$\mathscr{H}_\infty$ model reduction for continuous-time Markovian jump systems with incomplete statistics of mode information

Yanling Wei[a], Jianbin Qiu[a,*], Hamid Reza Karimi[b] and Mao Wang[a]

[a]The Research Institute of Intelligent Control and Systems, Harbin Institute of Technology, Harbin 150001, P. R. China;
[b]The Department of Engineering, Faculty of Engineering and Science, University of Agder, N-4898 Grimstad, Norway

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This paper investigates the problem of $\mathscr{H}_\infty$ model reduction for a class of continuous-time Markovian jump linear systems with incomplete statistics of mode information, which simultaneously considers the exactly known, partially unknown and uncertain transition rates. By fully utilising the properties of transition rate matrices, together with the convexification of uncertain domains, a new sufficient condition for $\mathscr{H}_\infty$ performance analysis is first derived, and then two approaches, namely, the convex linearisation approach and the iterative approach, are developed to solve the model reduction problem. It is shown that the desired reduced-order models can be obtained by solving a set of strict linear matrix inequalities (LMIs) or a sequential minimisation problem subject to LMI constraints, which are numerically efficient with commercially available software. Finally, an illustrative example is given to show the effectiveness of the proposed design methods.

Keywords: Markovian jump systems; model reduction; incomplete statistics of mode information; linear matrix inequality

1. Introduction

In recent decades, extensive research has been focused on Markovian jump linear systems (MJLSs) in both academic and industrial communities. The inspiration of these studies is that MJLSs can model different types of dynamic systems subject to random abrupt changes in their structures, e.g. fault-prone manufacturing systems, power systems, economics systems and networked control systems (NCSs) (Wang, Lam, and Liu 2004a; Qiu, Feng, and Yang 2009; Qiu, Feng, and Gao 2010; Shen, Wang, and Hung 2010; Wang, Liu, and Liu 2010; Niu, Ho, and Li 2011; Shen, Wang, Hung, and Chesi 2011) and so on. In essence, MJLSs are a special class of hybrid systems with finite-state operation modes, and the system mode switching is governed by a Markov process (Wang, Lam, and Liu 2004b). It is known that the transition rates (TRs) in the Markov process determine the behaviour and performance of systems, and a number of results on the analysis and synthesis of MJLSs have been obtained under the assumption that the mode transition information is perfectly known (Shi, Boukas, and Agarwal 1999; Xu, Chen, and Lam 2003; Shi, Xia, Liu, and Rees 2006; Niu, Ho, and Wang 2007; Fei, Gao, and Shi 2009; Wu, Shi, and Su 2012).

However, it is noted that not all the mode transition information can be easily measured in many practical systems. Even if the mode TRs could be measured, there exist measurement errors (also referred as TR uncertainties) inevitably. Therefore, in recent years there have been some results on the analysis and design of MJLSs with incomplete statistics of mode information (Boukas 2005; Xiong, Lam, Gao, and Ho 2005; Karan, Shi, and Kaya 2006; Xiong and Lam 2009; Zhang and Lam 2010; Zhang, He, Wu, and Zhang 2011; Wei, Qiu, Karimi, and Wang 2013a). To mention a few, in the context of continuous-time case, Boukas (2005) concerned the stability analysis problem for MJLSs with polytopic-type uncertain TRs; Xiong et al. (2005) considered the robust stabilisation problem for MJLSs with norm-bounded uncertain TRs; Zhang and Lam (2010) addressed the stability analysis and synthesis problems for MJLSs with partially unknown TRs. In parallel, the corresponding results for the discrete-time case can be found in Boukas (2009), Liu, Wang, and Wang (2011), Zhao, Zhang, Shen, and Gao (2011) and Wei, Wang, and Qiu (2013b).

On the other hand, mathematical modelling of complex physical systems often results in high-order models, which brings serious difficulties to the analysis and synthesis of the concerning systems (Birouche, Mourllion, and Basset 2012; Tahavori and Shaker 2013). Therefore, in practical applications it is desirable to approximate these high-order models by some simple lower-order models according to some given criteria. During the past decades, a number of approaches for model reduction have been developed, such as the Hankel norm approximation method (Zhou 1995; Gao, Lam, Wang, and Wang 2004), the $\mathscr{H}_2$ approach (Yan and Lam 1999), the $\mathscr{H}_\infty$ approach (Zhang, Huang, and Lam 2003; Gao, Lam, and Wang 2006; Wu and Zheng 2009)

*Corresponding author. Email: jbqiu@hit.edu.cn

and the $\mathscr{L}_2$-$\mathscr{L}_\infty$ approach (Lam, Gao, Xu, and Wang 2005). More recently, the linear matrix inequality (LMI) technique has been applied to deal with the model reduction problem for different classes of systems (Zhang et al. 2003; Lam et al. 2005; Gao et al. 2006; Wu and Zheng 2009). Specifically, Zhang et al. (2003) addressed the $\mathscr{H}_\infty$ model reduction problem for MJLSs with completely known TRs. However, to the authors' best knowledge, few results have been reported on the $\mathscr{H}_\infty$ model reduction for continuous-time MJLSs with incomplete statistics of mode information, which simultaneously considers the exactly known, partially unknown and uncertain TRs. This motivates us for the present study.

According to the issues mentioned above, in this paper, we will investigate the problem of $\mathscr{H}_\infty$ model reduction for a class of continuous-time MJLSs with incomplete statistics of mode information. The incomplete statistics of mode in-formation in the Markov stochastic process simultaneously takes into account the exactly known, partially unknown and uncertain TRs, which is a more practical scenario. By fully utilising the properties of the TRMs, together with the con-vexification of uncertain domains, a new $\mathscr{H}_\infty$ performance analysis criterion for MJLSs with incomplete statistics of mode information will be first derived. To solve the model reduction problem, two sharply different approaches will then be presented. The first approach is based on a lin-earisation procedure, which casts the model reduction into a convex optimisation problem. The second one, which is based on a decoupling technique and cone complemen-tarity linearisation (CCL) method (El Ghaoui, Oustry, and AitRami 1997; Qiu, Feng, and Gao 2011), casts the model reduction into a sequential minimisation problem subject to linear matrix inequality (LMI) constraints. An illustrative example will be provided to demonstrate the effectiveness of the proposed approaches.

Notations. The notations used throughout the pa-per are standard. $\mathbf{R}^n$ and $\mathbf{R}^{m\times n}$ denote, respectively, the n-dimensional Euclidean space, and the set of all $m \times n$ real matrices; $\mathbb{N}^+$ represents the set of positive integers; the notation $P > 0$ (≥ 0) means that P is real symmetric positive (semi-positive) definite; $\mathrm{Sym}\{A\}$ is the shorthand notation for $A + A^T$; $\mathbf{I}$ and $\mathbf{0}$ represent the identity matrix and a zero matrix, respectively; $(\Omega,\ \mathcal{F},\ \mathcal{P})$ denotes a com-plete probability space, in which Ω is the sample space, $\mathcal{F}$ is the σ algebra of subsets of the sample space, and $\mathcal{P}$ is the probability measure on $\mathcal{F}$; $\mathcal{E}[\cdot]$ stands for the mathematical expectation; $\|\cdot\|$ denotes the Euclidean norm of a vector or its induced norm of a matrix; signals that are square inte-grable over $[0, \infty)$ are denoted by $L_2[0, \infty)$ with the norm $\|\cdot\|_2$. Matrices, if their dimensions are not explicitly stated, are assumed to be compatible for algebraic operations.

2. Problem formulation and preliminaries

Consider the following class of Markovian jump linear systems (MJLSs) in a fixed complete probability space $(\Omega, \mathcal{F}, \mathcal{P})$,

$$(\Sigma): \quad \begin{aligned} \dot{x}(t) &= A(r(t))x(t) + B(r(t))u(t) \\ z(t) &= C(r(t))x(t) + D(r(t))u(t), \end{aligned} \tag{1}$$

where $x(t) \in \mathbf{R}^{n_x}$ is the state vector; $u(t) \in \mathbf{R}^{n_u}$ is the con-trol input vector which belongs to $L_2[0, \infty)$; $z(t) \in \mathbf{R}^{n_z}$ is the output vector. In (1), the process $\{r(t), t \geq 0\}$ is a continuous-time homogeneous Markov chain with right continuous trajectories and takes values in a finite set $\mathcal{I} := \{1, 2, \ldots, N\}$ with a transition rate matrix (TRM) $\Lambda := [\lambda_{ij}]_{N\times N}$ given by

$$\Pr\{r(t+h) = j \,|r(t) = i\} = \begin{cases} \lambda_{ij}h + o(h), & i \neq j \\ 1 + \lambda_{ii}h + o(h), & i = j, \end{cases}$$

where $h > 0$, $\lim_{h\to 0}(o(h)/h) = 0$, and $\lambda_{ij} \geq 0$, for $j \neq i$, is the transition rate (TR) from mode i at time t to mode j at time $t + h$, and $\lambda_{ii} = -\sum_{j=1, j\neq i}^{N} \lambda_{ij}$. In the sequel, for each possible $r(t) = i$, $i \in \mathcal{I}$, the system matrices of the ith mode are known and denoted by (A_i, B_i, C_i, D_i), which are real matrices with appropriate dimensions.

The TRs of the stochastic process in this paper are considered to be *uncertain and partially available*, i.e., the TRM $\Lambda = [\lambda_{ij}]_{N\times N}$ is assumed to belong to a given polytope P_Λ with vertices Λ_s, $s = 1, 2, \ldots, M$, $\mathrm{P}_\Lambda := \left\{\Lambda \middle| \Lambda = \sum_{s=1}^{M} \alpha_s \Lambda_s; \alpha_s \geq 0, \sum_{s=1}^{M} \alpha_s = 1\right\}$, where $\Lambda_s = [\lambda_{ij}]_{N\times N}$, $i, j \in \mathcal{I}$, are given TRMs containing unknown el-ements still. For instance, a system (Σ) with four operation modes, the TRM may be as,

$$\begin{bmatrix} \lambda_{11} & \tilde{\lambda}_{12} & \hat{\lambda}_{13} & \lambda_{14} \\ \hat{\lambda}_{21} & \hat{\lambda}_{22} & \lambda_{23} & \lambda_{24} \\ \tilde{\lambda}_{31} & \hat{\lambda}_{32} & \tilde{\lambda}_{33} & \hat{\lambda}_{34} \\ \lambda_{41} & \hat{\lambda}_{42} & \hat{\lambda}_{43} & \hat{\lambda}_{44} \end{bmatrix},$$

where the elements labelled with '$\hat{\ }$' and '$\tilde{\ }$' represent the unknown information and polytopic uncertainties on TRs, respectively, and the others are known TRs. For nota-tional clarity, $\forall i \in \mathcal{I}$, we describe $\mathcal{I} = \mathcal{I}_{\mathcal{K}}^{(i)} \cup \mathcal{I}_{\mathcal{UC}}^{(i)} \cup \mathcal{I}_{\mathcal{UK}}^{(i)}$ as follows,

$$\begin{aligned} \mathcal{I}_{\mathcal{K}}^{(i)} &= \{j : \lambda_{ij} \text{ is known}\}, \\ \mathcal{I}_{\mathcal{UC}}^{(i)} &= \{j : \tilde{\lambda}_{ij} \text{ is uncertain}\}, \\ \mathcal{I}_{\mathcal{UK}}^{(i)} &= \{j : \hat{\lambda}_{ij} \text{ is unknown }\}. \end{aligned} \tag{2}$$

Also, throughout this paper, we denote $\lambda_{\mathcal{K}}^{(i)} := \sum_{j\in\mathcal{I}_{\mathcal{K}}^{(i)}} \lambda_{ij}$ and $\lambda_{\mathcal{UC}}^{(is)} := \sum_{j\in\mathcal{I}_{\mathcal{UC}}^{(i)}} \tilde{\lambda}_{ij}^{(s)}$, where $\tilde{\lambda}_{ij}^{(s)}$ represents an uncertain TR in the sth polytope, $\forall s = 1, 2, \ldots, M$. For tractability reasons, we further restrict the unknown diagonal element $\hat{\lambda}_{ii}$ as $\lambda_{\mathcal{B}}^{(i)} \leq \hat{\lambda}_{ii}$ (Zhang and Lam 2010), where $\lambda_{\mathcal{B}}^{(i)}$ provides a lower bound for the unknown element $\hat{\lambda}_{ii}$, and satisfies $\lambda_{\mathcal{B}}^{(i)} \leq -\left(\lambda_{\mathcal{K}}^{(i)} + \lambda_{\mathcal{UC}}^{(is)}\right)$.

To approximate the original MJLS in (1), in this paper, we are interested in designing the following mode-dependent reduced-order model,

$$\begin{aligned}\dot{\hat{x}}(t) &= A_{ri}\hat{x}(t) + B_{ri}u(t)\\ \hat{z}(t) &= C_{ri}\hat{x}(t) + D_{ri}u(t),\end{aligned} \tag{3}$$

where $\hat{x}(t) \in \mathbf{R}^{n_r}(n_r < n_x)$, $\hat{z}(t) \in \mathbf{R}^{n_z}$, and $A_{ri} \in \mathbf{R}^{n_r \times n_r}$, $B_{ri} \in \mathbf{R}^{n_r \times n_u}$, $C_{ri} \in \mathbf{R}^{n_z \times n_r}$ and $D_{ri} \in \mathbf{R}^{n_z \times n_u}$ are the gains of the reduced-order models to be determined.

Define $\bar{x}(t) := \left[x^T(t)\ \hat{x}^T(t) \right]^T$, and $\bar{z}(t) := z(t) - \hat{z}(t)$. Then, by augmenting (1) and (3) the model error dynamics can be described as,

$$\begin{aligned}(\bar{\Sigma}):\ \dot{\bar{x}}(t) &= \bar{A}_i\bar{x}(t) + \bar{B}_iu(t)\\ \bar{z}(t) &= \bar{C}_i\bar{x}(t) + \bar{D}_iu(t),\end{aligned} \tag{4}$$

where

$$\bar{A}_i = \begin{bmatrix} A_i & \mathbf{0} \\ \mathbf{0} & A_{ri} \end{bmatrix},\ \bar{B}_i := \begin{bmatrix} B_i \\ B_{ri} \end{bmatrix},$$
$$\bar{C}_i = \begin{bmatrix} C_i & -C_{ri} \end{bmatrix},\ \bar{D}_i := D_i - D_{ri}. \tag{5}$$

Therefore, the purpose of this paper is to design a mode-dependent reduced-order model in the form of (3), such that the model error system $(\bar{\Sigma})$ in (4) with incomplete statistics of mode information is stochastically stable and the induced L_2-norm of the operator from $u(t)$ to the model error $\bar{z}(t)$ is less than γ, i.e., $\|\bar{z}(t)\|^2_{\mathcal{E}_2} := \mathcal{E}\left\{ \int_0^\infty \bar{z}^T(t)\bar{z}(t)\mathrm{d}t \right\} \le \gamma^2 \|u(t)\|_2^2 := \gamma^2 \int_0^\infty u^T(t)u(t)\mathrm{d}t$, under zero initial conditions for any non-zero $u(t) \in L_2[0, \infty)$.

Before ending the section, we give the following lemma on the $\mathscr{H}_\infty$ performance analysis of system (4) with *completely known* TRs, which will be used in the proof of our main results.

Lemma 2.1 (Zhang et al. 2003)**:** *For the MJLS* (4) *with completely known TRs and a given scalar* $\gamma > 0$*, if the following coupled inequalities*

$$\begin{bmatrix} \bar{A}_i^T P_i + P_i\bar{A}_i + \mathscr{P}_i + \bar{C}_i^T\bar{C}_i & P_i\bar{B}_i + \bar{C}_i^T\bar{D}_i \\ * & -(\gamma^2\mathbf{I} - \bar{D}_i^T\bar{D}_i) \end{bmatrix} < 0, \quad \forall i \in \mathcal{I}, \tag{6}$$

where $\mathscr{P}_i := \sum_{j=1}^N \lambda_{ij}P_j$*, have a feasible solution* $P = \{P_1, P_2, \ldots, P_N\}$ *with* $P_i > 0$*, then the MJLS* (4) *with completely known TRs is stochastically stable with an* $\mathscr{H}_\infty$ *performance* γ*.*

3. Main results

In this section, we will first derive the $\mathscr{H}_\infty$ performance analysis criterion for the model error system $(\bar{\Sigma})$ in (4) with incomplete statistics of mode information. Then, two sharply different approaches will be developed to solve the $\mathscr{H}_\infty$ model reduction problem formulated in the above section.

3.1. $\mathscr{H}_\infty$ performance analysis

In this subsection, by fully exploiting the properties of the transition rate matrix (TRM), together with the convexification of uncertain domains, a new $\mathscr{H}_\infty$ performance analysis criterion for the model error system $(\bar{\Sigma})$ in (4) with incomplete statistics of mode information is presented, which will play a key role in solving the $\mathscr{H}_\infty$ model reduction problem.

Proposition 3.1: *The model error system in* (4) *with incomplete statistics of mode information is stochastically stable with an* $\mathscr{H}_\infty$ *performance* γ *if there exist matrices* $P_i > 0$*, for each mode* $i \in \mathcal{I}$*, such that the following matrix inequalities hold,*

$$\Upsilon_{ij}^{(s)} := \begin{bmatrix} -\mathbf{I} & \bar{C}_i & \bar{D}_i \\ * & \bar{A}_i^T P_i + P_i\bar{A}_i + \bar{\mathscr{P}}_{ij}^{(s)} & P_i\bar{B}_i \\ * & * & -\gamma^2\mathbf{I} \end{bmatrix} < 0,$$
$$j \in \mathcal{I}_{\mathcal{UK}}^{(i)},\ s = 1, 2, \ldots, M, \tag{7}$$

where

$$\begin{aligned}\bar{\mathscr{P}}_{ij}^{(s)} &:= \begin{cases} \mathscr{P}_{\mathcal{K}}^{(i)} + \mathscr{P}_{\mathcal{UC}}^{(is)} - \left(\lambda_{\mathcal{K}}^{(i)} + \lambda_{\mathcal{UC}}^{(is)}\right)P_j,\ j \in \mathcal{I}_{\mathcal{UK}}^{(i)}, & \text{if } i \in \mathcal{I}_{\mathcal{K}}^{(i)} \cup \mathcal{I}_{\mathcal{UC}}^{(i)}, \\ \mathscr{P}_{\mathcal{K}}^{(i)} + \mathscr{P}_{\mathcal{UC}}^{(is)} + \lambda_{\mathcal{B}}^{(i)}P_i - \left(\lambda_{\mathcal{B}}^{(i)} + \lambda_{\mathcal{K}}^{(i)} + \lambda_{\mathcal{UC}}^{(is)}\right)P_j,\ j \in \mathcal{I}_{\mathcal{UK}}^{(i)}, & \text{if } i \in \mathcal{I}_{\mathcal{UK}}^{(i)}, \end{cases}\\ \mathscr{P}_{\mathcal{K}}^{(i)} &:= \sum\nolimits_{j\in\mathcal{I}_{\mathcal{K}}^{(i)}} \lambda_{ij}P_j,\ \mathscr{P}_{\mathcal{UC}}^{(is)} := \sum\nolimits_{j\in\mathcal{I}_{\mathcal{UC}}^{(i)}} \tilde{\lambda}_{ij}^{(s)}P_j,\\ \lambda_{\mathcal{K}}^{(i)} &:= \sum\nolimits_{j\in\mathcal{I}_{\mathcal{K}}^{(i)}} \lambda_{ij},\ \lambda_{\mathcal{UC}}^{(is)} := \sum\nolimits_{j\in\mathcal{I}_{\mathcal{UC}}^{(i)}} \tilde{\lambda}_{ij}^{(s)}.\end{aligned} \tag{8}$$

Proof: Based on Lemma 2.1, it is known that system (4) subject to completely known transition rates (TRs) is stochastically stable with an $\mathscr{H}_\infty$ performance γ if (6) holds. Since the diagonal elements in the TRM may be unknown, we shall divide the proof of Proposition 3.1 into two cases, that is, $i \in \mathcal{I}_{\mathcal{K}}^{(i)} \cup \mathcal{I}_{\mathcal{UC}}^{(i)}$ and $i \in \mathcal{I}_{\mathcal{UK}}^{(i)}$, respectively.

