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MODELING AND ANALYSIS OF THE SPREAD OF CARRIER DEPENDENT INFECTIOUS DISEASES WITH ENVIRONMENTAL EFFECTS

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In this paper, SIS and SIRS models for carrier dependent infectious diseases with immigration are proposed and analyzed by considering effects of environmental and human population related factors which are conducive to the growth of carrier population. In the modeling process, the density of carrier population is governed by a general logistic model. Further, it is assumed that the growth rate per capita and the modified carrying capacity of carrier population increase as the human population density increases. In each case, it is shown that the spread of an infectious disease increases as the carrier population density increases and the disease becomes more endemic due to immigration.

Keywords: Carrier; environmental discharges; immigration; susceptibles; infectives; logistic model.

1. Introduction

There are many carrier dependent infectious diseases which afflict human population around the world. However, the underdeveloped regions, which are situated in equatorial zones are most affected by such diseases. In villages, towns and cities of these regions, which are lacking in sanitation, various kinds of carriers, such as flies, ticks, mites, etc., are generally present in the environment. They transport bacteria of infectious diseases (cholera, dysentery, measles and gastroenteritis, etc.) from infectives to susceptibles and thus spread such diseases in human population indirectly.

The modeling and analyses of infectious diseases have been conducted by many scientists [1–4, 7, 10–12, 14–16]. A review regarding the study of infectious diseases has also been presented by Hethcote [9]. It is noted here that very little attention has been paid to the modeling and analysis of such diseases by considering the effect of carrier population, the density of which increases due to environmental factors like temperature, humidity, rain, vegetation, etc., in the habitat [5, 9]. It also increases due to the following environmental and human population related factors:

(i) Discharge of household wastes (garbage, trash, etc.) in residential areas.
(ii) Open drainage of sewage in residential areas.
(iii) Plantations of vegetation and hedges in residential areas and in parks.
(iv) Industries and transport systems producing wastes in residential areas.
(v) Open water storage tanks, ponds, etc.

It is pointed out that effects of these factors increase as the human population density increases, thus leading to further growth of the carrier population density. Therefore, to consider effects of such factors in modeling, it is assumed that the carrier population density is taken to be governed by a generalized logistic model. Further, the per capita growth rate and the modified carrying capacity of carrier population are assumed as functions of human population density and increase as the density of human population increases.

Further, it is noted that in most of the epidemic models, the total population size is assumed to be constant as birth and death rate are taken to be equal. But this does not happen if death rate caused by an infectious disease is significantly large, as in the case of cholera, etc. A similar situation also arises if there is no balance between incoming and outgoing population in the region under consideration [8]. Keeping this in view, models with demographic structures have been proposed and analyzed involving variation of total population size, which involve birth rate, death rate, immigration, etc. [8, 13, 17].

Thus, in this paper, SIS and SIRS models for carrier dependent infectious diseases with immigration are proposed and analyzed by considering environmental and human population related factors which are conducive to the growth of carrier population.

2. SIS Model with Constant Immigration

Let $N(t)$ be the total population density which is divided into two sub-classes: the susceptible class $X(t)$ and the infective class $Y(t)$. It is assumed that all susceptibles are affected by the carrier population of density $C(t)$, which is governed by a general logistic model.

In view of the above and by assuming simple mass action interaction, an SIS model is proposed as follows:

$$\begin{aligned} \dot{X}^{\&} &= A - \beta XY - \lambda XC + \nu Y - dX\,, \\ \dot{Y}^{\&} &= \beta XY + \lambda XC - (\nu + \alpha + d)Y\,, \\ \dot{N}^{\&} &= A - dN - \alpha Y\,, \\ \dot{C}^{\&} &= s(N)C - \frac{s_0 C^2}{L(N)} - s_1 C\,, \\ X + Y &= N\,, \\ X(0) = X_0 > 0,\ Y(0) &\geq 0,\ N(0) = N_0 > 0,\ C(0) = C_0 \geq 0\,. \end{aligned} \tag{2.1}$$

Here $(\dot{\ })$ shows the derivative w.r.t. time t.

In model (2.1), A is a constant immigration rate of human population from outside the region under consideration, d is natural death rate constant, β and λ are the transmission coefficients due to infective and carrier population respectively, α is the disease related death rate constant, ν is the recovery rate constant. The constant s_1 is the death rate coefficient of carriers due to natural factors as well as by control measures. We may note here that if the growth rate and death rate of carrier population are balanced, then it may tend to zero.

In (2.1), $s(N)$ is the growth rate per capita of the carrier population density such that $s(N) - s_1$ is its intrinsic growth rate as compared to the usual logistic model. In view of the assumption that the growth rate per capita increases as the human population density increases, we have,

$$s(0) = s_0 > 0 \quad \text{and} \quad s'(N) \geq 0\,, \tag{2.2}$$

where s_0 is the value of $s(N)$ when $N = 0$ and $(\ )'$ denotes the derivative of the function with respect to its argument.

