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Model predictive control of urban drainage systems: A review and perspective towards smart real-time water management

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ABSTRACT

Model predictive control (MPC) can be used to manage combined urban drainage systems more efficiently for protection of human health and the environment, but examples of operational implementations are rare. This paper reviews more than 30 years of partly heterogeneous research on the topic. We propose a terminology for MPC of urban drainage systems and a hierarchical categorization where we emphasize four overall components: the "receding horizon principle", the "optimization model", the "optimization solver", and the "internal MPC model". Most of the reported optimization models share the trait of a multiobjective optimization based on a conceptual internal MPC model. However, there is a large variety of both convex and non-linear optimization models and optimization solvers as well as constructions of the internal MPC model. Furthermore, literature disagrees about the optimal length of the components in the receding horizon principle. The large number of MPC formulations and evaluation approaches makes it problematic to compare different MPC methods. This review highlights methods, challenges, and research gaps in order to make MPC of urban drainage systems accessible for researchers and practitioners from different disciplines. This will pave the way for shared understanding and further development within the field, and eventually lead to more operational implementations.

1. Introduction

Urban drainage systems convey stormwater and wastewater out of cities, and are therefore key infrastructure elements in any modern society. Urban drainage

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systems are mostly composed of sewers in terms of pipes and tunnels but may also include channels and ditches. Along with clean water pipes, sewers are a key element in the "sanitary revolution", which is based on passive protection against health hazards by separating clean and dirty water and is considered among the most important medical milestones since 1840 (Ferriman, 2007). Sewers can be separate systems, meaning that stormwater and wastewater flows in distinct pipe systems, or combined systems that convey both types of water in the same pipes. Combined systems are predominant in old city centers in Europe and North America. In these systems, polluted water flows to a wastewater treatment plant (WWTP) before being discharged to the environment; this usually works well during dry weather and even during minor rain events. However, infiltration-inflow and rainy periods can cause flows to increase beyond the inlet capacity of WWTPs, leading to unavoidable bypass, while heavy rain storms can cause combined sewer systems to fill up and surcharge, leading to flooding of basements and streets and to combined sewer overflows (CSOs) into the environment through overflow structures (CSO structures). In both cases, diluted but untreated wastewater is released into the environment, which can have severe consequences. From an environmental perspective, the effects include eutrophication and oxygen depletion in receiving waters and toxic impacts on the aquatic environment (Lijklema et al., 1993). Furthermore, humans can come in contact with polluted water – for example, when bathing close to CSO outlets after rain storms or if extreme rain has caused flooding of streets and basement – and this constitutes a health risk.

Urban drainage systems were developed in the 1850s and their purpose at that time was to secure public hygiene and prevent flooding. From 1960s onwards, pollution loads and environmental impacts became a focus and WWTPs were expanded and upgraded to decrease the discharge of pollutants to natural water bodies. Since then, many governments and environmental protection agencies have implemented regulations to reduce the frequency and magnitude of CSO events, mainly through expansion of the pipe systems and construction of storage basins. On top of stricter legislation, recent research shows that cities are growing and becoming denser, while many parts of the world are expected to receive more intense rainstorms in the next decades (Arnbjerg-Nielsen et al., 2013; Kaspersen et al., 2017; Sørup et al., 2016). These developments force us to rethink how to manage stormwater and wastewater in the future. There are three different means of addressing the challenges of increased runoff and stricter legislation:

- Preventing stormwater from entering the urban drainage system, typically by using locally placed stormwater control measures (SCM) that utilize a combination of the hydrological processes storage, infiltration, evapotranspiration, and delayed runoff.
- Expanding existing structures in the combined sewer system, including pipes, basins, and overflow structures.
- Implementing more advanced control strategies for the combined sewer system, based on actuators such as pumps and moveable gates.

Several authors have recently advocated the use of advanced control strategies as a measure to improve the performance of urban drainage systems (García et al., 2015; Mollerup, 2015; Mollerup et al., 2012; Ocampo–Martinez and Puig, 2010; Vezzaro et al., 2014b). *Smart cities* is an emerging concept in which cities develop from being static to flexible systems and where most information can be monitored, transferred, stored, and finally used to facilitate a more intelligent real-time management of the city as a whole. By focusing on the latter of the three means listed previously, a smart city strategy enables sewer systems to evolve from being passive to active adaptive units that can respond differently depending on the given situation (Kerkez et al., 2016). Real-time control (RTC) can be used to make the sewer systems "smart" by using system observations and numerical modeling to enhance the use of the existing systems. This reduces the need for investments in extra storage volume (Eggimann et al., 2017), which is an attractive factor in densely populated areas with limited space for new constructions (Gelormino and Ricker, 1994; Mollerup, 2015).

One way of performing advanced RTC of urban drainage systems is by applying model predictive control (MPC). MPC for combined sewer systems is an adaptive control strategy in which the optimal control is recalculated recursively as new information about the state of the sewer system and new rainfall forecasts become available. The amount of literature published within this field has increased during the past 5 years, produced by authors from disciplines such as civil, chemical, and environmental engineering, as well as hydrology and meteorology, computer science, and control engineering. This has led to a lot of innovation, but also to linguistic uncertainty and ambiguity, which means that a literature review is both timely and appropriate. The present paper aims to provide an overview of methods and tools for performing MPC within the field of urban drainage and the main benefits that can be achieved. This is expected to lay a foundation for a more efficient progress towards MPC of urban drainage systems becoming a mature technology in the decades to come. The review will focus on MPC implemented for combined sewer systems and mostly disregard literature considering integrated control of sewer systems, WWTPs, and receiving water bodies. After this introduction, Section 2 provides an overview of concepts and terms used in the MPC literature related to combined urban drainage systems and proposes a consistent terminology. Sections 3–6 go into details regarding the four most important elements of MPC: the receding horizon principle (Section 3), optimization models (Section 4), optimization solvers (Section 5), and internal MPC models (Section 6). Section 7 addresses key considerations when evaluating and implementing MPC and discusses the terminology and research gaps, before conclusions are drawn in Section 8.

2. Methodology and overview of the field

2.1. Literature study

This review is based on a total of 113 references from 1983 to 2018, of which more than 60 percent are addressing MPC of urban drainage systems. Some of these
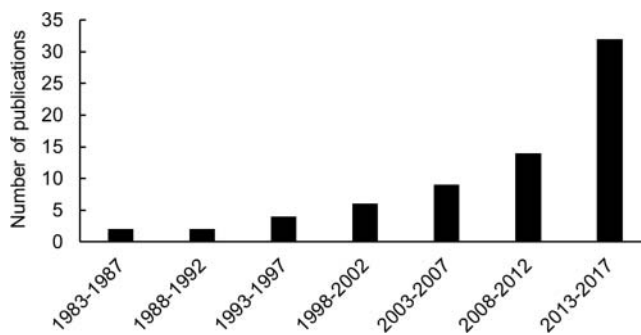
Figure 1. Number of publications addressing MPC of urban drainage systems per 5 years.

references were previously known internally in the author group, whereas others emerged during invaluable discussions with professionals in the industry and academia. The remaining references have been found in a systematic literature search conducted using DTU Findit[1] and Scopus[2] as search engines and a combination of search terms related to MPC and urban drainage, such as "model predictive control", "receding horizon control", or "rolling horizon control" together with "urban drainage" or "sewer". We also consulted the reference lists of the reviewed publications in order to obtain additional literature. Scopus was used to find literature citing some of the key publications, which revealed newer publications. Figure 1 shows the distribution of the publication years for the MPC literature used in this review. The number of publications has been increasing steadily over the past four decades and the number of publications on MPC of urban drainage systems has more than doubled during the past 5 years, which again highlights the relevance and timeliness of this review.

2.2. Control of urban drainage systems in general

2.2.1. Passive control and RTC

Control of urban drainage systems can be done either by passive control or RTC. In passive control, diversion elements such as weirs, gates, and valves are controlled by fixing each of them to a certain static setting. This setting is considered permanent but can be adjusted to a better setting if, for example, an offline model-based optimization of the system is performed (Vitasovic, 2006). In RTC, actuators, including movable diversion elements and pumps, are controlled by converting real-time measurements from the system into operational decisions by rules and algorithms of varying complexity. This requires the installation of sensors and controllers in the sewer system, together with the implementation of a telemetry system and a supervisory control and data acquisition (SCADA) system (Campisano et al., 2013; Cembrano et al., 2004; Puig et al., 2009). The literature on RTC of

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[1]Search engine by the Technical University of Denmark, http://findit.dtu.dk/
[2]Search engine by Elsevier, https://www.elsevier.com/solutions/scopus

urban drainage systems is abundant. Beeneken et al. (2013), García et al. (2015), Schilling (1989), Schütze et al. (2004), and Schütze and Muschalla (2013) provide a good starting point for understanding RTC in more general terms, whereas Schütze et al. (2008) suggested a procedure for assessing the RTC potential for a given system.

RTC consists of control loops where a controller changes the manipulated variable of an actuator with either continuous or discrete settings based on the difference between the set-point value and controlled variable (Campisano et al., 2013; Schütze et al., 2003, 2004). The sensor can be placed at the actuator site or further away. Table 1 describes these different control terms in more detail.

There are several ways of performing control and Table 2 categorizes control methods into degree of control, degree of automation, physical extension, system-wise extension, RTC strategies, and timing of input. RTC can generally be divided into heuristic and optimization-based control (García et al., 2015). Heuristic approaches can be appealing because the resulting control seems rational; however, it can be difficult to obtain an optimal solution in this manner (Cen and Xi, 2009; García et al., 2015; Marinaki and Papageorgiou, 1999; Mollerup et al., 2013; Papageorgiou, 1983, 1988). Optimal control is also difficult to achieve with passive control due to the dynamic loading of the system; this is especially evident during spatially and time-wise unevenly distributed rainfall (García et al., 2015; Löwe et al., 2016; Marinaki and Papageorgiou, 1998, 1999, 2001). Urban drainage systems are most often controlled by passive control, rule-based (local) control, or manually by an operator. However, better control can usually be achieved by using global RTC (Cen and Xi, 2009; Giraldo et al., 2010; Leirens et al., 2010; Löwe et al., 2016; Marinaki and Papageorgiou, 1998, 1999; Ocampo-Martinez and Puig, 2009a; Ocampo-Martinez et al., 2008; Papageorgiou, 1983, 1988, Pleau et al., 1996, 2005, Rauch and Harremoës, 1996, 1999). Hence, it is likely that suboptimal control is currently implemented in many places.

Table 1. RTC terms. Physical devices in ***italic and bold*** and their modeling counterparts in *italic*.

Terms	Description
Sensors	Monitor system states, such as flow, water level, or water quality.
Set-points	The desired state values for a certain place in the sewer system, such as a downstream pipe flow.
Controlled variables	The variables that should obtain a certain set-point.
Actuators	Controllable devices, such as pumps, gates, weirs, and valves.
Manipulated variables	The variables that can be changed actively in the control, such as a pump rate.
Controllers	Devices such as the programmable logic controller (PLC) and remote terminal unit (RTU) that adjust the actuators based on sensor values. These are the hardware on which different "software" controllers/algorithms can be implemented.
PID controller	A common controller for varying the settings of the actuator continuously is the proportional-integral-derivative (PID) controller.
Two-point controller	A common controller for discrete settings is the two-point (on/off) controller that has only the option of being "on" or "off" (for example, for a pump) or "open" or "closed" (for example, for a gate).

Table 2. Categorization of control methods.

Category	Control method	Description
Degree of control	*Passive control*	Diversion elements are fixed to a static setting.
	Real-time control (RTC)	The settings of actuators are changed dynamically based on real-time measurements from the system.
Degree of automation	*Manual*	An operator adjusts the actuators in the system.
	Supervisory	Actuators are adjusted automatically but the set-points or the direct settings of the actuator are specified/approved by an operator/supervisory system.
	Automatic	The entire system is operated automatically.
Physical extension	*Local*	The control is performed independently for each actuator based on measurements from the immediate surroundings.
	Global	The control is based on observations throughout the system and all actuators are regulated at once from a global perspective.
System-wise extension	*System-wide control*	Global control only considering the urban drainage system.
	Integrated (system-wide) control	Global control considering several subsystems alongside the urban drainage system; for example, wastewater treatment plants and receiving water bodies.
	Plant-wide control	Control only considering the wastewater treatment plant.
RTC strategies	*Heuristic control*	The control is based on experience, which includes fuzzy-logic control, static rules optimized offline (rule-based control), or systems that are controlled manually by an operator.
	Optimization-based control	The control is modeled as a dynamic optimization problem, which includes linear-quadratic regulators, evolutionary strategies, MPC and population dynamics-based control.
Timing of input	*Reactive control*	The control is determined only based on measurements.
	Predictive control	The control is determined based on predictions of the future system state.

2.2.2. Operational goals for RTC

There are many possible reasons for implementing control and the resulting operational goals can be quantified using different metrics. In a system-wide urban drainage context, advanced RTC will most often be performed in order to minimize one or more of the following:

  – CSO to the environment (measured in water volumes, pollutant loads, resulting oxygen concentration in recipient, damage cost, or risk).
  – Flooding of the urban landscape (measured in water volumes, pollutant loads, damage cost, or risk).
  – Energy consumption or operational cost (measured in energy usage or cost).
  – Wear and tear of actuators to increase their lifespan (measured in usage, settings variation, or cost).

The first two of the listed operational goals will probably have higher priority than the latter two. Furthermore, Campisano et al. (2013) and Vitasovic (2006) suggested avoiding sediment deposition in sewer systems; managing flows in case of, for example, construction work or equipment failures; and managing the flow to the WWTP as potential operational goals.

2.2.3. Impediments to implementation of RTC

RTC has been used in urban drainage systems for more than 40 years, but the development of more sophisticated methods has been limited due to unreliable sensors, actuators, and communication systems, together with insufficient computational power. Many of these limitations have now been overcome to such a degree that it is possible to implement more efficient control strategies (including MPC). In addition, there is an increased interest in automated operation of urban drainage systems (Meseguer and Quevedo, 2017) stemming from a stronger organization of utilities due to corporatization of the water sector and improved rainfall forecasts using weather radar and numerical weather prediction (NWP) models. Despite this, the use of MPC as a control strategy within urban drainage is limited to a few examples of real-life operations and concepts that are ready for implementation (see Fiorelli et al., 2013; Pleau et al., 2005; Vezzaro and Grum, 2014), which indicates that we still face challenges that limit both the implementation and further development of MPC techniques. These challenges include organizational issues such as the lack of trust from operators (Vitasovic, 2006) or lack of cooperation between planning and operation departments; proper choice of control equipment (Campisano et al., 2013); adaption to the limited computational power and uncertainty of the rain input that requires a mathematically rigorous formulation of the control problem to facilitate an efficient solution (Dong et al., 2017); and linguistic uncertainties, which make it difficult to exchange knowledge and experiences across research institutes, industries, and disciplines.

2.3. The basic principles of MPC

MPC was first described theoretically in the 1960s, but not applied until the 1970s (Qin and Badgwell, 2003). The fact that the underlying MPC concept is relatively easy to understand has led to the expansion of its application in some industries (Maciejowksi, 2002). It has been used across a large variety of technical fields, including the production of pulp and paper, food processing, in chemical plants, and in the automotive and aerospace industries (Qin and Badgwell, 2003).

Overall, MPC is optimization in a discretely forward moving time window and consists of two key aspects: the **receding horizon principle**, which means that the optimization of the control actions is repeated recursively within a finite time horizon; and **optimization**, which involves choosing the best sequence of control actions possible within this horizon. This optimization is composed of an optimization model (which describes the optimization problem), an internal MPC model (which models the dynamics of the relevant parts of the urban drainage system), and an optimization solver (which performs the actual optimization of the control actions).

In MPC, the system states are computed by the internal MPC model from the time of forecast and into the future by incorporating input predictions in the form of rain, runoff, and/or sewage from other parts of the sewer system (Fig. 2). The
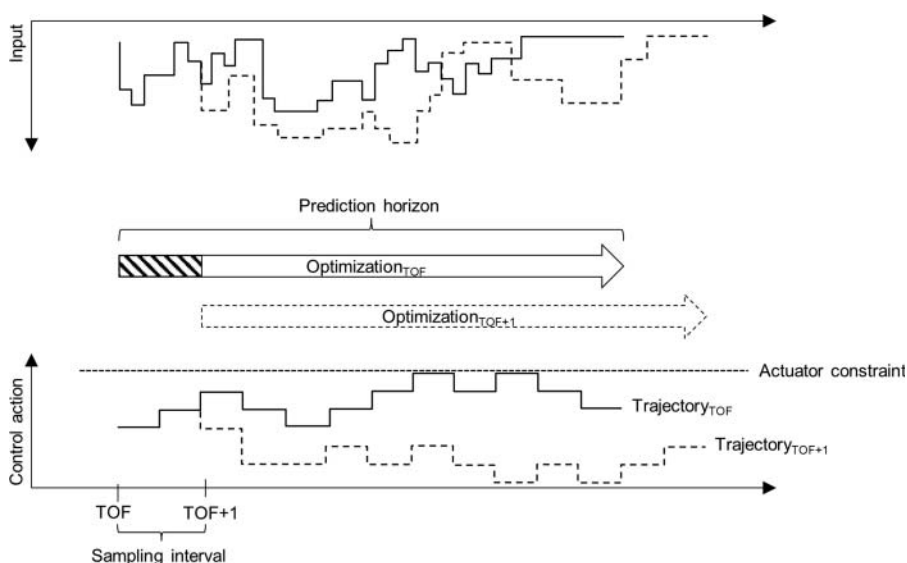
Figure 2. The concept of MPC, including the receding horizon principle and optimization. The terms "prediction horizon" and "sampling interval" are explained in Section 3. TOF = time of forecast.

operational goals are defined in the optimization model, on which basis the optimization solver computes the control actions. Only the first part of the optimized control is carried out in reality; in the meantime, a new optimization is performed. New information about the system state and input can be incorporated at every reoptimization.

In principle, MPC is capable of dealing with complex, multivariable systems, of including transport times, and of considering operational as well as physical constraints, such as actuator and pipe flow limitations. Hence, MPC is suitable as a global (system-wide) control scheme, but can also be applied locally. MPC is especially advantageous as a control strategy for large and complex urban drainage systems with multiple overflow structures, multiple WWTPs, and a complex network of actuators and storage basins distributed in different parts of the sewer system, where constraints on flows, volumes, and water levels need to be respected, and in case of a heterogeneously distributed rainfall (Cembrano et al., 2004; Gelormino and Ricker, 1994; Ocampo-Martinez and Puig, 2009a, 2010; Pleau et al., 2005; Puig et al., 2009). By applying input predictions, MPC is capable of anticipating problems arising from a limited capacity of structures; thus, the control becomes proactive (Duchesne et al., 2001, 2004; García et al., 2015; van Overloop et al., 2008; Puig et al., 2009; Schütze et al., 2004).

2.4. Linguistic uncertainty in MPC literature

MPC was used by industry long before it was used in academia (Maciejowksi, 2002). Consequently, this has led to linguistic uncertainty within MPC of urban

drainage systems. The lack of common terminology and inconsistency in which information is provided to the reader makes it difficult to obtain a full understanding of the applied MPC methods and the experiences gained from using them. This has slowed progress in the field, made collaboration with other fields more challenging, and made it more difficult to implement MPC in real-life operations.

Terminology is contextual and sometimes influenced by both the spatial and temporal scale of the considered system; that is, it becomes profession- or application-area-oriented. An example of a contextual term is the term "system-wide control" (Table 2). The focus of this review is the sewer system; thus, "system-wide" refers to the entire sewer system of a city, town, or urban area. In other systems – which may include receiving waters and WWTPs – the meaning of "system-wide" is extended and the urban drainage system will be perceived as a single subsystem. Examples of profession-influenced terms are the use of "virtual tanks" and "disturbances", which are applied in some of the reviewed literature, but clearly originate from other technical fields than urban drainage. The term "virtual tanks" probably has its origin in process engineering and, in an urban drainage context, is usually referred to as "linear reservoirs", inspired by hydrology. Similarly, the term "disturbances" is widely used within control engineering, although rain, runoff and sewage are not considered as disturbances in urban drainage, but rather as the driving force of all the occurring processes; therefore, we denote this as "input" or "forcing". In some of the reviewed literature, "input" is contrarily used for the control actions, because these often act as inputs to the controllers in the system. Hence, it can be difficult, but all the more important, to reach a common understanding of the applied terms within each field. Table 3 clarifies the terms that are used in this review, their definitions, and how they relate to the terms used across the reviewed literature. The terms are divided into general terms, terms related to the receding horizon principle, and terms about optimization. The individual terms are explained and discussed in more detail in the table and/or in the following sections.

2.5. Categorization tree for MPC

Figure 3 shows a categorization tree for the different components of MPC. Both the receding horizon principle and optimization contain multiple sub-components, which are highly interconnected. For example, the objectives chosen to reflect the operational goals in the optimization model will set certain requirements for the internal MPC model. The internal MPC model may be either a complex non-linear model or a simple linear model. This choice, together with the choice of optimization model, will then place restrictions on the chosen optimization solver, and the computational time of the optimization and the available forecast quality will together affect the setup of the receding horizon principle. Therefore, it is not trivial to assess these components individually and make generalizations, although this is what we have

Table 3. Overview of terminology used in this review and in the literature and definitions of the term.

	Term used in this review	Alternative terms used in literature
General terms	**Control strategy or control scheme** *The overall control such as passive control, rule-based control, model predictive control, etc.*	
	Passive control *All diversion elements are fixed to a static setting*	No control, static control, fixed control, and static management
	Model predictive control (MPC) *Consists of a receding horizon principle and an optimization of the control actions*	Model-based predictive control, receding horizon control, moving horizon optimal control, optimization with rolling horizon, certainty equivalent open-loop feedback control and global predictive control, and global optimal control explicitly for global MPC
	MPC technique, MPC method, or MPC scheme *The specific MPC set-up*	
	Control actions, manipulated variables, or optimization variables *The optimized future control which will form a control trajectory (also called "control signal")*	
	Input model *Provides the input forecasts to the internal MPC model*	Disturbance model
	High-fidelity (HiFi) model *Detailed, distributed model replicating reality with high fidelity*	System model, hydraulic model, realistic simulation model, physically-based model, simulator, hydraulic sewer routing model, hydraulic simulation model, and distributed urban drainage model (DUDM)
Receding horizon principle	**Receding horizon principle** *The recursive re-optimization of the control in a forward-moving time window, including the prediction horizon, forecast horizon, control horizon, sampling interval, and setting duration*	
	Prediction horizon *Interval in which the internal MPC model is run and for which the objective function calculates the cost of a given control*	Optimization horizon, operational time horizon, optimization window, optimization time horizon, and output horizon
	Forecast horizon *Interval in which input forecasts are available. Hereafter, a predefined input algorithm is applied.*	
	Control horizon *Interval in which the control can be altered by the optimization. Hereafter, a predefined control strategy is applied.*	
	Collective horizon *The collective name when the prediction, forecast, and control horizon are of equal length*	
	Sampling interval *Interval between the control re-optimization*	Sampling period, control period, control interval, sampling time, repetition period, optimization time step, sampling time interval, control step, sampling control period, and control sample time
	Setting duration *The length of each control action in the control trajectory*	Control period, control interval, control horizon, aggregation horizon, and blocks

Table 3. (*Continued*)

	Term used in this review	Alternative terms used in literature
Optimization	**Optimization** *Consist of an optimization model, internal MPC model, and optimization solver* **Optimization model** *Consist of an objective function, constraints, and optimization variables* **Objective function** *Function that quantifies the objectives in the control optimization* **Penalties** *Used to prioritize between objectives in the objective function* **Optimization solver** *The implementation of the algorithm that searches for the optimal solution* **Internal MPC model** *The model used for predicting the system dynamics. Has a model structure and uses inputs and initial conditions* **Inputs** *The driving force of the model, for example, rain or flow forecasts* **Initial conditions** *The start values for the internal MPC model at each new control re-optimization*	 Cost function and performance criterion Weights Optimizer, solver Simplified model, operational model, sewer system model, system model, prediction model, control model, control-oriented model, optimal control model, simplified network model, phenomenological simulator, surrogate model, and simulation model Disturbances, external inflows, loadings, nowcast (in meteorological contexts), and forcing

attempted in this review. The color scheme applied in Fig. 3 acts as a reading guide for the remaining part of the review.

3. The receding horizon principle

The receding horizon principle is a recursive optimization in a forward-moving time window and this window can, according to Rauch and Harremoës (1999), be broken down to three time horizons: the prediction horizon, the forecast horizon, and the control horizon. They also defined the sampling interval (sampling time), and we additionally see the need to define the setting duration as an additional time window. These five time windows are elaborated below.

The "prediction horizon" denotes the period in which the internal MPC model is run into the future (see Fig. 4), and the value of the objective function is calculated for this entire interval; thus, the performance of a given set of control actions is quantified. There might not be reliable input forecasts for the entire prediction horizon and the period in which the inputs are trusted is denoted as the "forecast horizon". A predefined algorithm may give the input for the remaining part of the prediction horizon (for example, by letting the inflow fade from the last forecasted value to dry weather flow; see Section 6.2). Furthermore, the prediction horizon might be so long that the required computational time for the optimization of the control actions exceeds what is possible in a real-time framework. The actual period in which the control actions are optimized is denoted as the "control
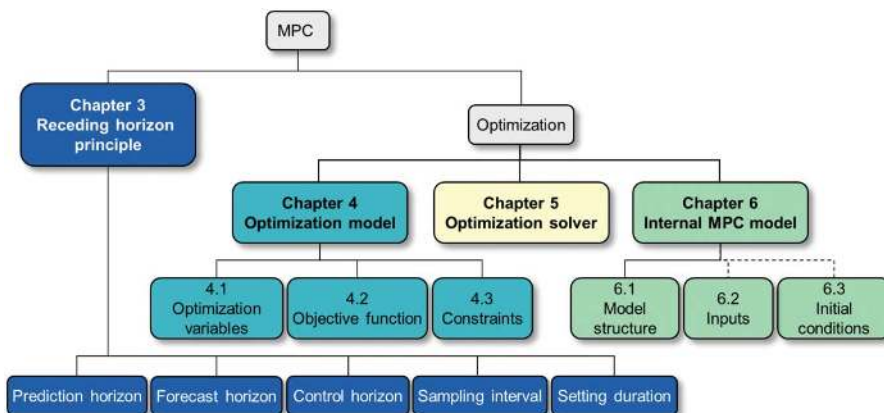
Figure 3. Hierarchical categorization of MPC components. A solid line indicates that a component consists of subcomponents, while a dashed line indicates the presence of an external subcomponent. The color scheme can be used as a reading guide for the remaining part of the paper.

horizon", whereas predefined control actions (such as not changing the control actions from the last value within the control horizon) are used to calculate the value of the objective function for the remaining part of the prediction horizon (Maciejowksi, 2002; Rauch and Harremoës, 1999).

The optimization of the control actions is repeated after a specified period – called the "sampling interval" (Fig. 4) – and only the control actions within this period are actually carried out. At each control reoptimization, observations from the system are taken into account as well as new rain and/or runoff forecasts when available (Joseph-Duran et al., 2014b; Papageorgiou, 1988). The sampling interval is also often seen to represent the frequency with which the control actions change value (Rauch and Harremoës, 1999). However, some studies have made a distinction between the length of the sampling interval and this frequency, which we define as the "setting duration". Thus, the setting duration describes the level of detail of the control trajectory.

3.1. Time windows in the context of MPC

The receding horizon principle requires the selection of the length of the five time windows. Ideally, the prediction horizon should be long enough to cover all consequences of any conducted control. For system-wide control of urban drainage systems, it should be long enough for the water to propagate through the system and for the basins to empty. When it is computationally too expensive to let the prediction horizon cover basin emptying explicitly, the effect of a sufficiently long prediction horizon can be emulated by adding a penalty in the objective function that is dependent on the remaining volume in the basins or on the equal filling of basins, as applied by, for example, Cembrano et al. (2004), Cen and Xi (2009), Fiorelli and
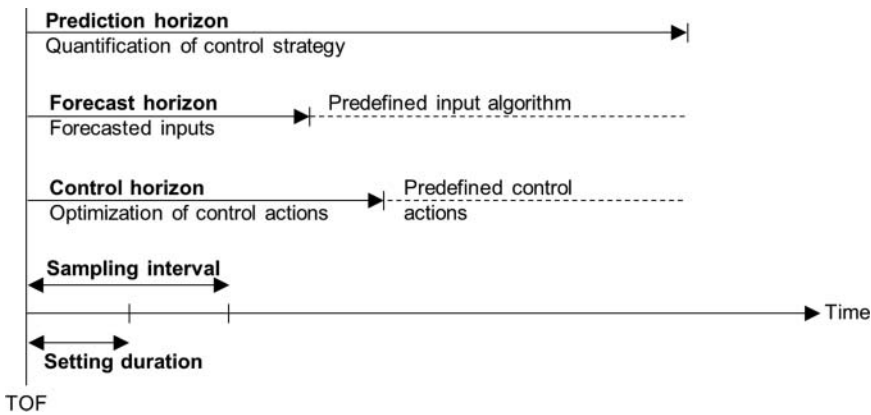
Figure 4. Time windows used in the receding horizon principle. The lengths of the time windows are only an example. Here, the sampling time is twice as long as the setting duration, as in Fig. 2. Inspired by Rauch and Harremoës (1999).

Schutz (2009), Gelormino and Ricker (1994), and Pleau et al. (1996); see Section 4.2.1. The length of the forecast horizon is bound by the reliability of the rain or runoff forecasts, whereas the control horizon is constrained by the computational demands imposed by the optimization (hereunder, the length of the time windows, the size of the optimization problem, and the choice of optimization solver).

Most of the reviewed literature does not distinguish the length of the prediction, forecast, and control horizons. The length chosen to cover all three horizons will become a trade-off between the availability of reliable forecasts and computational power, on one hand, and the extent to which the effects of the control are quantified, on the other. This trade-off can, in a worst-case scenario, lead to "myopic control", meaning that the control is only optimal within the considered horizon, but might lead to a very poor control in the longer run (Cembrano et al., 2004; Duchesne et al., 2001, 2004; Papageorgiou, 1988; Puig et al., 2009; Rauch and Harremoës, 1999). It is most often seen that only one linguistic term is applied collectively for all three horizons. Usually, the term "prediction horizon" or "control horizon" is used, which makes it difficult to understand the details of the applied MPC scheme. In the present review, we have denoted this as the "collective horizon" to underline that the three horizons are not distinguished and have been merged into one.

The length of the sampling interval must take into account the computational power and the sampling interval of the telemetry system, while the setting duration is bound by the computational power and the speed at which the settings of the actuators can change in reality (Cembrano et al., 2004; Puig et al., 2009).

3.2. Experiences from the literature

The length of the time windows will be dependent on many factors, which are input-, case-, and modeling-specific.

A sampling interval in the range of 1 to 10 min is normally applied in the reviewed literature, with 5 min being the most commonly used interval. The setting duration is seen to be both shorter than the sampling interval in order to get more detailed control (Joseph-Duran et al., 2013a; Papageorgiou, 1988) and longer to save computational time (Cen and Xi, 2009; Gelormino and Ricker, 1994; Giraldo et al., 2010). The most extreme version of the latter case is the optimization of a constant trajectory throughout the entire horizon, implemented in, for example, Meneses et al. (2018), Mollerup (2015) and Vezzaro and Grum (2014). This means that only one control action value is calculated for the entire control horizon instead of a time-varying trajectory of control actions. Courdent et al. (2015) used a varying setting duration that became coarser with time. The setting duration is often not reported explicitly in the reviewed literature, probably because the authors use the modeling time step (which is often not given either) or the sampling interval as the setting duration, but omit this information.

A value in the range of 30–120 min is normally used for the collective horizon, where 30 min (also supported by Meseguer and Quevedo (2017)) or 120 min are the most common values. These lengths coincide with the forecast ability of many radar nowcast products (Thorndahl et al., 2017), which is probably why longer forecasts are normally not applied. Courdent et al. (2015) used rain from a NWP model as input to the internal MPC model, allowing for a collective horizon of 13 hr.

Fiorelli et al. (2013), Pleau et al. (2005), and Vezzaro and Grum (2014) are the only reviewed papers to report that MPC is either in or ready for operation; they used sampling intervals of 10, 5, and 2 min, respectively, setting durations of 10, 5, and 120 min, and all applied a collective horizon of 2 hr.

3.3. Significance on performance

Figure 5 shows how the collective horizon, sampling interval, and setting duration may differ from each other. In theory, a long collective horizon and a short sampling interval and setting duration will increase the performance of the model (Rauch and Harremoës, 1999); however, only a few studies have investigated how much this improves the performance. Gelormino and Ricker (1994) found that it is possible to apply a longer collective horizon without increasing the problem size when using a setting duration that is larger than the sampling interval, and that this does not have a significant influence on the total CSO volume. In addition, the performance was not increased significantly by increasing the collective horizon from 10 to 200 min, a finding that was explained by the exclusion of transport time in the model (that is, the internal MPC model was too simple to gain from and increased collective horizon). Mollerup (2015) obtained a decreasing performance as the collective horizon was increased and explained this with the choice of having a constant trajectory. This constant trajectory will represent a best "average" control and increasing the horizon length will decrease the model's ability to adjust to the
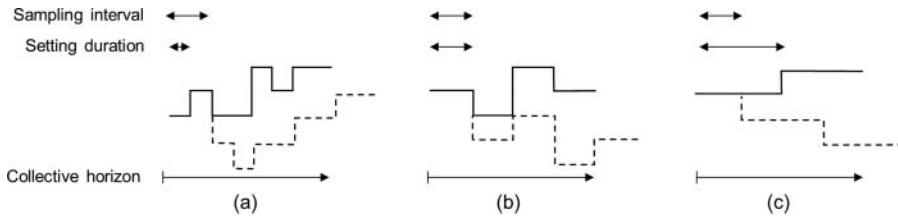
Figure 5. The difference between the collective horizon, sampling interval, and setting duration. In (a) the sampling interval is longer than the setting duration; in (b) they are equally long; and in (c) the sampling interval is shorter.

fast dynamics associated with rain events. Duchesne et al. (2003), Marinaki and Papageorgiou (2001), Nelen (1992), and Papageorgiou (1988) all included both transport time and time-varying control actions and found that a longer collective horizon did, indeed, influence the results positively. However, Nelen (1992) and Papageorgiou (1988) stated that there is an upper limit above which the performance does not increase significantly (60 and 100 min, respectively). Likewise, there is a lower limit under which myopic control is present (25 min found by Papageorgiou (1988)). Marinaki and Papageorgiou (2001) found that the largest effect is not obtained by increasing the length of the collective horizon, but by going from passive control to MPC, and the model performance is more sensitive to changes in the length of the sampling interval (here, 3–9 min) than the length of the collective horizon (here 1–4 hr). Furthermore, Duchesne et al. (2003) found that the increase in performance due to a longer collective horizon is most pronounced when surcharged pipes are not allowed. Allowing surcharged pipes made it possible to apply a shorter collective horizon without negatively affecting the performance, as pressurized flow travels faster through the system; however, they also found that surcharged pipes cause constraint violations.

4. Optimization models

The task of the optimization model is to identify the best control trajectory for each actuator during the upcoming control horizon. The best set of control trajectories (one for each actuator) is the one that optimizes the objectives within the given constraints. The general formulation of an optimization model is (for an in-depth reference, see Boyd and Vandenberghe (2009)):

$$
\begin{aligned}
&\min \; J(\mathbf{u}) \\
&\text{subject to } f_i(\mathbf{u}) \leq b_i, \; i = 1, \ldots, m
\end{aligned}
\tag{1}
$$

where $J(\mathbf{u})$ is the objective function for the optimization variables $\mathbf{u}$, whereas $f_i(\mathbf{u}) \leq b_i$ are the constraint inequalities, where $f_i(\mathbf{u})$ is the constraint function representing the

constrained control or system output at index i and b_i is the bound for the constraint. This description of the optimization model is very general and the following sections aim to describe how researchers engaged in urban drainage have dealt with selecting optimization variables, objective function, and constraints.

4.1. Optimization variables

The optimization variables are gathered in a vector **u**, which holds the control actions for the upcoming control horizon for all actuators in the sewer system. Thus, optimization variables, control actions, and manipulated variables are all terms for the same concept. The control trajectory for a single actuator is a sequence of control actions within the control horizon; each is as long as the setting duration. In the reviewed literature, the control trajectory consists of a sequence of flows, which acts as set-points for actuators in a local control loop controlled by, for example, a proportional-integral-derivative (PID) controller (see Section 7.2 for more details).

The solving time of the optimization model increases with the number of optimization variables. Especially for non-linear optimization (see Section 4.4), it is important to keep the number of optimization variables as low as possible, as the solving time for these generally grows fast with an increasing number of variables. A common approach to reduce the number of optimization variables is to force the control action to remain the same during a time span, which is larger than the sampling time, as discussed in Section 3.2. In control theory, this is known as *blocking*, as the control action is blocked from moving.

4.2. Objective functions

The objective function quantifies the cost of a set of control trajectories by evaluating how future outputs or control actions in the system deviate from the desired values, which are denoted "references". This requires a definition of the references, a model of how the state of the system evolves (the internal MPC model), and a way of penalizing the deviation from the reference values.

4.2.1. Operational goals

The first step in designing an objective function is to define operational goals. Some objectives directly target the desired operational goals, whereas others act indirectly. The most commonly used objectives are listed in Table 4, of which CSO minimization is by far applied the most.

Fiorelli et al. (2013), Fiorelli and Schutz (2009), and Gillé et al. (2008) have listed pros and cons for the first three objectives in Table 4 and offered suggestions for improvement. They concluded that the efficiency of CSO minimization is highly dependent on the quality of the input forecasts and that this objective is especially useful when CSO mitigation is not equally important at

all CSO locations. They also found that meeting the WWTP inflow capacity can cause the outflow from basins to oscillate and that it can be difficult to set one value for the WWTP capacity. Instead, they suggested limiting the rate of change of the outflows (objective 5 in Table 4) and having a dynamically varying value for the WWTP capacity. These studies, furthermore, concluded that obtaining equally filled storage basins can be a disadvantage when the basins are located far apart, have very different emptying times, or when most of the storage basins are full.

Some less common objectives include:

- Adherence to specific flow references (that is, a desired flow at a specific location in the network) (Cembrano et al., 2004) or a certain basin filling degree (Joseph-Duran et al., 2014b; Mollerup et al., 2016).
- Minimization of the operational costs by, for example, minimizing the control action itself (Farahani et al., 2017; García et al., 2015; Ocampo-Martinez et al., 2007, 2008). This can also be applied to on/off control by using discrete states (Leirens et al., 2010).
- Minimization of the monetary cost related to a CSO event. This can also include the uncertainty related to the forecasted input and thus the risk associated with a CSO event, which was applied in the Dynamic Overflow Risk Assessment (DORA) method (Courdent et al., 2015; Löwe et al., 2016; Meneses et al., 2018; Vezzaro and Grum, 2012, 2014; Vezzaro et al., 2013, 2014a, 2014b).
- Preservation of system behaviors that are not included in the internal MPC model, such as ensuring that overflows can only occur from filled basins and that the set-points of local controllers are respected (Pleau et al., 1996) or including transport time in pipes (Fiorelli and Schutz, 2009; Fiorelli et al., 2013).
- Duration of overflow (Farahani et al., 2017).
- Optimization of water quality parameters, such as the dissolved oxygen concentration. Water volumes are often used to represent this objective, but Rauch and Harremoës (1999, 1998, 1996) showed that the water

Table 4. Commonly applied objectives, operational goals, and associated metrics.

Objective	Operational goal	Metric
1) Minimize CSO	Directly targets CSO minimization.	Volumes
2) Maximize the use of the WWTP capacity	Indirectly targets flooding and CSO minimization by minimizing the amount of water stored in basins; hereby, preparing for the next rain event.	Volumes
3) Distribute water to obtain equally filled basins	Indirectly targets flooding and CSO minimization.	Volumes
4) Minimize flooding	Directly targets flooding minimization.	Volumes
5) Minimize changes in the control action by penalizing the rate of change	Directly targets the increase in actuator lifespan.	Usage
6) Minimize the water volume in the basins	Indirectly targets flooding and CSO minimization by minimizing the amount of water stored in basins; hereby, preparing for the next rain event.	Volumes

quality objective cannot necessarily be represented by water volumes; however, it is difficult to include water quality aspects due to the complexity of the required models and the scarcity of data for model calibration (Duchesne et al., 2003, 2004). The models will often be integrated models and are, therefore, outside the scope of this review. However, Vezzaro et al. (2014b) indirectly included water quality aspects in a system-wide control model by using a time-dynamic overflow cost as penalty. Mahmoodian et al. (2017) also included water quality in a strict system-wide setting by modeling the concentration of pollutants in the sewer system and using objectives 1–3 from Table 4 in the objective function, with both volumes and concentrations (with uncertainty estimation) as metrics.

4.2.2. Quantifying deviations from objectives

The next step in designing the objective function is to develop measures to quantify the deviation from the reference (that is, the desired value that should be obtained, or "tracked"). This deviation can be quantified at a specific location as either the absolute (Eq. 2), the squared (Eq. 3), or the maximum (Eq. 4) deviation from the reference:

$$\sum_{i=1}^{H_p} P_i \, | \hat{y}_i - r_i | \tag{2}$$

$$\sum_{i=1}^{H_p} P_i \left(\hat{y}_i - r_i \right)^2 \tag{3}$$

$$\max_{1 \le i \le H_p} \left(P_i \, | \hat{y}_i - r_i | \right) \tag{4}$$

$\hat{y}_i$ denotes the predicted output value (for example, a CSO volume) at time i and r_i is the reference value. In Eqs. 2 and 3, the deviations are summed over the prediction horizon H_p and weighed over time by the penalty P_i.

Linear terms (Eq. 2) can be used when the objective is a linear function of the variable (Gelormino and Ricker, 1994) or if the total sum should be minimized (Ocampo-Martinez et al., 2008). Quadratic terms (Eq. 3) can indicate that this objective is more important than a linear term (Cembrano et al., 2004) and will, to a larger extent, cause the penalized deviations to be distributed over space and time, making the operation more smooth (Darsono and Labadie, 2007). Finally, the infinity norm (Eq. 4) can be used to target peak minimization (Ocampo-Martinez et al., 2008). Most studies have either applied entirely linear or entirely quadratic terms in the objective function, although some (Cembrano et al., 2004; Farahani et al., 2017; Gelormino and Ricker, 1994; Mailhot et al., 1999; Ocampo-Martinez et al., 2008) applied a mixture.

4.2.3. Prioritization between objectives

In most cases, the objective function consists of multiple individual objectives. This case is referred to as a "multi-objective" or "multi-goal" optimization. The individual objectives may be conflicting, and there exist several ways of indicating which of the included objectives that are most important to comply with. *Scalarization* turns a multiobjective optimization problem into a single-objective optimization problem by assigning different penalties, or weights, to different objectives (Ocampo-Martinez et al., 2008; Vitasovic, 2006). The *lexicographic* approach avoids defining penalties by ranking the individual objectives in advance. The optimization model is then solved considering only the most important objective at first, and then proceeds to lower-ranking objectives (Ocampo-Martinez et al. (2008)). Neither do approaches that calculate the Pareto frontier need predefined weights on the individual objectives, as an entire set of solutions that spans "the space of weight combinations" is calculated (Deb, 2001). This section focuses mostly on scalarization, as this is the most commonly applied method in the reviewed literature.

Eq. 5 shows an example of a scalarized objective function, which is assembled from purely quadratic terms. The first term relates to output deviations and the second term relates to the control actions.

$$
J(\hat{\mathbf{u}}) = \sum_{i=1}^{H_p} \sum_{j=1}^{N_y} P_{y,j}(\hat{y}_{i,j} - r_{y,j})^2
$$

$$
+ \sum_{i=0}^{H_p-1} \left(\sum_{j=1}^{N_u} P_{u,j}(\hat{u}_{i,j} - r_{u,j})^2 + \sum_{j=1}^{N_{\Delta u}} P_{\Delta u,j}\Delta\hat{u}_{i,j}^2 \right) \tag{5}
$$

$\hat{y}_{i,j}$ are the predicted outputs (controlled variables), whereas $\hat{u}_{i,j}$ are the control actions (manipulated variables) and $\Delta\hat{u}_{i,j}$ are the rate of change of control actions. At each setting duration i, the deviations from the reference values, $r_{y,j}$ and $r_{u,j}$, and the rate of change, $\Delta\hat{u}_{i,j}$, are calculated and scaled by the penalties $P_{y,j}$, $P_{u,j}$ and $P_{\Delta u,j}$. These are summed over the length of the prediction horizon, H_p, for all N_y considered outputs and up to H_p-1 for the N_u considered control actions and $N_{\Delta u}$ control actions where the rate of change of the control actions is taken into account. In many cases, the predicted output $\hat{y}_{i,j}$ has a one-to-one correspondence to the predicted state; for example, "reservoir volume" is often used as state variable and is also an output of interest.

Penalties are often chosen through a trial-and-error process and it is generally observed that flooding and CSO volumes are given the highest penalties. It is possible to give the same type of objective the same penalty value, but the penalties can also be used to distinguish between, for example, different WWTPs or different CSOs with regard to receiving water sensitivity (Courdent et al., 2015; Duchesne

et al., 2004; Fiorelli and Schutz, 2009; Fiorelli et al., 2013; Joseph-Duran et al., 2014b; Löwe et al., 2016; Patry, 1983; Pleau et al., 1996, 2005; Vezzaro and Grum, 2014).

A common way of prioritizing is to use a factor of 10 between the penalty values (see, for example, Joseph-Duran et al., 2014a; Marinaki and Papageorgiou, 1998; Ocampo-Martinez and Puig, 2010). However, the model performance is only sensitive to changes in the penalties until a certain level, after which the benefits of additional tuning will be insignificant (Mollerup, 2015; Vezzaro and Grum, 2014). If metrics have different order of magnitude, such as flows and volumes, it can be difficult to make a clear prioritization between them by using penalties, as highlighted by Mahmoodian et al. (2017). Ideally, the assigned penalties should be based on the sensitivity of the receiving waters or other case-specific considerations; hence, the penalties could be determined as a relative weighing performed by the urban water managers (Duchesne et al., 2004; Vezzaro and Grum, 2014). This, however, may be difficult if the objectives are not directly physically relatable.

Some studies use dynamically variable penalties. Courdent et al. (2015), Gelormino and Ricker (1994) and Pleau et al. (1996, 2005) defined penalties on the CSO volume that decrease over the prediction horizon in order to postpone CSO, with the reasoning that the far future is more uncertain than the near future. Furthermore, some authors vary the penalties from one optimization to the next depending on the input and system state; for example, the amount of rain (Fiorelli et al., 2013; Giraldo et al., 2010). The penalties can also be changed when something happens that changes the cost of deviating from the reference value for the remainder of an event; for example, at a specific CSO location after an overflow has already occurred or when the first flush volume has passed through the WWTP (Patry, 1983; Vezzaro et al., 2014a). Courdent et al. (2015) not only changed the penalty values, but also constructed different "modes" with individual objective functions, which were applied based on the system state and the forecasted rain. Pleau et al. (2005) also used input-dependent modes to shift between passive control (for dry weather periods) and MPC (for wet-weather conditions). The trial-and-error approach for choosing penalties can be time-consuming and adapting penalties to changes in the system can be difficult.

An alternative to scalarization is to use a lexicographic approach, where penalties are avoided and the prioritization between the different objectives instead is made a priori. Ocampo-Martinez et al. (2008) compared scalarization to the lexicographic approach for the Barcelona catchment and found that the lexicographic approach mostly gave a reduction in CSO volume and increase in water volume treated at the WWTP.

Yet another alternative is to apply multiobjective optimization algorithms based on Pareto dominance. The advances in using Pareto dominance optimization for water management problems is reported by Nicklow et al. (2010). These algorithms provide a set of Pareto (or non-dominated) solutions according to the trade-offs between the different objectives. The advantage of using Pareto

dominance optimization is that one does not have to specify preferences (or penalties) to any of the objectives. Instead, one can choose a preferred solution among the Pareto optimal solutions by balancing objectives and one can also bring in additional information that is not explicitly included in the optimization. A significant drawback is the computational requirements, which may be excessive when many objectives are considered. In addition, it can be difficult to interpret the set of Pareto solutions in order to choose a preferred solution, especially when many objectives are included. These drawbacks are particularly challenging for real-time optimization, and we are not aware of any publications on use of Pareto optimization for MPC of urban drainage systems; however, Fiorelli et al. (2013) used Pareto optimization on historical events to investigate and choose penalties for a scalarized objective function MPC.

4.3. Hard and soft constraints

Constraints can restrict the optimization variables and the future system outputs and thus represent limitations of the considered system. For a sewer system, obvious physical constraints are minimum and maximum basin volumes, flows at certain locations, flow through an actuator, and maximum rate of change of the flow through an actuator. Formulating the constraints might be straightforward in case the constraint represents, for example, a maximum pump rate. Without using a high-fidelity (HiFi) model it may, however, be difficult to assess bounds on future system outputs, such as the maximum flow capacity of a pipe, and some actuator capacities, such as the maximum flow through gates. In general, the constraints can be written as:

$$\text{Constraints on outputs: } \mathbf{y}_{min} \leq \mathbf{y} \leq \mathbf{y}_{max} \tag{6}$$

$$\text{Constraints on control actions: } \mathbf{u}_{min} \leq \mathbf{u} \leq \mathbf{u}_{max} \tag{7}$$

$$\text{Constraints on rate of change of control action: } \Delta\mathbf{u}_{min} \leq \Delta\mathbf{u} \leq \Delta\mathbf{u}_{max} \tag{8}$$

Each of the three boxed inequalities in Eqs. 6–8 can be split in two "less-or-equal" inequalities, which fit the general formulation of the constraints in Eq. 1.

The constraints in Eqs. 6–8 are "hard", which means that they cannot be violated in the optimization model. If no solution exists that satisfies all constraint inequalities, the optimization model is infeasible. Gelormino and Ricker (1994) described how their model became infeasible in dry-weather situations because inaccurate inflow predictions led to negative storage volumes. They dealt with this by simply removing the lower bound on the storage volume (essentially allowing negative volumes).

Another approach is to allow constraint violation by turning hard constraints into "soft" constraints. Taking the upper limit on the output as an example, a slack

variable, s_i, is added to the upper bound in each of the constraint inequalities as

$$\hat{y}_i \leq y_{max} + s_i, \ \ i = 0, \ldots, H_p \tag{9}$$

This means that the vector of control trajectories is augmented by the slack variables; thus, the constraint can be violated. The size of the constraint violation enters the optimization model as an optimization variable. This violation is then penalized by adding the term $\Sigma_{i=1}^{H_p} \Sigma_{j=1}^{N_s} P_{s,j} s_{i,j}$ to the objective function (Eq. 5), where $s_{i,j}$ is the exceedance/undershoot of the N_s soft constraints, and $P_{s,j}$ is the penalty. Both Duchesne et al. (2003) and Pleau et al. (2005) relaxed the constraints and thus allowed for their violation. It will often not make sense to give $P_{s,j}$ a physical meaning, so it must be tuned to make up for the inadequacy of the internal MPC model. In these cases, the inclusion of soft constraints should be the last resort.

4.4. Convex versus non-linear programs

There are basically two ways of formulating the optimization model: either the optimization model is formulated as a convex program (convex optimization) or as a non-linear program (non-linear optimization). The major reason for choosing a convex program is that it is fast to solve, with a guarantee of finding the global optimum; however, the shortcoming is that the system dynamics must be profoundly simplified in order to adhere to the convexity. The major reason for choosing a non-linear optimization program is its ability to take a non-linear model of the system dynamics into account. The downside is that a non-linear program can handle fewer optimization variables, requires more computation time, and may be "trapped" in a local optimum without finding the global optimum. The trade-off is, therefore, between a detailed and efficient optimization of highly approximated system dynamics and a slower, coarse optimization based on more detailed system dynamics. As we will outline in detail in the upcoming sections, and further discuss in Section 7.1, there is no unique answer to which type of optimization model to prefer.

4.4.1. The convexity criterion

The optimization model is a convex program if both the constraint and objective functions are convex. Optimization models where either the objective or constraint functions are not convex are non-linear programs (Boyd and Vandenberghe, 2009). For both the objective and constraint functions, the convexity criterion reads:

$$f_j(\alpha \mathbf{u}_1 + \beta \mathbf{u}_2) \leq \alpha f_j(\mathbf{u}_1) + \beta f_j(\mathbf{u}_2) \ \ \ \mathbf{u}_1, \mathbf{u}_2 \epsilon R^n, \ \alpha + \beta = 1, \ \alpha \geq 0, \ \beta \geq 0 \tag{10}$$

For an optimization model for a sewer system, the functions f_j (one for the objective function and numerous for the constraints) are constructed using the system dynamics described by the internal MPC model. $f_j : R^n \rightarrow R$ is a mapping of a control trajectory $\mathbf{u}$ to a single number. The dimension (n) of the control trajectory is the number of control actions inside the control horizon (control horizon divided by setting duration) multiplied by the number of locations with a controlled flow. The convexity criterion states that the value of the function for any linear combination of control trajectories ($f_j(\alpha\mathbf{u_1} + \beta\mathbf{u_2})$) must be less than or equal to the linear combination of the individual function values ($\alpha f_j(\mathbf{u_1}) + \beta f_j(\mathbf{u_2})$). The coefficients α and β used to create the linear combinations must be non-negative and sum up to unity.

4.4.2. Convex programs

Linear programs (LP) and quadratic programs (QP) are well-known subsets of convex programs. Linearity is a special case of the convexity criterion (Eq. 10). If the internal MPC model dynamics are linear, then all future outputs are linear functions of the control actions and bounding the future output from below or above will consequently result in linear inequalities. Therefore, a typical and easy way to obtain convex constraint functions is to define linear system dynamics in the internal MPC model.

A convex program is computationally much more efficient to solve than a non-linear program. Therefore, convex optimization programs can deal with large amounts of optimization variables (tens to hundreds of thousands), which implies that the control trajectory can be optimized with a fine-grained time resolution. Using a convex program also ensures that the global optimum is found. The drawback of a convex program is that it can be difficult to formulate convex constraints when the dynamics of the system change at thresholds. Such a shift in the flow domain can be very difficult to include in the internal MPC model when formulating a convex program (see Section 6). Therefore, many authors investigate non-linear programs.

4.4.3. Non-linear programs

Non-linear constraint or objective functions result in a non-linear program. Non-linear constraint functions are obtained if the internal MPC model is non-linear. In case the non-linear internal MPC model is described as a mixed logical dynamical (MLD) system (see Section 6.1), a mixed integer program (MIP) is obtained, which can either be a mixed integer linear program (MILP) or a mixed integer quadratic program (MIQP).

Non-linear programs have one major advantage: they can embrace non-linear system dynamics. The introduction of non-linearities allows for a more detailed system description, leading to an enhanced model performance. However, by introducing non-linearities into the model, the optimization may converge towards a local optimum. Therefore, the enhanced performance should be weighed against

the loss of optimality occurring because a global optimum is not guaranteed (Joseph-Duran et al., 2014b; Marinaki and Papageorgiou, 1998; Ocampo-Martinez et al., 2007, 2008; Papageorgiou, 1988) and against the increased computational time (Ocampo-Martinez and Puig, 2009a). Furthermore, it is important that the improvements obtained from including non-linearities are large compared to the errors stemming from the uncertain rainfall–runoff predictions (Ocampo-Martinez et al., 2008). In the original formulation of DORA (Vezzaro and Grum, 2014), the non-linearities stem from an abrupt exceedance of basin volumes (leading to overflow) and the inclusion of input uncertainty, and the application of a genetic algorithm here reduces the risk of getting trapped in a local optimum.

5. Optimization solvers

The *optimization solver* is the piece of software, which implements the *algorithm* that solves the optimization model. The algorithm is the "recipe for solving", and different implementations of the same algorithm may exhibit differences in performance, both with respect to finding the optimum and with respect to computational cost. The reviewed literature reports the use of solver implementations ranging from commercial products (for example, Joseph-Duran et al. (2014c)) over open-source solvers (for example, Vezzaro and Grum (2014)) to implementations made by the authors themselves (for example, Zimmer et al. (2015)). Optimization algorithms and their implementation in optimization solvers is a scientific field in itself and much too comprehensive to treat thoroughly in this review.

The optimization solver is an essential part of MPC because it governs how the internal MPC model and the objective function are used to find the desired optimum of the optimization model. The same optimization model can be solved by different solvers with very different computational cost, and sometimes also with different outcome. The most important factor when deciding for a solver is whether to pose the optimization problem as convex or non-linear. In the following, we outline the characteristics of solving convex and non-linear programs, respectively.

A convex program can be solved in a computationally efficient manner by using the interior point algorithm by Nesterov and Nemirovskii (1994). This algorithm can handle tens or even hundreds of thousands of optimization variables and still be computationally feasible.

Non-linear programs are typically solved by simulation–optimization based algorithms like genetic algorithms or gradient decent based algorithms. These algorithms require repeated simulations of the system dynamics with candidate sets of optimization variables. In most cases, it will be too time consuming to base the optimization on simulations with a detailed hydraulic model, and simplified models must be derived (see the discussion about internal MPC models in Section 6). Even with simplified (though non-linear) system dynamics, all simulation–optimization based algorithms have the large computational burden as their weak spot.

Optimization algorithms that assume continuous gradients (for example, gradient decent methods) may experience difficulties in case of rapid changes in system state, where the gradient becomes discontinuous. Heuristic optimization algorithms (for example, genetic algorithms and simulated annealing) handle these types of discontinuities better, but represent more computational challenges. Genetic algorithms have been widely used within water management optimization (see the review by Nicklow et al. (2010)), and Zimmer et al. (2015) tested different genetic algorithms for MPC of urban drainage systems.

Convex and non-linear optimization algorithms also treat the constraints on outputs (for example, a limit on the future values of the basin volume) differently. Constraints on outputs indirectly limit the possible choices of control trajectories, as some choices will lead to constraint violations. A convex optimization program forces the constraints on outputs to be formulated as a convex function of the initial conditions and the control trajectory. This will tell the interior point algorithm to limit the search space to combinations of control actions that fulfill the constraint inequalities, without the need for first running the candidate control trajectory through a simulation. A simulation–optimization based algorithm cannot take constraints on outputs into account when choosing the next candidate control trajectory, because the output is not an explicit function of the initial conditions and the control trajectory. Therefore, it is not known whether the candidate control trajectory fulfills the output constraints until a simulation has been run.

6. Internal MPC models

The internal MPC model represents the dynamics of the urban drainage system and is used to predict the future state of the system, given the initial conditions of the system, the input such as rainfall and sewage, and the control actions. It is important that the internal MPC model in combination with the optimization model runs fast enough to allow the optimization solver to optimize the control actions within the available time frame. In sewer systems, flow can be described by the Saint–Venant equations, which are composed by a continuity equation and a momentum equation (see, for example, Butler and Davies, 2011; Chow et al., 1988). These give a detailed, non-linear mathematical description of the flow in the urban drainage system and are used in HiFi models, together with the hydraulic equations for the different hydraulic structures. There is a broad consensus on the application of the HiFi models (for example, MIKE URBAN or SWMM) for design and analysis purposes within the urban drainage community (Elliott and Trowsdale, 2007), but they are too complex to be used as internal MPC models. Instead, simpler models are used; however, there is much less consensus on how to construct lumped-conceptual models and even less on internal MPC models. Finding an appropriate model description that captures the important dynamics of the system is one of the most important tasks when designing RTC for urban drainage

systems, as this accuracy is directly reflected in the quality of the optimized control (Joseph-Duran et al., 2014a, 2014b, Ocampo-Martinez and Puig, 2009a, 2010; Papageorgiou, 1983; Pleau et al., 1996; Schütze et al., 2004). This trade-off between computational cost and the accuracy of the model is quantified by Mollerup (2015), who showed that more detailed models have better model performances, but also longer computational times.

It is not a trivial task to simplify reality, or a HiFi model, down to an internal MPC model that is computationally feasible yet sufficiently accurate. Such a task includes considerations about important physical structures and processes, operational goals, whether the program is convex or not, and the applied optimization solver.

6.1. Model structure

In the reviewed literature, the internal MPC models range from rather detailed models to conceptual models that are constructed by analyzing the layout of the sewer system and then deciding on a set of model elements that can be used as representatives. The latter category is by far the most used approach. A list of model elements has been compiled from the reviewed literature and listed in Table 5. Actuators are conceptually embedded in these model elements, where gates, weirs, and valves, for example, are implicitly part of the complex diversion elements, while pumps, for example, can be represented as the manipulated outflow from a basin.

The model elements shown in Table 5 can be used to represent different complex phenomena that occur in the urban drainage system, for example, when the sewer network gradually fills up and reaches its capacity, hereby going from free flow to pressurized flow. These complex phenomena include internal overflows, external overflows, and backwater effects. The term "internal overflow" is usually used for overflows that take place inside the sewer system; however, the same mechanism can be used to model flooding that, after a period of storage above ground, returns to the sewer system; as these two are not always separable in the literature, we have denoted both as internal overflows. "External overflow" will be used as term for the CSOs and flooding that does not return to the sewer system, whereas the term "backwater effects" describes the accumulation of water upstream in the sewer system due to, for example, a downstream flow limitation.

Table 6 provides an overview of the model elements used in the reviewed literature and the linearity of both the overall model and each model element. The table also notes whether the phenomena of backwater effects and internal and external overflows are included. Some of the applied MPC methods, including the model elements and phenomena, are not described thoroughly in the reviewed literature; thus, it either requires expert knowledge within control theory or additional information to understand the methods in depth. Hence, Table 6 is constructed based

Table 5. Description of model elements used in internal MPC models.

Model element	Description
Real basins	Storage tanks.
Pipes	Used to represent transport time and/or the flow capacity of the sewer system.
Collectors (also called "interceptor pipes" or "trunk sewers")	Large pipes that can be modeled to account for the in-line storage capacity.
Simple pipe junctions	Summation or diversion locations without any passive elements or actuators.
Complex diversion elements	Diversion locations with passive elements or actuators; for example, overflow structures (internal and external weirs), gates, or valves.
Linear reservoirs (also called "virtual tanks")	Used to represent a large part of the sewer network; thus, the volume of water stored in the linear reservoirs represents the water volume stored in the pipes in this part of the system.
Wastewater treatment plants	Included to represent the maximum inflow capacity; however, these are only rarely used as individual elements; thus, they are left out in the remainder of this paper.

on our best interpretation of what has been done and should merely serve as a guideline for inspiration. The table also visualizes that an urban drainage system can be conceptualized in many different ways, even within the same group of authors. The differences between linear and non-linear internal MPC models will be elaborated in the following.

6.1.1. Linear internal MPC models

As stated earlier, convex constraint functions can be obtained by formulating the internal MPC model in a linear way, and most of the linear elements in Table 6 can be formulated by a collection of discrete-time state-space models:

$$\mathbf{x}(k+1) = \mathbf{A}\mathbf{x}(k) + \mathbf{B}_\mathrm{u}\mathbf{u}(k) + \mathbf{B}_\mathrm{d}\mathbf{d}(k)$$

$$\mathbf{y}(k) = \mathbf{C}\mathbf{x}(k) \tag{11}$$

where $\mathbf{x}$ is a vector of the states in the system, $\mathbf{u}$ is a vector of the control actions, $\mathbf{d}$ is the input that cannot be controlled, $\mathbf{y}$ is the system outputs, while $\mathbf{A}$, $\mathbf{B}_\mathrm{u}$, $\mathbf{B}_\mathrm{d}$, and $\mathbf{C}$ are the coefficient matrices and k is the modeling time step (García et al., 2015; Maciejowksi, 2002; Marinaki and Papageorgiou, 1998). Modeling of the different elements are summarized in Table 7, while a more thorough description can be found in the individual papers listed in Table 6.

An external overflow can be modeled by a linear model by defining a manipulated flow out of a given element. This flow enters the optimization model as optimization variables with a relatively large penalty in the objective function such that the overflow only occurs when no other option exists (Fiorelli and Schutz, 2009; Fiorelli et al., 2013; Gelormino and Ricker, 1994). Overflows can also implicitly be modeled linearly, either by including slack variables as done by Ocampo-Martinez et al. (2008) or by including a specific overflow term in the objective function, as

Table 6. Overview of the model elements included in the internal MPC models for some of the reviewed literature. The use of an element is indicated with √ and the ability to model backwater effects (backwat.), and internal (int.) and external (ext.) overflows is noted. The linearity of the internal MPC model is also shown.