Case (i): $i \in \mathcal{I}_{\mathcal{K}}^{(i)} \cup \mathcal{I}_{\mathcal{UC}}^{(i)}$.

In this case, $i \in \mathcal{I}_{\mathcal{K}}^{(i)} \cup \mathcal{I}_{\mathcal{UC}}^{(i)}$ implies that λ_{ii} is known or uncertain, then it is straightforward that $\lambda_{\mathcal{K}}^{(i)} + \lambda_{\mathcal{UC}}^{(is)} \le 0$.

First, we consider the case that $\lambda_{\mathcal{K}}^{(i)} + \lambda_{\mathcal{UC}}^{(is)} < 0$.

Noticing that with incomplete statistics of mode information, the term $\sum_{j=1}^{N} \lambda_{ij} P_j$ in (6) can be treated as,

$$\begin{aligned}\sum_{j=1}^{N} \lambda_{ij} P_j &= \sum_{j\in\mathcal{I}_{\mathcal{K}}^{(i)}} \lambda_{ij} P_j + \sum_{j\in\mathcal{I}_{\mathcal{UK}}^{(i)}} \hat{\lambda}_{ij} P_j \\ &\quad + \sum_{j\in\mathcal{I}_{\mathcal{UC}}^{(i)}} \left(\sum_{s=1}^{M} \alpha_s \tilde{\lambda}_{ij}^{(s)}\right) P_j \\ &= \mathscr{P}_{\mathcal{K}}^{(i)} + \left(-\lambda_{\mathcal{K}}^{(i)} - \lambda_{\mathcal{UC}}^{(is)}\right) \sum_{j\in\mathcal{I}_{\mathcal{UK}}^{(i)}} \frac{\hat{\lambda}_{ij}}{-\lambda_{\mathcal{K}}^{(i)} - \lambda_{\mathcal{UC}}^{(is)}} P_j \\ &\quad + \sum_{s=1}^{M} \alpha_s \mathscr{P}_{\mathcal{UC}}^{(is)}, \qquad (9)\end{aligned}$$

where

$$\begin{aligned}&\mathscr{P}_{\mathcal{K}}^{(i)} := \sum\nolimits_{j\in\mathcal{I}_{\mathcal{K}}^{(i)}} \lambda_{ij} P_j,\ \mathscr{P}_{\mathcal{UC}}^{(is)} := \sum\nolimits_{j\in\mathcal{I}_{\mathcal{UC}}^{(i)}} \tilde{\lambda}_{ij}^{(s)} P_j, \\ &\lambda_{\mathcal{K}}^{(i)} := \sum\nolimits_{j\in\mathcal{I}_{\mathcal{K}}^{(i)}} \lambda_{ij},\ \lambda_{\mathcal{UC}}^{(is)} := \sum\nolimits_{j\in\mathcal{I}_{\mathcal{UC}}^{(i)}} \tilde{\lambda}_{ij}^{(s)}, \qquad (10)\end{aligned}$$

and the elements $\hat{\lambda}_{ij}$, $j \in \mathcal{I}_{\mathcal{UK}}^{(i)}$, are unknown; and $\sum_{s=1}^{M} \alpha_s \tilde{\lambda}_{ij}^{(s)}$, $\forall j \in \mathcal{I}_{\mathcal{UC}}^{(i)}$ represents the uncertain elements in the polytopic uncertainty description.

Since $0 \le \alpha_s \le 1$, $\sum_{s=1}^{M} \alpha_s = 1$, and $0 \le \frac{\hat{\lambda}_{ij}}{-\lambda_{\mathcal{K}}^{(i)} - \lambda_{\mathcal{UC}}^{(is)}} \le 1$, $\sum_{j\in\mathcal{I}_{\mathcal{UK}}^{(i)}} \left(\frac{\hat{\lambda}_{ij}}{-\lambda_{\mathcal{K}}^{(i)} - \lambda_{\mathcal{UC}}^{(is)}}\right) = 1$, (9) can be rewritten as,

$$\begin{aligned}\sum_{j=1}^{N} \lambda_{ij} P_j &= \sum_{s=1}^{M} \alpha_s \sum_{j\in\mathcal{I}_{\mathcal{UK}}^{(i)}} \frac{\hat{\lambda}_{ij}}{-\lambda_{\mathcal{K}}^{(i)} - \lambda_{\mathcal{UC}}^{(is)}} \\ &\quad \times \left(\mathscr{P}_{\mathcal{K}}^{(i)} + \mathscr{P}_{\mathcal{UC}}^{(is)} - \left(\lambda_{\mathcal{K}}^{(i)} + \lambda_{\mathcal{UC}}^{(is)}\right) P_j\right). \qquad (11)\end{aligned}$$

Thus, for $0 \le \alpha_s \le 1$ and $0 \le \hat{\lambda}_{ij} \le -\left(\lambda_{\mathcal{K}}^{(i)} + \lambda_{\mathcal{UC}}^{(is)}\right)$, the left-hand side (LHS) of inequality (6) can be rewritten as,

$$\begin{aligned}\text{LHS(6)} = \sum_{s=1}^{M} \alpha_s \sum_{j\in\mathcal{I}_{\mathcal{UK}}^{(i)}} \frac{\hat{\lambda}_{ij}}{-\lambda_{\mathcal{K}}^{(i)} - \lambda_{\mathcal{UC}}^{(is)}} \Upsilon_{ij}^{(s)},\ j \in \mathcal{I}_{\mathcal{UK}}^{(i)}, \\ s = 1, 2, \ldots, M, \qquad (12)\end{aligned}$$

where

$$\begin{aligned}\Upsilon_{ij}^{(s)} &:= \begin{bmatrix} -\mathbf{I} & \bar{C}_i & \bar{D}_i \\ * & \bar{A}_i^T P_i + P_i \bar{A}_i + \bar{\mathscr{P}}_{ij}^{(s)} & P_i \bar{B}_i \\ * & * & -\gamma^2 \mathbf{I} \end{bmatrix}, \\ \bar{\mathscr{P}}_{ij}^{(s)} &:= \mathscr{P}_{\mathcal{K}}^{(i)} + \mathscr{P}_{\mathcal{UC}}^{(is)} - \left(\lambda_{\mathcal{K}}^{(i)} + \lambda_{\mathcal{UC}}^{(is)}\right) P_j. \qquad (13)\end{aligned}$$

Then, (6) holds if and only if $\Upsilon_{ij}^{(s)} < 0$ in (12).

Second, we consider the case that $\lambda_{\mathcal{K}}^{(i)} + \lambda_{\mathcal{UC}}^{(is)} = 0$.

In fact, if $\lambda_{\mathcal{K}}^{(i)} + \lambda_{\mathcal{UC}}^{(is)} = 0$, then all the elements in the ith row of the vertices Λ_s, $s = 1, 2, \ldots, M$ are completely known. For this case, the second term of the right-hand side of (9) is not involved and (7) can be obtained by following a similar line as above with $\bar{\mathscr{P}}_{ij}^{(s)} := \mathscr{P}_{\mathcal{K}}^{(i)} + \mathscr{P}_{\mathcal{UC}}^{(is)}$.

Based on the above discussions, it is known that in the presence of unknown elements $\hat{\lambda}_{ij}$, $j \neq i$, inequality (6) is equivalent to (7).

Case (ii): $i \in \mathcal{I}_{\mathcal{UK}}^{(i)}$.

We first consider the case that $\hat{\lambda}_{ii} < -\left(\lambda_{\mathcal{K}}^{(i)} + \lambda_{\mathcal{UC}}^{(is)}\right)$.

Equivalently, for this case the term $\sum_{j=1}^{N} \lambda_{ij} P_j$ in (6) can be expressed as,

$$\begin{aligned}\sum_{j=1}^{N} \lambda_{ij} P_j &= \sum_{j\in\mathcal{I}_{\mathcal{K}}^{(i)}} \lambda_{ij} P_j + \hat{\lambda}_{ii} P_i + \sum_{j\in\mathcal{I}_{\mathcal{UK}}^{(i)}, j\neq i} \hat{\lambda}_{ij} P_j \\ &\quad + \sum_{j\in\mathcal{I}_{\mathcal{UC}}^{(i)}} \left(\sum_{s=1}^{M} \alpha_s \tilde{\lambda}_{ij}^{(s)}\right) P_j \\ &= \mathscr{P}_{\mathcal{K}}^{(i)} + \hat{\lambda}_{ii} P_i - \left(\hat{\lambda}_{ii} + \lambda_{\mathcal{K}}^{(i)} + \lambda_{\mathcal{UC}}^{(is)}\right) \\ &\quad \times \sum_{j\in\mathcal{I}_{\mathcal{UK}}^{(i)}, j\neq i} \frac{\hat{\lambda}_{ij}}{-\hat{\lambda}_{ii} - \lambda_{\mathcal{K}}^{(i)} - \lambda_{\mathcal{UC}}^{(is)}} P_j \\ &\quad + \sum_{s=1}^{M} \alpha_s \mathscr{P}_{\mathcal{UC}}^{(is)}, \qquad (14)\end{aligned}$$

where $\mathscr{P}_{\mathcal{K}}^{(i)} := \sum_{j\in\mathcal{I}_{\mathcal{K}}^{(i)}} \lambda_{ij} P_j$, and $\mathscr{P}_{\mathcal{UC}}^{(is)} := \sum_{j\in\mathcal{I}_{\mathcal{UC}}^{(i)}} \tilde{\lambda}_{ij}^{(s)} P_j$.