Similarly, $L(N)$ is the modified carrying capacity of the carrier population and its value as compared to the usual logistic model is: $L(N)[\frac{s(N)-s_1}{s_0}]$. We assume here that this modified carrying capacity increases with human population density, so we have,

$$L(0) = L_0 > 0 \quad \text{and} \quad L'(N) \geq 0\,, \tag{2.3}$$

where L_0 is the value of $L(N)$ when $N = 0$.

From the last equation of (2.1), (2.2) and (2.3), it is noted that even in absence of human population related factors, the carrier population density increases in its natural environment and it tends to $L_0(1 - \frac{s_1}{s_0})$ which may become zero if $s_1 \to s_0$.

3. Equilibrium Analysis

To analyze the model (2.1), we consider the following reduced system (since $X+Y = N$),

$$\begin{aligned}
\dot{Y}^{\&} &= \beta(N-Y)Y + \lambda(N-Y)C - (\nu+\alpha+d)Y\,,\\
\dot{N}^{\&} &= A - dN - \alpha Y\,,\\
\dot{C}^{\&} &= s(N)C - \frac{s_0 C^2}{L(N)} - s_1 C\,.
\end{aligned} \tag{3.1}$$

The results of equilibrium analysis are given in the following theorem:

Theorem 3.1. *There exist following three equilibria of the system* (3.1),

(i) $E_0 = (0, \frac{A}{d}, 0)$.

(ii) $E_1 = (\bar{Y}, \bar{N}, 0)$ *which exists if* $\beta A - d(\nu+\alpha+d) > 0$ *where* $\bar{Y} = \frac{\beta\bar{N}-(\nu+\alpha+d)}{\beta}$ *and* $\bar{N} = \frac{1}{\alpha+d}\{A + \frac{\alpha}{\beta}(\nu+\alpha+d)\}$.

(iii) $E_2 = (Y^*, N^*, C^*)$.

Proof. The existence of E_0 or E_1 is obvious. The nontrivial equilibrium point $E_2 = (Y^*, N^*, C^*)$ is given by the solution of the following set of equations,

$$\beta Y^2 + \{(\nu + \alpha + d) - \beta N + \lambda C\}Y - \lambda NC = 0\,, \tag{3.2}$$

$$Y = \frac{A - dN}{\alpha}\,, \tag{3.3}$$

$$C = \frac{\{s(N) - s_1\}L(N)}{s_0}\,; \quad s_0 > s_1\,. \tag{3.4}$$

Eliminating Y from (3.2) and (3.3), we note that N^* should satisfy $F(N^*) = 0$, where,

$$F(N) = \frac{\beta}{\alpha^2}(A - dN)^2 + \frac{(A - dN)}{\alpha}\{(\nu + \alpha + d) - \beta N + \lambda C\} - \lambda NC\,, \tag{3.5}$$

and C is given by (3.4). □

It is easy to observe that, $F(0) > 0$ and $F(A/d) < 0$. Therefore, there exists a root N^* of $F(N) = 0$ in $0 < N < A/d$. We also find that, $F'(N) < 0$ in $0 < N < A/d$. Hence, there exists a unique root N^* given by $F(N^*) = 0$. Knowing the value of N^*, the values of Y^* and C^* can be computed from Eqs. (3.3) and (3.4) as $N^* < A/d$.

It may be noted from (3.4) that $\frac{dC}{dN} > 0$ in view of (2.2) and (2.3). Hence C increases as N increases. Also, from (3.2) we get $\frac{dY}{dC} > 0$. Therefore, it is clear that the density of infective population increases as the density of carrier population increases.

4. Stability Analysis

Now, we study stability of equilibria E_0, E_1 and E_2. The local stability results of these equilibria are stated in the following theorem:

Theorem 4.1. *The equilibria E_0 and E_1 are locally unstable and the equilibrium E_2 is locally asymptotically stable provided*

$$a_2 a_1 - a_0 > 0\,, \tag{4.1}$$

where,

$$a_2 = \beta Y^* + \frac{\lambda N^* C^*}{Y^*} + d + \frac{s_0 C^*}{L(N^*)}\,,$$

$$a_1 = \left(d + \frac{s_0 C^*}{L(N^*)}\right)\left(\beta Y^* + \frac{\lambda N^* C^*}{Y^*}\right) + \frac{d s_0 C^*}{L(N^*)} + \alpha(\beta Y^* + \lambda C^*)\,,$$

$$a_0 = \frac{\alpha s_0 C^*}{L(N^*)} \cdot (\beta Y^* + \lambda C^*) + \left(\beta Y^* + \frac{\lambda N^* C^*}{Y^*}\right)\frac{d s_0 C^*}{L(N^*)}$$
$$+ \alpha\lambda(N^* - