Modeling Agent's Preferences Based on Prospect Theory

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Abstract

It is well known that human preferences in decisions under risk do not always complies with expected utility theory (EUT). In fact, there are several effects that are inconsistent with basic tenets of EUT. Alternative theories have been proposed and perhaps the most well studied is Prospect Theory (PT). Recent work showed experimental results that support the idea that financial professionals may behave according to PT and violate EUT. Meanwhile, some argue that economy needs agent-based modeling, because it may be a better way to help guide financial policies than mathematical models. If financial professional behave according to PT in markets, then agent-based modeling needs PT based agents. Our idea is creating trading agents based on PT to simulate a market. However, the creation of an artificial agent based on PT as originally proposed is very hard and limited to two outcome prospects. We propose an agent model based on an extension of PT called Smooth Prospect Theory (SPT). We used this model to create agents to populate an artificial market with SPT and EUT agents. It was used to predict real market behavior for short periods. SPT agents provided more accurate predictions in crisis periods than EUT agents.

Introduction

One real-world problem that is not often addressed in social simulations is the fact that human beings do not make decisions under risk strictly based on expected utility. In fact, some alternative models are available, as Prospect Theory (PT). This theory was proposed by Kahneman and Tversky (Kahneman and Tversky 1979) and it can be seen as alternative to model and describe human decision making under risk. Their authors claim that several observed behaviors cannot be predicted or explained by expected utility theory.

Abdellaoui, Bleichrodt and Kamoun (Abdellaoui, Bleichrodt, and Kammoun 2013) showed experimental results that support the idea that financial professionals may behave according to prospect theory and violate expected utility maximization. They performed experiments where financial professional chose between prospects. In a 2009 essay, Farmer and Foley (Farmer and Foley 2009) argued for the use of agent-based models in economics, because they could be a better way to help guide financial policies than traditional mathematical models. If financial professional behave according to prospect theory in financial markets, the agent-based modeling needs PT based agents.

Our idea is creating artificial trading agents based on Prospect Theory (PT) and to simulate an artificial market populated with such agents. If investor's behavior is consistent with PT, such simulations may provide results closer to historical data from real markets.

The Prospect Theory and our agent model are described in section . We used a complex environment based on stock market, which is explained in section as test environment. The agent built for this environment is explained in section . The performed experiments and their results are presented and discussed in section . Finally, we point some conclusions and some remaining questions in section .

Prospect Theory and Agent Modeling

Prospect Theory is an alternative description model of human decision making under risk for expected utility that explains some pervasive effects that violate expected utilty theory (Kahneman and Tversky 1979). As example of such effects, people usually underweight outcomes, which are merely probable in comparison with outcomes that are obtained with certainty. Another effect pointed by Kahneman and Tversky, describes the observed preference in their experimental studies with human beings for guaranteed small gains over uncertain large gains, and conversely for uncertain large losses over small certain losses, called reflection effect. PT distinguishes two phases in the choice process: editing and evaluation. The editing phases yields a simpler representation of offered prospects. The second phase evaluates and selects the highest value prospect among the edited prospects (Kahneman and Tversky 1979).

The original PT (Kahneman and Tversky 1979) deals only with simple prospects, which have at most two non-zero outcomes. However, it is desirable to establish a foundation for complex prospects, which may have more than two non-zero outcomes or even continuous distributions. Furthermore, the editing phase of original PT is not well defined (Rieger and Wang 2008). Such phase makes the original form of PT really hard to be used in a computational model.

In fact, there are some proposals to extend PT for complex prospects as: Cumulative Prospect Theory (Tversky and Kahneman 1992) and Smooth Prospect Theory (Rieger and Wang 2008). These two proposals can be used for complex prospects, however there are empirical evidence that suggests that the original PT may predict some behavior that cannot be predicted by CPT, namely: violation of stochastic dominance and the fact that distinctive outcomes receive more weights than aggregated outcomes (Rieger and Wang 2008). Therefore, we chose Smooth Prospect Theory (SPT), despite some criticism about such theory (Amit Kothiyal and Wakker 2011).