Similarly, it follows from $0 \le \alpha_s \le 1$, $\sum_{s=1}^{M} \alpha_s = 1$, and $0 \le \frac{\hat{\lambda}_{ij}}{-\hat{\lambda}_{ii} - \lambda_{\mathcal{K}}^{(i)} - \lambda_{\mathcal{UC}}^{(is)}} \le 1$, $\sum_{j\in\mathcal{I}_{\mathcal{UK}}^{(i)}, j\neq i} \frac{\hat{\lambda}_{ij}}{-\hat{\lambda}_{ii} - \lambda_{\mathcal{K}}^{(i)} - \lambda_{\mathcal{UC}}^{(is)}} = 1$ that,

$$\begin{aligned}\sum_{j=1}^{N} \lambda_{ij} P_j &= \sum_{s=1}^{M} \alpha_s \sum_{j\in\mathcal{I}_{\mathcal{UK}}^{(i)}, j\neq i} \frac{\hat{\lambda}_{ij}}{-\hat{\lambda}_{ii} - \lambda_{\mathcal{K}}^{(i)} - \lambda_{\mathcal{UC}}^{(is)}} \\ &\quad \times \left[\mathscr{P}_{\mathcal{K}}^{(i)} + \mathscr{P}_{\mathcal{UC}}^{(is)} + \hat{\lambda}_{ii} P_i - \left(\hat{\lambda}_{ii} + \lambda_{\mathcal{K}}^{(i)} + \lambda_{\mathcal{UC}}^{(is)}\right) P_j\right]. \qquad (15)\end{aligned}$$

Correspondingly, for this case we can rewrite the left-hand side of the inequality (6) as

$$\begin{aligned}\text{LHS(6)} = \sum_{s=1}^{M} \alpha_s \sum_{j\in\mathcal{I}_{\mathcal{UK}}^{(i)}, j\neq i} \frac{\hat{\lambda}_{ij}}{-\hat{\lambda}_{ii} - \lambda_{\mathcal{K}}^{(i)} - \lambda_{\mathcal{UC}}^{(is)}} \Upsilon_{ij}^{(s)}, \\ j \in \mathcal{I}_{\mathcal{UK}}^{(i)},\ j \neq i,\ s = 1, 2, \ldots, M, \qquad (16)\end{aligned}$$

where

$$\Upsilon_{ij}^{(s)} := \begin{bmatrix} -\mathbf{I} & \bar{C}_i & \bar{D}_i \\ * & \bar{A}_i^T P_i + P_i \bar{A}_i + \bar{\mathscr{P}}_{ij}^{(s)} & P_i \bar{B}_i \\ * & * & -\gamma^2 \mathbf{I} \end{bmatrix},$$

$$\bar{\mathscr{P}}_{ij}^{(s)} := \mathscr{P}_{\mathcal{K}}^{(i)} + \mathscr{P}_{\mathcal{UC}}^{(is)} + \hat{\lambda}_{ii} P_i - \left(\hat{\lambda}_{ii} + \lambda_{\mathcal{K}}^{(i)} + \lambda_{\mathcal{UC}}^{(is)}\right) P_j. \tag{17}$$

It follows from (16) that (6) is equivalent to,

$$\Upsilon_{ij}^{(s)} < 0, \; j \in \mathcal{I}_{\mathcal{UK}}^{(i)}, \; j \neq i. \tag{18}$$

For numerical tractability, by introducing a lower bound $\lambda_{\mathcal{B}}^{(i)}$ for the unknown element $\hat{\lambda}_{ii}$, we have,

$$\lambda_{\mathcal{B}}^{(i)} \leq \hat{\lambda}_{ii} < -\lambda_{\mathcal{K}}^{(i)} - \lambda_{\mathcal{UC}}^{(is)}, \tag{19}$$

which implies that $\hat{\lambda}_{ii}$ may take any value in $\left[\lambda_{\mathcal{B}}^{(i)}, -\lambda_{\mathcal{K}}^{(i)} - \lambda_{\mathcal{UC}}^{(is)} + \epsilon\right]$ for some sufficiently small $\epsilon < 0$. Then $\hat{\lambda}_{ii}$ can be further written as a convex combination as follows,

$$\hat{\lambda}_{ii} = -\kappa \lambda_{\mathcal{K}}^{(i)} - \kappa \lambda_{\mathcal{UC}}^{(is)} + \kappa \epsilon + (1-\kappa) \lambda_{\mathcal{B}}^{(i)}, \tag{20}$$

where $0 \leq \kappa \leq 1$. Since $\hat{\lambda}_{ii}$ in (20) depends on κ linearly, and therefore (18) only needs to be satisfied for $\kappa = 0$ and $\kappa = 1$, that is, (18) holds if and only if the following inequalities in (21)–(22) simultaneously hold,

$$\Upsilon_{ij}^{(s)} < 0, \; j \in \mathcal{I}_{\mathcal{UK}}^{(i)}, \; j \neq i, \tag{21}$$

where $\Upsilon_{ij}^{(s)}$ is defined in (17) with $\bar{\mathscr{P}}_{ij}^{(s)} = \mathscr{P}_{\mathcal{K}}^{(i)} + \mathscr{P}_{\mathcal{UC}}^{(is)} - \left(\lambda_{\mathcal{K}}^{(i)} + \lambda_{\mathcal{UC}}^{(is)}\right) P_i + \epsilon (P_i - P_j)$, and

$$\Upsilon_{ij}^{(s)} < 0, \; j \in \mathcal{I}_{\mathcal{UK}}^{(i)}, \; j \neq i, \tag{22}$$

where $\Upsilon_{ij}^{(s)}$ is defined in (17) with $\bar{\mathscr{P}}_{ij}^{(s)} = \mathscr{P}_{\mathcal{K}}^{(i)} + \mathscr{P}_{\mathcal{UC}}^{(is)} - \left(\lambda_{\mathcal{K}}^{(i)} + \lambda_{\mathcal{UC}}^{(is)}\right) P_j + \lambda_{\mathcal{B}}^{(i)} (P_i - P_j)$.

Since ϵ is small enough, (21) holds if and only if

$$\Upsilon_{ij}^{(s)} < 0, \; j \in \mathcal{I}_{\mathcal{UK}}^{(i)}, \; j \neq i, \tag{23}$$

where $\Upsilon_{ij}^{(s)}$ is defined in (17) with $\bar{\mathscr{P}}_{ij}^{(s)} = \mathscr{P}_{\mathcal{K}}^{(i)} + \mathscr{P}_{\mathcal{UC}}^{(is)} - \left(\lambda_{\mathcal{K}}^{(i)} + \lambda_{\mathcal{UC}}^{(is)}\right) P_i$, which is implied by (22) when $j = i, \; j \in \mathcal{I}_{\mathcal{UK}}^{(i)}$. Hence, the inequality (6) can be replaced by (7) in the context $\forall j \in \mathcal{I}_{\mathcal{UK}}^{(i)}$.

On the other hand, the case $\hat{\lambda}_{ii} = -\left(\lambda_{\mathcal{K}}^{(i)} + \lambda_{\mathcal{UC}}^{(is)}\right)$ means that all the elements in the ith row of the vertices Λ_s, $s = 1, 2, \ldots, M$ are completely known. For this case, the third term of the right-hand side of (14) is not involved and by following a similar line as above, (7) can be obtained with $\bar{\mathscr{P}}_{ij}^{(s)} := \mathscr{P}_{\mathcal{K}}^{(i)} + \mathscr{P}_{\mathcal{UC}}^{(is)} - (\lambda_{\mathcal{K}}^{(i)} + \lambda_{\mathcal{UC}}^{(is)}) P_i$.

In summary, with the presence of unknown and uncertain elements in the TRM, one can readily conclude that the error system (4) is stochastically stable with an $\mathscr{H}_\infty$ performance γ if (7) holds. The proof is completed. □

Remark 1: By fully exploiting the properties of the TRMs, together with the convexification of uncertain domains, a novel $\mathscr{H}_\infty$ performance analysis criterion is presented for the MJLS (4) with incomplete statistics of mode information in Proposition 3.1. The incomplete statistics of mode information simultaneously considers the exactly known, partially unknown and polytopic-type uncertain TRs, and thus is more general for practical scenarios. It is worth mentioning that the condition in (7) is no loss of generality, since the lower bound $\lambda_{\mathcal{B}}^{(i)}$, of $\hat{\lambda}_{ii}$ is allowed to be arbitrarily small. It is also noted that there exist product terms between the Lyapunov matrices and system matrices in the condition (7), which bring some difficulties in the solutions of model reduction problem. By applying some decoupling techniques, in the following, two sharply different approaches to solve the $\mathscr{H}_\infty$ model reduction problem will be proposed.

In the sequel, based on the new $\mathscr{H}_\infty$ performance analysis criterion presented in Proposition 3.1, we will give the model reduction synthesis results in the presence of incomplete statistics of mode information.

