

Technical report 03-016

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A. Hegyi, B. De Schutter, and H. Hellendoorn, "Model predictive control for optimal coordination of ramp metering and variable speed limits," *Transportation Research Part C*, vol. 13, no. 3, pp. 185–209, June 2005.

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*This report can also be downloaded via https://pub.deschutter.info/abs/03_016.html

Model Predictive Control for Optimal Coordination of Ramp Metering and Variable Speed Limits

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Abstract:

This paper discusses the optimal coordination of variable speed limits and ramp metering in a freeway traffic network, where the objective of the control is to minimize the total time that vehicles spend in the network. Coordinated freeway traffic control is a new development where the control problem is to find the combination of control measures that results in the best network performance. This problem is solved by model predictive control, where the macroscopic traffic flow model METANET is used as the prediction model. We extend this model with a model for dynamic speed limits and for mainstream origins. This approach results in a predictive coordinated control approach where variable speed limits can prevent a traffic breakdown and maintain a higher outflow even when ramp metering is unable to prevent congestion (e.g., because of an on-ramp queue constraint). The use of dynamic speed limits significantly reduces congestion and results in a lower total time spent.

Since the primary effect of the speed limits is the limitation of the main-stream flow, a comparison is made with the case where the speed limits are replaced by main-stream metering. The resulting performances are comparable. Since the range of flows that main-stream metering and dynamic speed limits can control is different, it is concluded that the choice between the two should be primarily based on the traffic demands.

Keywords:

model predictive control, optimal control, ramp metering , variable speed limits, main-stream metering, coordinated traffic control

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1 Introduction

1.1 *Coordination and prediction are necessary*

The steadily increasing number and length of traffic jams on freeways has led to the use of several dynamic traffic management measures all over the world such as on-ramp metering, dynamic routing and the provision of congestion information. Usually these measures operate based on local data (occupancy, intensity or speed measurements). However, considering the effect of the measures on the network level has many advantages compared to local control. E.g., solving a local congestion only, may have as consequence that the vehicles run faster into another (downstream) congestion, whereas still the same amount of vehicles have to pass the bottleneck (with a given capacity), and so the average travel time on the network level will still be the same. Another reason for considering the effects of control on the network level, is that in a dense network a local control measure can have effects on more distant parts of the network: an improved flow may cause congestion somewhere else in the network or a reduced flow may prevent congestion somewhere else in the network. Furthermore, if dynamic origin-destination (OD) data is available, control on the network level can take advantage of the predictions of the flows in the network. Local controllers are not able to use OD information because the flows arriving at the local controller depend on the actions of other controllers elsewhere in the network, which are unknown. E.g., during peak hours the density on the mainstream (freeway) can be so high that the queue on an on-ramp spills back to the surface streets of the city, whereas (pro-active, coordinated) metering of upstream on-ramps could reduce the density of the main-stream flow and prevent spill back of the on-ramp queue. Another source of degraded network performance is that congestion may block traffic flows on routes that do not pass the bottleneck (or incident location), such as a freeway with a congested off-ramp where the vehicles that want to leave the freeway block the mainstream traffic.

To solve these problems a control strategy on the network level is needed. In this context, ‘network level’ means the network-wide coordination of control measures, i.e., the measures are operated based on global data. Besides using global data, prediction of the network evolution is also needed to achieve optimal network control, since the effect of a control measure on more distant locations will only be visible after some time. To predict the effects of a control measure several techniques can be used, such as case-based reasoning in Sadek and Demetsky (1999), rule-based systems in Cuena et al. (1995), or model-based prediction in Alessandri et al. (1998, 1999). In this paper we use for the predictions the macroscopic traffic flow model METANET described in Kotsialos et al. (1999); Papageorgiou et al. (1990b); Kotsialos et al. (2002).

To find the optimal combination of control measures (control inputs) we apply a

model predictive control (MPC) framework, see Camacho and Bordons (1995); García et al. (1989); Maciejowski (2002). MPC is an optimal control method applied in a rolling horizon framework. Optimal control is successfully applied by Kotsialos et al. (1999, 2001, 2002) to coordinate or integrate traffic control measures. Both optimal control and MPC have the advantage that the controller generates control signals that are optimal according to a user-supplied objective function. However, MPC has some important advantages over the traditional optimal control. First, optimal control has an *open-loop* structure, which means that the disturbances (in our case: the traffic demands) have to be completely and exactly known before the simulation, and the traffic model has to be very accurate to ensure sufficient precision for the whole simulation. MPC operates in *closed-loop* which means that the traffic state and the current demands are regularly fed back to the controller, and the controller can take disturbances (here: demand prediction errors) into account and correct for prediction errors resulting from model mismatch. Second, adaptivity is easily implemented in MPC, because the prediction model can be changed or replaced during operation. This may be necessary when traffic behavior significantly changes (e.g., in case of incidents, changing weather conditions, lane closures for maintenance). Third, for MPC a shorter prediction horizon is usually sufficient, which reduces complexity, and make the real-time application of MPC feasible. An example of the application of the MPC framework for integrated speed limit control can be found in Breton et al. (2002).

The MPC framework requires an objective function that expresses the performance of the traffic network (as a function of a given control input). The objective function we use is the total time spent (TTS) by the vehicles in the network, and to get a smooth control signal we will add a small term that penalizes abrupt changes in the control signal.

1.2 Ramp metering, dynamic speed limits and main-stream metering

In this paper we consider traffic networks that are controlled by ramp metering and variable speed limits. We demonstrate that speed limits can complement ramp metering, when the traffic demand is so high that ramp metering is not efficient anymore. In addition, we will compare the coordinated control of ramp metering and speed limits with the case where the speed limits are replaced by main-stream metering.

Ramp metering can be used for two different purposes. First, when traffic is dense, ramp metering can prevent a traffic breakdown by adjusting the metering rate such that the density remains below the critical value. Second, when drivers try to bypass congestion on a freeway by taking a local road (rat running), ramp metering can increase travel times and discourage the use of the bypass, see Middelham (1999). Several field and simulation studies have shown the effectiveness of ramp metering

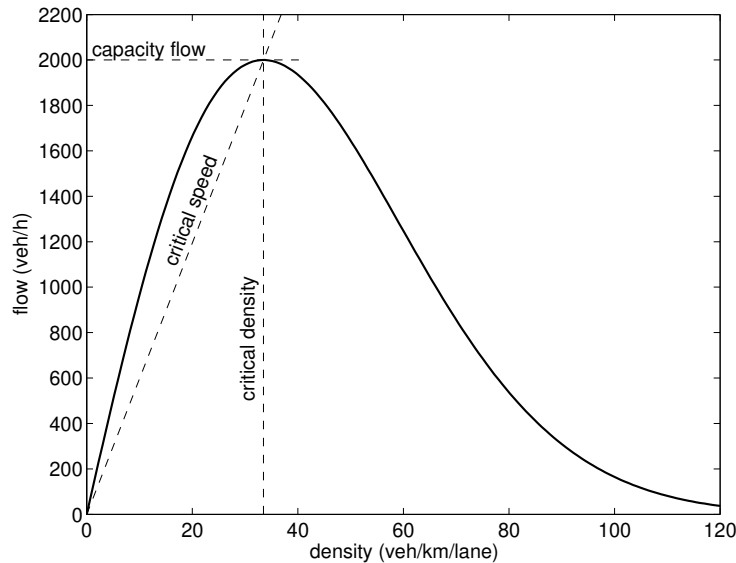


Fig. 1. A typical example of the fundamental diagram. The critical speed is the speed that corresponds to maximum flow and critical density.

for the first purpose, see Cambridge Systematics, Inc. (2001); Papageorgiou et al. (1990a, 1997); Taale and Middelham (2000). Generally a 0–5 % increase in the flow and a 0–10 % increase in the speed is achieved. In a simulation study Kotsialos et al. (2001) reported a decrease of 20–30 % in the TTS by optimal coordinated ramp metering on the Amsterdam ring road. In this paper we focus on the first objective of ramp metering, i.e., improving the traffic flow.

In literature basically two views on the use of speed limits can be found. The first, in Alessandri et al. (1998, 1999); Di Febbraro et al. (2001); Hoogen and Smulders (1994); Smulders (1990), emphasizes the homogenization effect, whereas the second, in Chien et al. (1997); Lenz et al. (1999, 2001), is more focused on the prevention of traffic breakdown. The idea of homogenization is that speed limits reduce the speed differences between vehicles which is expected to result in a higher (and safer) traffic flow. The homogenization approach typically uses speed limits that are close to, but above the critical speed (the speed that corresponds to the maximal flow, see Figure 1). The traffic breakdown prevention approach focuses more on preventing too high densities, and it also allows lower than critical speed limits. Opposed to the homogenization approach, this approach can also resolve existing jams. The results of the field study by Hoogen and Smulders (1994) indicate that the effect of homogenization on freeway performance is negligible; however, a positive safety effect can be expected. To the authors' best knowledge there are currently no published results available of experiments in connection with using speed limits to prevent traffic breakdown.

While ramp metering limits the flow at the entrances of the freeway, main-stream metering limits the flow on the freeway itself. The technical implementation is similar to ramp metering: by choosing the relative green time in the red-green cy-

cle the number of vehicles that may pass is controlled. Because of the similarity with on-ramp metering the same models are used for main-stream metering as for ramp metering. Simulation studies that include main-stream metering are presented in Kotsialos et al. (2001); Elloumi et al. (1997).

1.3 Control methods

Several control methods are used in literature to find a control law for speed control, such as multi-layer control in Papageorgiou (1983), sliding-mode control in Lenz et al. (1999, 2001), or optimal control in Alessandri et al. (1998, 1999). In Di Febbraro et al. (2001) the optimal control is approximated by a neural network in a rolling horizon framework. The authors mention on-line optimization (instead of the static control law represented by the neural network) as an option, but they prefer the neural network approach because of its speed. In this paper we demonstrate the feasibility of on-line optimization. The advantage of this approach is that it is able to adapt to changing traffic conditions.