The **Smooth Prospect Theory** (SPT) (Rieger and Wang 2008) incorporates the editing phase into the calculation and avoids the unclear part of the original form of PT. SPT deals with several possible outcomes per prospect and even continuous probability distribution. The SPT value is computed for each prospect and the highest value prospect is selected. The SPT value of a discrete prospect L with an arbitrary number of outcomes x_i and respective probabilities p_i is given by equation 1. In this paper, we use only discrete prospects, despite SPT can also be used for continuous distributions (Rieger and Wang 2008).

$$\frac{\sum_{i=1}^{n} w(p_i)v(x_i)}{\sum_{i=1}^{n} w(p_i)} \quad (1)$$

where the value function v(x) is chosen as:

$$v(x) = \begin{cases} x^{\alpha} & x \geq 0 \\ -\lambda(-x^{\beta}) & x < 0 \end{cases} \quad (2)$$

and $\lambda \approx 2.25$ is called "loss-aversion" coefficient and α, β are the risk-attitudes parameters for gains and losses. Furthermore, the weighting function is defined as:

$$w(p) := \frac{p^{\gamma}}{(p^{\gamma} + (1-p)^{\gamma})^{1/\gamma}} \quad (3)$$

and γ is the parameter that reflect the amount of over or underweighting (Rieger and Wang 2008). SPT can be also used for continuous distributions (Rieger and Wang 2008), however we use in this paper the formulation for the discrete distribution for simplicity.

SPT Agent Model

An agent model defines the action selected by the agent at each step of time according to his perception of the environment, his internal state and his preferences. Given a finite set of possible actions θ, an expected utility agent would select action i $\in \theta$ with the highest expected utility among all possible actions. However, the SPT agent sees each possible action as one prospect l with several possible outcomes and a distribution probability function. The SPT value may be calculated for prospects with discrete distributions as stated in equation 1 and the highest SPT prospect is selected. Since there is a one to one relationship between prospect and action, the action is also defined by the chosen prospect . In order to define his action, the agent needs to define a prospect for each possible action. Therefore, we define a **Prospect Construction Phase**, where the prospects are defined. Such phase takes place before the evaluation phase and provides the information needed for it (see figure 1).

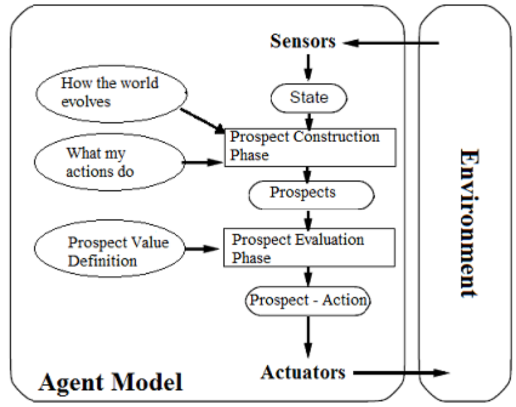

Figure 1: SPT Agent Model

The **prospect construction phase** produces a set of prospects, each one with a probability distribution function over outcomes and a linked action from θ set. This phase is strongly dependent of the problem domain. Prospect construction may be a simple task or very complex task according to the problem domain. We created implementation of the prospect construction phase for two problem domains: a very simple case and a complex case. In the simple case, the agent is required to choose among lotteries with a finite set of possible outcomes and their respective probabilities. Simulated experiments for such scenario are presented and discussed in section . The example of complex domain is the artificial stock market described in section . We present trading agents based on SPT for such domain in section .

Simulated Experiments in Choosing Prospects

We built an agent according to the proposed SPT agent model and used it in a set of twelve choosing problems. Such problems are described by Kahneman and Tversky in their remarkable paper (Kahneman and Tversky 1979). In each problem, the participant chooses between two prospects presented to him. Each prospect is described by pair of the value of the outcome and outcome's probability. For instance, the prospect (-2500:0.3,2400:0.6) means that there is 0.3 chance of losing 2500 dollars, 0.6 chance of winning 2400 dollars and 0.1 (=1.0-0.3-0.6) chance of keeping the current balance. The SPT agent's results are a closer to the people's preferences than EUT agent's results as shown in table 1. SPT presented only four discrepancies against six discrepancies presented by EUT agent. It is worth to say that this agents do not use the prospect construction phase, because the prospects are already provided by each problem.