3.2. *Model reduction via convex linearisation approach*

In this subsection, by a linearisation procedure, the model reduction problem described in the above section will be cast into a convex optimisation problem. The following theorem presents a sufficient condition for the existence of admissible reduced-order models.

Theorem 3.2: *Consider MJLS* (1) *with incomplete statistics of mode information and reduced-order model* (3). *The model error system* (4) *is stochastically stable with a guaranteed $\mathscr{H}_\infty$ performance γ, if there exist positive-definite symmetric matrices $P_i = \begin{bmatrix} P_{i(1)} & E P_{i(2)} \\ * & P_{i(2)} \end{bmatrix} \in \mathbf{R}^{(n_x+n_r)\times(n_x+n_r)}$, $E := \begin{bmatrix} \mathbf{I}_{n_r} & \mathbf{0}_{n_r \times (n_x - n_r)} \end{bmatrix}^T$, and matrices $\bar{A}_{ri} \in \mathbf{R}^{n_r \times n_r}$, $\bar{B}_{ri} \in \mathbf{R}^{n_r \times n_u}$, $C_{ri} \in \mathbf{R}^{n_z \times n_r}$, and $D_{ri} \in \mathbf{R}^{n_z \times n_u}$, for each mode $i \in \mathcal{I}$, such that the following LMIs hold,*

$$\begin{bmatrix} -\mathbf{I} & C_i & -C_{ri} & D_i - D_{ri} \\ * & \Psi_{1ij}^{(s)} & \Psi_{2ij}^{(s)} & P_{i(1)} B_i \\ * & * & \Psi_{3ij}^{(s)} & P_{i(2)} E^T B_i \\ * & * & * & -\gamma^2 \mathbf{I} \end{bmatrix} < 0,$$

$$j \in \mathcal{I}_{\mathcal{UK}}^{(i)}, \; s = 1, 2, \ldots, M, \tag{24}$$

where

$$\left.\begin{array}{l}\Psi_{1ij}^{(s)} := \Gamma_1 + \mathscr{P}_{\mathcal{K}}^{(1i)} + \mathscr{P}_{\mathcal{UC}}^{(1is)} - \big(\lambda_{\mathcal{K}}^{(i)} + \lambda_{\mathcal{UC}}^{(is)}\big)P_{j(1)},\ j \in \mathcal{I}_{\mathcal{UK}}^{(i)} \\ \Psi_{2ij}^{(s)} := \Gamma_2 + E\big(\mathscr{P}_{\mathcal{K}}^{(2i)} + \mathscr{P}_{\mathcal{UC}}^{(2is)}\big) - \big(\lambda_{\mathcal{K}}^{(i)} + \lambda_{\mathcal{UC}}^{(is)}\big)EP_{j(2)},\ j \in \mathcal{I}_{\mathcal{UK}}^{(i)} \\ \Psi_{3ij}^{(s)} := \mathtt{Sym}\{\bar{A}_{ri}\} + \mathscr{P}_{\mathcal{K}}^{(2i)} + \mathscr{P}_{\mathcal{UC}}^{(2is)} - \big(\lambda_{\mathcal{K}}^{(i)} + \lambda_{\mathcal{UC}}^{(is)}\big)P_{j(2)},\ j \in \mathcal{I}_{\mathcal{UK}}^{(i)}\end{array}\right\} \text{if } i \in \mathcal{I}_{\mathcal{K}}^{(i)} \cup \mathcal{I}_{\mathcal{UC}}^{(i)},$$

$$\left.\begin{array}{l}\Psi_{1ij}^{(s)} := \Gamma_1 + \mathscr{P}_{\mathcal{K}}^{(1i)} + \mathscr{P}_{\mathcal{UC}}^{(1is)} + \lambda_{\mathcal{B}}^{(i)}P_{i(1)} - \big(\lambda_{\mathcal{B}}^{(i)} + \lambda_{\mathcal{K}}^{(i)} + \lambda_{\mathcal{UC}}^{(is)}\big)P_{j(1)},\ j \in \mathcal{I}_{\mathcal{UK}}^{(i)} \\ \Psi_{2ij}^{(s)} := \Gamma_2 + E\big(\mathscr{P}_{\mathcal{K}}^{(2i)} + \mathscr{P}_{\mathcal{UC}}^{(2is)}\big) + \lambda_{\mathcal{B}}^{(i)}EP_{i(2)} - \big(\lambda_{\mathcal{B}}^{(i)} + \lambda_{\mathcal{K}}^{(i)} + \lambda_{\mathcal{UC}}^{(is)}\big)EP_{j(2)},\ j \in \mathcal{I}_{\mathcal{UK}}^{(i)} \\ \Psi_{3ij}^{(s)} := \mathtt{Sym}\{\bar{A}_{ri}\} + \mathscr{P}_{\mathcal{K}}^{(2i)} + \mathscr{P}_{\mathcal{UC}}^{(2is)} + \lambda_{\mathcal{B}}^{(i)}P_{i(2)} - \big(\lambda_{\mathcal{B}}^{(i)} + \lambda_{\mathcal{K}}^{(i)} + \lambda_{\mathcal{UC}}^{(is)}\big)P_{j(2)},\ j \in \mathcal{I}_{\mathcal{UK}}^{(i)}\end{array}\right\} \text{if } i \in \mathcal{I}_{\mathcal{UK}}^{(i)},$$

$$\begin{aligned}&\Gamma_1 := \mathtt{Sym}\{P_{i(1)}A_i\},\\ &\Gamma_2 := E\bar{A}_{ri} + A_i^T EP_{i(2)},\ E := \big[\mathbf{I}_{n_r}\ \ \mathbf{0}_{n_r\times(n_x-n_r)}\big]^T,\\ &\mathscr{P}_{\mathcal{K}}^{(1i)} := \sum\nolimits_{j\in\mathcal{I}_{\mathcal{K}}^{(i)}} \lambda_{ij}P_{j(1)},\ \mathscr{P}_{\mathcal{K}}^{(2i)} := \sum\nolimits_{j\in\mathcal{I}_{\mathcal{K}}^{(i)}} \lambda_{ij}P_{j(2)},\\ &\mathscr{P}_{\mathcal{UC}}^{(1is)} := \sum\nolimits_{j\in\mathcal{I}_{\mathcal{UC}}^{(i)}} \tilde{\lambda}_{ij}^{(s)}P_{j(1)},\ \mathscr{P}_{\mathcal{UC}}^{(2is)} := \sum\nolimits_{j\in\mathcal{I}_{\mathcal{UC}}^{(i)}} \tilde{\lambda}_{ij}^{(s)}P_{j(2)}.\end{aligned} \tag{25}$$

Moreover, if the above conditions have a set of feasible solutions $(P_{i(1)}, P_{i(2)}, \bar{A}_{ri}, \bar{B}_{ri}, C_{ri}, D_{ri})$*, then an admissible* n_r*-order approximation model in the form of (3) can be obtained as,*

$$A_{ri} = P_{i(2)}^{-1}\bar{A}_{ri},\ B_{ri} = P_{i(2)}^{-1}\bar{B}_{ri},\ C_{ri} = C_{ri},\ D_{ri} = D_{ri}. \tag{26}$$

Proof: By Proposition 3.1, the model error system (4) is stochastically stable with an $\mathscr{H}_\infty$ performance γ, if for each mode $i \in \mathcal{I}$, there exist positive-definite symmetric matrices P_i such that (7) holds. Now, for simplicity in model reduction synthesis procedure, we first specify the Lyapunov matrices P_i in (7) as,

$$P_i = \begin{bmatrix} P_{i(1)} & EP_{(2)} \\ * & P_{(3)} \end{bmatrix}, \tag{27}$$

where $E := \big[\mathbf{I}_{n_r}\ \ \mathbf{0}_{n_r\times(n_x-n_r)}\big]^T$, $P_{i(1)} \in \mathbf{R}^{n_x\times n_x}$, $P_{(2)} \in \mathbf{R}^{n_r\times n_r}$ and $P_{(3)} \in \mathbf{R}^{n_r\times n_r}$. Then, similar to Gao et al. (2006) and Wei et al. (2013a), performing a congruent transformation to P_i by $\mathrm{diag}\{\mathbf{I}_{n_x}, P_{(2)}P_{(3)}^{-1}\}$, yields,

$$\begin{bmatrix} P_{i(1)} & EP_{(2)}P_{(3)}^{-1}P_{(2)}^T \\ * & P_{(2)}P_{(3)}^{-1}P_{(2)}^T \end{bmatrix} := \begin{bmatrix} P_{i(1)} & E\underline{\underline{\bar{P}_{(2)}}} \\ * & \underline{\underline{\bar{P}_{(2)}}} \end{bmatrix}. \tag{28}$$

Thus, *without loss of generality*, we can directly specify the Lyapunov matrices as,

$$P_i = \begin{bmatrix} P_{i(1)} & E\underline{\underline{P_{i(2)}}} \\ * & \underline{\underline{P_{i(2)}}} \end{bmatrix}. \tag{29}$$

It is noted that in this way the matrix variables $P_{i(2)}$ *are set as Markovian and can be absorbed directly by the gain variables* A_{ri} *and* B_{ri} *by introducing*

$$\bar{A}_{ri} = P_{i(2)}A_{ri},\ \bar{B}_{ri} = P_{i(2)}B_{ri},\ i \in \mathcal{I}. \tag{30}$$

Now, substituting the Lyapunov matrix P_i given in (29) into (7), together with consideration of the matrices defined in (30), leads to (24) exactly. In other words, (24) is a sufficient condition for (7) with the Lyapunov matrices P_i shown in (29).

