS design.

Building model

Mathematical model of a building is a basis for MPC implementation. The most popular building modeling framework consists of using resistor-capacitor (RC) network to model thermodynamic processes in buildings [8], [18], [19]. The basic idea of this methodology is to represent building elements (or complete zones) with as few thermal circuit elements as possible. The resulting linear dynamic models are established as simple, computationally efficient and accurate enough models.

Due to the hourly resolution of the weather forecast given for the building location by Croatian Meteorological and Hydrological Service, a model of the 9th floor of the FER building built on RC principles is discretized with an hourly sampling time T_s. Forecast of solar irradiance given at a time instant k is cumulative solar irradiance received on a unit surface within one sampling interval. Forecast of the outdoor air temperature T_o is given as predicted value at k. To utilize this information, influence of outdoor air temperature is discretized by employing first-order hold while the rest of the system is discretized by zero-order hold. The resulting discrete system is as follows:

$$\mathbf{x}_{k+1} = \mathbf{A}\mathbf{x}_k + \mathbf{B}_u\mathbf{u}_k + \mathbf{B}_d^1 T_{o,k} + \mathbf{B}_d^2 T_{o,k+1} + \mathbf{B}_d^* \mathbf{d}_k,$$
$$\mathbf{y}_{k+1} = \mathbf{C}\mathbf{x}_{k+1}, \qquad (1)$$

where $\mathbf{x}_k \in \mathbb{R}^n$ is the system state vector, $\mathbf{u}_k \in \mathbb{R}^q$ is the thermal energy input to each of q controllable zones from k to $k+1$, $T_{o,k} \in \mathbb{R}^1$ is outdoor air temperature, $\mathbf{d}_k \in \mathbb{R}^p$ is disturbance input (solar irradiance, internal gains, etc.) and $\mathbf{y}_k \in \mathbb{R}^m$ is the vector of room temperatures at time step k. Matrices $\mathbf{A}$, $\mathbf{B}_u$, $\mathbf{B}_d^1$, $\mathbf{B}_d^2$ and $\mathbf{B}_d^*$ are matrices of appropriate dimensions.

MPC problem formulation

The resulting performance of the control system with employed MPC depends completely on the MPC problem formulation. A special care needs also to be paid to the fact that the computed controls are implemented in a receding horizon fashion. The MPC uses the dynamic model of the building and information on the future disturbance profiles to predict future building behavior and, based on these predictions, computes the optimal control input trajectory. Predicted states and outputs along the prediction horizon $N \in \mathbb{Z}$ are conveniently written as:

$$\mathbf{Y} = \boldsymbol{\alpha}\mathbf{x}_{t|t} + \boldsymbol{\beta}\mathbf{U} + \boldsymbol{\gamma}_1\mathbf{D}_1 + \boldsymbol{\gamma}_2\mathbf{D}_2, \tag{2}$$

where $\mathbf{Y}$ is a stack of future outputs:

$$\mathbf{Y} = [\mathbf{y}_{t+1|t}^T \quad \mathbf{y}_{t+2|t}^T \quad \dots \quad \mathbf{y}_{t+N|t}^T]^T, \tag{3}$$

$\mathbf{U}$ is a stack of future inputs:

$$\mathbf{U} = [\mathbf{u}_{t|t}^T \quad \mathbf{u}_{t+1|t}^T \quad \dots \quad \mathbf{u}_{t+N-1|t}^T]^T, \tag{4}$$

$\mathbf{D}_1$ and $\mathbf{D}_2$ are stacks of future disturbances:

$$\begin{aligned}\mathbf{D}_1 &= [T_{o,t|t} \quad T_{o,t+1|t} \quad \dots \quad T_{o,t+N|t}]^T,\\ \mathbf{D}_2 &= [\mathbf{d}_{t|t}^T \quad \mathbf{d}_{t+1|t}^T \quad \dots \quad \mathbf{d}_{t+N-1|t}^T]^T,\end{aligned} \tag{5}$$

and $\boldsymbol{\alpha}$, $\boldsymbol{\beta}$, $\boldsymbol{\gamma}_1$ and $\boldsymbol{\gamma}_2$ are matrices based on the discrete building model matrices (Eq. 1). Notation $\mathbf{y}_{t+k|t}$ denotes predicted rooms temperature at time $t+k$, obtained by applying the input sequence $\mathbf{U}$ to the system starting from current state $\mathbf{x}_{t|t}$.