Literature	Internal MPC model linearity	Real basin		Pipe		Collector		Simple pipe junction		Complex diversion element		Linear reservoir	
		Linear	Non-linear	Linear	Non-linear	Linear	Non-linear	Linear	Non-linear	Linear	Non-linear	Linear	Non-linear
Cui et al. (2015)	Linear	√						√					
Fiorelli and Schutz (2009) and Fiorelli et al. (2013)	Linear	√ext.											
Gillé et al. (2008) and Mahmoodian et al. (2017)	Linear	√ext.		√									
Ocampo-Martinez et al. (2008)	Linear	√			√int./ext.							√ext.	
Mailhot et al. (1999)	Linear									√int.			
Vazquez et al. (1997)	Linear	√ext.								√int./ext.			
Pleau et al. (1996)	Linear	√int./ext.		√									
Gelormino and Ricker (1994)	Linear			√								√ext.	
Papageorgiou (1983)	Linear			√				√					
Farahani et al. (2017), Ocampo-Martinez et al. (2013) and Ocampo-Martinez and Puig (2010, 2009a)	Non-linear	√int./ext.			√int./ext.			√			√int./ext.		√int./ext.
Courdent et al. (2015), Löwe et al. (2016), Vezzaro and Grum (2014, 2012), and Vezzaro et al. (2014a, 2014b, 2013)	Non-linear		√ext.										
Mollerup (2015) and Mollerup et al. (2016)	Non-linear		√ext.	√						√			
Joseph-Duran et al. (2014d)	Non-linear		√	√			√int./ext.				√int./ext.		
Joseph-Duran et al. (2014a)	Non-linear		√	√			√int./ext.				√int./ext.		
Joseph-Duran et al. (2014b)	Non-linear		√	√							√int./ext.		
Joseph-Duran et al. (2013b)	Non-linear		√int./ext.	√					√int./ext.		√int./ext.		√int./ext.
Beak et al. (2013)	Non-linear		√int./ext.	√				√					
Joseph-Duran et al. (2013a, 2013c)	Non-linear		√	√				√			√int./ext.		
Mollerup et al. (2012)	Non-linear		√ext.								√int./ext.		√int./ext.
Leirens et al. (2010)	Non-linear		√ext.	√							√ext.		
Cen and Xi (2009)	Non-linear		√ext.					√					
Ocampo-Martinez et al. (2007)	Non-linear		√					√					
Cen and Xi (2007) and Marinaki and Papageorgiou (1999, 1998)	Non-linear		√ext.		√int./ext.			√			√int.		√int./ext.
Duchesne et al. (2004, 2003, 2001) and Pleau et al. (2005)	Non-linear					√backwat.							
Marinaki and Papageorgiou (2001)	Non-linear		√int./ext.	√									
Papageorgiou (1988)	Non-linear		√	√							√ext.		

done by Mailhot et al. (1999). In these two cases, the phenomena are no longer embedded in the internal MPC model; thus, they are not shown in Table 6. Additionally, one may question whether they represent reality sufficiently well as the overflow water never leaves the internal MPC model. Backwater effects are disregarded in all linear models.

6.1.2. Non-linear internal MPC models

Most internal and external overflows, such as the flow over a weir, are inherently non-linear and depend on the state of the system. Therefore, it can be useful to make a more realistic model description of these phenomena by including non-linearities in the internal MPC model. In most cases, the

Table 7. Model techniques applied for different linear model elements.

Model element	Modeling technique
Real basins	Modeled by simple mass balances where the discharge from the basins is manipulated. Here, an example from Ocampo-Martinez et al. (2007) is given: $$v_i(k+1) = v_i(k) + \Delta t\big((q_i^{in}(k) - q_i^{out}(k)\big)$$ where v_i is the volume for basin i at time k, Δt is the sampling interval while q_i^{in} and q_i^{out} are the inflow and outflow.
Pipes	May only represent a flow capacity to enable the modeling of an overflow, or include travel time by using a flow equation where the outflow of the pipe is depending linearly on the inflow at previous modeling time steps. Here an example from Joseph-Duran et al. (2014a) is shown where the current flow is based on the travel time, k_i, and a weighing factor, a_i: $$q_i^{out}(k) = a_i q_i^{in}(k - k_i) + (1 - a_i)q_i^{in}(k - k_i - 1)$$ The travel time will depend on the flow, and Fiorelli et al. (2013) recommend using full flowing pipes for calibrating travel times. The time delay can also be included in the objective function as in Fiorelli and Schutz (2009); thus, it is not an element in itself and it is in this case not shown in Table 6.
Simple pipe junctions	Modeled as simple summations (merging flow) or diversions into predefined ratios (splitting flow). Ocampo-Martinez and Puig (2009a) provided an example of a merging flow: $$q_i^{out} = \sum_{i=0}^{n} q_i^{in}$$ where n is the number of inflows. The splitting flow can be modeled by applying a predefined partitioning of the flow (Joseph-Duran et al., 2014d; Ocampo-Martinez and Puig, 2009a).
Complex diversion elements	Modeled as a mass balance in Ocampo-Martinez et al. (2008), where it is denoted a "redirection gate": $$q_i^{out}(k) = Q_i(k) + \sum_j u_i^j(k)$$ where q_i^{out} is the outflow from the basin that flows to the diversion element, j is an index over all manipulated flows coming from the diversion element, and Q_i is a flow path having a limited flow capacity. Exceedance of this capacity leads to an overflow event.
Linear reservoirs	Modeled as a mass balance where the outflow from the linear reservoirs is linearly dependent on the volume (see, for example, Chow et al., 1988). Here, an example from Ocampo-Martinez et al. (2007) is shown: $$v_i(k+1) = v_i(k) + \Delta t \phi_i S_i P_i(k) + \Delta t\big(q_i^{in}(k) - q_i^{out}(k)\big)$$ $$q_i^{out}(k) = \beta_i v_i(k)$$ where Δt is the sampling interval, the second term on the right side is a simple input model with ϕ_i as the runoff coefficient, S_i as the surface area of the catchment and P_i as the rain intensity, and β_i as the volume/flow conversion coefficient.

equations representing the model elements from Table 7 are simply expanded to include the non-linear overflow description. The following aims to provide an overview of different techniques for dealing with the non-linearities that arise when including non-linear phenomena. More information can be found in the papers listed in Table 6.

The elements representing internal and external overflow phenomena can be modeled in a non-linear way by using hybrid model predictive control (HMPC), where the system is described as a combination of continuous and discrete dynamics. HMPC systems include the MLD systems, linear complementary (LC) systems, min-max-plus scaling (MMPS) systems, and piecewise affine (PWA) systems. MLD systems have been applied in numerous papers and are often constructed by adding logical conditions to the otherwise linear descriptions shown in Table 7, which determine whether the capacity of the element has been exceeded. This enables the system dynamics to be modeled by both a continuous part and a logical part, where the latter makes the modeling of non-linear state depending phenomena possible (Joseph-Duran et al., 2013b, 2014b; Ocampo-Martinez et al., 2007). In order to give a better understanding of the MLD systems, two (slightly modified) examples of overflows are taken from Ocampo-Martinez et al. (2007). The first example is the modeling of a diversion element with a passive weir, which can be formulated as

$$
q_i^{out}(k) \;=\; \begin{cases} q_i^{in}(k) \text{ if } q_i^{in}(k) \leq \overline{q}_i^{out} \\ \overline{q}_i^{out} \qquad \text{otherwise} \end{cases}
$$

$$
q_i^{overflow}(k) \;=\; q_i^{in}(k) - q_i^{out}(k) \tag{12}
$$

where $q_i^{out}(k)$ is the outflow, $q_i^{in}(k)$ is the inflow, $\overline{q}_i^{out}$ is the capacity of the pipe while $q_i^{overflow}(k)$ is the overflow. The second example is directly relatable to the linear formulation for linear reservoirs in Table 7. Now, an overflow will occur from the storage basin when the storage capacity of the basin, $\overline{v}_i$, is reached. Therefore, an extra overflow term is added to the mass balance:

$$
v_i(k+1) = v_i(k) + \Delta t \big(q_i^{in}(k) - q_i^{out}(k) - q_i^{overflow}(k) \big)
$$

$$
q_i^{out}(k) = \begin{cases} \beta_i \overline{v}_i \text{ if } v_i(k) \geq \overline{v}_i \\ \beta_i v_i(k) \quad \text{otherwise} \end{cases}
$$

$$
q_i^{overflow}(k) = \begin{cases} \dfrac{v_i(k) - \overline{v}_i}{\Delta t} \text{ if } v_i(k) \geq \overline{v}_i \\ 0 \qquad \text{otherwise} \end{cases} \tag{13}
$$

If the overflow represents an internal overflow, it is normally either directed straight to another part of the system (see, for example, Ocampo-Martinez and Puig, 2010), or the water is stored in a fictive basin to represent street flooding from where it can return to the network (see, for example, Joseph-Duran et al., 2014a).

Ocampo-Martinez and Puig (2010) introduced piecewise linear function-based (PWLF) MPC in order to exclude the logical variables that lead to computational cost issues in the MLD systems. The PWLF-based model was shown to comply with the time constraints, but led to a less optimal control than the MLD modeling approach. The internal and external overflows can also be modeled by non-linear smoothing, by applying a constraint branching algorithm and by using the general disjunctive programming framework, as Joseph-Duran et al. (2014b) did.

Duchesne et al. (2001, 2003, 2004) and Pleau et al. (2005) included backwater effects in their internal MPC model by modeling pipe flow with the Saint–Venant equations, while Duchesne et al. (2001, 2003, 2004) also allowed surcharge. Those studies used different approximations of the Saint–Venant equations (kinematic wave model, diffusion wave model, and the full dynamic wave model) based on the system states; thus, the non-linear phenomena were not modeled by expanding the elements in Table 7. Duchesne et al. (2001, 2003, 2004) used 18 pipes to represent the ~17 km long collector and the model is still not considered to be a full HiFi model. Much of the literature disregards the use of the Saint–Venant equations; however, Duchesne et al. (2004) stated that their model runs at least 10,000 times faster than the real flow process and Pleau et al. (2005) mentioned that the optimization problem on average is solved in less than 1 min, allowing the internal MPC model to be used in an MPC context. Leirens et al. (2010) also applied the Saint–Venant equations to describe the flow in pipes, but they stated that this is a costly process and that future work should look at the practical time constraints.

Zimmer et al. (2015) used an internal model which had been derived as a tabulated metamodel of a SWMM model by Zimmer et al. (2013). Thus, this model was not based on the elements in Table 7. The metamodel was able to model domain shifts from free flow to pressurized flow and ran about twice as fast as the corresponding SWMM model.

6.2. Inputs

Input to the internal MPC model includes rain, runoff, and/or sewage from other part of the urban drainage system. Some studies (including Cembrano et al., 2004; Ocampo-Martinez et al., 2013; Ocampo-Martinez and Puig, 2010; and Puig et al., 2009) transformed measured rain data from rain gauges into flow data directly in the internal MPC model. However, most studies have excluded the part of the sewer system not affected by changes in the control from the internal MPC model. Instead, the flows entering the internal MPC model from these parts of the network are represented by flow time series taken either from historically measured
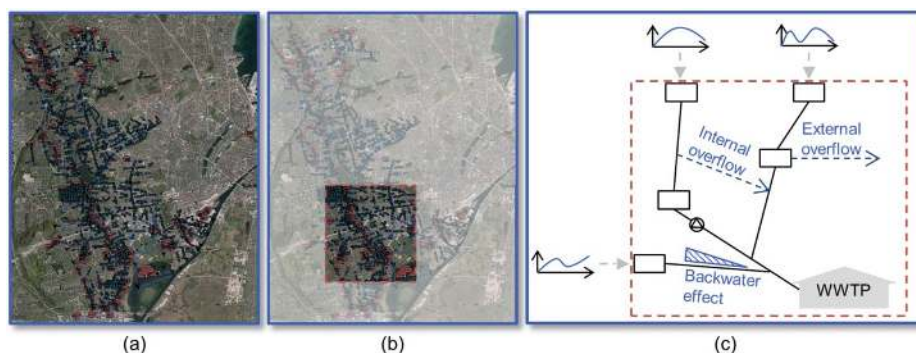
Figure 6. Relation between a HiFi model and an internal MPC model and its input, exemplified for a catchment in Copenhagen, Denmark. (a) The HiFi model used as starting point. (b) The relevant part of the HiFi model containing the sewer system affected by the control optimization, which may contain backwater effects and internal and external overflows. (c) The conceptual, internal MPC model of the relevant part of the sewer system with flow input from measurements or generated by a (simplified or HiFi) input model.

flow (for example, Gelormino and Ricker, 1994) or generated by input models that transform rain data to flow data (see Fig. 6). When doing the initial design and setup of an MPC controller, the inflow time series are either measured or generated by an offline input model. Of course, this approach can only be used for offline testing and not in a real-time operation where real-time input models are needed to produce the inflows to the internal MPC model based on meteorological data.

6.2.1. Meteorological data and rain forecasts

Rainfall observations are the most important input when modeling urban runoff and, therefore, also for MPC of urban drainage systems. It is most common to use rain gauge data as input when doing offline historical simulations. This is partly due to historical reasons and data availability, but also because rain gauge data usually result in more accurate runoff predictions than radar data, even for detailed distributed models that can utilize the spatially distributed information in radar data (see, for example, Goormans and Willems (2013)). However, data from networks of rain gauges are not well suited for generating the rainfall forecasts required for operational MPC applications and, therefore, operational MPC applications are likely to be based on rainfall forecasts either from extrapolated radar rainfall estimates (Thorndahl et al., 2017), called nowcasts among meteorologists, or from NWP models (Courdent et al., 2018). Generally, radar nowcasts perform better for short time horizons up to a couple of hours, whereas NWP is required for longer time horizons (Thorndahl et al., 2013). In a review of multiple studies, McMillan et al. (2012) found that it is to be expected that radar rainfall data can be 30–50 percent off compared to the rain rate on hourly scale if radar data is the only source of rain data. On shorter time scales, this error is much larger because the error of radar rainfall estimates grows significantly when the spatial and temporal resolution is increased (Seo and Krajewski, 2010).

Most MPC studies apply ex-post rainfall forecasts (that is, historical measurements used as forecasts; Beven and Young (2013)) from either rain gauges or radar over the forecast horizon (for example, Fiorelli et al., 2013; Gelormino and Ricker, 1994; Joseph-Duran et al., 2014b; Marinaki and Papageorgiou, 2001; Ocampo-Martinez et al., 2008; Vezzaro et al., 2014b), while a few have applied synthetic data (hypothetical rain series or Chicago Design Storms) (for example, Mollerup, 2015; Papageorgiou, 1988). To the best of our knowledge, the only paper that clearly states that actual rainfall forecast data has been used when evaluating MPC is Löwe et al. (2016). Furthermore, the rainfall input can either be spatially distributed as applied by, for example, Joseph-Duran et al. (2014b), Löwe et al. (2016), and Pleau et al. (2005), or homogeneously distributed in space as applied by, for example, Fiorelli et al. (2013), Mollerup et al. (2016), and Vezzaro and Grum (2014); however, this information is often omitted.

6.2.2. Input models and input predictions

The separation between input model and internal MPC model requires the determination of the boundaries between the two models (the red boundary in Fig. 6). This is not a trivial task due to overflow structures, backwater effects, and state-dependent changes in the dynamics. In theory, real-time input models can be either HiFi models or simplified models, but HiFi models will often be computationally too expensive. Nevertheless, most of the reviewed literature either uses flow time series obtained from offline HiFi models to generate input to the internal MPC model, for example Joseph-Duran et al. (2013a, 2013c) or does not state the origin of the flow time series in detail. Simplified input models are rarely encountered in the reviewed MPC literature, but can be found in some studies (for example, Fiorelli et al., 2013; Joseph-Duran et al., 2014b; Löwe et al., 2016; Vezzaro and Grum, 2014). However, to our knowledge only Löwe et al. (2016) used the input model directly in a simulated MPC operation. This indicates that the focus in the reviewed literature has been on the internal MPC models only, which underlines how far we are from an actual implementation where real-time input models are vital. The construction of input models is a research field in itself and is beyond the scope of this review.