Our Simulated Artificial Market

In previous work (Castro and Parsons 2013), we used a simulated artificial market populated by heterogeneous trading agents to predict the price behavior of an asset. We used the same simulated artificial market in this work. In our artificial market, we compare the price defined by our artifi-

	Prospect A	Prospect B	EUT	P	SPT
1	(2500:0.33, 2400:0.66)	(2400:1)	**A**	B	B
2	(2500:0.33)	(2400:0.34)	A	A	A
3	(4000:0.8)	(3000:1)	**A**	B	**A**
3N	(-4000:0.8)	(-3000:1)	**B**	A	**B**
4	(4000:0.2)	(3000:0.25)	A	A	A
4N	(-4000:0.2)	(-3000: 0.25)	B	B	B
7	(3000:0.9)	(6000:0.45)	A	A	**B**
7N	(-3000:0.9)	(-6000: 0.45)	**A**	B	**A**
8	(3000: 0.002)	(6000: 0.001)	**A**	B	B
8N	(-3000: 0.002)	(-6000: 0.001)	A	A	A
11	(1000:0.5)	(500:1)	**A**	B	B
12	(-1000:0.5)	(-500:1)	A	A	A

Table 1: Choosing Problems and Preferences observed by Expected Utility Theory (EUT) Agents, People (P) and Smooth Prospect Theory (SPT) Agents. In problem 11, the agent receives 1000 then chooses between prospects A and B. In problem 12, the agent receives 2000 then chooses between prospects.

cial society, a set of heterogeneous trading agents (section), with actual prices (or **external price**) obtained from a real stock exchange. The price predicted by the artificial market, $\overline{P_t}$, is determined by the buy and sell orders given by the set of trader agents present in stock market that acts as a continuous double auction. The clearing process is performed by the Four heap algorithm described in (Wurman, Walsh, and Wellman 1998). The predicted price (or **internal price**) for a given instant of time is defined as the average of all transaction prices weighted by the volume of each transaction. That way, one agent that gives a higher volume order is more relevant to the market price formation than other agent that submits small volume orders (Phelps 2007b; 2007a). We define **market specification** as the set of agents and their possessions (asset and money).

In order to compare among artificial market specifications, we need to define formally what is a better description of the real market. For a given time period, we define the **session prediction error** (E) as the sum of the quadratic difference of internal ($\overline{P_t}$) and external (P_t) price. If one artificial market specification M provides a smaller session error (E), than another artificial market specification, then we may say that artificial market specification M is a better description or predictor than the other. We use an algorithm based on hill climbing algorithm to adjust the artificial market specifications in order to reduce this prediction error, as detailed in section .

Trading Agents

We implemented four types of trading agents — **fundamentalist** traders, who have a fixed idea of the value of a good based on historical data, **technical** traders, who trade when the price trend alters, **market maker** traders who provides liquidity to the market and **SPT** trader, who is based on smooth prospect theory. The first three types are detailed in a previous paper (Castro and Parsons 2013). The SPT trader is described in section .

Prediction Evaluation

The difference between the price defined by the simulated transactions, that we call *internal price* and the price observed in the corresponding instant t at the real market , the so called *external price*, is the prediction error for a given instant of time t. However, the prediction error of a period of time is much more relevant than just one moment to state that one artificial market specification is better adapted than other one. Therefore, we need to define formally what a better prediction in a defined time period, in order to make possible the comparisons among artificial market specifications. For a given time period, we define the **session prediction error** (E), as stated in eq. 4, where $\overline{P_t}$ refers to the price predicted by one artificial market in instant t, while P_t refers to the price observed in the real market. If one artificial market specification M provides a smaller session error (E), than another artificial market specification, then we may say that artificial market M is a better description or predictor than the other.