The most frequent MPC problem formulation for temperature control in buildings consists of a simplistic minimization of energy consumption with respect to the temperature constraints set by the end-users and physical limitations of actuators [8]:

$$\begin{aligned}\min_{\mathbf{U}} \quad & J(\mathbf{U}, \mathbf{x}_{t|t}, \mathbf{D}_1, \mathbf{D}_2)\\ \text{s.t.} \quad & \mathbf{SP} - \boldsymbol{\Delta} \leq \mathbf{Y} \leq \mathbf{SP} + \boldsymbol{\Delta},\\ & \mathbf{P}_{\text{min}} \leq \mathbf{U} \leq \mathbf{P}_{\text{max}},\end{aligned} \tag{6}$$

where:

$$J(\mathbf{U}, \mathbf{x}_{t|t}, \mathbf{D}_1, \mathbf{D}_2) = \sum_{k=0}^{N-1} |\mathbf{R}_{t+k|t}\mathbf{u}_{t+k|t}|_1, \tag{7}$$

and $\mathbf{SP}$ is a stack of future set-points profiles per zone:

$$\mathbf{SP} = [\mathbf{SP}_{t+1|t}^T \quad \mathbf{SP}_{t+2|t}^T \quad \dots \quad \mathbf{SP}_{t+N|t}^T]^T. \tag{8}$$

Operator $|.|_1$ denotes L_1 norm and $\mathbf{P}_{\text{min}}$ and $\mathbf{P}_{\text{max}}$ are limitations on power inputs $\mathbf{U}$. Set-points $\mathbf{SP}_{t+k|t} \in \mathbb{R}^m$ and allowed deviations from them along the prediction horizon $\boldsymbol{\Delta} \in \mathbb{R}^{N \cdot m}$, are defined by the end-users for each of m zones. $\mathbf{R}_{t+k|t} \in \mathbb{R}^{q \times q}$ is a weighting matrix typically set to identity matrix of appropriate size. Formulations like these handle users temperature constraints as hard constraints, which often results in infeasibility, especially when sign change of $\mathbf{U}$ is not possible (only heating or only cooling available).

To solve this problem, temperature constraints are "softened" by introducing them into the cost function through slack variables $\boldsymbol{\sigma}_{t+k|t} \in \mathbb{R}^m$ with large weights $\mathbf{G}_{t+k|t} \in \mathbb{R}^{m \times m}$ (e.g. $\mathbf{G}_{t+k|t} = 10^6\mathbf{I}_{m\times m}, \forall k$). The resulting optimization problem is as follows:

$$\begin{aligned}\min_{\mathbf{U},\boldsymbol{\Sigma}} \quad & J(\mathbf{U}, \mathbf{x}_{t|t}, \mathbf{D}_1, \mathbf{D}_2, \boldsymbol{\Sigma})\\ \text{s.t.} \quad & \mathbf{SP} - \boldsymbol{\Delta} - \boldsymbol{\Sigma} \leq \mathbf{Y} \leq \mathbf{SP} + \boldsymbol{\Delta} + \boldsymbol{\Sigma},\\ & \mathbf{P}_{\text{min}} \leq \mathbf{U} \leq \mathbf{P}_{\text{max}},\\ & \boldsymbol{\Sigma} \geq \mathbf{0},\end{aligned} \tag{9}$$

where:

$$\begin{aligned}J(\mathbf{U}, \mathbf{x}_{t|t}, \mathbf{D}_1, \mathbf{D}_2, \boldsymbol{\Sigma}) = & \sum_{k=0}^{N-1} |\mathbf{R}_{t+k|t}\mathbf{u}_{t+k|t}|_1\\ & + \sum_{k=1}^{N} |\mathbf{G}_{t+k|t}\boldsymbol{\sigma}_{t+k|t}|_1,\end{aligned} \tag{10}$$

with $\boldsymbol{\Sigma}$ defined as:

$$\boldsymbol{\Sigma} = [\boldsymbol{\sigma}_{t+1|t}^T \quad \boldsymbol{\sigma}_{t+2|t}^T \quad \dots \quad \boldsymbol{\sigma}_{t+N|t}^T]^T. \tag{11}$$