Most of the models used in literature represent the speed limits by a factor that downscales the fundamental diagram. This can give too optimistic results (see Section 3.2.2), and therefore we will introduce another equation to model the effect of speed limits.

For a review of the literature on coordinated traffic control we refer to Kotsialos et al. (2002).

The contributions of this paper can be summarized as follows:

- We apply the MPC framework to freeway traffic control problems, which has certain advantages compared to optimal control.
- We integrate ramp metering and speed limit control which results in improved performance.
- We extend the METANET model with a speed limit and main-stream metering model; with a model for main-stream origin links (as opposed to on-ramps); and introduce a boundary condition for the virtual downstream density of the last segment of a link that connects to a destination.

This paper is organized as follows. In Section 2 the problem and the basic idea of the solution of the coordinated ramp metering and speed control is discussed. In Section 3 the basic ingredients of model predictive control are introduced, and the prediction model including the extensions is presented. The proposed control method is applied to a benchmark problem in Section 4. The conclusions are stated in Section 5. Finally, some topics for further research are discussed in Section 6.

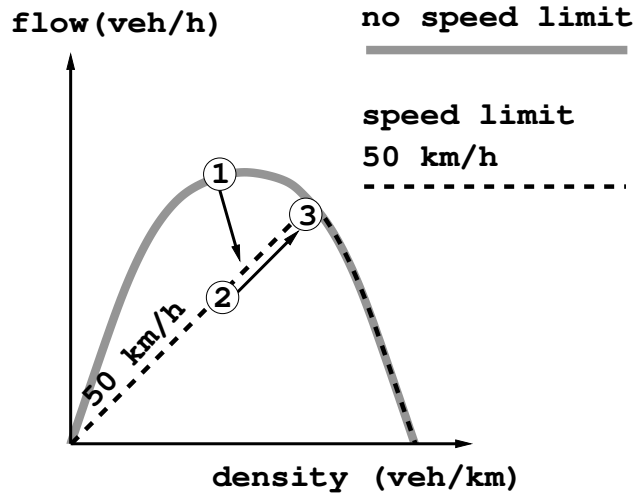


Fig. 2. When traffic on the main road is in state 1, then it is nearly unstable and even a small flow from the on-ramp can cause a breakdown. Speed limits change the state from 1 to somewhere between 2 and 3, and change the shape of the fundamental diagram from the solid gray line to the dashed black line. The decreases of flow creates some space for the on-ramp traffic.

2 Problem description

It is clear that ramp metering is only useful when traffic is not too light (otherwise ramp metering is not needed) and not too dense (otherwise breakdown will happen anyway). This region is on the stable (left) side of the fundamental diagram, and close to the top (see state 1 in Figure 2), where a breakdown can happen. In practice there often is a constraint present regarding the operation of the ramp metering device, which has as a consequence that the minimum flow from the on-ramp is higher than zero¹. Even these minimal flows can cause a breakdown when the traffic on the mainstream is dense, which then results in a reduced outflow and increased travel times.

In this paper we consider a situation where speed limits are imposed on the mainstream traffic while the on-ramp is metered (see Figure 3). The main idea is that when ramp metering is unable to prevent congestion, variable speed limits could prevent a breakdown by limiting the inflow into the area where the traffic breakdown starts. The speed limits change the shape of the fundamental diagram (see Figure 2) and reduce the outflow of the controlled segment. Suppose the traffic state on the freeway is 1. When a speed limit is applied, the speed drops and the density increases, so the traffic state will be somewhere between 2 and 3. However, because of the high traffic demand (state 1) the state will approach state 3, the capacity of the new fundamental diagram. Since this flow is lower than the capacity of

¹ Examples of such constraints are: a minimum metering rate or a maximum queue length on the on-ramp. Both constraints serve usually to prevent too long queues on the on-ramp, which could block other traffic streams on the secondary road network.

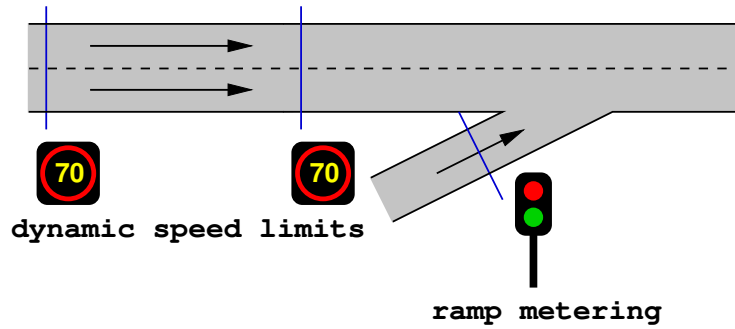


Fig. 3. We consider a combination of speed limits and ramp metering as control measures.

the freeway without speed limit, there will be some space left to accommodate the traffic from the on-ramp and a breakdown is prevented. Consequently, the density in the on-ramp area on the freeway remains low and the outflow remains high.

It is important to note that the congestion after a breakdown usually has a flow that is 5–10 % lower than the available capacity (the so called capacity-drop phenomenon). So any control method that improves the flow will at best achieve an improvement of approximately 5–10 %. However, Papageorgiou et al. (1998) showed in an example that in a traffic network an increase of outflow of 5 % may result in a decrease of the total time spent in the network of 20 %. This effect can be explained by the fact that the number of vehicles in the network is equal to the accumulated net inflow of the network (where the net inflow is the difference between the inflow and the outflow). However, the outflow is lower when there is congestion, so the queue grows faster, and consequently congestion will last longer, and the outflow will be low for a longer time (the time that the queue needs to dissolve). This is why one should try to prevent or postpone a breakdown as much as possible. A drawback of the above approach is that the mainstream flow will also decrease. But if the control is optimized properly, this flow drop will be always less than or equal to the flow drop of a breakdown, since otherwise breakdown would be the optimal situation. Another point of criticism could be that the approach keeps the controlled network congestion free, but at the cost of creating congestion at the entrances of the network. This is only partially true, because the controller will indeed delay the traffic sometimes to prevent a breakdown in the network, but afterwards the flow will be higher than if the breakdown would have occurred. So, the inflow of the controlled part of the network will be decreased by the speed limits only for a short period of time. Unfortunately this still can cause congestion in upstream sections. A remedy could be to extend size of the network with as many (uncontrolled) upstream sections as necessary to cover the congested area. In this way the congestion caused by the speed limits will not spill back to the mainstream origin queue and the congestion dynamics can be taken into account by the controller. Second, the network that is considered (i.e., evaluated and controlled) can be chosen larger, because the traffic is apparently so dense that the effects of the control reach beyond the bounds of the actual network.

3 Approach

3.1 Model Predictive Control

To solve the problem of coordination of speed limits and ramp metering we apply the model predictive control scheme in a rolling horizon framework, see Camacho and Bordons (1995); García et al. (1989); Maciejowski (2002). In this control scheme a discrete-time model is used to predict the future behavior of the process. The goal of the controller is to find the control signals that result in an optimal process (traffic) behavior. To express performance an objective function is defined and the control signals that optimize (in our case: minimize) this function are found via optimization.

The control is applied in a rolling horizon scheme: at each time instant k a new optimization is performed over the prediction horizon $[k, \dots, k + N_p - 1]$, and only the first value of the resulting control signal (the control signal for time instant k) is applied to the process (see Figure 4). The next time instant $k + 1$ this procedure is repeated. To reduce complexity and improve stability often a control horizon N_c ($\leq N_p$) is introduced, and after the control horizon has been passed the control signal is taken to be constant. So there are two loops: the rolling horizon loop and the optimization loop inside the controller. The loop inside the controller of Figure 4 is executed as many times as needed to find the optimal control signals at time instant k , for given N_p , N_c , traffic state and expected demand. The loop connecting the controller and the traffic system is performed once for each k and provides the state feedback to the controller. Recall that this feedback is necessary to correct for (the ever present) prediction errors, and disturbance rejection (compensation

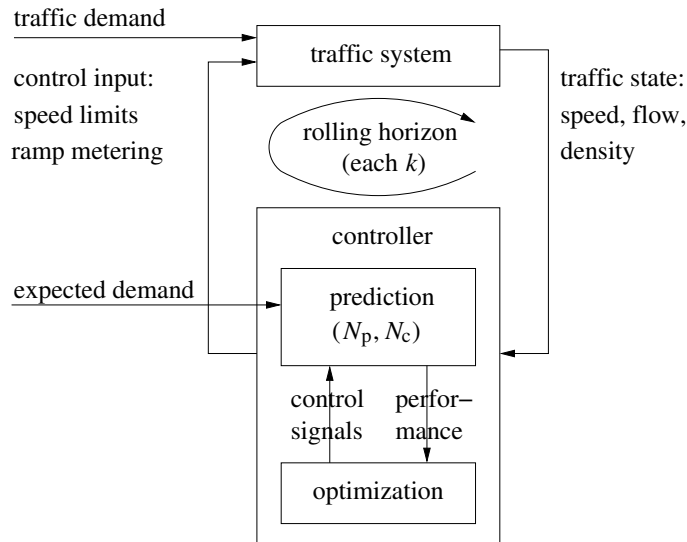


Fig. 4. Schematic view of the model predictive control (MPC) structure.

for unexpected traffic demand variations). The advantage of this rolling horizon approach is that it results in an on-line adaptive control scheme that allows us to take changes in the system or in the system parameters into account by regularly updating the model of the system. In conventional MPC heuristic tuning rules have been developed to select appropriate values for N_p and N_c .

One of the main parameters of MPC is the length of the prediction horizon N_p , the number of samples for which the behavior model is predicted. One should choose N_p long enough to include all relevant system dynamics in the prediction, but a too large N_p unnecessarily increases the computational demand.

When optimizing large networks, the computational complexity may become too high. In these cases the network should be separated into some subnetworks that are controlled by separate MPC controllers. The subnetworks should be chosen such that the interaction between them is small as possible. To handle the remaining interaction (e.g., occasionally overflowing queues) adequately structures such as hierarchical control or agent based control can be used.

3.2 Prediction model

The MPC procedure includes a prediction of the network evolution as a function of the current state and a given control input. For this prediction we use the destination independent METANET model from Kotsialos et al. (1999); Papageorgiou et al. (1990b); Kotsialos et al. (2002) with two extensions: a model for speed limits, and new model for mainstream origins (as opposed to on-ramp origins).

3.2.1 Original METANET model

We refer to Kotsialos et al. (2002) for a full description of the METANET model. For the sake of self-containedness we repeat the basic METANET model here.

The METANET model represents a network as a directed graph with the links (indicated by the index m) corresponding to freeway stretches. Each freeway link has uniform characteristics, i.e., no on-ramps or off-ramps and no major changes in geometry. Where major changes occur in the characteristics of the link or in the road geometry (e.g., on-ramp or an off-ramp), a node is placed. Each link m is divided into N_m segments (indicated by the index i) of length L_m (see Figure 5). Each segment i of link m is characterized by the *traffic density* $\rho_{m,i}(k)$ (veh/km/lane), the *mean speed* $v_{m,i}(k)$ (km/h), and the *traffic volume* or *outflow* $q_{m,i}(k)$ (veh/h), where k indicates the time instant $t = kT$, and T is the time step used for the simulation of the traffic flow (typically $T = 10$ s).

The following equations describe the evolution of the network over time. The out-

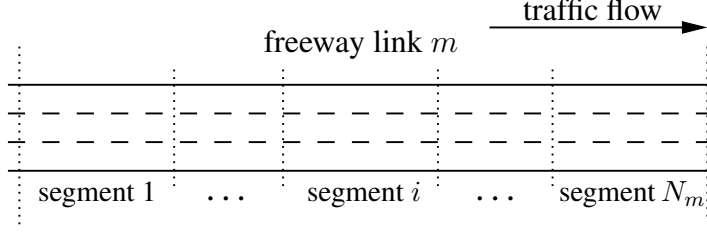


Fig. 5. In the METANET model a freeway link is divided into segments.

flow of each segment is equal to the density multiplied by the mean speed and the number of lanes on that segment (denoted by λ_m):

$$q_{m,i}(k) = \rho_{m,i}(k) v_{m,i}(k) \lambda_m . \quad (1)$$

The density of a segment equals the previous density plus the inflow from the upstream segment, minus the outflow of the segment itself (conservation of vehicles):

$$\rho_{m,i}(k+1) = \rho_{m,i}(k) + \frac{T}{L_m \lambda_m} (q_{m,i-1}(k) - q_{m,i}(k)) . \quad (2)$$

While equations (1) and (2) are based on physical principles and are exact, the equations that describe the speed dynamics and the relation between density and the desired speed are heuristic. The mean speed at time instant $k+1$ equals the mean speed at time instant k plus a relaxation term that expresses that the drivers try to achieve a desired speed $V(\rho)$, a convection term that expresses the speed increase (or decrease) caused by the inflow of vehicles, and an anticipation term that expresses the speed decrease (increase) as drivers experience a density increase (decrease) downstream:

$$\begin{aligned} v_{m,i}(k+1) = & v_{m,i}(k) + \frac{T}{\tau} \left(V(\rho_{m,i}(k)) - v_{m,i}(k) \right) + \\ & \frac{T}{L_m} v_{m,i}(k) (v_{m,i-1}(k) - v_{m,i}(k)) - \\ & \frac{\vartheta T}{\tau L_m} \frac{\rho_{m,i+1}(k) - \rho_{m,i}(k)}{\rho_{m,i}(k) + \kappa} , \end{aligned} \quad (3)$$

where τ , ϑ and κ are model parameters, and with

$$V(\rho_{m,i}(k)) = v_{\text{free},m} \cdot \exp \left[-\frac{1}{a_m} \left(\frac{\rho_{m,i}(k)}{\rho_{\text{crit},m}} \right)^{a_m} \right] , \quad (4)$$

with a_m a model parameter, and where the free-flow speed $v_{\text{free},m}$ is the average speed that drivers assume if traffic is freely flowing, and the critical density $\rho_{\text{crit},m}$ is the density at which the traffic flow is maximal.

Origins are modeled with a simple queue model. The length of the queue equals

the previous queue length plus the demand² $d_o(k)$, minus the outflow $q_o(k)$:

$$w_o(k+1) = w_o(k) + T(d_o(k) - q_o(k)) .$$

The outflow of the origin depends on the traffic conditions on the mainstream and, for the metered on-ramp, on the ramp metering rate³ $r_o(k)$, where $r_o(k) \in [0, 1]$. More specifically, $q_o(k)$ is the minimum of three quantities: the available traffic in time period k (queue plus demand), the maximal flow that could enter the free-way because of the mainstream conditions, and the maximal flow allowed by the metering rate:

$$q_o(k) = \min \left[d_o(k) + \frac{w_o(k)}{T}, Q_o \cdot r_o(k), Q_o \left(\frac{\rho_{\max,m} - \rho_{m,1}(k)}{\rho_{\max,m} - \rho_{\text{crit},m}} \right) \right] , \quad (5)$$

where Q_o is the on-ramp capacity (veh/h) under free-flow conditions and $\rho_{\max,m}$ (veh/km/lane) is the maximum density of link m , and m is the index of the link to which the on-ramp is connected.

In order to account for the speed drop caused by merging phenomena, if there is an on-ramp then the term

$$-\frac{\delta T q_o(k) v_{m,1}(k)}{L_m \lambda_m (\rho_{m,1}(k) + \kappa)} \quad (6)$$

is added to (3), where δ is a model parameter.

When there is a lane drop, the speed reduction due to weaving phenomena is expressed by:

$$-\frac{\phi T \Delta \lambda \rho_{m,N_m}(k) v_{m,N_m}^2(k)}{L_m \lambda_m \rho_{\text{crit},m}} , \quad (7)$$

where $\Delta \lambda = \lambda_m - \lambda_{m+1}$, the number of lanes being dropped, and ϕ is a model parameter.

The coupling equations to connect links are as follows. Every time there is a major change in the link parameters or there is a junction or a bifurcation, a node is placed between the links. This node provides the incoming links with a (virtual — when there are more leaving links —) downstream density, and the leaving links with an inflow and a (virtual — when there are more entering links —) upstream speed.

² Just as in Kotsialos et al. (1999, 2001); Papageorgiou et al. (1990a) we assume that the demand is independent of any control actions taken in the network.

³ For an unmetered on-ramp we also can use (5) by setting $r_o(k) \equiv 1$.

The flow that enters node n is distributed among the leaving links according to

$$Q_n(k) = \sum_{\mu \in I_n} q_{\mu, N_\mu}(k) \quad , \quad (8)$$

$$q_{m,0}(k) = \beta_n^m(k) \cdot Q_n(k) \quad , \quad (9)$$

where $Q_n(k)$ is the total flow that enters the node at time k , I_n is the set of links that enter node n , $\beta_n^m(k)$ are the turning rates (the fraction of the total flow through node n that leaves via link m), and $q_{m,0}(k)$ is the flow that leaves node n via link m .

When node n has more than one leaving link, the virtual downstream density $\rho_{m, N_{m+1}}(k)$ of entering link m is given by

$$\rho_{m, N_{m+1}}(k) = \frac{\sum_{\mu \in O_n} \rho_{\mu,1}^2(k)}{\sum_{\mu \in O_n} \rho_{\mu,1}(k)} \quad ,$$

where O_n is the set of links leaving node n .

When node n has more than one entering link, the virtual upstream speed $v_{m,0}(k)$ of leaving link m is given by

$$v_{m,0}(k) = \frac{\sum_{\mu \in I_n} v_{\mu, N_\mu}(k) \cdot q_{\mu, N_\mu}(k)}{\sum_{\mu \in I_n} q_{\mu, N_\mu}(k)} \quad .$$

3.2.2 Extensions

Since the original METANET model does not describe the effect of speed limits we have extended the desired-speed equation (4) to incorporate speed limits.

In some publications the effect of the speed limit is expressed by scaling down the desired speed (a function of density) by $v_{\text{control}}/v_{\text{free},m}$. This changes the whole speed-density diagram, also for the states where the speed would otherwise be lower than the value of the speed limit. This means, for example, that if the free flow speed is 120 km/h and the displayed speed limit is 100 km/h then it is assumed that the speed and flow of the traffic are reduced even when the vehicles are traveling at 80 km/h. Furthermore, scaling down the desired speed also reduces the capacity, while there is no reason to assume that a speed limit *above* the critical speed (speeds where the flow has not reached capacity yet) would reduce the capacity of the road. These assumptions are rather unrealistic, and they exaggerate the effect of speed limits. However, to get a more realistic model for the effects of the speed limits, we assume that the desired speed is the minimum of the following two quantities: the desired speed based on the experienced density, and the desired

speed caused by the speed limit displayed on the variable message sign (VMS):

$$V(\rho_{m,i}(k)) = \min \left(v_{\text{free},m} \cdot \exp \left[-\frac{1}{a_m} \left(\frac{\rho_{m,i}(k)}{\rho_{\text{crit},m}} \right)^{a_m} \right], (1 + \alpha)v_{\text{control},m,i}(k) \right), \quad (10)$$

where $v_{\text{control},m,i}(k)$ is the speed limit imposed on segment i , link m , at time k , and $1 + \alpha$ is the non-compliance factor that expresses that drivers usually do not fully comply with the displayed speed limit and their target speed is usually higher than what is displayed.⁴

To express the different nature of a mainstream origin link o compared to a regular on-ramp (the queue at a mainstream origin is in fact an abstraction of the sections upstream of the origin of part of freeway network that we are modeling), we use a modified version of (5) with another flow constraint, because the inflow of a segment (and thus the outflow of the mainstream origin) can be limited by an active speed limit or by the actual speed on the first segment (when either of them is lower than the speed at critical density). We assume that the maximal flow equals the flow that follows from the speed-flow relationship from (1) and (4) with the speed equal to the speed limit or the actual speed on the first segment, whichever is smaller. So if o is the origin of link μ , then we have

$$q_o(k) = \min \left[d_o(k) + \frac{w_o(k)}{T}, q_{\text{lim},\mu,1}(k) \right],$$

where $q_{\text{lim},\mu,1}(k)$ is the maximal inflow determined by the limiting speed in the first segment of link μ :

$$q_{\text{lim},\mu,1}(k) = \begin{cases} \lambda_\mu \cdot v_{\text{lim},\mu,1}(k) \cdot \rho_{\text{crit},\mu} \left[-a_\mu \ln \left(\frac{v_{\text{lim},\mu,1}(k)}{v_{\text{free},m}} \right) \right]^{\frac{1}{a_\mu}} & \text{if } v_{\text{lim},\mu,1}(k) < V(\rho_{\text{crit},\mu}) \\ q_{\text{cap},\mu} & \text{if } v_{\text{lim},\mu,1}(k) \geq V(\rho_{\text{crit},\mu}), \end{cases}$$

where

$$v_{\text{lim},\mu,1}(k) = \min(v_{\text{control},\mu,1}(k), v_{\mu,1}(k))$$

is the speed that limits the flow, and

$$q_{\text{cap},\mu} = \lambda_\mu V(\rho_{\text{crit},\mu}) \rho_{\text{crit},\mu}$$

is the capacity flow.

⁴ Data from the Dutch freeways shows that when the speed limits are not enforced the average speed is approximately 10% higher than what is displayed ($\alpha = 0.1$), and when they are enforced the average speed is approximately 10% lower than what is displayed ($\alpha = -0.1$).

In addition assumptions are made regarding the boundary conditions for the upstream speed and downstream density. When there is no entering link (but a mainstream origin) we assume that the speed of the (virtual) entering link equals the speed of the first segment

$$v_{m,0}(k) = v_{m,1}(k) . \quad (11)$$

When there is no leaving link (but a mainstream destination) we assume that the density of the (virtual) leaving link equals the density of segment N_μ in free flow, and equals the critical density in congested flow:

$$\rho_{\mu,N_\mu+1}(k) = \begin{cases} \rho_{\mu,N_\mu}(k) & \text{if } \rho_{\mu,N_\mu}(k) < \rho_{\text{crit},\mu} \\ \rho_{\text{crit},\mu} & \text{if } \rho_{\mu,N_\mu}(k) \geq \rho_{\text{crit},\mu} . \end{cases} \quad (12)$$

This is a reasonable abstraction of density behavior in the unmodeled downstream segments when there is no downstream bottleneck.

3.3 Objective function

The model predictive control algorithm finds the control signals $r_o(j)$ and $v_{\text{control},m,i}(j)$ for $j \in \{k, \dots, k + N_c - 1\}$ that minimize the following objective function:

$$J(k) = T \sum_{j=k}^{k+N_p-1} \left\{ \sum_{m,i} \rho_{m,i}(j) L_m \lambda_m + \sum_o w_o(j) \right\} + \sum_{j=k}^{k+N_c-1} \left\{ a_{\text{ramp}} \sum_{o \in O_{\text{ramp}}} (r_o(j) - r_o(j-1))^2 + a_{\text{speed}} \sum_{(m,i) \in I_{\text{speed}}} \left(\frac{v_{\text{control},m,i}(j) - v_{\text{control},m,i}(j-1)}{v_{\text{free},m}} \right)^2 \right\} , \quad (13)$$

where O_{ramp} is the set of indexes o of those on-ramps where ramp metering is present, and I_{speed} is the set of pairs of indexes (m, i) of the links and segments where speed control is present. This objective function contains two terms for the TTS (one term for the mainstream traffic and one term for the on-ramp queues), and two terms that penalize abrupt variations in the ramp metering and speed limit control signals respectively. The last two terms are weighted by the non-negative weight parameters a_{ramp} and a_{speed} .

The utilized numerical algorithm to solve the optimization problem is the MATLAB implementation of the SQP algorithm (fmincon). This optimization method has the advantage that the constraints can be formulated explicitly, without the use of a penalty term in the objective function.

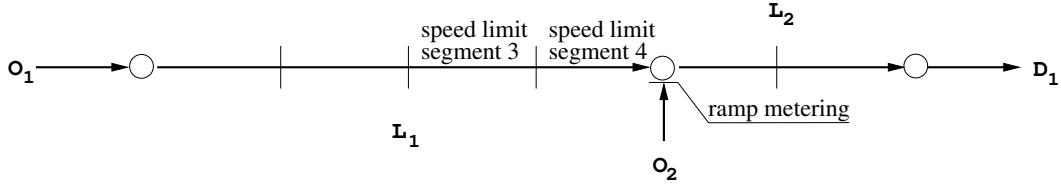


Fig. 6. The benchmark network includes two sections with speed limits, and a metered on-ramp.

3.4 Tuning of N_p and N_c

The tuning rules to select appropriate values for N_p and N_c that have been developed for conventional MPC cannot be straightforwardly applied the traffic flow control framework presented above. However, based on heuristic reasoning we can determine an initial guess for these parameters.

N_p should be larger than the typical travel time from the controlled segments to the exit of the network, because if we take the prediction horizon N_p shorter than the typical travel time in the network, the effect of the vehicles that are influenced by the current control measure and — as a consequence — have an effect on the network performance before they exit the network, will not be taken into account. Furthermore, a control action may affect the network state (by improved flows, etc.) even when the actually affected vehicles have already exited the network. On the other hand, N_p should not be too large because of the computational complexity of the MPC optimization problem. So based on this reasoning we select N_p to be about the typical travel time in the network. For the control horizon N_c we will select a value that represents a trade-off between the computational effort and the performance.

In order to illustrate the control framework presented above we will now apply it to a simple traffic network.

4 A benchmark problem

4.1 Network and scenarios

The benchmark network for the experiments was chosen as simple as possible (see Figure 6), since we want to focus on the relevant points of the approach presented in this paper. The network consists of a mainstream freeway with two speed limits, and a metered on-ramp. The second speed limit is included to have more control over the state (speed, density) in the segment that is just before the on-ramp. The network considered consists of two origins (a mainstream and an on-ramp), two

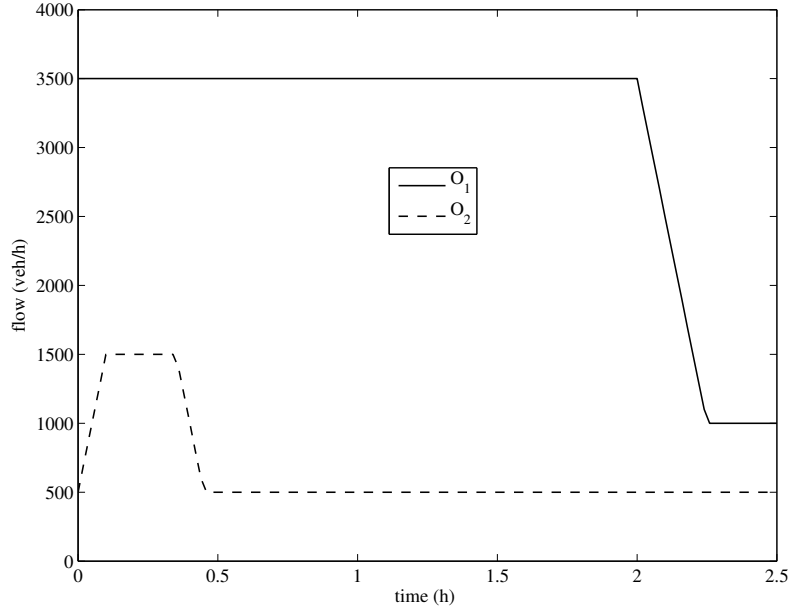


Fig. 7. The demand scenario considered in the simulation experiments.

freeway links, and one destination. O_1 is the main origin and has two lanes with a capacity of 2000 veh/h each. The freeway link L_1 follows with two lanes, and is 4 km long consisting of four segments of 1 km each. Segments 3 and 4 are equipped with a VMS where speed limits can be set. At the end of L_1 a single-lane on-ramp (O_2) with a capacity of 2000 veh/h is attached. Link L_2 follows with two lanes and two segments with length of 1 km each, and ends in destination D_1 with unrestricted outflow. We assume that the queue length at O_2 may not exceed 100 vehicles, in order to prevent spill-back to a surface street intersection.

We use the network parameters as found in Kotsialos et al. (1999): $T = 10$ s, $\tau = 18$ s, $\kappa = 40$ veh/km/lane, $\vartheta = 60$ km²/h, $\rho_{\max} = 180$ veh/km/lane, $\delta = 0.0122$, $a_1 = a_2 = 1.867$ and $\rho_{\text{crit}} = 33.5$ veh/km/lane.

Furthermore, we assume that the desired speed is 10% higher than the displayed speed limit, that is $\alpha = 0.1$; and we set the signal variance penalty weights a_{ramp} and a_{speed} to 0.4.

The controller sampling time is chosen to be 1 minute. During one sampling interval the control signals are assumed to be constant.