$$E = \sum_{t=1}^{N} (\overline{P_t} - P_t)^2 \tag{4}$$

Market Adjustment Process

We use the fact that traders with higher volume have more relevance to the market price formation to adjust the market specification (i.e., the set of the agents and their assets) to fit data previously observed in real markets. For simplicity, each agent type has just one instance, and it trades one specific share quantity at each round. The adjustment process defines the share quantities of each one of the agents (fundamentalist, technical, SPT and market making agents). The objective function is the **session prediction error** (E). It is very hard to know a priori how a change in one of the specification parameters may affect the predicted price $\overline{P_t}$ or the session error (E). Therefore, we used a general search method to find minimum points of the objective function E. It is the random-start gradient descendent method, which is variant of the common hill climbing methods (Russell and Norvig 2003). It uses a new random starting point at each time, it finds a local minimum for the objective.

Building Trading Agents based on Prospect Theory

One trading agent needs to define his buy, sell or hold orders for a specific financial asset at each moment. The SPT trading agent defines one prospect for each possible action in his prospect construction phase. Then, he finds the highest SPT value prospect as explained in section and select the action linked to that prospect. This prospect construction phase is described in section . In order to simplify the problem, we defined that the outcomes are limited as detailed in section .

Prospect Construction Phase for Trading Agents

Auctions can be seen as a decision making process under risk, including continuous double auctions as observed in stock market. Furthermore, each trading agent must be able to make price prediction and use it to define one order $\theta_t \in$ [-M,M] at a given moment t. The value M is the maximum number of shares that can be bought or sold by the agent in one cycle. Positive values of θ mean a buy order, while negative θ means a sell order and $\theta = 0$ means keep the current position. The agent order and market behavior will define agent's outcome. Such outcome is the difference between the position at time t and the next time, after the order θ_t is executed. This outcome may be calculated as stated in equation 5, where P_t refers to the price, M_t is the amount of money, Q_t is the number of shares at time t. We assume that orders will be always executed at the market price P_{t+1}. Each order defines changes in Q_{t+1} and the market behavior defines the real value of P_{t+1}.

$$\begin{aligned} Outcome = (M_t - P_t * \theta_t) + (Q_t + \theta_t) * P_{t+1} \\ -[M_t + P_t * Q_t] \qquad (5) \\ Outcome = (P_{t+1} - P_t) * (Q_t + \theta_t) \end{aligned}$$

The market price P_{t+1}cannot be defined a priori, but it can be estimated by the trading agents ($\overline{P_{t+1}}$), so we can calculate $\overline{P_{t+1}} - P_t$ and then estimate the possible outcomes. Any order may bring different outcomes according to the market price in the next round P_{t+1}. In order to establish prospects given the possible orders, we would need to determine the probabilities given each possible outcome considering all possible orders. The future price $\overline{P_{t+1}}$ is a continuous value and θ_t is dependent of the trading strategy (certainly non-linear) and market state, so the outcome is itself a continuous non-linear function which would require a probability density function to represent the associated probabilities. We initially intend to use a simple approach based on the assumption that the price P_{t+1} is a random variable with a Gaussian probability distribution. It is easy to see that the outcome (eq. 5) is a linear function of P_{t+1}. Let x be the outcome, so the density probability function p(x) can be given by equation 6, where σ is the standard deviation and μ the expected value or $\overline{P_{t+1}}$.

$$p(x) = \frac{\exp^{-(P_{t+1}-\mu)^2/2\sigma^2}}{\sigma * \sqrt{2 * \pi}} \qquad (6)$$

Let $a_t = Q_t + \theta_t$ and $b_t = P_t(Q_t + \theta_t)$, so we can rewrite equation 5 to determine a new expression for P_{t+1} and using it in equation 6, we may find an expression for the distribution probability function p(x) for the outcome x given by equation 7, where a_t and b_t are known at time t. It can be used for the calculation of SPT as stated in equation 1.

$$p(x) = \frac{\exp^{-(x-b_t-a_t\mu)^2/2a_t^2\sigma^2}}{\sigma * \sqrt{2 * \pi}} \qquad (7)$$