In a real-life operation, the input models will produce input predictions. These predictions will be uncertain but the effects of this uncertainty are theoretically reduced by the recursive control optimization if new inputs are obtained and initial conditions in the internal MPC model are updated for each sampling interval (Fiorelli et al., 2013; Marinaki and Papageorgiou, 1998, 1999, 2001; Papageorgiou, 1983, 1988; Schütze et al., 2004). Overly uncertain input forecasts are not useful; therefore, some of the reviewed literature investigated the effect of having a forecast horizon shorter than the prediction horizon (thus, having incomplete input information). Here, the ex-post flow is applied within the forecast horizon and a predefined input algorithm is subsequently used to compute the flow in the remaining part of the prediction horizon. In the most extreme cases, the forecast

horizon is set to 0; thus, the forecasts for the entire prediction horizon are based on the predefined input algorithm. Examples of prolongation schemes are to:

- Let the input linearly fade to zero from the present value (Vazquez et al., 1997).
- Let the input increase or decrease for the next one or two modeling time steps with the average slope of the last four modeling time steps and then let it decrease to dry weather flow during the next 30 min (Papageorgiou, 1988).
- Let the input constantly equal the present value. Here, the present value can either represent the true state (Fiorelli and Schutz, 2009; Fiorelli et al., 2013; Gelormino and Ricker, 1994; Ocampo-Martinez et al., 2008) or the uncertainty of the state can be included (Fiorelli and Schutz, 2009; Löwe et al., 2016).
- Use the three last modeling time steps to predict the input for the next 20 min by linear regression and subsequently let the inflow decrease to dry weather flow over the next 20 min (Marinaki and Papageorgiou, 2001).
- Set the input equal to five times the dry weather flow (Fiorelli et al., 2013).

The reviewed literature has contradictory conclusions regarding the effect that the quality of the input has on the control performance; this may be due to the following reasons:

1) The use of a predefined input algorithm will lead to a worse input quality and thus also a worse control; however, it is a subjective matter how much the control can acceptably deteriorate before it can be concluded that the input quality is important.
2) The importance of accurate forecasts will be affected by factors such as the spatial distribution and size of the rain event, the size of the catchment, and the methodology and accuracy of the applied predefined input algorithms. This is supported by Fiorelli et al. (2013) and Rauch and Harremoës (1999).

Due to forecast uncertainties, there is a risk of taking action because the model anticipates an input that will never take place (Raso et al., 2014); thus, it is beneficial to include the forecast uncertainty in the optimization of the control, as also shown by Vezzaro et al. (2014a). The input uncertainty can be included in the control optimization itself by, for instance, incorporating the fact that the far future is more uncertain than the near future in the objective function, as was described in Section 4.2.3. However, this only includes a fixed decrease in the trust in forecast over time. Farahani et al. (2017) also include input uncertainty in the objective function, but in a time-varying manner that allows for the optimization of the "worst case scenario". A complete representation of the time-varying trust in forecasts can be included by, for example, applying stochastic gray-box models (Breinholt et al., 2011, 2012) or by wrapping an uncertainty propagation layer around the existing deterministic models in the form of Monte Carlo simulations (Schütze et al., 2004). An example of using gray-box models to incorporate the uncertainty of the forecasts in the control optimization is given by Löwe et al. (2016). Here, the

uncertainty of the runoff forecast is estimated by taking into account the uncertainty regarding rainfall data, model structure, and system observations.

6.3. Initial conditions

The internal MPC model can be kept in touch with reality (or at least affected by it) by updating the initial conditions at each sampling interval. The recursive updating can reduce the negative effects of the simplicity of the model structure and ensure that the internal MPC model is kept on track; this is also known as "re-aligning" (Maciejowksi, 2002). Few studies in the reviewed literature have updated initial conditions from either online measurements or HiFi models that run in real-time:

– Measured data can be used as new initial conditions when observations relate directly to states in the model, as done by Courdent et al. (2015), and in the more general case by using data assimilation methods such as the Kalman filter, as done by Löwe et al. (2016), Pleau et al. (1996), and Vezzaro et al. (2013), or a variational state estimation approach, as suggested by Joseph-Duran et al. (2015, 2014c). Puig et al. (2009) did not update the initial conditions and instead used measurements from the system to autocalibrate the internal MPC model parameters at every time step.

– The initial states of the internal MPC model can also be taken directly from a HiFi model after the HiFi model has been run with the MPC optimized control for one sampling interval, as done by Cembrano et al. (2004), Joseph-Duran et al. (2014a), and Marinaki and Papageorgiou (2001), under the assumption that the HiFi model reflects reality sufficiently well. HiFi models themselves are not usually updated with measurements, but it can be done (Borup et al., 2014).

7. Discussion

As we have shown, MPC is a complex matter and there are many choices to make regarding the specific MPC method, operational implementation, and linguistics. It is also important to document likely improvements before investing in MPC. This section will highlight some key considerations that should be addressed when dealing with MPC in urban drainage, and answer the following questions:

1) How do I choose between a convex and a non-linear program?
2) How does MPC interact with other control layers, such as local PID controllers?
3) How big improvements can be gained from implementing MPC?
4) Can you compare different MPC methods?
5) Which research gaps and challenges do we face next?

7.1. Convex versus non-linear programs – an essential question

Deciding between a convex and a non-linear program is crucial because it governs many other connected aspects of MPC. This should be settled early in the process

as it can be difficult, or at least time-consuming, to change later, and the choice will put restrictions on the applied optimization solver. Roughly speaking, the choice is between the two following options:

– Deciding a fine-resolution control trajectory based on simplified model dynamics (convex program).
– Making a coarse-resolution control trajectory – maybe even an average decision for the entire control horizon – based on a more detailed description of the system dynamics and/or the uncertainty (non-linear program).

There is a trade-off between the time resolution of the control trajectory and the degree of detail in the model of the system dynamics. Obviously, detailed optimization makes no sense if the system dynamics are too simple, and detailed system dynamics will not save the day if the optimization model is too coarse. The flow chart in Fig. 7 is based on the routes that the reviewed publications have taken from control problem to optimal control actions.

Often, a non-linear internal MPC model is selected for one of two reasons. It may include (1) complex inherently non-linear system dynamics such as flow over thresholds, as has been encountered in a large proportion of the reviewed literature (for example, Joseph-Duran et al., 2014a; Marinaki and Papageorgiou, 2001; Ocampo-Martinez et al., 2007). It may also include (2) non-linear uncertainty considerations in the control optimization, as in DORA (Vezzaro and Grum, 2014) regarding input uncertainty and by Mahmoodian et al. (2017) regarding uncertainty of modeled concentrations. Uncertainty can also be included in a convex formulation, but this has not been encountered in the reviewed literature and it is, therefore, not shown in Fig. 7. The choice of a non-linear program due to a complex dynamics description requires a reduction in the number of optimization variables in order to stay within the time constraints in a real-time application, which could limit how large a physical system can be considered in the optimization and/or how detailed the control trajectory can be. Even with a limited number of optimization variables, the solving is still computationally more demanding than it is for a convex program (Boyd and Vandenberghe, 2009).

Choosing a linear internal MPC model to describe the system dynamics provides linear (and thereby convex) constraint functions. The benefit of a convex program is that it can be solved very quickly, even for tens of thousands of optimization variables, which facilitates a fine resolution of the control trajectory. Also, the optimization solver algorithm of a convex program explicitly takes constraints on future outputs into account. The drawback is that the system dynamics must be simplified to the extent that constraint and objective functions are explicit (and convex) functions of the optimization variables. The difficult part here is the formulating of the important non-linear phenomena (for example, overflow thresholds) in linear models in a manner that reasonably represents reality.
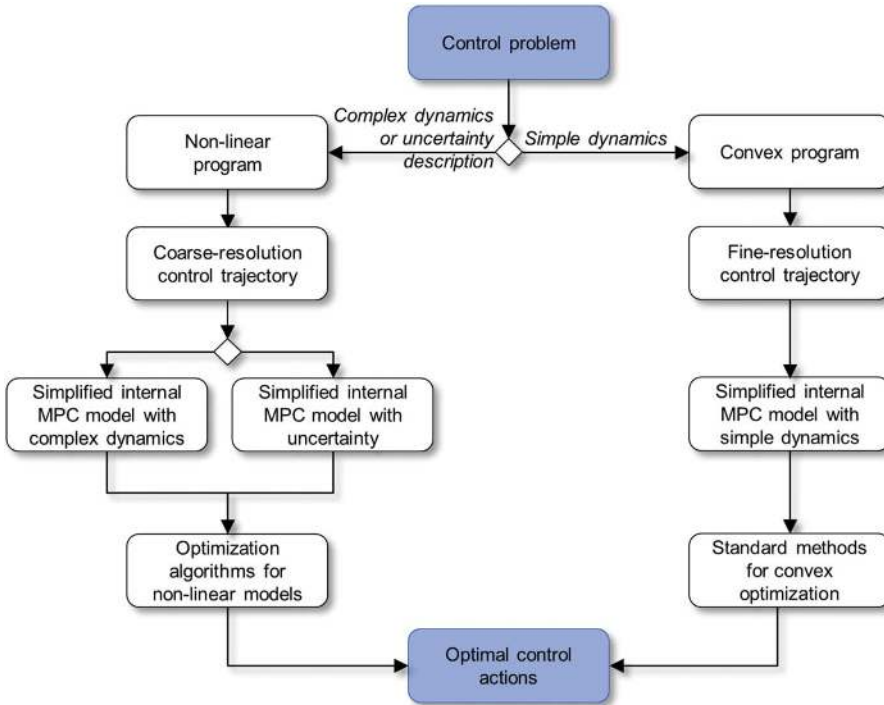
Figure 7. Different routes to take from control problem to optimal control actions.

7.2. Control hierarchies

Having a clear strategy for making the control setup compatible with a real-world implementation is obviously important. Figure 8 shows a slightly modified version of the time-scale dependent control hierarchy proposed by Mollerup et al. (2017). This hierarchy breaks down the overall control problem into four distinct hierarchical levels that can be managed independently and have clear communication interfaces.

The hierarchical layers are distinguished based on differences in time scale of the process dynamics and possible control actions taking place, and each layer takes care of different aspects of the control problem. Layer 4 considers the management of objectives; thus, it defines overall objectives and constraints for the operation of the sewer system, which may, for example, change seasonally or depend on forecasted rainfall intensities (see, for example, Courdent et al., 2015; Fiorelli et al., 2013; Giraldo et al., 2010). Layer 3 takes care of the system-wide or global optimization; thus, it finds the optimal control based on operation objectives and constraints (defined at Layer 4) for the entire urban drainage system and determines the overall set-points that are sent to the controllers at the coordinating Level 2. This level is typically decentralized and considers both interaction between different local control loops, as well as local flow and water level constraints. Layer

Figure 8. Time-scale dependent control hierarchy for sewer systems. The indication of control strategies used at each layer in the hierarchy is slightly modified from Mollerup et al. (2017) based on the terminology and structure developed in this review.

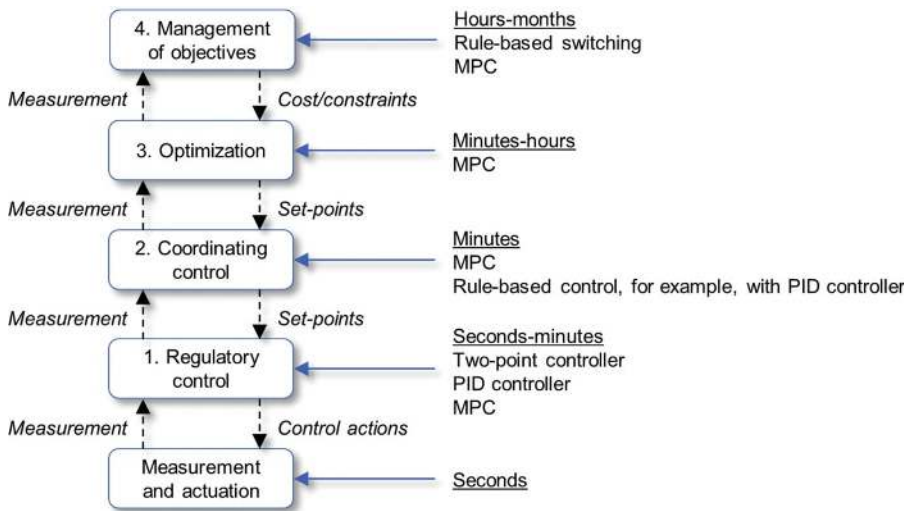1 takes care of the regulatory control; thus, it ensures that the set-points received from higher layers are followed.

Mollerup et al. (2017) placed MPC at the second layer. In practice MPC is often placed in a merged second and third layer, where it computes the overall global (system-wide) set-points. Local controllers placed in the first, regulatory control layer turn these local set-points determined by the system-wide MPC, such as a gate flow, into settings for the individual actuators, such as the gate opening. However, we argue that MPC could also be used both as local controller in Layer 1 and for overall management of objectives in Layer 4.

Most practical implementations of RTC in sewer systems include only Layer 1; that is, only local rule-based control loops, which may be robust in operation but very difficult to configure to achieve optimal system-wide control (Mollerup et al., 2013). Failing to distinguish the different hierarchical layers and allowing the overall MPC optimization at any layer above the first to influence actuators directly can lead to control systems that are vulnerable to, for example, communication failures between central and local control stations and are, therefore, unacceptable to operators. Thus, including a regulatory control layer in advanced control systems based on a hierarchy, as outlined in Fig. 8, will increase robustness to communication failure where predefined control rules can take over locally. In case this also fails, actuators can automatically be set to predefined, fixed levels. The need for a combination of local and global control schemes is acknowledged in the MPC literature (see, for example, Duchesne et al., 2004; Fiorelli et al., 2013; Frier et al., 2013; García et al., 2015; Joseph-Duran et al., 2014b; Mollerup et al., 2017; Ocampo-Martinez et al.,

2008; Papageorgiou, 1983; Pleau et al., 2005). However, the lower control levels are only rarely included in the model setup and evaluation in the reviewed literature.

Fail-safety measures are applied in the MPC implementations reported by Fiorelli et al. (2013), Frier et al. (2013), and Pleau et al. (2005), where the control system automatically shifts to a different mode in case of irregular behavior. The control system described by Frier et al. (2013) is set up such that the fallback to a lower level occurs automatically, whereas operator intervention is required to increase the level again (Bassø, 2016). Guerra et al. (2007) and Ocampo-Martinez and Puig (2009b) constructed fault-tolerant control that can be applied in the MPC concept, making the control scheme more robust and able to detect and account for faults in system components such as the actuators.

Mollerup (2015) and Mollerup et al. (2016) described a methodology for designing the different layers in a sewer system control hierarchy based on temporal decomposition. However, operational goals might also differ depending on the considered spatial scale, so it can be beneficial to establish a spatial decomposition of the control system. This approach was investigated by Zamora et al. (2010), who discussed how local control loops can be coordinated.

7.3. Obtained MPC performances in literature

Table 8 shows how MPC performs as reported in the reviewed literature. The decrease in CSO and flooding volume are listed together with the increase in flow to the WWTP and decrease in the monetary cost related to a CSO event (CSO cost). Table 8 also lists the number of events that the performance evaluations are based on, together with the location, areal extent, number of controlled actuators, length of time windows (collective horizon, sampling interval, setting duration), and whether the overall control problem is solved using a linear, quadratic, or non-linear program. The table also notes the investigated control scheme and the baseline control strategy.

Table 8 shows that MPC is mostly investigated for European or North American cities, and the studies include between one and 685 rain events to evaluate the performance. Most commonly, 10 or fewer actuators are MPC-controlled, with the highest two numbers being 24 and 47. All of the listed references use CSO volume as an evaluation measure, whereas only 25 percent use the remaining three measures (flood volume, flow to WWTP, and CSO cost). Table 8 shows the performance as an interval (illustrating the variation for different rain events) and/or an accumulated value for a series of rain events (that is, the total amount of CSO and flood volume, etc.), depending on the information given in the reviewed papers. In general, the studies minimize the volume of CSO and flooding and the cost of CSO while maximizing the volume treated at the WWTP. The MPC schemes sometimes lead to a complete avoidance of overflow or flooding for individual events and, therefore, achieve a 100 percent decrease, although some of the schemes also perform worse than the baseline control for some events. The latter is seen in two

Table 8. MPC performance reported by the reviewed literature. A performance span for all applied rain events is stated together with the overall improvement for these events (written in []), if possible. Only one of the two is shown if information is scarce or only one rain event is considered.