To examine the effect of the combination of variable speed limits and ramp metering a typical demand scenario is considered (see Figure 7): The mainstream demand has a constant, relatively high level and a drop after 2 hours to a low value in 15 minutes. The demand on the on-ramp increases to near capacity, remains constant for 15 minutes, and decreases finally to a constant low value. The scenario was chosen such that the final (and steady) state of the network is the same for all scenarios (which means that all final densities are the same for all scenarios). This enables

us to compare the TTS of the different scenarios. Under the given demand scenario the ‘no-control’, the ‘ramp metering only’ and the ‘coordinated speed limits and ramp metering’ cases are compared. Since the control optimizes TTS, the relevant quantity for comparing the following simulations is TTS.

Since one of the effect of the speed limits is that it holds back the demand, the comparison of ramp metering in combination with main stream metering instead of speed limits is also interesting. However, using main-stream metering has some disadvantages compared to speed limits:

- (1) Dynamic speed limit displays are already installed on most freeways⁵, while main-stream metering requires the installation of new equipment.
- (2) Main-stream metering operates on only one location, while speed limits can operate on many consecutive segments on the freeway. When limiting the flow on one location only, the risk of creating a shock wave is higher, while coordinated speed limits can prevent the creation of shock waves (see Hegyi et al. (2004)).
- (3) When main-stream metering is active, the maximum flow is bounded by minimum red (and amber) times to approximately 75 % of the freeway capacity, so there is no possibility to take control actions that result in flows in the range of 75 %–100 % of the freeway capacity.

An advantage of main-stream metering is that it can limit the flow much more than speed limits, what can be useful in very severe situations. As a comparison the simulations of using main-stream metering instead of speed limits are included in Section 4.3.

4.2 Results

The results of the no-control case are shown in Figure 8. As the demand increases on the on-ramp, the density on the main-stream on-ramp section (segment 1 of link 2) also increases and creates a congestion that propagates through link 1 and grows outside (upstream) the network. This causes a queue at origin 1 of approximately 150 vehicles. When the demand on the on-ramp drops to 500 veh/h the densities of segment 1 of link 2 up to segment 1 of link 1 gradually decrease but remain at a relatively high value (approximately 50 veh/km/lane). Finally, when the mainstream demand drops, the congestion dissolves and the network reaches steady state. The TTS is 1460.0 veh.h.

In Figure 9 we can see that ramp metering only performs well (density in the on-ramp segment, link 2 segment 1, remains around 40 veh/km/lane, and the outflow of the network is high) until the maximum queue length is reached. After that point the

⁵ In The Netherlands, but also in many other countries.

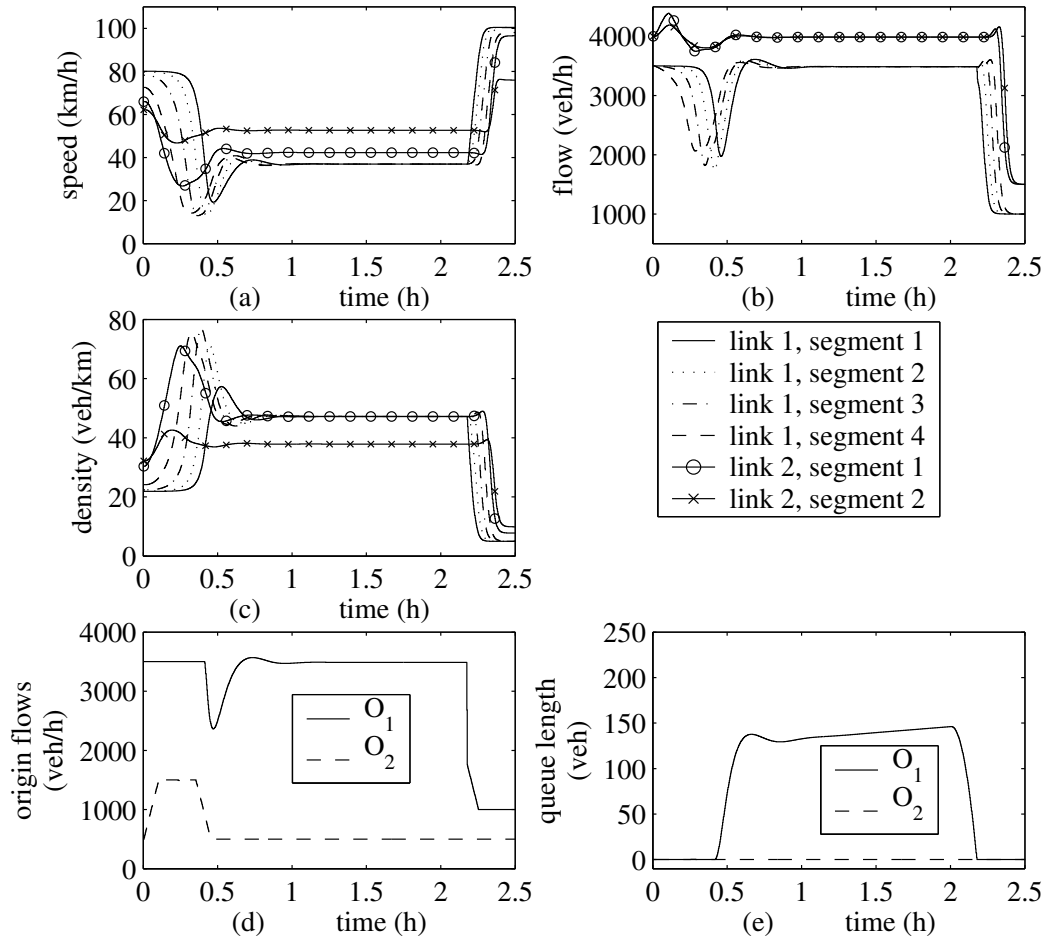


Fig. 8. The simulation for the no-control case. The high demand on the on-ramp causes a congestion that remains existing until the main-stream demand drops to a low value.

performance breaks down in the ‘ramp metering only’ case, but in the ‘coordinated speed limits and ramp metering’ case the speed limits become active at this moment (see Figure 10). The speed limits reduce the inflow of the critical segment and cause a lower density (around 40 veh/km/lane) which enables a higher outflow. The TTS in the ‘ramp metering only’ case was 1382.6 veh.h which is an improvement of 5.3 % compared to the ‘no control’ case. The TTS in the ‘coordinated speed limits and ramp metering’ case is 1251.0 veh.h, which is an improvement of 14.3 %.

So, we can conclude that the speed limits substantially improve the network performance.

For the MPC-controlled cases the optimal prediction horizon was found to be approximately $N_p = 42$, corresponding to 7 minutes, which is in the order of the typical travel time through the network (4 km/40 km/h). Shorter prediction horizons did not take the whole response of the system into account and result in insufficient control actions. Longer prediction horizons tend to take the future demand too much into account, which degrades the performance. When the difference $N_p - N_c$

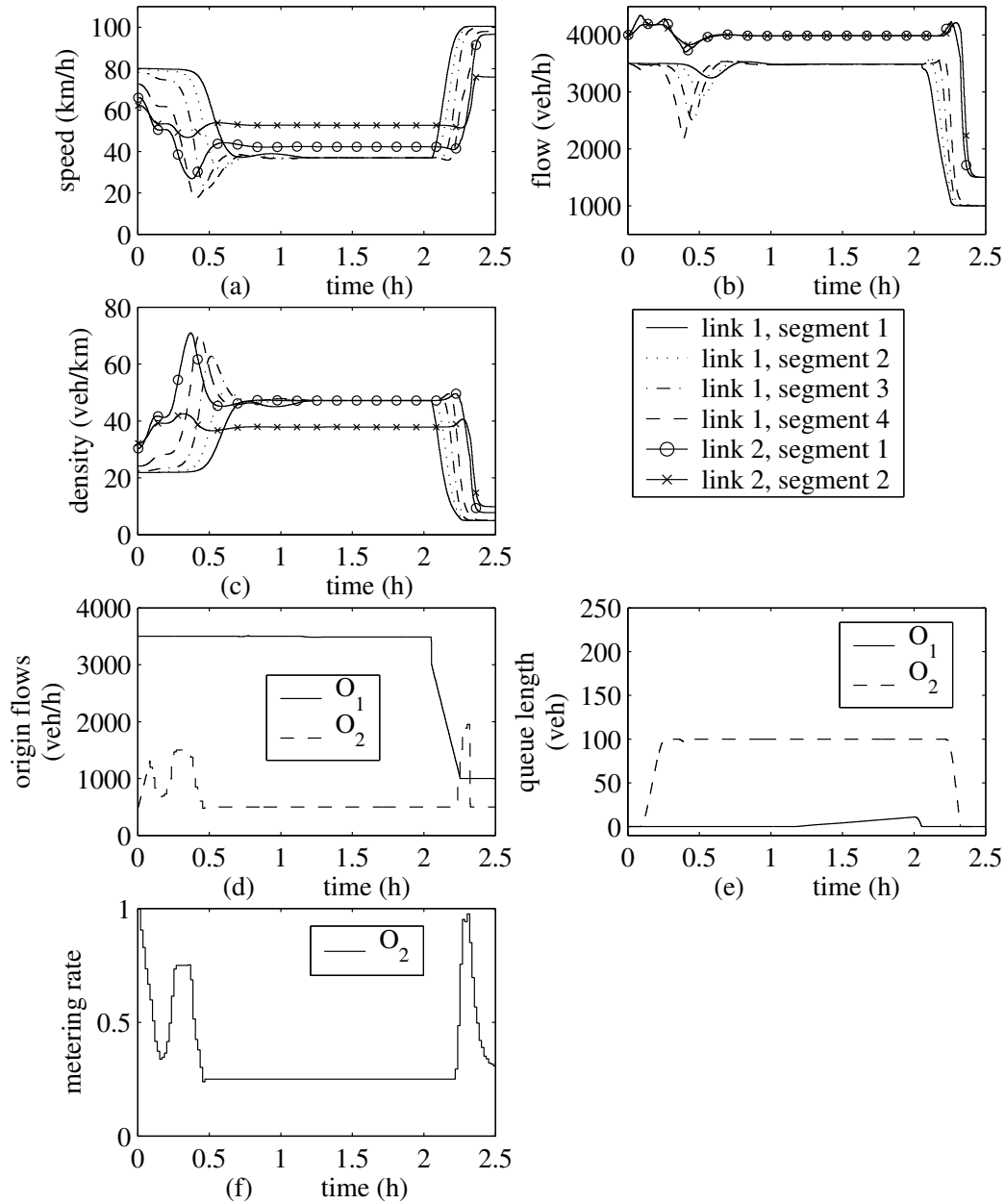


Fig. 9. The simulation results for the ‘ramp metering only’ case. When the density approaches the critical density (33.5 veh/km), the ramp metering gradually switches on, and keeps the flow high (around 4200 veh/h). After the queue length on the on-ramp has reached the maximum queue length (100 veh), the ramp metering is forced to let the complete demand enter the freeway. This causes a congestion, and hence results in a reduced outflow. The queue at O_1 is slowly increasing until the main-stream demand drops.

was kept constant (corresponding to 2 minutes) further increase of N_p caused only a small decrease of the TTS. A control horizon $N_c = 3$ (corresponding to 3 minutes) was sufficient for the ‘ramp metering only’ case, for the ‘coordinated speed limits and ramp metering’ case a control horizon of $N_c = 5$ (corresponding to 5 minutes) was necessary. Longer control horizons tended to result in more variance

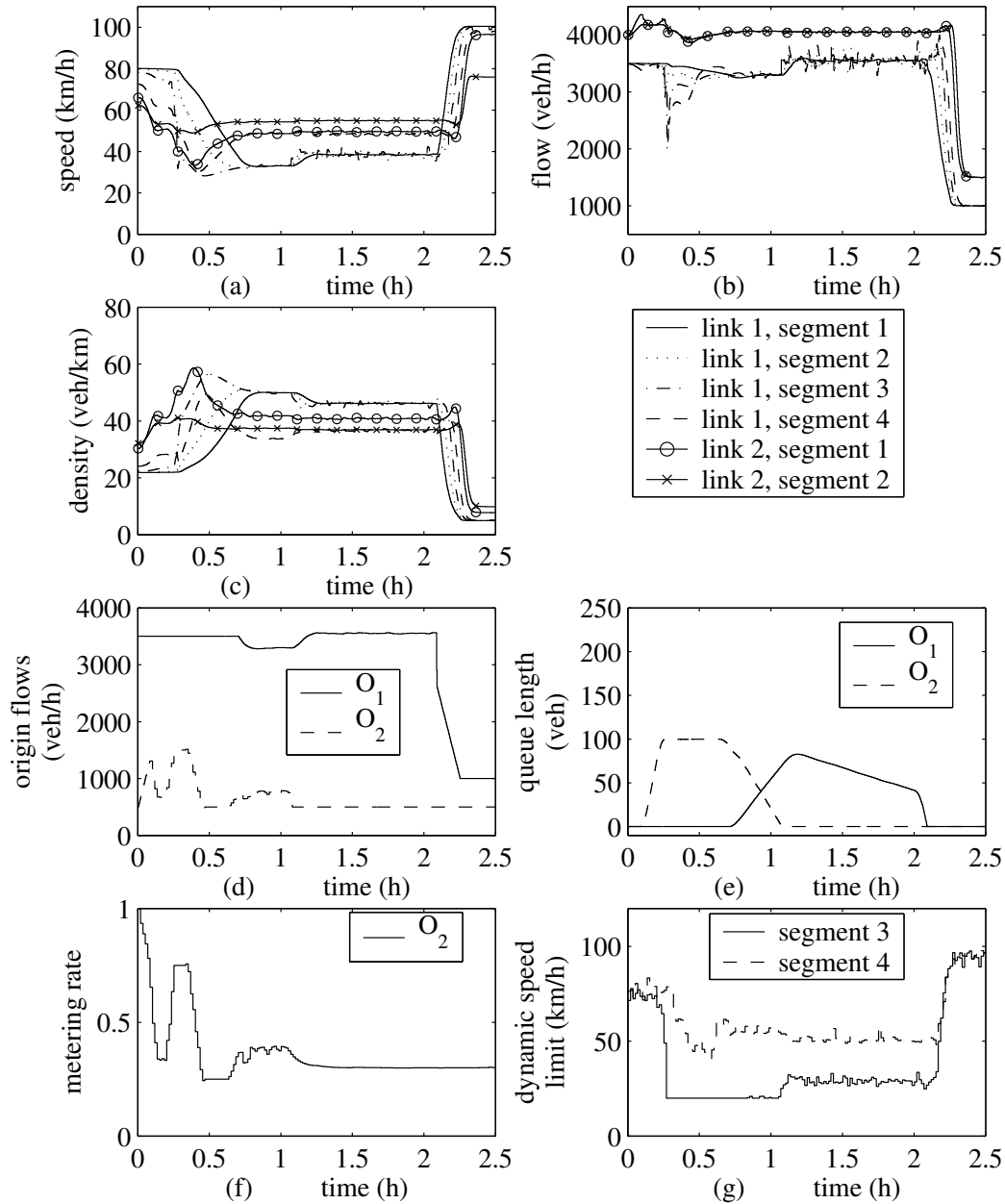


Fig. 10. The results for the ‘coordinated speed limits and ramp metering’ case. In the beginning the ramp metering signal is similar to that of the ‘ramp metering only’ case, but when the density becomes too high (which endangers the high outflow), the speed limits become active and reduce the inflow from the mainstream. This initially causes a longer queue on the mainstream, but the outflow is kept higher and both queues (O_1 and O_2) are decreasing even before the main-stream demand drops.

in the control signals.

The computation times were typically around 6 seconds and maximal 10 seconds for one MPC iteration in the ‘coordinated speed limits and ramp metering’ case (which is the most demanding case). Since the controller sampling time is 1 minute

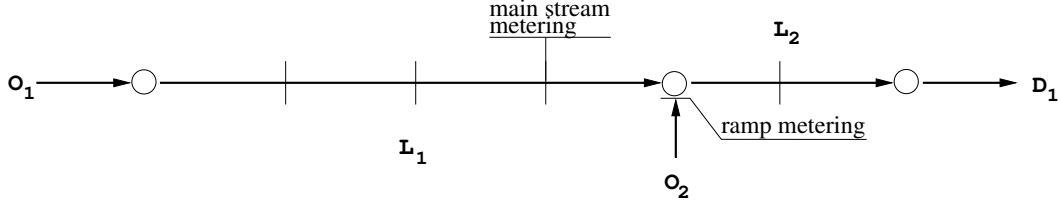


Fig. 11. The benchmark network with main-stream metering and a metered on-ramp.

this is fast enough for real-time implementation.

4.3 Ramp metering combined with main-stream metering

Since the primary effect of speed limits in this study is that they reduce the flow, it is interesting to compare the ‘coordinated speed limits and ramp metering’ case with the scenario that the speed limits are replaced by main-stream metering (‘coordinated main-stream metering and ramp metering’). In this section we present a main-stream metering model and the simulation result of applying main-stream metering to the benchmark network of Section 4.

4.3.1 Main-stream metering model

Main-stream metering on segment i of link m is modeled as a restriction on the outflow of the segment as follows

$$q_{m,i}(k) = \min(r_{\text{msm}}(k)Q_m, q_{\text{orig},m,i}(k)) \quad (14)$$

where Q_m is the nominal capacity⁶ of link m , $r_{\text{msm}}(k)$ is the main-stream metering signal where $r_{\text{msm}}(k) \in [0, 1]$, and $q_{\text{orig},m,i}(k)$ is the outflow of segment i of link m that follows from (1).

If $q_{m,i}(k) < q_{\text{orig},m,i}(k)$ the speed of the segment has to be adapted accordingly:

$$v_{m,i}(k) = \begin{cases} v_{\text{orig},m,i}(k) \frac{q_{m,i}(k)}{q_{\text{orig},m,i}(k)} & \text{if } q_{m,i}(k) < q_{\text{orig},m,i}(k) , \\ v_{\text{orig},m,i}(k) & \text{otherwise,} \end{cases} \quad (15)$$

where $v_{\text{orig},m,i}(k)$ is the speed that follows from (3).

4.3.2 Simulation

In this section we show the results of the simulations with coordinated main-stream metering and ramp metering for three different boundary conditions. The layout of the network is shown in Figure 11. The main-stream metering is chosen to operate on segment 3 of link 1, because that is the segment on which the speed limits have the largest effect (see Section 4.2).

In the first simulation the lower bound LB_{msm} of the main-stream metering signal is chosen such that it results in a flow equivalent to the lower bound of the speed limits used in the simulations for the ‘coordinated speed limits and ramp metering’ case, $LB_{\text{msm}} = 0.62$ (see Figure 12). The interpretation of the main-stream metering signal behavior is very similar to the speed limit in the ‘coordinated speed limits and ramp metering’ case: main-stream metering becomes active when ramp metering is unable to keep the on-ramp segment of the freeway congestion free. The TTS is 1241.4 veh.h, which means an improvement of 15 % compared to the ‘no-control’ case.

In the second case the lower bound of the main-stream metering was set to a low value (0.2) (see Figure 13). This allows to fully utilize the advantage of main-stream metering over speed limits: main-stream metering can limit the flow much more. The TTS in this case is 1206.7 veh.h which is an improvement of 17.4 %. This performance is — as can be expected — better than in the first case.

When main-stream metering is active, the maximum flow is bounded by minimum red (and amber) times to approximately 75 % of the freeway capacity, so there is no possibility to take control actions that result in flows in the range of 75 %–100 % of the freeway capacity. Therefore, we include a third simulation, where we take into account the on/off switching of the main-stream metering. So, the outflow of segment 3 of link 1 is unconstrained when main-stream metering is off, and is limited to $UB_{\text{msm}} \cdot q_{\text{cap},m,i}$ (we take here 75 % of the nominal capacity of the freeway: $UB_{\text{msm}} = 0.75$) when main-stream metering is on. To approximate the on/off switching a similar approach is used as in Hegyi et al. (2004): the real-valued control signal $r_{\text{msm}}(k)$ that results from one MPC iteration is rounded as

$$r_{\text{msm,rounded}}(k) = \begin{cases} 1 & \text{if } (1 + UB_{\text{msm}})/2 \leq r_{\text{msm}}(k) \\ UB_{\text{msm}} & \text{if } (1 + UB_{\text{msm}})/2 > r_{\text{msm}}(k) \geq UB_{\text{msm}} \\ r_{\text{msm}}(k) & \text{otherwise,} \end{cases} \quad (16)$$

where $UB_{\text{msm}} \in [0, 1]$ represents the fastest metering rate, achieving the maximum flow during main-stream metering and $r_{\text{msm,rounded}}(k)$ is applied to the traffic process. See Figure 14 for the simulation results. The TTS is 1224.4 veh.h which is

⁶ We prefer not to use $q_{\text{cap},m} = \lambda_m V(\rho_{\text{crit},m}) \rho_{\text{crit},m}$, which is the capacity following from the fundamental diagram, because equations (1), (2), (3), (4) also allow higher flows than $q_{\text{cap},m}$. We chose Q_m to be 5 % higher than $q_{\text{cap},m}$ based on simulations.

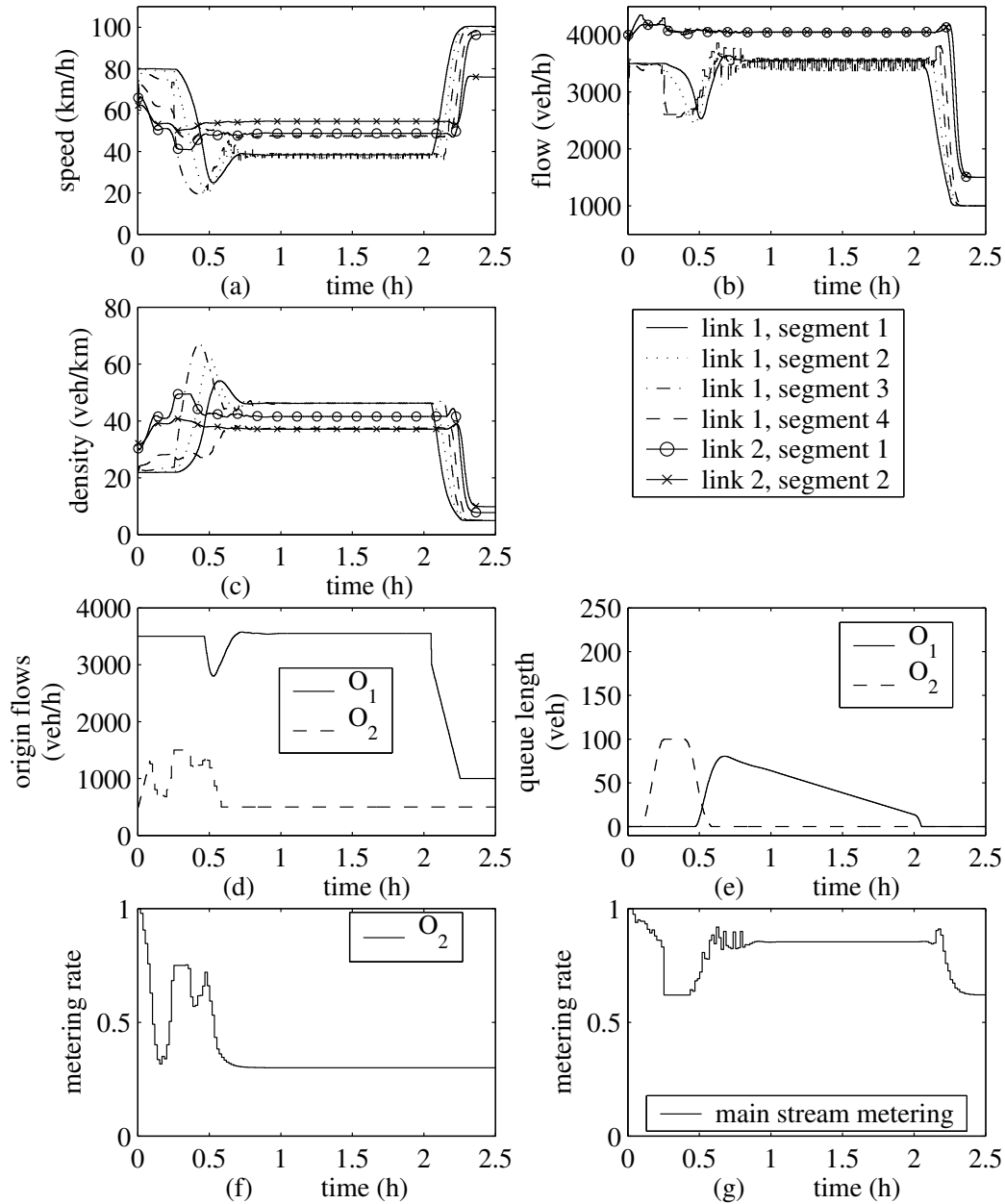


Fig. 12. The results for the ‘coordinated main-stream metering and ramp metering’ case. The behavior of the main-stream metering is similar to behavior of the speed limits in the ‘coordinated speed limits and ramp metering’ case.

an improvement of 16.1 %. This performance is very good. However, the controller oscillates (between switching on and off the main-stream metering) when the optimal metering rate would be somewhere in $[UB_{msm}, 1]$ (compare with the second case in Figure 13(g)). Such a frequent on/off switching is undesirable, because it may create shock waves and it may be unsafe. Therefore, in cases where only a limited flow reduction is necessary, speed limits are recommended.

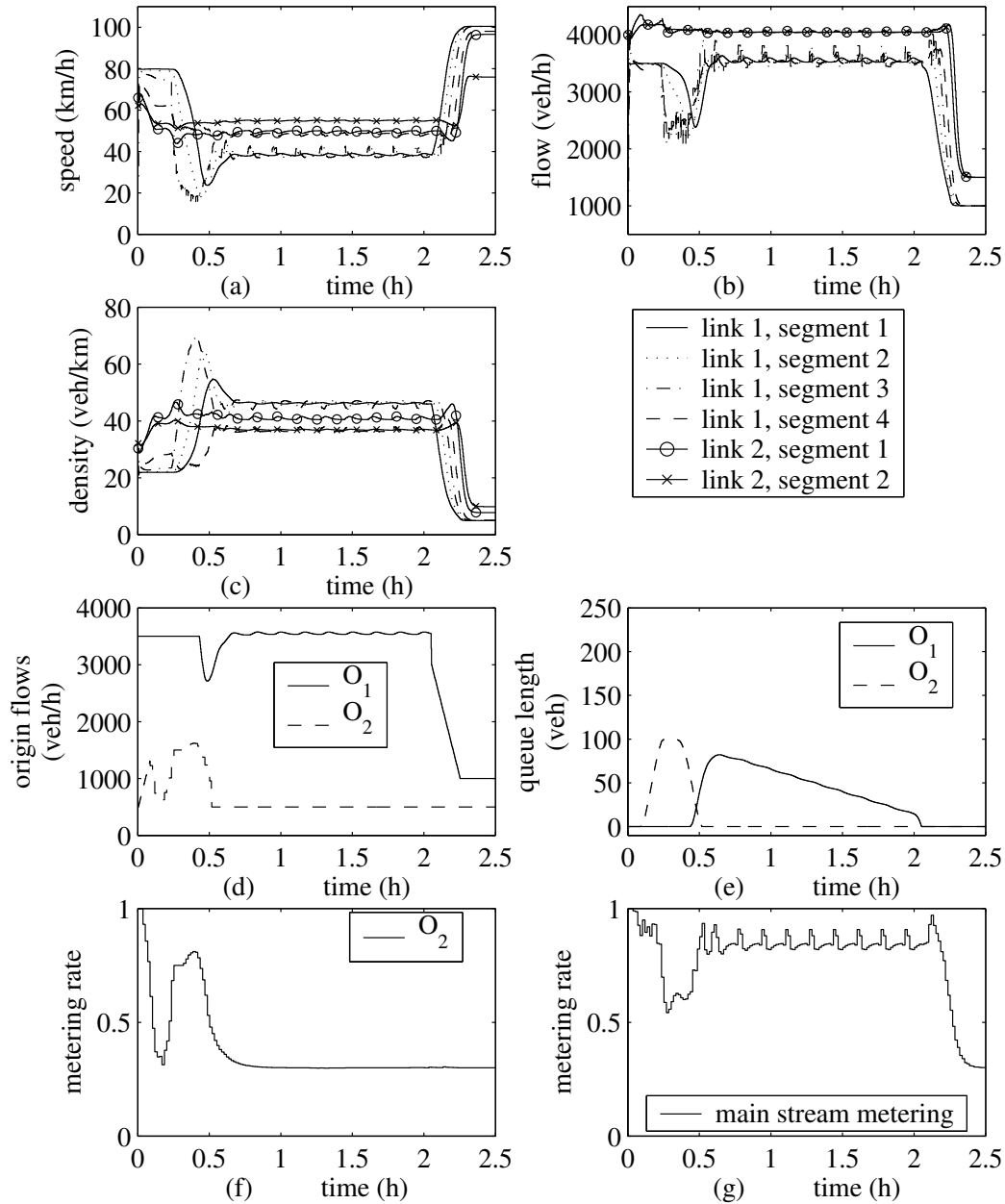


Fig. 13. The results for the ‘coordinated main-stream metering and ramp metering’ case with lower bound of $r_{\text{msm}}(k)$ set to 0.2.