Finite Outcomes

Since we assume that the price P_{t+1} is a random variable with a Gaussian probability distribution and therefore the outcome (eq. 5) is also a random variable. Each prospect would have infinite possible outcomes and could be calculated by SPT for continuous distributions, as explained in section . In order to avoid such complexity, we decided to define a finite set of possible outcomes. As each order θ_t is limited to [-M,M] and if we assume that P_{t+1} is limited to [0,2 $* P_t$], it is easy to verify using equation 5 that the outcome x is limited to interval $[-P_t(Q_t + M), P_t(Q_t + M)]$. If we adopt a step ϵ in (0,1), we limit the number of possible outcomes to $2 * P_t(Q_t + M)/\epsilon$ for each prospect. Furthermore, we limit the number of prospect to three sell (-M), hold (0) and buy (M). We believe that using these simplistic but reasonable assumptions, it is possible to construct a meaningful prospect to each action and calculate a SPT value for it.

Simulated Experiments in an Artificial Stock Market

We have performed some simulated experiments in a simulated artificial market (section) populated by traditional (fundamentalist, technical or market maker) agents and SPT agents. The results are compared to a market composed only by traditional agents in order to test our simple model and evaluate the quality of predictions using real market data that includes nine years of Intel stock prices between 2003 to 2011 from Nasdaq exchange. The simulations were performed considering periods with high price volatility (crisis) and low price volatility (non-crisis periods). We implemented our trading agents using an adapted version of auction simulator called JASA (Phelps 2007b).

Year	VAR	SCE	TRA	SPT	Best
2003	542.3	CR	344.3	337.6	SPT
2004	365.1	CR	501.0	491.5	SPT
2005	167.7	NCR	407.1	295.5	SPT
2006	284.1	NCR	289.3	640.4	TRA
2007	269.9	NCR	349.0	1.139.8	TRA
2008	1116.6	CR	263.4	367.6	TRA
2009	542.1	CR	449.8	444.2	SPT
2010	254.5	NCR	296.3	292.5	SPT
2011	292.6	NCR	234.5	322.2	TRA

Table 2: Smallest errors achieved by traditional (TRA) and SPT agents per year's simulation. The variance (VAR) observed for each year is used to classify the scenario (SCE) as crisis situation (CR) or non-crisis situation (NCR) .

The simulation results are presented on table 2. The variance of arithmetic returns were calculated for each year and used to classify it as crisis or non-crisis (*SCE column*). High variance indicates crisis and low variance indicates non-crisis year. We present the smallest session error for each year achieved by traditional (TRA) and SPT agents in columns TRA and SPT, respectively.

The predictions done by SPT agents presented better performance (smaller errors) than traditional agents on three of four crisis situations, but only two better performance of five possible for non-crisis scenarios. Therefore, one may argue that in crisis periods, the predictions made by artificial market populated with PT based agents presented behavior closer to reality than traditional agents. Furthermore, this observation supports the idea that in crisis period trading agent strategies may be influenced by psychological biases as described in PT (Kahneman and Tversky 1979).

Conclusions

Agent based modeling (ABM) may become a better way to guide financial policies, than traditional models according to some researchers (Farmer and Foley 2009). However, several problems may be identified in this approach. For instance, human beings do not make risky decisions strictly based on expected utility theory (EUT) as usually assumed in ABM. In fact, some alternative descriptive models are available, as Prospect Theory (PT). Recent work (Abdellaoui, Bleichrodt, and Kammoun 2013) showed experimental results that support the idea that financial professionals may behave according to PT and violate EUT.

We used an extension of PT called Smooth Prospect theory (SPT) to develop an agent model. Such model uses a prospect construction phase that creates a one-to-one relationship between prospect and agent's action. This model was used to build a trading agent to verify if artificial markets populated with this kind of agent may achieve better prediction performance. Those agents were used in several simulated experiments.

The experiments showed that the artificial market populated with SPT agents performs better in crisis periods than artificial market without SPT agents. This observation supports the idea that in crisis period trading agent strategies are more influenced by psychological biases as described in PT (Kahneman and Tversky 1979). In the future, we intend to try to find market scenarios where prospect based agents, as proposed here, may be a better description model than EUT agents.

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