On the other hand, $P_i > 0$ implies that $P_{i(2)}$ is nonsingular. Thus, the reduced-order model can be constructed by (26). This completes the proof. □

Remark 2: Theorem 3.2 provides a sufficient condition for the solvability of $\mathscr{H}_\infty$ model reduction synthesis problem for the MJLS (1) with incomplete statistics of mode information. A desired reduced-order model can be determined by solving the following convex optimisation problem,

Problem MRLA (model reduction via linearisation approach).

Minimise γ *subject to* (24) *for* $P_{i(1)}$, $P_{i(2)}$, $\bar{A}_{ri}$, $\bar{B}_{ri}$, C_{ri}, D_{ri}, $i \in \mathcal{I}$.

Remark 3: It is noted that in order to obtain the strict LMIs-based conditions in Theorem 3.2, a relaxation matrix E is imposed in the Lyapunov matrices P_i, $i \in \mathcal{I}$. This structural constraint inevitably brings some degree of design conservatism. To reduce the design conservatism, in the following subsection, we will resort to an iterative approach to solve the model reduction problem.

3.3. Model reduction via iterative approach

As mentioned in the previous subsection, the design conservatism of Theorem 3.2 is mainly induced by a structural constraint on the Lyapunov matrices. Therefore, for reduced-order model synthesis purpose and to lessen the design conservatism, it shall be useful to eliminate the products of the system matrices with Lyapunov matrices. *Nevertheless, it is worth pointing out that due to the incomplete statistics of transition information, the indices* $i \in \mathcal{I}$,

$j \in \mathcal{I}_{UK}^{(i)}$ and $s = 1, 2, \ldots, M$ are simultaneously involved in (7) in Proposition 3.1. Thus, the celebrated elimination lemma (or called projection lemma) (Zhang et al. 2003; Wu and Zheng 2009) cannot be utilised to eliminate the coupling between the system matrices and Lyapunov matrices in (7). In other words, for MJLSs in (1) with incomplete statistics of mode information, the reduced-order models (3) cannot be obtained by the projection approach as proposed in Gao et al. (2004), Zhang et al. (2003) and Wu and Zheng (2009). To this end, inspired by Shaked (2001) and Qiu, Feng, and Gao (2012), in this subsection another decoupling technique will be used to separate the Lyapunov matrices from the system matrices. The result is summarised in the following theorem.

Theorem 3.3: *Consider MJLS in* (1) *with incomplete statistics of mode information and reduced-order model in the form of* (3)*. Given a scalar $\delta > 0$, the model error system in* (4) *is stochastically stable with an $\mathscr{H}_\infty$ performance γ, if there exist positive-definite symmetric matrices $\{P_i, X_i\} \in \mathbf{R}^{(n_x+n_r)\times(n_x+n_r)}$, and matrices $A_{ri} \in \mathbf{R}^{n_r \times n_r}$, $B_{ri} \in \mathbf{R}^{n_r\times n_u}$, $C_{ri} \in \mathbf{R}^{n_z\times n_r}$, and $D_{ri} \in \mathbf{R}^{n_z\times n_u}$, such that*

$$\begin{bmatrix} -\delta X_i & \mathbf{0} & \mathbf{I}+\delta\bar{A}_i & \delta\bar{B}_i \\ * & -\mathbf{I} & \bar{C}_i & \bar{D}_i \\ * & * & -\delta^{-1}P_i + \bar{\mathscr{P}}_{ij}^{(s)} & \mathbf{0} \\ * & * & * & -\gamma^2\mathbf{I} \end{bmatrix} < 0,$$
$$j \in \mathcal{I}_{UK}^{(i)},\ s = 1, 2, \ldots, M, \qquad (31)$$

$$P_i X_i = \mathbf{I},\ i \in \mathcal{I}, \qquad (32)$$

where

$$\bar{A}_i := \begin{bmatrix} A_i & \mathbf{0} \\ \mathbf{0} & A_{ri} \end{bmatrix},\ \bar{B}_i := \begin{bmatrix} B_i \\ B_{ri} \end{bmatrix},\ \bar{C}_i := \begin{bmatrix} C_i & -C_{ri} \end{bmatrix},\ \bar{D}_i := D_i - D_{ri},$$
$$\bar{\mathscr{P}}_{ij}^{(s)} := \begin{cases} \mathscr{P}_{\mathcal{K}}^{(i)} + \mathscr{P}_{UC}^{(is)} - \left(\lambda_{\mathcal{K}}^{(i)} + \lambda_{UC}^{(is)}\right)P_j,\ j \in \mathcal{I}_{UK}^{(i)}, & \text{if } i \in \mathcal{I}_{\mathcal{K}}^{(i)} \cup \mathcal{I}_{UC}^{(i)}, \\ \mathscr{P}_{\mathcal{K}}^{(i)} + \mathscr{P}_{UC}^{(is)} + \lambda_{\mathcal{B}}^{(i)}P_i - \left(\lambda_{\mathcal{B}}^{(i)} + \lambda_{\mathcal{K}}^{(i)} + \lambda_{UC}^{(is)}\right)P_j,\ j \in \mathcal{I}_{UK}^{(i)}, & \text{if } i \in \mathcal{I}_{UK}^{(i)}, \end{cases} \qquad (33)$$

with $\mathscr{P}_{\mathcal{K}}^{(i)}$, $\mathscr{P}_{UC}^{(is)}$, $\lambda_{\mathcal{K}}^{(i)}$ and $\lambda_{UC}^{(is)}$ defined in (8).

Proof: Similarly, it follows from Proposition 3.1 that if we can show (7), then the claim result follows. To this end, rewrite the inequality in (7) as,

$$\begin{bmatrix} -\mathbf{I} & \bar{C}_i & \bar{D}_i \\ \bar{C}_i^T & \bar{\mathscr{P}}_{ij}^{(s)} & \mathbf{0} \\ \bar{D}_i^T & \mathbf{0} & -\gamma^2\mathbf{I} \end{bmatrix} + \mathtt{Sym}\left\{ \begin{bmatrix} \mathbf{0} \\ P_i \\ \mathbf{0} \end{bmatrix} \begin{bmatrix} \mathbf{0} & \bar{A}_i & \bar{B}_i \end{bmatrix} \right\} < 0,$$
$$j \in \mathcal{I}_{UK}^{(i)},\ s = 1, 2, \ldots, M. \qquad (34)$$

Note that

$$\begin{bmatrix} \mathbf{0} \\ P_i \\ \mathbf{0} \end{bmatrix} \begin{bmatrix} \mathbf{0} & \bar{A}_i & \bar{B}_i \end{bmatrix} = \begin{bmatrix} \mathbf{I} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & P_i & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{I} \end{bmatrix} \begin{bmatrix} \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \bar{A}_i & \bar{B}_i \\ \mathbf{0} & \mathbf{0} & \mathbf{0} \end{bmatrix},\ \begin{bmatrix} \mathbf{I} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & P_i & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{I} \end{bmatrix} > 0. \qquad (35)$$

Then, it is easy to see that there exists a sufficiently small positive scalar δ such that the following inequality implies (34),

$$\delta \begin{bmatrix} \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \bar{A}_i & \bar{B}_i \\ \mathbf{0} & \mathbf{0} & \mathbf{0} \end{bmatrix}^T \begin{bmatrix} \mathbf{I} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & P_i & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{I} \end{bmatrix} \begin{bmatrix} \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \bar{A}_i & \bar{B}_i \\ \mathbf{0} & \mathbf{0} & \mathbf{0} \end{bmatrix}$$
$$+\,\mathrm{LHS}(34) < 0,\ i \in \mathcal{I},\ j \in \mathcal{I}_{UK}^{(i)},\ s = 1, 2, \ldots, M. \qquad (36)$$

It is noted that for any matrices P, A and a positive scalar δ, we have the following equality,

$$PA + A^TP + \delta A^TPA = (\mathbf{I} + \delta A)^T \times (\delta^{-1}P)(\mathbf{I} + \delta A) - \delta^{-1}P. \qquad (37)$$

Then, rewriting the inequality (36) based on (37) and by Schur complement twice, we have

$$\begin{bmatrix} -\delta P_i^{-1} & \mathbf{0} & \mathbf{I}+\delta\bar{A}_i & \delta\bar{B}_i \\ * & -\mathbf{I} & \bar{C}_i & \bar{D}_i \\ * & * & -\delta^{-1}P_i + \bar{\mathscr{P}}_{ij}^{(s)} & \mathbf{0} \\ * & * & * & -\gamma^2\mathbf{I} \end{bmatrix} < 0,$$
$$j \in \mathcal{I}_{UK}^{(i)},\ s = 1, 2, \ldots, M. \qquad (38)$$

Setting $X_i := P_i^{-1}$, it is easy to see that (38) is equivalent to (31) and (32). This completes the proof. □

Remark 4: Theorem 3.3 provides another sufficient condition for testing the solvability of $\mathscr{H}_\infty$ model reduction synthesis for MJLS (1) with incomplete statistics of mode information. It is noted that the condition in Theorem 3.3 is not strict LMIs-based due to the matrix equality in (32). However, with the cone complementarity linearisation (CCL) technique (El Ghaoui et al. 1997; Qiu et al. 2011), we can resolve this nonconvex feasibility problem by formulating it into a sequential optimisation problem subject to LMI constraints. The basic idea of CCL algorithm is that if the LMI $\left[\begin{smallmatrix} P & \mathbf{I} \\ \mathbf{I} & X \end{smallmatrix}\right] \geq 0$ is feasible with the $n \times n$ matrix variables $P > 0$ and $X > 0$, then $\mathrm{Trace}(PX) \geq n$, and $\mathrm{Trace}(PX) = n$ if and only if $PX = \mathbf{I}$.

Based on the above discussions and using a cone complementarity technique, the nonconvex feasibility problem given in (31) and (32) is converted into the following nonlinear minimisation problem that involves LMI conditions.