Source	Location	Area [km²]	Events [no.]	Actuators [no.]	Col. hor.	Samp. int.	Set. dur.	Program	Decreased CSO vol.	Decreased flood vol.	Increased flow to WWTP	Decreased CSO cost	Investigated control scheme	Baseline control scheme
Duchesne et al. (2004)	Laval, Canada	17.3	23	7	20	5	Unkn.	Non-linear	7				MPC (no storage in manh.)	Loc. con.
									35				MPC (10 m storage in manh.)	
Pleau et al. (1996)	Quebec, Canada	Unkn.	1	8	60	5	Unkn.	Quadratic	86				MPC (no noise)	Pas. con. (gates set to DWF cap.)
									75				MPC (noise, no KF)	
													MPC (noise, KF)	
Mailhot et al. (1999)		Unkn.	56	6	Unkn.	Unkn.	Unkn.	Non-linear	80				MPC	Pas. con. (current strategy)
Pleau et al. (2005)		500	7	5	120	5	Unkn.	Non-linear	30-100 [60]				MPC	Pas. con. (current strategy)
									55-100 [77]					
Leirens et al. (2010)	Bogotá, Colombia	Unkn.	1	6	300	60	60	Non-linear	40				MPC	Loc. + pas. con. (pump when almost full basins, gates open)
Giraldo et al. (2010), Zamora et al. (2010)		2.3	1	6	300	60	60	Non-linear	>50				MPC	
Vezzaro and Grum (2012, 2014)	Aarhus, Denmark	2.8 imp.	30	9	120	2	120	Non-linear	-70-20-[24]			-45-70 [4]	Loc. con. (eq. filled basins)	Pas. con. **
									5-100 [37]				MPC (no forec.)	
									5-100 [45]				MPC (determ. forec. unc.)	

Reference	Location							Type					
Rauch and Harremoës (1996)	Copenhagen, Denmark	Unkn.	3	5	Unkn.	Unkn.	Unkn.	Non-linear	0-50			MPC	Pas. con. (WWTP design cap.)
Rauch and Harremoës (1998)		10.2 imp.	11	4	Unkn.	Unkn.	Unkn.	Non-linear	2-100 [17]			MPC	Pas. con. **
Rauch and Harremoës (1999)		10.2 imp.	1	5	Unkn.	Unkn.	Unkn.	Non-linear	15			MPC	Pas. con. (gates open)
Vezzaro et al. (2013)		76 (23 imp.)	2	8	120	Unkn.	120	Non-linear	-19-42 [26]		-15-48 [30]	MPC (det. forec. unc.)	MPC (stoch. forec. unc.)
Vezzaro et al. (2014b)			13	8	120	2	120	Non-linear	60			MPC (stoch. forec. unc.)	Loc. con. **
Courdent et al. (2015)		16.6 imp.	1	4	Varies	2	Varies	Non-linear	-1.4		11	MPC (long forec., 13 hours)	MPC (short forec., 2 hours)
Löwe et al. (2016)		76	98	7	120	2	120	Non-linear	62		54	MPC (known rain, unc.)	MPC (no forec.)
									76		70	MPC (known rain, no unc.)	
									57		43	MPC (forec. rain, unc.)	
									57		54	MPC (forec. rain, no unc.)	
Mollerup et al. (2016)		3.2	1 year	3	5	5	5	Non-linear	19			Reg. con.	Loc. con. (current strategy)
									10			MPC (opt. con.)	
									11			MPC (reg. and opt. con.)	
Meneses et al. (2018)*	Lundtofte, Denmark	5.84 imp.	46	17	120	2	120	Non-linear	8		6	MPC	Pas. con. (basin expansion)
									1		3	MPC	Loc. con. (opt. gate settings)
Vazquez et al. (1997)*	Saverne, France	Unkn.	685	Unkn.	120	4	Unkn.	Linear	0-100		0-14	MPC	Pas. con. **
Marinaki and Papageorgiou (2001)*	Obere Iller, Germany	Unkn.	1	11	120	3	Unkn.	Non-linear	78			MPC	Pas. con. (opt. gate settings)
Fiorelli and Schutz (2009), Gillé et al. (2008)	Heiderscheidergrund, Luxembourg	Unkn.	2 days 1 month	24	120	10	Unkn.	Quadratic	46			MPC	Pas. con. **
									13				
Fiorelli et al. (2013)		Unkn.	4	24	120	10	Unkn.	Quadratic	0-45 [12]			MPC	Pas. con. (German guidelines)
Mahmoodian et al. (2017)		Unkn.	1	2	Unkn.	Unkn.	Unkn.	Non-linear	14			MPC (quantitative con.)	MPC (qualitative con.)
Cembrano et al. (2004)	Barcelona, Spain	Unkn.	2	2	30	5	5	Non-linear	2-5 [3]	26	7-10 [8]	Pas. con. (basins, gates open)	Pas. con. (no basins, current strategy)
									2-5 [3]	31	7-14 [10]	MPC (basins)	Pas. con. (gates open)
Ocampo-Martinez et al. (2007)		22.6	10	4	30	5	Unkn.	Non-linear	-5-11 [3]	0-100 [14]	-3-2 [1]	MPC	
Ocampo-Martinez et al. (2008)			15	7	30	5	Unkn.	Quadratic	-1-100 [5]	-233-11 [-2]	0-8 [2]	MPC	MPC (scalarization)

(Continued on next page)

Table 8. (*Continued*)

Source	Location	Area [km²]	Events [no.]	Actuators [no.]	Time windows [min] Col. hor.	Samp. int.	Set. dur.	Program	Improvement [%] Decreased CSO vol.	Decreased flood vol.	Increased flow to WWTP	Decreased CSO cost	Investigated control scheme	Baseline control scheme
Puig et al. (2009)*		Unkn.	1	Unkn.	7	Unkn.	Unkn.	Non-linear	18		50		MPC (lexicographic appr.)	Loc. con. (current strategy)
Ocampo-Martinez and Puig (2009a, 2010)		22.6	5	4	300	5	Unkn.	Non-linear	-1-8 [1] / 1-11 [3]	0-18 [13] / 0-16 [14]	-1-2 [0] / 1-2 [2]		MPC / MPC (PWLF model) / MPC (hybrid model)	Pas. con. **
Ocampo-Martinez et al. (2013)		Unkn.	3	4	30	5	Unkn.	Non-linear	9-18 [10]	2-28 [15]	15-38 [25]		MPC	Loc. con. (current strategy)
Joseph-Duran et al. (2014c)*		Unkn.	4	Unkn.	Unkn.	Unkn.	Unkn.	Non-linear	31-83	60-95	20-124		MPC	Pas. con. **
Joseph-Duran et al. (2013a)*		26	2	Unkn.	30	5	Unkn.	Non-linear	43-95 [51]	94-97 [97]	137-215 [161]		MPC	
Joseph-Duran et al. (2014b)*		Unkn.	4	Unkn.	30	5	5	Non-linear	31-92	96-97	64-108		MPC	
Gelormino and Ricker (1994)	Seattle, USA	Unkn.	10	47	10-200	10	Unkn.	Quadratic	13-52 [25]				MPC	Heu. con. (current strategy)
Cen and Xi (2007)		Unkn.	2	Unkn.	Unkn.	1	10	Non-linear	59-63 [62] / 35-40 [38]				MPC	Pas. con. (gates open) / Pas. con. (opt. gate settings)
Beak et al. (2013)		Unkn.	1	Unkn.	20	1	5	Non-linear	41				MPC	Pas. con (gates open)

*The performance of both control schemes is obtained from the same HiFi model. **The exact settings for control are not stated. Appr. = approach, cap. = capacity, col. hor. = collective horizon, con. = control, determ. = deterministic, eq. = equally, forec. = forecast, imp. = impervious, loc. = local, manh. = manhole, opt. = optimizing/optimized, pas. = passive, reg. = regulatory, samp. int. = sampling interval, set. dur. = setting duration, stoch. = stochastic, unc. = uncertainty, unkn. = unknown, vol. = volume.

cases: (1) if a more important objective is used with a higher penalty and/or (2) when multiple rain events are used to evaluate the MPC scheme. The use of multiple rain events will increase the probability of facing a rain situation that the MPC scheme cannot manage well; thus, applying multiple rain events may result in an overall worse performance than applying only one or a few rain events. None of the reviewed studies test how the MPC method performs in cases where the baseline control strategy does not give an overflow. This is a clear limitation as it is important to document whether the MPC create, for example, a CSO even though the baseline control strategy does not.

Some references use other objectives to optimize the control actions, which are not listed in Table 8. Mahmoodian et al. (2017) considered pollutant loads directly and achieved a reduction in the overflow pollutant mass of 14 percent compared to optimizing overflow volumes only. Some references (for example, Fiorelli et al., 2013; Marinaki and Papageorgiou, 2001; Ocampo-Martinez et al., 2008), include the rate of change of control actions in the objective function, but a corresponding evaluation measure has not been encountered in literature; thus, the trade-off between a rate-of-change constraint and other control objectives have not been properly evaluated.

The spatial scale of the investigated catchments ranges from very small catchments of a few square kilometers to large catchments in the range of hundreds of square kilometers, spanning the major part of a city. Only 40 percent of the listed references give information about the spatial scale of the investigated urban drainage system, but a slight positive correlation between average CSO reduction and catchment size is evident from these data.

7.4. Challenges in comparing MPC schemes

In this section, we highlight the factors that we believe are most important when comparing different MPC schemes. Implementing MPC involves many choices, all of which may significantly affect the performance of the applied MPC method. Some of these choices relate to the details of the specific MPC method, while others relate to the evaluation of the method, including comparison with the baseline control strategy, whether the computational cost is considered or not, and which case study and rain data are used. The fact that the reviewed studies made different choices regarding these evaluation factors makes it very difficult to compare the different MPC methods; thus, the improvement percentages reported in Table 8 should be interpreted with care.

7.4.1. Evaluation model
It may be necessary to implement and test an MPC scheme on a real system in order to convince practitioners that it actually works and should be trusted in the continuation, but this is not possible when the purpose is to benchmark different control strategies against each other. Reality cannot be rerun; thus, the

performance of the simulated MPC scheme must be compared with another simulated (baseline) control strategy by using models. These control strategies should ideally be compared using models with identical underlying structures that represent the dynamics of the actual system with sufficient accuracy.

Some of the reviewed literature states that the baseline control scheme and investigated MPC method have been evaluated using different models (for example, Ocampo-Martinez et al., 2007; Ocampo-Martinez and Puig, 2010). Other publications have not described the applied baseline evaluation model (for example, Cembrano et al., 2004; Giraldo et al., 2010; Joseph-Duran et al., 2014a), and it is therefore unclear whether both control schemes are evaluated using the internal MPC model (as done by, for example, Fiorelli et al., 2013; Mollerup et al., 2016; Ocampo-Martinez et al., 2008) or whether the baseline control is evaluated using a HiFi model while the investigated scheme is evaluated using the internal MPC model. Borsanyi et al. (2008) propose a benchmark methodology framework and recommend using an RTC model validated by a HiFi model for both baseline control and investigated RTC method. However, we presume that the simplified internal MPC model is likely to overestimate its own performance, which would yield an unrealistic picture of the performance. Therefore, the most optimal comparison of control schemes is obtained when they are both evaluated using a HiFi model by running the internal MPC model and HiFi model simultaneously, although this is performed in only a few of the reviewed publications (see publications marked with an asterisk (*) in Table 8).

The reviewed literature uses passive, local, or heuristic control as a baseline. However, it is sometimes unclear if the baseline control is a simulation of the real system control and if not, whether the chosen baseline control scheme then has been through an offline optimization or is just chosen arbitrarily (for example, Duchesne et al., 2004; Rauch and Harremoës, 1998; Vezzaro and Grum, 2012). It is important that the baseline control is properly defined and described when benchmarking the MPC performance.

7.4.2. Computational cost, available time, and accuracy

In order to make a fair comparison between MPC methods, it is essential to know whether they comply with the time constraints of an operational implementation. Some of the case studies listed in Table 8 only deal with a subset of the real system, and the MPC method's scalability to larger problems should be considered when evaluating the computational cost. The computational cost is not always stated in the reviewed literature, which makes it hard to judge the applicability of the MPC method in an operational setting.

There is a trade-off between the accuracy in the description of the system dynamics in the internal MPC model and the computation time it takes to solve an optimization model that encloses these dynamics. The two extremes in the reviewed literature are the papers by Zimmer et al. (2015) and Joseph-Duran et al. (2014c). In the high end with respect to computation time,

Zimmer et al. (2015) reported computation times in the order of 0.5 to 1.5 hr for solving a 1-hr control horizon problem using genetic algorithms. In the low end, Joseph-Duran et al. (2014c) solved a mixed-integer linear program for a 40-min control horizon in, on average, less than 1 sec. Zimmer et al. (2015)'s optimization model had in total 16 optimization variables compared to almost 10,000 optimization variables used by Joseph-Duran et al. (2014c), but nevertheless the time for solving the optimization model with only 16 variables is orders of magnitude higher. Zimmer et al. (2015) used a highly accurate (though still simplified) model of the system dynamics. This model (described by Zimmer et al. (2013)) had a simulation time that was approximately 50 percent of the simulation time of the corresponding HiFi model (SWMM), and the computation time for one model realization alone was 3 to 6 sec. The MLD used by Joseph-Duran et al. (2014c) can probably not resemble the dynamics quite as accurately, and this might lead to suboptimal control signals. However, the impact of less accurate dynamics can be reduced by frequent recurrence of the MPC optimization, as also discussed in Section 6.3. On the other hand, the computation time found by Zimmer et al. (2015) was of the size of the control horizon. Although the efficiency of their optimization could probably be improved by further parallelization of the genetic algorithm implementation, it will produce a long lag time between the time of forecast and the actual implementation of the control.

7.4.3. Case study used for evaluation

It can be useful to test the MPC method on a small system with only few controllable devices in order to obtain a proof-of-concept and valuable information about the control mechanisms. However, it is necessary to implement MPC on a larger system in order to assess the true performance potential and the actual computational cost of the applied MPC scheme. Mollerup et al. (2016) encountered a situation where a lower-level control scheme performed better than MPC and this was explained by, for example, the use of spatially homogeneous rain and a limited complexity and physical extent of the case study (the internal MPC model covered a 3 × 5 km area).

The obtained MPC results also depend on the specific layout of the case study, such as the steepness of the pipe system, the available storage capacity, the location of actuators, etc. Borsanyi et al. (2008) recommended the construction of two virtual test cases with different properties and RTC potentials for benchmarking RTC strategies in a benchmark methodology framework. Furthermore, the applied case study will determine the operational goals of the control. Obviously, only control strategies with similar operational goals can be compared as supported by Borsanyi et al. (2008).

Mauricio-Iglesias et al. (2015) and Mollerup et al. (2015) investigated ways of making a more optimal pairing between the actuators (manipulated variables) and the measurements (controlled variables) in the regulatory layer by self-optimizing

control and singular value decomposition, respectively, whereas Duchesne et al. (2003) investigated how the MPC performance is affected by the location of the controllable devices in the system. Overall, they found that it is important to consider these aspects in order to obtain a robust control with a high performance.

7.4.4. Rain data used for evaluation

It is important to evaluate the performance of the applied MPC scheme using a long historical record of rainfall events, as the system will perform differently under different hydraulic loadings, even if the total rain depth of the events is the same (Vezzaro and Grum, 2012). This is also recommended in the benchmark methodology framework proposed by Borsanyi et al. (2008), where one year long historical rain series from European cities with different climatic archetypes are applied. The performance enhancement obtained by applying the MPC strategy will deteriorate for large rain events as the sewer system fills up; thus, there is no capacity left in the system to control (Duchesne et al., 2004; Gelormino and Ricker, 1994; Nelen, 1992; Rauch and Harremoës, 1996). It is also expected that MPC will reach its highest potential when there is a large spatial variability of the rain, as this will enable the MPC strategy to, for example, empty storage basins in a more efficient way than what can be done with passive or local control. Thus, the applications that assume homogenously distributed rain are likely to obtain a lower performance improvement. This will be especially evident for sewer systems with a large spatial extent. The benefits of spatially distributed rain information are discussed by Nelen (1992).

In some cases, myopic control may result from the use of rainfall input products for the MPC scheme with a shorter forecast horizon than the relevant system dynamics. Ex-post forecasts are often used in the reviewed literature, which neglects uncertainty and favors MPC compared to other control schemes that are not using rainfall forecasts. Therefore, the rainfall data used as forecasted rainfall in the evaluation should differ from the rainfall used as true rainfall for the baseline control scheme. However, we only found one study (Löwe et al., 2016) that used real forecasts for evaluating MPC.

7.5. Suggestions for future research

MPC is not a new technique within urban drainage, but there are still areas of dispute and research gaps, as García et al. (2015) also recognized. A major research gap concerns how to gain a better overview of which MPC methods are suitable for different conditions, but this avenue of study remains stalled due to the points listed in Section 7.4. In order to overcome some of the issues with the non-comparability of the MPC methods, it could be possible to select a set of benchmarking case studies that provide MPC researchers with a HiFi model, possibly predefined operational goals and rainfall time series; enabling the comparison of different MPC methods in respect to both computational costs and performance across

research groups. The results from such an approach would be case-specific and the conclusions may not necessarily be transferable to other case studies. However, it would form an initial platform that makes it possible to extract further knowledge of the possibilities and limitations of different MPC methods, including, for example, how to represent different structures of the sewer system in the internal MPC model. In addition, we found that some of the most important areas for further research regard the applied input and its uncertainty, data assimilation in both the input and internal MPC models, the operational implementation, and alternative applications for MPC in the context of smart cities. We also propose an alternative way of interpreting "MPC" and underline the importance of consistent reporting of technical MPC implementation details and evaluation results.

7.5.1. Input and its uncertainty

It is necessary to investigate the relation between the quality of rainfall forecasts and the resulting control performance in order to clarify the importance of accurate input models. Input models are a research field in themselves and current research includes the use of rainfall forecasts from both radar and NWP models. Radar nowcasting may provide lead times of 30–120 min, whereas NWPs may provide lead times of several hours and even days, representing temporal scales for a range of operational objectives; this has promise of optimization objectives that have not yet been fully explored. The current clear distinction between radar nowcasts and NWP is, however, likely to vanish in the near future as the NWP's becomes faster. This facilitates a real-time merging between radar data and NWP models (Korsholm et al., 2015), meaning that the nowcast is likely to be produced by an NWP model. Future research should aim at reducing the impact of the uncertainty from radar and NWP forecast products, for example, by updating the input model (see Section 7.5.2), and on making the spatial and temporal resolution of the rainfall forecast products fine enough to be used effectively in an urban setting.

Rainfall forecasts and input models will always be uncertain and in case the input quality is found to be important for the performance of MPC, this uncertainty information should be used in the control optimization to enhance the decision-making. The DORA algorithm (Vezzaro and Grum, 2014) has already shown that uncertainty can be efficiently included in MPC optimization. The use of input uncertainty in MPC has been studied for controlling river and reservoir systems (for example, Raso et al., 2014; van Overloop et al., 2008). However, it may not be straightforward to apply these methods in an urban drainage context because the physical systems are different; urban drainage systems generally include more thresholds and non-linearities than natural water systems.

7.5.2. Data assimilation

Only a few papers have actually updated their internal MPC models in real time with either measurements or results from a HiFi model. Joseph-Duran et al.

(2014d) and Marinaki and Papageorgiou (2001) updated the initial conditions of the internal MPC model with results from a HiFi model run in real time, while Joseph-Duran et al. (2015), Löwe et al. (2016), Pleau et al. (1996), and Vezzaro et al. (2013) assimilated measurements into the internal MPC model. It is also possible to assimilate measurements into HiFi models in real time (Borup et al., 2014), which could be used to update the initial conditions of the internal MPC model, but this has not yet been encountered in the MPC literature for urban drainage systems. Examples of autocalibration or updating of initial conditions with measurements can also be found for input models (see, for example, Breinholt et al. (2011) and Pedersen et al. (2016)), which will diminish the uncertainty induced by the rainfall data. This uncertainty might be reduced by forcing the input models up to the time of forecast with rain gauge data instead of using radar and NWP products. This does, however, result in temporal displacements in the hydrological response because data from a point is assigned to a large area. This temporal displacement counteracts data assimilation to such an extent that even uncertain, low quality radar rainfall estimates may be the better choice compared to rain gauge data once system observations are assimilated into the model (Borup et al., 2013). The choices of rainfall product and data assimilation method are thus entangled, and it is still unresolved in literature how to best solve this issue.

7.5.3. Operational implementation of MPC

Two aspects are important considering the operational implementation:

1) **Practical implementation:** It should be investigated how MPC can be robustly and efficiently implemented in real-world systems. This includes the development of procedures for spatial decomposition of the control (especially important for implementations in large systems), which should be applied in combination with time-scale dependent decomposition, as discussed in Section 7.2. In addition, the possibility of using decomposition to reduce the computational cost could be further investigated. The regulatory control layer is not taken into account in any of the reviewed publications and a poor regulatory layer will ultimately disrupt the performance of a well-functioning MPC implementation; thus, it is also important to consider this lower layer in an operational implementation. Acquiring rain data in the right temporal resolution, spatial scale and quality in real time is also an essential part of going into operation. This is not a trivial task even though prototype systems to facilitate this do exist (Hill et al., 2011). Likewise, the construction of operational input models is crucial.

2) **Operator acceptance:** As stated previously, one further impediment for the implementation of advanced control strategies is the lack of trust from operators. The control strategy obtained by the MPC may at times be contraintuitive for the operator. Combined with a lack of trust, this can result in the operator switching from automatic to manual control. Therefore, we need a stronger focus on how to make the operators feel

confident working with advanced, automated algorithms. The research may include (1) higher involvement of the local operators in the development of the MPC strategy and possibly the incorporation of their experiences in the control strategy; (2) better training of the operators; (3) development of proper control dashboards with enough information about the underlying control optimization to support the best possible decision making and/or; (4) a transition period with real-time, but off-line, MPC operation where the operators can compare the current control with MPC or the construction of pilot-scale facilities where MPC can be compared to other control strategies online. Research in this field requires not only the knowledge from MPC specialists but also the collaboration with experts from a range of other fields such as social scientists and people working with human–computer interfaces.

7.5.4. Alternative applications of MPC

Until now, MPC of urban drainage systems has mainly been about controlling water volumes and loads; however, most applications that are using a reactive control could benefit from MPC. An example of such an alternative use of MPC is Liu et al. (2016) who experimented with using MPC for dosing chemicals into the sewer system to mitigate sulfide-induced corrosion. The emerging smart cities concept will furthermore pave the way for new applications of MPC, including controlling urban drainage systems with a view to ensuring bathing-quality water in harbor areas, controlling inlets to the sewer system in case of surface flooding of the urban terrain, controlling interactions between the sewer system and stormwater control measures installed to retain stormwater locally, and controlling traffic in case of flood risk. The operational goals can be expanded from only considering damage control and economic optimization to also include amenity values, giving the control a completely new dimension.

7.5.5. Alternative interpretation of MPC

We would like to take this opportunity to suggest a reassessment on how MPC is interpreted. The vast majority of the reviewed MPC literature uses schemes that apply a dynamic optimization with an objective function to optimize the control and describes this as a vital part of MPC; thus, most people consider MPC to belong to the subgroup of optimization-based control strategies (see Table 2). However, the linguistic term MPC only refers to control that is determined based on model predictions; thus, in principle MPC could also cover heuristic or rule-based control that build on model predictions, but do not include a dynamic optimization. Although no examples of this have been found in the literature on system-wide control of urban drainage systems, there are examples of rule-based MPC without dynamic optimization in an integrated control context. For example, Sharma et al. (2013) described the potential for switching of a WWTP into wet-

Table 9. Checklist for writing MPC papers within urban drainage. Example literature is given where [1] Vezzaro and Grum (2012) is a conference article containing less information than the journal article [2] Vezzaro and Grum (2014). ✓ means that the information is included, (✓) that it is partly included, ✗ that it is missing, and n.r. that it is not relevant information in this specific paper.

		Example literature	
	Information	1	2
Receding horizon principle	Is the length of prediction horizon, control horizon, forecast horizon, sampling interval, setting duration and modeling time step stated?	✗	(✓)
	If the forecast horizon is shorter than the prediction horizon, which input data has then been used for the remaining prediction horizon?	n.r.	n.r.
	If the control horizon is shorter than the prediction horizon, which predefined control has then been used for the remaining prediction horizon?	n.r.	n.r.
Optimization model and solver	What are the operational goals and objectives, how is the deviation from the reference values quantified, and how are the objectives prioritized?	✓	✓
	What is the number of decision variables per sampling interval?	✗	✗
	What is the applied optimization solver and the computational cost?	✗	✗
Internal MPC model	Which model elements are included, how are they modeled and are they linear or non-linear?	✓	✓
	Which phenomena are represented in the model elements and how are they modeled?	✓	✓
	Has the internal MPC model been calibrated (and how)?	n.r.	n.r.
	Are the initial states updated from measurements, a HiFi model or not at all?	✗	✗
	Is historical rain, design rain, hypothetical rain, or rainfall forecast applied as input?	✓	✓
	Is the input spatially distributed?	✓	✓
	Does the conversion from rain to flow take place in a simple input model, a HiFi model, or in the internal MPC model itself, and are the inputs generated online or offline?	(✓)	✓
	If a simple input model is used, is it then validated?	✗	✓
MPC performance	Is the MPC optimized control transferred to a HiFi model for performance calculations?	✗	✗
	Is the baseline control taken from a HiFi model, is it the current or a fictive control strategy and has this strategy been optimized before comparison with MPC?	✗	✗
	What is the case study area and its physical extent and how many rain events have been applied?	✓	✓

weather mode based on runoff models driven by radar rainfall forecasts. By requiring dynamic optimization with an objective function to be a part of the MPC concept, we not only risk limiting future applications of MPC, but we also make it difficult to categorize control algorithms that are based on model predictions but do not fit into the current MPC definition. Nevertheless, we acknowledge that the interpretation of MPC is already well-founded, not only within urban drainage, but in many other areas. We, therefore, suggest that a prefix can be added to the MPC term as an indication of the exclusion of dynamic optimization. For instance, "rule-based MPC" can be introduced as a term when static rules

optimized offline are used in combination with forecast information instead of dynamic optimization.

7.5.6. Consistent reporting

We have created a checklist (Table 9) to encourage the inclusion of comprehensive information about the MPC implementation in future publications. Although this checklist is certainly not complete, it does contain the elements that we find most important and most often lacking in the literature. As examples, we have included two references and marked which information they have included and which they have left out. We would also like to stress the need to use a clear and consistent terminology throughout both the individual papers and the urban drainage field as a whole to make it easier to understand new literature and to avoid misconceptions and misunderstandings.

8. Conclusions

Research in MPC of urban drainage systems started in the early 1980s, but has increased significantly in the past 5 years, as judged by the published literature. We conclude that MPC of urban drainage systems may play an essential role in the "smart water cities" of the future, where sewage infrastructure evolves from being passive to adaptive units that can proactively respond differently depending on the given situation. However, only a few instances have been reported of MPC methods that are either ready for implementation or are already in operation. Computational costs, uncertainty of input forecasts, lack of consensus on best practice within MPC, a confusing MPC terminology at the interface between many disciplines, and the lack of institutional capacity are all factors that continue to impede implementation and further research in the field. In this paper, we propose a unifying terminology in order to establish a common ground for understanding, which we hope will be used or at least challenged by other authors. Furthermore, we propose a hierarchical categorization of MPC for urban drainage systems and emphasize four overall components that are subject to in-depth analysis in the review:

– Five distinct time windows ideally play roles in the **receding horizon principle** for urban drainage systems, where the optimization of control actions is repeatedly carried out within a finite time horizon. The prediction horizon (in which the objective function is calculated), the forecast horizon (in which reliable inputs are available), and the control horizon (in which control actions are optimized) are not distinguished clearly in the literature; thus, we refer to them jointly as the "collective horizon". In the reviewed literature, the collective horizon is generally in the order of 30–120 min. This coincides with the typical forecast ability of many radar-based rainfall forecast products, but is not long enough to represent all the relevant system dynamics. The sampling interval (1–10 min

in literature) defines how often the optimization of control actions is repeated, and the setting duration (5–120 min in literature) defines how often the control actions change values. There seems to be no consensus in the literature regarding how these time windows are best determined in practice, considering the size of the control problem and computational demands.

– The **optimization model** identifies the best control trajectory for each actuator during the upcoming control horizon and consists of an objective function (defining the operational goals of the optimization), a set of optimization variables (control actions for all actuators in the system), and a set of constraints (boundaries for the optimization variables and future system outputs). Much of the literature optimizes the control actions by using an objective function based on volumes and/or flows. CSO minimization is most often targeted but other objectives are also used.

– The **optimization solver** is the piece of software that obtains the optimal control trajectory, and the choice of optimization solver depends highly on the convexity of the optimization problem. Optimization solvers are only discussed briefly as we acknowledge that it is too comprehensive to treat thoroughly in this review; however, we encourage others to undertake this task.

– The **internal MPC model** should capture the dynamics of the relevant parts of the urban drainage system. All reviewed papers use simplified models either in a non-linear or convex optimization, due to the computational burden involved when applying HiFi models; however, there is little consensus about how to conceptualize reality or a detailed HiFi model into an internal MPC model that is computationally feasible but still sufficiently accurate. Convex optimization often makes use of linear discrete-time state-space models and represents overflows either in the internal MPC model by using slack variables or as a specific overflow term in the objective function. Flow over weirs are often modeled using mixed logical systems in non-linear optimization. Some studies have omitted important details about the internal MPC model, along with considerations of realistic input and input models representing the part of urban drainage system not affected by changes in the control. Many studies have used ex-post rainfall forecasts in combination with a HiFi model to construct inflow time series, even though this is not feasible in an operational setup. The degree to which the updating of initial conditions of the internal MPC model improves the MPC performance is not reported in the literature, even though this updating is normally an essential part of MPC.

MPC of an urban drainage systems is a complex matter and many mutually interdependent choices are required. Below, we highlight four key considerations based on our review:

- The formulation of the optimization model is a key choice from the beginning and involves a trade-off between the number of optimization variables and the degree of detail in the internal MPC model. Convex programs that use linear internal models and constraint functions can be solved explicitly and quickly, but there is no consensus yet on how to formulate important non-linear phenomena such as overflow thresholds in a manner that represents reality reasonably well. non-linear optimization allows the inclusion of overflows, flooding, etc. in the internal MPC model, but it is computationally more demanding and perhaps, in some cases, infeasible for real-time applications.

- There is a need to decompose an overall control system into different layers based on the time scale of the process dynamics and control setup. Here we propose distinguishing between four layers, each of which can be designed to take care of different aspects of the control. Spatial decomposition determining how local and global control can support each other also appears necessary, but is not dealt with in the literature, apart from the acknowledgement of local PID controllers.

- MPC has mostly been reported for European and North American cities, covering small (a few square kilometers) to very large (hundreds of square kilometers) catchments and having from 10 or fewer to almost 50 actuators. Studies generally show a reduction of CSO volumes, flooding, and CSO cost while maximizing the volume sent to the treatment plant. Performance evaluations are mostly based on very few or even synthetic events and only few studies have evaluated the performance based on long-term simulations covering a year or more. This study shows that MPC for CSO mitigation may work better for larger catchments and that it is efficient for small and medium-sized events where storage is available for optimization, whereas large events are virtually unaffected by MPC.

- It is difficult to compare different MPC methods as these are documented using different evaluation schemes, input, case studies, etc., all of which affect the obtained model performance. MPC performance evaluations should ideally be based on simulations with a detailed HiFi model that fully represents the relevant system features and dynamics and can be used to simulate both the considered control strategy and a baseline strategy used for comparison. However, these details are often left out of the reviewed literature; thus, it is not always clear whether the evaluation model is the same as the internal MPC model, which would likely overestimate the performance of MPC. Performance evaluations should also use rainfall input data that realistically represent intensity variations and spatial variability for a range of hydraulic loadings.

Finally, we list important areas for further research related to MPC of urban drainage systems:

- A better overview of which MPC methods are suitable for different conditions is needed. Selecting a set of benchmarking may help overcome some of the issues with non-comparability of MPC methods.
- Including uncertainty in the optimization of the control actions can lead to a more robust control and is an almost untouched area within urban drainage.
- There is a distinct lack in the MPC literature on urban drainage systems of methods for assimilating on-line observations to initialize internal MPC models or input models.
- The final steps towards implementing MPC in real-time operational systems, such as clarifying which input models should be used, how MPC should interact with other control schemes in both temporal and spatial control hierarchies, and how to overcome operator reluctance, are missing, which shows that MPC is still far from being a mature technology that is ready for standard implementation.
- The smart cities concept may lead to a range of new operational goals where MPC can be instrumental in changing urban drainage systems from passive conventional infrastructure systems to proactive adaptive systems based on a high level of automation. These include securing bathing water quality, controlling the interaction between sewers and surface flooding, controlling the interaction with local stormwater control measures, and controlling traffic in case of flood risk.
- We provide a checklist to encourage the inclusion of comprehensive information about the MPC implementation and performance evaluation in future publications.

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