In most of the situations, occupants want the exact temperature to the one they have chosen on the zone thermostat. This can be obtained by defining the MPC problem as a classic reference tracking problem:

$$\begin{aligned}\min_{\mathbf{U}} \quad & J(\mathbf{U}, \mathbf{x}_{t|t}, \mathbf{D}_1, \mathbf{D}_2)\\ \text{s.t.} \quad & \mathbf{P}_{\text{min}} \leq \mathbf{U} \leq \mathbf{P}_{\text{max}},\end{aligned} \tag{12}$$

where:

$$\begin{aligned}J(\mathbf{U}, \mathbf{x}_{t|t}, \mathbf{D}_1, \mathbf{D}_2) = & \sum_{k=0}^{N-1} |\mathbf{R}_{t+k|t}\mathbf{u}_{t+k|t}|_1\\ & + \sum_{k=1}^{N} |\mathbf{Q}_{t+k|t}(\mathbf{y}_{t+k|t} - \mathbf{SP}_{t+k|t})|_1,\end{aligned} \tag{13}$$

and $\mathbf{Q}_{t+k|t} \in \mathbb{R}^{m \times m}$ is a weighting matrix. This MPC formulation, combined with a receding horizon strategy, often results in either minimum energy performance at the cost of completely disregarded temperature comfort or in permanent set-point following with disregarded energy consumption. A compromise between the two options is made through the mentioned weighting matrices.

To tackle the opposing criteria of reference following and energy saving, weighting matrices $\mathbf{R}_{t+k|t}$ and $\mathbf{Q}_{t+k|t}$ have to be chosen in a way which enables smart switching between these two requirements based on predicted disturbance profiles. Optimization cost J presented in this paper is comprised of two terms, J_1 and J_2. Term J_1 is related to minimization of energy consumption:

$$J_1(\mathbf{U}, \mathbf{x}_{t|t}, \mathbf{D}_1, \mathbf{D}_2) = \sum_{k=0}^{N-1} |\mathbf{R}_{t+k|t}\mathbf{u}_{t+k|t}|_1, \tag{14}$$

with $\mathbf{R}_{t+k|t}$ set to identity matrix $\mathbf{I}_{q\times q}$. Temperature demands of the end-users are forced by the term J_2:

$$\begin{aligned}J_2(\mathbf{U}, \mathbf{x}_{t|t}, \mathbf{D}_1, \mathbf{D}_2, \boldsymbol{\Sigma}_1, \boldsymbol{\Sigma}_2, \eta) = & \sum_{k=1}^{N} |\mathbf{G}_{1,t+k|t}\boldsymbol{\sigma}_{1,t+k|t}|_1\\ & + \sum_{k=1}^{N} |\mathbf{G}_{2,t+k|t}\boldsymbol{\sigma}_{2,t+k|t}|_1\\ & + \eta \cdot \sum_{k=1}^{N} |\mathbf{Q}_{t+k|t}(\mathbf{y}_{t+k|t} - \mathbf{SP}_{t+k|t})|_1,\end{aligned} \tag{15}$$

where $\eta \geq 0 \in \mathbb{R}^1$ is arbitrary weighting coefficient. Asymmetric slack variables $\boldsymbol{\Sigma}_1$ and $\boldsymbol{\Sigma}_2$ defined as in Eq. (11) guarantee minimal temperature requirements, i.e. different weighting factors $\mathbf{G}_{1,t+k|t} \neq \mathbf{G}_{2,t+k|t}$ can be used for penalizing upper and lower limit temperature constraints violation. J_2 can be interpreted as four-segmented convex PieceWise Affine (PWA) penalty function (Fig. 2).

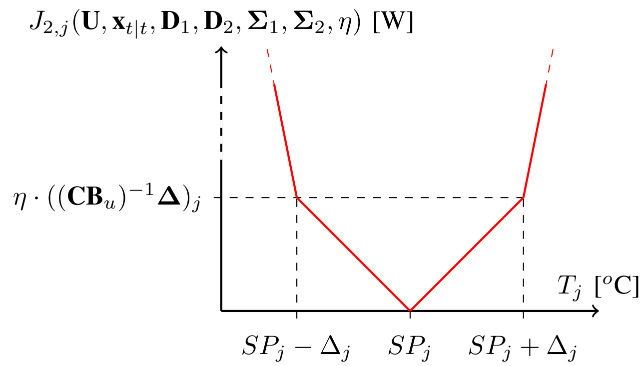

Fig. 2. Convex PWA penalty function for the j^{th} zone and $N = 1$.

To be comparable, both parts of optimization cost have to be expressed in the same units. Since J_1 is defined in watts, weighting matrix $\mathbf{Q}_{t+k|t}$ is utilized to convert temperature related cost part J_2 from degrees Celsius to watts. Building model (Eq. 1) is linear so the amount of energy which can be saved by allowing the zone temperature to slide below the set-point during the heating season or above the set-point during the cooling season, under the same weather conditions, is linear function of system dynamics. Sensitivity of the energy consumption to the zones temperature is defined as:

$$\frac{\partial \mathbf{U}}{\partial \mathbf{Y}} = \frac{\partial(\boldsymbol{\beta}^{-1}(\mathbf{Y} - \boldsymbol{\alpha}\mathbf{x}_{t|t} - \boldsymbol{\gamma}_1\mathbf{D}_1 - \boldsymbol{\gamma}_2\mathbf{D}_2))}{\partial \mathbf{Y}} = \boldsymbol{\beta}^{-1}. \quad (16)$$

Matrix $\boldsymbol{\beta}^{-1}$ is lower bidiagonal matrix with all elements on the main diagonal equal to $(\mathbf{CB}_u)^{-1}$ and to the $-(\mathbf{B}_u^{-1}\mathbf{AC}^{-1})$ on the secondary diagonal. By setting weighting matrices along the horizon to:

$$\mathbf{Q}_{t+k|t} = \begin{bmatrix} (\mathbf{CB}_u)^{-1} \\ -(\mathbf{B}_u^{-1}\mathbf{AC}^{-1}) \end{bmatrix}, \quad k = 1, \ldots, N-1, \quad (17)$$

and $\mathbf{Q}_{t+N|t} = (\mathbf{CB}_u)^{-1}$, J_2 is converted from degrees to watts. Final optimization cost is thus:

$$\begin{aligned} J(\mathbf{U}, \mathbf{x}_{t|t}, \mathbf{D}_1, \mathbf{D}_2, \mathbf{\Sigma}_1, \mathbf{\Sigma}_2, \eta) &= J_1(\mathbf{U}, \mathbf{x}_{t|t}, \mathbf{D}_1, \mathbf{D}_2) \\ &+ J_2(\mathbf{U}, \mathbf{x}_{t|t}, \mathbf{D}_1, \mathbf{D}_2, \mathbf{\Sigma}_1, \mathbf{\Sigma}_2, \eta), \end{aligned} \quad (18)$$

Weighting factor η determines the importance of reference tracking with respect to the minimization of energy consumption (Fig. 3).

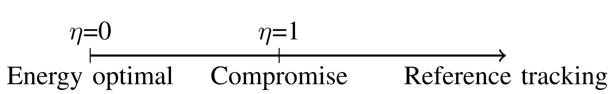

Fig. 3. Dependence between weighting factor η and system performance.

For $\eta = 1$ both energy consumption and reference tracking have the same weights so the controller will decide what is best from the energy viewpoint. For $\eta > 1$ bigger weight is set on reference tracking so the system will slide from the set-point when it is unavoidable or when this will results with considerable energy savings. Final optimization problem written in compact form is as follows:

$$\begin{aligned} \min_{\mathbf{U}, \mathbf{\Sigma}_1, \mathbf{\Sigma}_2} \quad & J(\mathbf{U}, \mathbf{x}_{t|t}, \mathbf{D}_1, \mathbf{D}_2, \mathbf{\Sigma}_1, \mathbf{\Sigma}_2, \eta) \\ \text{s.t.} \quad & \mathbf{SP} - \mathbf{\Delta} - \mathbf{\Sigma}_1 \le \mathbf{Y} \le \mathbf{SP} + \mathbf{\Delta} + \mathbf{\Sigma}_2 \\ & \mathbf{P}_{\min} \le \mathbf{U} \le \mathbf{P}_{\max}, \\ & \mathbf{\Sigma_1} \ge \mathbf{0}, \\ & \mathbf{\Sigma_2} \ge \mathbf{0}. \end{aligned} \quad (19)$$

With such a criterion, in the heating season, solar irradiance influence that can result in overheating is heavily penalized, which adversely forces the system to minimize overheating, i.e. to use the free energy from outdoors starting from the lower edge of the allowed range. Effectively, the actuators are controlled such that the lower bound of the temperature range is reached prior to the stream of free energy from outdoors. In the cooling season system is forced to quit cooling the zone when free cooling can be utilized.

III. Simulation Results

Simulation setup

The test-case studied in this paper is 9th floor of the FER building (Fig. 4). Overall studied area of the test-site is about 700 m^2 large.

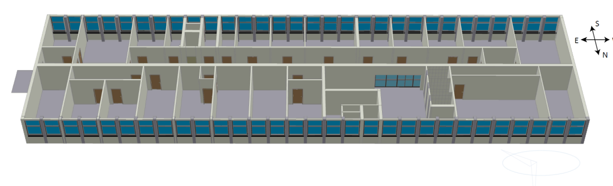

Fig. 4. 3D drawing of the 9th floor of FER skyscraper building.

All controllers are employed to control directly the thermal powers required to achieve the desired temperature behavior. Fan-coils have very fast dynamics, i.e. every feasible power can be achieved in negligible time, so in this study lower level controller required to calculate direct control actions to fan-coils, required for real implementation of PI and MPC algorithms, is not considered. Instead, it is assumed that power references can be tracked perfectly.

Data used for external conditions (outdoor air temperature and solar irradiances) are historical measurements from the year 2014 taken on a meteorological station close to the FER building. Weather disturbances are, in such a setup, assumed to be perfectly forecasted, and all other disturbances are neglected. For real building implementations, with a lot of uncertain and unpredictable disturbances, their compensation is performed by introducing an estimator into the control loop. Optimization horizon is 24 h long with hourly sampling time. Hysteresis and PI controller are continuous-time controllers operated in standard closed loop fashion.

It is assumed that at a certain moment only heating or only cooling is available. This corresponds to standard two pipe implementation of heating/cooling system present in many buildings. The cooling season starts at 1st June and lasts until 1st October. The building operates in two working modes: daily mode, from 6 to 18 hours, during which temperature requirements of the end-users are set to 24 ^{o}C for both seasons, and night mode, from 18 to 6 hours. During night mode ($t + k$ is within the interval from 18 to 6 hours) reference following part is omitted from the temperature related cost function part J_2. This implies $\mathbf{Q}_{t+k|t} = \mathbf{0}_{m\times m}$ for $t + k$ within night mode interval and $\mathbf{Q}_{t+k|t}$ defined as in Eq. (17) otherwise. Allowed deviations during night mode are set to $\Delta_j^* = 6$ ^{o}C, which effectively ensures the minimum temperature of $SP_j - \Delta_j^*$ to prevent the building from cooling down too much during the heating season. In the cooling season, night mode additionally implies unavailable cooling powers, i.e. $\mathbf{P}_{\text{min}} = \mathbf{P}_{\text{max}} = 0$.

Simulations are performed within MATLAB environment [16]. Optimization problems are solved by using YALMIP [20] and CPLEX [21].

Comfort metrics

Average deviation (AD) from the set-point is calculated as the ratio of the sum of all the deviation amounts during daily mode and overall number of samples during daily mode, where M is number of measurements per zone:

$$AD = \frac{1}{mM}\sum_{t=0}^{M} |\mathbf{y}_{t|t} - \mathbf{SP}_{t|t}|_1. \tag{20}$$

Results

Comparison of overall energy consumption during the whole year of 2014, for different types of control, different flexibilities Δ and different weights η is given in Fig. 5. All saving percentages in the Fig. 5 are calculated in relation to the PI controller. Energy savings achievable by replacing the RXC controller with presented MPC with the same temperature constraints and $\eta = 1$ are up to 30% in the cooling season and up to 3% in the heating season. For the case with $\eta = 0$ savings percentages are even higher, but at the cost of totally disregarded users comfort. By choosing η user can define does he want to be energy efficient and save the energy or does he strictly wants the reference following behavior. Obtained numbers show large potential of the presented MPC formulation, especially because for $\eta \geq 1$ energy savings are not obtained at the account of totally violated users comfort. Moreover, following from Fig. 6 it may be observed that users comfort is improved, i.e. average deviations of MPC are smaller than the ones of hysteresis controller. The results show that for the presented case study energy savings can be increased beyond the 20% without becoming worse in average set-point deviation than hysteresis controller. Figure 7 shows a comparison of time responses of two selected building zones controlled by the hysteresis controller and the MPC during one week in November 2014 with $\Delta = 0.2$ ^{o}C and $\eta = 1.1$.

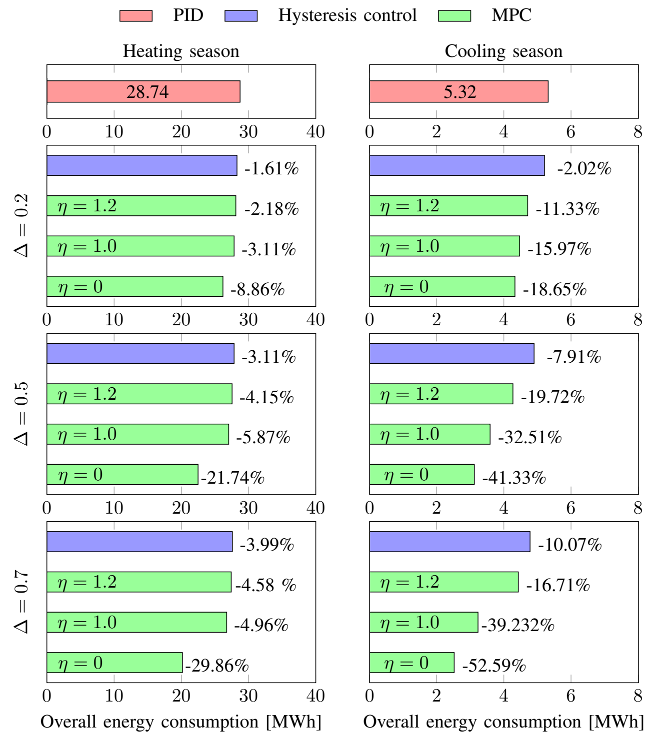

Fig. 5. Comparison of the overall energy consumption of different controllers in 2014.

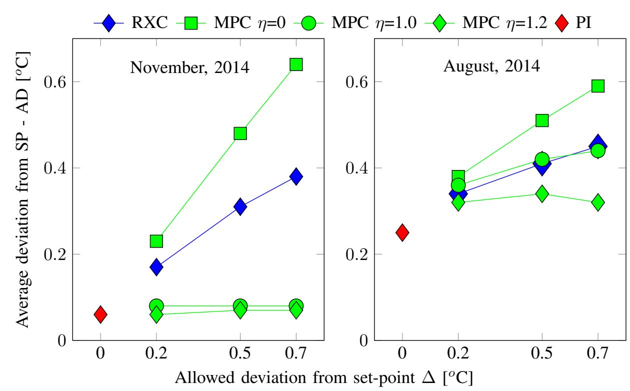

Fig. 6. Average deviation from SP for different types of controllers.