Problem MRIA (model reduction via iterative approach).

Minimise Trace $\left(\sum_{i=1}^{N} P_i X_i\right)$ subject to (31) and

$$\begin{bmatrix} P_i & \mathbf{I} \\ * & X_i \end{bmatrix} \geq 0, \ \forall i \in \mathcal{I}. \tag{39}$$

Then, the suboptimal performance of γ can be found by the following algorithm. The convergence of this algorithm is guaranteed in terms of similar results in El Ghaoui et al. (1997) and Qiu et al. (2011).

Algorithm MRIA: Suboptimal performance of γ

Step 1. Choose a sufficiently large initial $\gamma > 0$ and a small positive scalar δ, such that there exists a feasible solution to (31) and (39). Set $\gamma_0 = \gamma$.

Step 2. Find a feasible set $\left(P_i^{(0)}, X_i^{(0)}, A_{ri}^{(0)}, B_{ri}^{(0)}, C_{ri}^{(0)}, D_{ri}^{(0)}, \forall i \in \mathcal{I}\right)$ that satisfies the conditions in (31) and (39). Set $q = 0$.

Step 3. Solving the following LMI problem over the variables P_i, X_i, A_{ri}, B_{ri}, C_{ri} and D_{ri},

$$\text{minimise trace}\left(\sum_{i=1}^{N}\left(P_i^{(q)} X_i + P_i X_i^{(q)}\right)\right)$$
$$\text{subject to (31) and (39).} \tag{40}$$

Set $P_i^{(q+1)} = P_i$ and $X_i^{(q+1)} = X_i$.

Step 4. Substituting the gains A_{ri}, B_{ri}, C_{ri} and D_{ri} obtained in Step 3 into (7) and if the LMIs in (7) are feasible with respect to the variables P_i, $i \in \mathcal{I}$, then set $\gamma_0 = \gamma$ and return to Step 2 after decreasing γ to some extent. If (7) are infeasible within the maximum number of iterations allowed, then exist. Otherwise, set $q = q + 1$, and go to Step 3.

Remark 5: It is noted that the Algorithm MRIA involves a tuning parameter δ. When the scalar δ is given, the problem MRIA can be easily solved by the CCL technique. The issue that one then faces is how to find a scalar such that (31) and (39) have feasible solutions. A simple way to address the tuning issue is by the trial-and-error method. Another possible way is to utilise some numerical optimisation-search algorithms, such as the program *fminsearch* in the optimisation toolbox of MATLAB, genetic algorithm, etc. It has been demonstrated that these optimisation-search procedures are indeed efficient for the LMIs-based parameter-tuning problems (Shaked 2001; Qiu et al. 2012).

Remark 6: It is noted that the conditions in Theorem 3.2 are convex, and thus can be readily solved with commercially available software. The design conservatism of Theorem 3.2 mainly comes from the structural constraint of Lyapunov matrices P_i in (29). In the iterative approach, the conditions given in (31) and (32) are equivalent to the corresponding performance analysis results given in Proposition 3.1. This is the main advantage of Theorem 3.3 over Theorem 3.2. However, the computation cost involved in Algorithm MRIA is also much larger than that involved in Theorem 3.2 (MRLA), especially when the number of iterations increases.

4. An illustrative example

In this section, we will present an example to demonstrate the effectiveness of the proposed approaches.

Consider a continuous-time Markovian jump linear system (MJLS) in (1) with four modes, and the system parameters are given as follows,

$$\left[\begin{array}{c|c} A_1 & B_1 \\ \hline C_1 & D_1 \end{array}\right] = \left[\begin{array}{cccc|c} -2 & 3 & -1 & 1 & -2.5 \\ 0 & -1 & 1 & 0 & 1.3 \\ 0 & 0 & -3 & 12 & 1.6 \\ 0 & 0 & 0 & -4 & -3.4 \\ \hline 1.2 & 0.5 & 1.7 & 1.2 & 0.5 \end{array}\right],$$

$$\left[\begin{array}{c|c} A_2 & B_2 \\ \hline C_2 & D_2 \end{array}\right] = \left[\begin{array}{cccc|c} -1 & 2 & -1 & 1 & -1.5 \\ 0 & -3 & 1 & 0 & 1.2 \\ 0 & 0 & -3 & 4 & 1.6 \\ 0 & 0 & 0 & -5 & -2 \\ \hline 1 & 0.5 & 1.2 & 1 & 0.5 \end{array}\right],$$

$$\left[\begin{array}{c|c} A_3 & B_3 \\ \hline C_3 & D_3 \end{array}\right] = \left[\begin{array}{cccc|c} -3 & 1 & -1 & 1 & -1 \\ 0 & -2 & 1 & 0 & 0.8 \\ 0 & 0 & -5 & 2 & 1.3 \\ 0 & 0 & 0 & -3 & -1.4 \\ \hline 0.8 & 0.4 & 1.2 & 1.2 & 0.5 \end{array}\right],$$

$$\left[\begin{array}{c|c} A_4 & B_4 \\ \hline C_4 & D_4 \end{array}\right] = \left[\begin{array}{cccc|c} -2.5 & 1 & -1 & 1 & -1.6 \\ 0 & -3 & 1 & 0 & 1.3 \\ 0 & 0 & -4 & 6 & 1 \\ 0 & 0 & 0 & -6 & -2.4 \\ \hline 1.3 & 0.5 & 1.5 & 0.8 & 0.5 \end{array}\right].$$

Four different cases for the transition rate matrix (TRM) are given in Table 1, where the elements labelled with '$\hat{\ }$' and '$\tilde{\ }$' represent the unknown and uncertain transition rates (TRs), respectively. *Specifically, the Case 1, Case 2, Case 3 and Case 4 stand for the completely known TRs, incomplete statistics of mode information (including known, partially unknown and uncertain TRs), partially unknown TRs, and completely unknown TRs, respectively.*

For Case 2 shown in Table 1, it is assumed that the uncertain TRs comprise two vertices Λ_s, $s = 1, 2$, where the vertices for the second row $\Lambda_{s(2)}$, $s = 1, 2$, are given by

$$\Lambda_{1(2)} = \left[\hat{\lambda}_{21} \ -2 \ 1.2 \ \hat{\lambda}_{24}\right],$$
$$\Lambda_{2(2)} = \left[\hat{\lambda}_{21} \ -1 \ 0.3 \ \hat{\lambda}_{24}\right],$$

Table 1. Four different TRMs.

Case 1: Completely known TRM	Case 2: Incomplete TRM1
$\begin{bmatrix} -1.3 & 0.2 & 0.8 & 0.3 \\ 0.3 & -1.3 & 0.5 & 0.5 \\ 0.1 & 0.9 & -2.5 & 1.5 \\ 0.4 & 0.2 & 0.6 & -1.2 \end{bmatrix}$	$\begin{bmatrix} -1.3 & 0.2 & \hat{\lambda}_{13} & \hat{\lambda}_{14} \\ \hat{\lambda}_{21} & \tilde{\lambda}_{22} & \hat{\lambda}_{23} & \hat{\lambda}_{24} \\ 0.1 & \hat{\lambda}_{32} & -2.5 & \hat{\lambda}_{34} \\ \hat{\lambda}_{41} & 0.2 & 0.6 & \hat{\lambda}_{44} \end{bmatrix}$
Case 3: Incomplete TRM2	**Case 4: Completely unknown TRM**
$\begin{bmatrix} -1.3 & 0.2 & \hat{\lambda}_{13} & \hat{\lambda}_{14} \\ \hat{\lambda}_{21} & \hat{\lambda}_{22} & \hat{\lambda}_{23} & \hat{\lambda}_{24} \\ 0.1 & \hat{\lambda}_{32} & -2.5 & \hat{\lambda}_{34} \\ \hat{\lambda}_{41} & 0.2 & 0.6 & \hat{\lambda}_{44} \end{bmatrix}$	$\begin{bmatrix} \hat{\lambda}_{11} & \hat{\lambda}_{12} & \hat{\lambda}_{13} & \hat{\lambda}_{14} \\ \hat{\lambda}_{21} & \hat{\lambda}_{22} & \hat{\lambda}_{23} & \hat{\lambda}_{24} \\ \hat{\lambda}_{31} & \hat{\lambda}_{32} & \hat{\lambda}_{33} & \hat{\lambda}_{34} \\ \hat{\lambda}_{41} & \hat{\lambda}_{42} & \hat{\lambda}_{43} & \hat{\lambda}_{44} \end{bmatrix}$

and the other rows in the two vertices are defined with the same elements, that is,

$$\begin{aligned} \Lambda_{s(1)} &= \begin{bmatrix} -1.3 & 0.2 & \hat{\lambda}_{13} & \hat{\lambda}_{14} \end{bmatrix}, \\ \Lambda_{s(3)} &= \begin{bmatrix} 0.1 & \hat{\lambda}_{32} & -2.5 & \hat{\lambda}_{34} \end{bmatrix}, \\ \Lambda_{s(4)} &= \begin{bmatrix} \hat{\lambda}_{41} & 0.2 & 0.6 & \hat{\lambda}_{44} \end{bmatrix}, \ s = 1, 2. \end{aligned}$$

Furthermore, we restrict the unknown diagonal element $\hat{\lambda}_{44}$ with a lower bound $\lambda_{\mathcal{B}}^{(4)} = -3$ in Case 2; set $\lambda_{\mathcal{B}}^{(2)} = -4$ and $\lambda_{\mathcal{B}}^{(4)} = -3$ in Case 3; and also assign $\lambda_{\mathcal{B}}^{(1)} = -2$, $\lambda_{\mathcal{B}}^{(2)} = -4$, $\lambda_{\mathcal{B}}^{(3)} = -3$ and $\lambda_{\mathcal{B}}^{(4)} = -3$ a priori for Case 4, respectively.

