# Model Reduction Techniques for Computing Approximately Optimal Solutions for Markov Decision Processes

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**Abstract**

We have recently developed a method for solving implicit (or factored) Markov decision processes (MDPs) with very large state spaces. This method reduces a large implicit MDP to a family of related smaller explicit MDP's which we call an "approximate MDP". An approximate MDP defines a set of (exact) MDPs by specifying upper and lower bounds on the transition probabilities and rewards. We have also developed algorithms that operate on approximate MDPs to estimate value functions and find approximately optimal policies.

Our MDP reduction algorithm uses the property of $\epsilon$-homogeneity as a basis for aggregating states of a large implicit MDP. Intuitively, this property enables us to group together states that behave approximately the same under all or some subset of policies. Borrowing from recent work on model minimization in computer-aided software verification, we have developed an algorithm that takes a factored representation of an MDP and an $0 \leq \epsilon \leq 1$ and computes an approximate MDP whose states are $\epsilon$-homogeneous sets of states in the original MDP. In the case in which $\epsilon = 0$, the upper and lower bounds in the approximate MDP are identical and the approximate MDP yields optimal solutions. In the best of cases, the size of the state space for the resulting approximate MDP can be exponentially smaller than the state space for the original, but, in general, there need not be any reduction. In the case in which $\epsilon > 0$, the reduced process is an approximate MDP with nontrivial bounds. By varying epsilon we can trade time and space (specifically in terms of the size of the corresponding state space) for solution quality.

Several papers relating to this work can be found on the web at the above cited website.