IV. CONCLUSIONS

Possible gains of Model Predictive Control (MPC) in building climate energy savings depend largely on the formulation of the MPC optimization problem. The most important criterion for the problem formulation is to guarantee recursive feasibility to enable real system application. Another important aspect is care about users comfort. Users are usually most comfortable when the temperature in the zone is exactly on the value they have chosen at a thermostat. The definition of the MPC problem presented in this article enables users to define the desired temperature and their own comfort region. Temperature drifts from the set-point only when this is economically justifiable or when there is no possibility to compensate

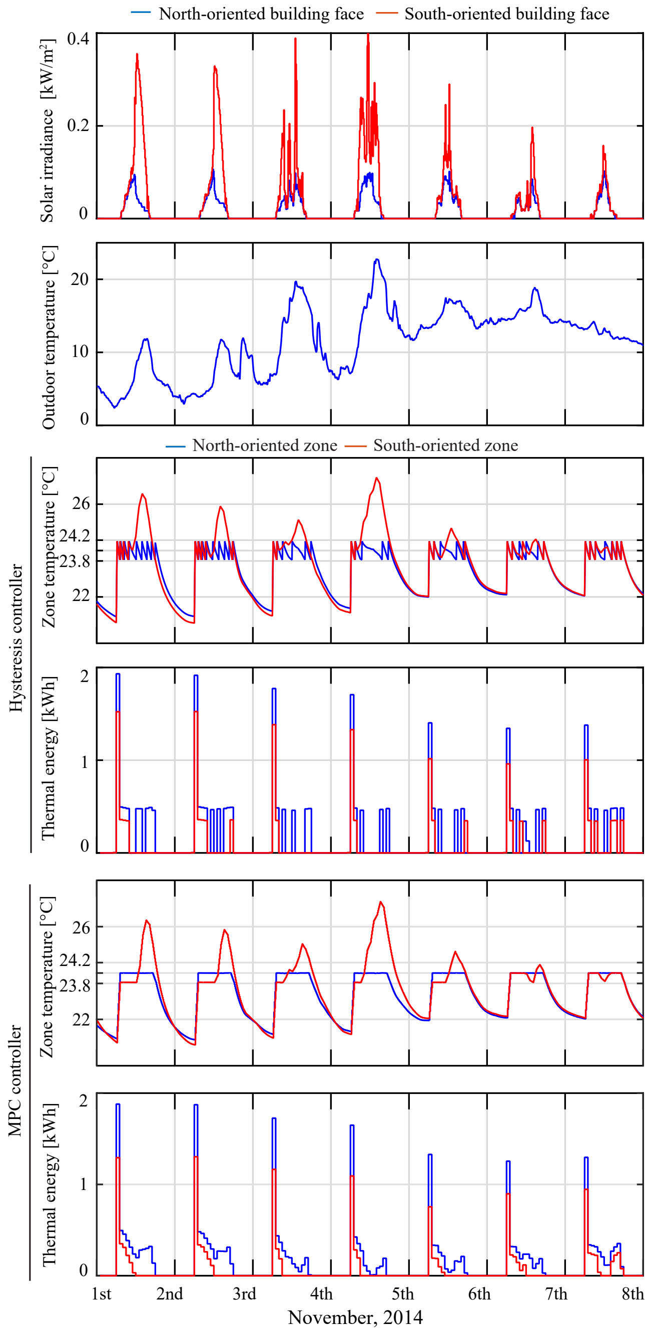

Fig. 7. Comparison of time responses of two selected building zones controlled by hysteresis controller and MPC during one week of heating season for $\Delta = 0.2$ ^{o}C and $\eta = 1.1$.

the effects of external disturbances. In this study, advanced options like peak shaving, variable energy prices, flexible night regime, etc. are intentionally left out to show that even for a simple case, properly designed MPC can significantly outperform conventional controllers without compromising users comfort. Moreover, it is shown that the users comfort is improved. The expected savings are even much higher with MPC's full potential exploited in zone control, especially in terms of accounted hour-to-hour variable energy prices and in terms of coordination with central heating/cooling medium production.

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