5 Conclusions

We have applied model predictive control to traffic networks with variable speed limits and ramp metering, to achieve optimal coordination between these measures. Speed limits proved to be useful when ramp metering was unable to keep the on-ramp segment of the freeway congestion free. This idea was illustrated by a simple example network, where the cases ‘ramp metering only’ and ‘coordinated ramp metering and speed limits’ were compared for a typical demand scenario. For both

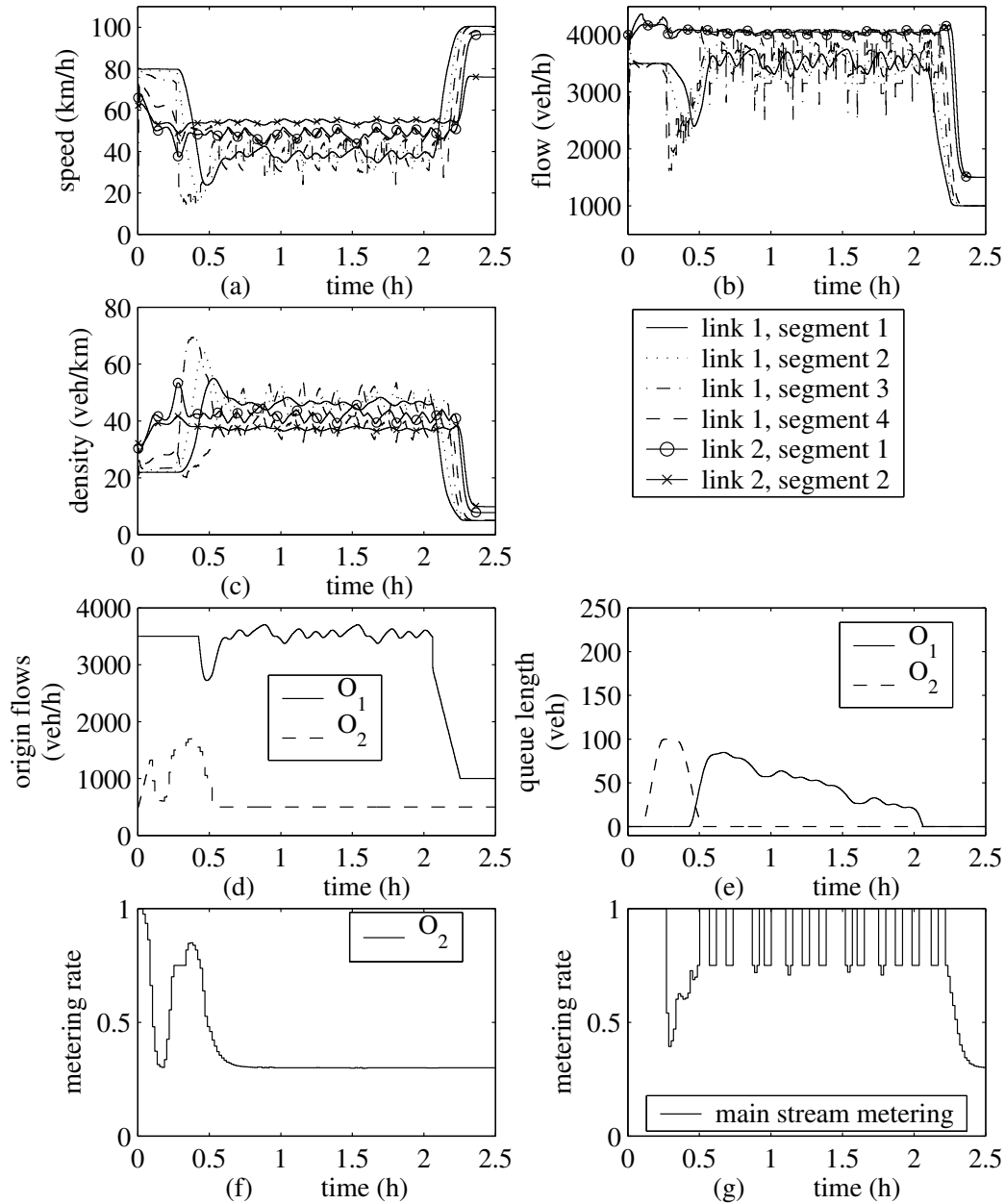


Fig. 14. The results for the ‘coordinated main-stream metering and ramp metering’ case with lower bound of $r_{\text{msm}}(k)$ set to 0.2, and taking the maximal possible flow into account when main-stream metering is on. The controller shows oscillatory behavior when the optimal metering rate is between the upper bound and 1.

cases the control signal was optimized such that the total time spent in the network is minimal. We found that the coordinated case results in a network that has a higher outflow and a significantly lower total time spent.

The interpretation of these result is that the choice between speed limits and main-stream metering should be made based on the demands on the on-ramp and the freeway. If the speed limits can limit the flow sufficiently, i.e., the flow corresponding to the

lowest speed limits is such that the ramp flow and the main flow can be accommodated, then speed limits are preferred. If not, main-stream metering should be used, which can limit the flow much more. The preference for speed limits is motivated by the advantages of speed limits compared to main-stream metering. First, the maximum flow for main-stream metering is limited to approximately 75 % of the nominal capacity of the freeway when main-stream metering is on. This can cause oscillatory behavior when only a lighter flow limitation is necessary. Second, main-stream metering limits the flow only at one location which may cause shock waves. Opposed to this, speed limits can limit the flow more gradually, and since there are often installed more speed limit signs on freeway stretch they can prevent shock waves. A possible approach that utilizes the advantages of both speed limits and main-stream metering would be to use both measures and switch between the two, depending on the severity of the traffic congestion.

6 Further research

Several topics for further research are related to modeling. In (10) we have modeled the effect of a speed limit as a change in the desired speed of the drivers. We have implicitly assumed that this change occurs for all the drivers in the segment instantaneously, which may not reflect reality, because only those drivers will change their desired speed who have entered the segment after the speed limit was changed and who have seen the speed limit sign. For any speed limit model that is chosen, verification based on real data is needed.

Another topic for future research is the study of other macroscopic traffic flow models, such as the gas kinetic model of Helbing (1997) or Hoogendoorn and Bovy (2000) for MPC traffic control applications.

In our simulations we have assumed that the real world coincides with the model used by the controller to make predictions, and that the disturbances (demands) are exactly known. In practice, the model of the controller is always different from the real traffic system, and the disturbances are only partially known. Model predictive control is known to perform well when this kind of differences occurs, see Maciejowski (2002). A demonstration of this feature would be useful. Furthermore, in the future the standard approaches of MPC that ensure stability will be studied.

We have also assumed that drivers are able to follow speed limits that have real values. However, in practice the displayed speed limits are integer multiples of some speed limit step size (typically 10 or 20 km/h, or 5 or 10 mph). The effects of using discretized speed limits is studied in Hegyi et al. (2004). The main conclusion of this paper is that rounding the speed limits to the nearest multiple of 10 km/h results in practically the same performance as in the case of real speed limit values.

Since upstream of many on-ramps an off-ramp is located, and on-ramp queues often block these off-ramps, it would be interesting to also include off-ramps in the benchmark network. In this way the effect of integrated control of speed limits and ramp-metering on off-ramp blocking could be investigated. In practice the blocking of off-ramp traffic is a typical source of network performance degradation; the theoretical improvement that can be achieved by preventing off-ramp blocking is published in Papageorgiou and Kotsialos (2002).

In the objective function in this paper the same weight was used for the time that vehicles spend on the on-ramp as the time that they spend on the freeway. By putting a higher weight on one of the terms, a preference can be expressed for long-distance or short-distance traffic. E.g., weighting the time spent on the freeway more, will result in a shorter average travel time on the freeways, but a longer waiting time on the on-ramps. It can be expected that this will encourage the short-distance drivers to choose another route.

Another interesting and relevant topic for future research is the study of larger networks, such as ring roads around cities or areas where several cities are connected by freeways, and where OD demands go typically through the whole network. Also considering other control measures in addition to speed limits and ramp metering (such as peak-lanes, route information, tidal flow, etc.) is an important topic for future research. An important aspect of the study of larger networks will be the trade-off between efficiency and optimality.

Whichever traffic flow model is chosen, the model has to be calibrated and validated with real data. In the future, the effectiveness of model predictive control for optimal coordination of speed limits and ramp metering with model parameters extracted from real, measured data will be studied. It is expected that the model predictive coordination of control measures will improve performance for any set of model parameters that result in a model that can reproduce the capacity-drop phenomenon.

Acknowledgments

This research was supported by the Traffic Research Center (AVV) of the Dutch Ministry of Transport, Public Works and Water Management, by the NWO-CONNEKT project ‘Advanced multi-agent control and information for integrated multi-class traffic networks’ (AMICI), by the project ‘Multi-agent control of large-scale hybrid systems’ (DWV.6188) of the Dutch Technology Foundation STW, and by the TU Delft spearhead program ‘Transport Research Centre Delft: Towards Reliable Mobility’.

We would like to thank Prof. M. Papageorgiou of the Technical University of Crete (Chania, Crete, Greece) for his interesting and useful comments and suggestions

on the first version of this paper.

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