The objective is to design a reduced-order model of the form (3) to approximate the above system such that the model error system (4) is stochastically stable with an $\mathscr{H}_\infty$ performance index γ. By solving the problems MRLA and MRIA with $\delta = 0.1$ and the maximum number of itera-tions allowed as 100, a detailed comparison between the minimum $\mathscr{H}_\infty$ performance indices $\gamma_{\min}$ obtained based on Theorems 3.2 and 3.3 is summarised in Table 2. By inspection of Table 2, it is easy to see that the results based on Theorem 3.3 (MRIA) are much less conservative than those based on Theorem 3.2 (MRLA). It is also shown from Tables 1 and 2 that the more the information on TRs is available, the better $\mathscr{H}_\infty$ performance can be obtained, which is effective to reduce the design conservatism. Therefore, the introduction of the uncertain TRs is meaningful.

Specifically, for $n_r = 3$, we obtain $\gamma_{\min} = 1.4164$ by Theorem 3.2 with incomplete TRM1 shown in Table 1, and the three-order model parameters are given by,

$$\left[\begin{array}{c|c} A_{r1} & B_{r1} \\ \hline C_{r1} & D_{r1} \end{array}\right] = \left[\begin{array}{ccc|c} -1.7242 & 2.4467 & -0.7379 & 2.6112 \\ 0.0017 & -1.0005 & 0.9994 & -1.3006 \\ 3.5045 & -6.6357 & 0.1266 & -0.2661 \\ \hline -1.5486 & 0.1635 & -2.0128 & 0.3667 \end{array}\right],$$

$$\left[\begin{array}{c|c} A_{r2} & B_{r2} \\ \hline C_{r2} & D_{r2} \end{array}\right] = \left[\begin{array}{ccc|c} -1.2273 & 1.3075 & -0.6212 & 2.0471 \\ 0.1000 & -3.1191 & 0.9180 & -1.4547 \\ 0.6898 & -2.2215 & -2.2220 & -2.2403 \\ \hline -1.0315 & 0.0796 & -1.4729 & 0.3911 \end{array}\right],$$

$$\left[\begin{array}{c|c} A_{r3} & B_{r3} \\ \hline C_{r3} & D_{r3} \end{array}\right] = \left[\begin{array}{ccc|c} -2.9838 & 0.4674 & -0.7342 & 1.4153 \\ 0.1116 & -2.2021 & 0.9262 & -0.8209 \\ 0.0825 & -0.7079 & -4.3823 & -1.2055 \\ \hline -0.8386 & 0.0698 & -1.5773 & 0.2912 \end{array}\right],$$

$$\left[\begin{array}{c|c} A_{r4} & B_{r4} \\ \hline C_{r4} & D_{r4} \end{array}\right] = \left[\begin{array}{ccc|c} -2.7213 & 0.5360 & -0.6519 & 1.9938 \\ 0.2009 & -3.1982 & 0.8568 & -1.8801 \\ 0.1847 & -3.5380 & -2.6115 & -1.4419 \\ \hline -1.2610 & -0.0789 & -1.7249 & 0.3993 \end{array}\right].$$

For $n_r = 3$, we obtain $\gamma = 1.0$ by solving the Algo-rithm MRIA after 100 iterations with incomplete TRM1 shown in Table 1, and the three-order model parameters are

Table 2. Comparison of minimum $\mathscr{H}_\infty$ performance for different TRMs.

	Three-order		Two-order		One-order	
TRMs	Theorem 3.2 (MRLA)	Theorem 3.3 (MRIA)	Theorem 3.2 (MRLA)	Theorem 3.3 (MRIA)	Theorem 3.2 (MRLA)	Theorem 3.3 (MRIA)
Case 1	1.1472	0.4	3.1683	0.9	3.1886	0.9
Case 2	1.4164	1.0	3.4504	1.2	3.4759	1.2
Case 3	1.6002	1.2	3.5248	1.4	3.5497	1.4
Case 4	2.8666	2.1	4.9436	2.8	4.9575	2.8

given by,

$$\left[\begin{array}{c|c} A_{r1} & B_{r1} \\ \hline C_{r1} & D_{r1} \end{array}\right] = \left[\begin{array}{ccc|c} -2.6346 & 2.4270 & 0.2987 & 3.4541 \\ 0.6704 & -1.9250 & 0.6497 & -1.8063 \\ 7.6536 & -7.7100 & -3.4463 & -1.9041 \\ \hline -1.6845 & 0.6684 & -1.6363 & 0.4908 \end{array}\right],$$

$$\left[\begin{array}{c|c} A_{r2} & B_{r2} \\ \hline C_{r2} & D_{r2} \end{array}\right] = \left[\begin{array}{ccc|c} -2.0600 & 1.3113 & -0.0737 & 1.6033 \\ 0.9626 & -4.3074 & 0.3832 & -1.4598 \\ 2.4065 & -2.9088 & -3.4451 & -1.7387 \\ \hline -1.5378 & 0.4070 & -1.1007 & 0.5312 \end{array}\right],$$

$$\left[\begin{array}{c|c} A_{r3} & B_{r3} \\ \hline C_{r3} & D_{r3} \end{array}\right] = \left[\begin{array}{ccc|c} -3.0769 & 0.2218 & -0.0270 & 1.1196 \\ -0.1415 & -3.2393 & 0.4244 & -1.0786 \\ 1.2613 & -1.2557 & -5.3077 & -1.4461 \\ \hline -1.3818 & 0.6491 & -1.1270 & 0.5125 \end{array}\right],$$

$$\left[\begin{array}{c|c} A_{r4} & B_{r4} \\ \hline C_{r4} & D_{r4} \end{array}\right] = \left[\begin{array}{ccc|c} -3.7483 & 0.9741 & 0.0087 & 1.9017 \\ 0.5611 & -4.7360 & 0.5143 & -1.6656 \\ 3.7021 & -4.2059 & -4.4138 & -1.1339 \\ \hline -1.7264 & 0.3602 & -1.3931 & 0.5426 \end{array}\right].$$

The feasible solutions for the other cases are omitted for brevity.

In order to further illustrate the effectiveness of the designed approximation models, simulations have been carried out. Specifically, choose the zero initial condition and the input $u(t) = 0.1e^{-0.01t+1}\cos(0.02t)$. With the above obtained approximation models under Case 2 in Table 1 and given one possible realisation of the Markovian jumping mode, the output trajectories of the original system and approximation models obtained based on Theorem 3.2 (MRLA) and Theorem 3.3 (MRIA) are depicted in Fig-ures 1 and 2, respectively. It can be clearly observed from the simulation curves that, despite the incomplete transi-tion descriptions in the TRM, the obtained reduced-order models approximate the original system very well.

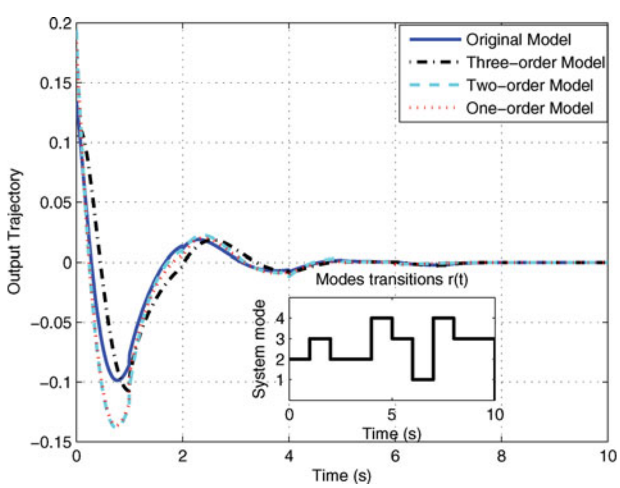

Figure 1. Output trajectories of the original system and reduced-order models with incomplete TRM1 based on Theorem 3.2 (MRLA).

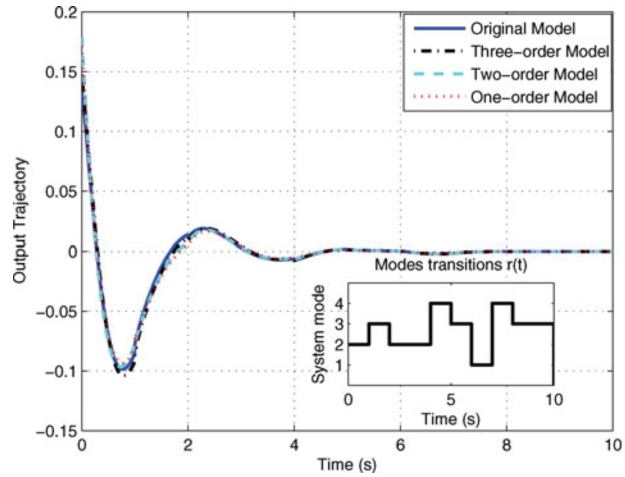

Figure 2. Output trajectories of the original system and reduced-order models with incomplete TRM1 based on Theorem 3.3 (MRIA).

5. Conclusions

This paper has addressed the problem of $\mathscr{H}_\infty$ model reduction for a class of continuous-time Markovian jump linear systems with incomplete statistics of mode information, which simultaneously involves the exactly known, partially unknown and uncertain transition rates. By fully utilising the properties of the transition rate matrices, together with the convexification of uncertain domains, a new $\mathscr{H}_\infty$ performance analysis criterion has been derived. Two sharply different approaches, namely, the convex linearisation approach and iterative approach, have been developed to solve the model reduction problem. It has been shown that the desired reduced-order models can be obtained by solving a set of strict linear matrix inequalities (LMIs) or a sequential minimisation problem subject to LMI constraints. An illustrative example has been provided to demonstrate the effectiveness of the proposed approaches.

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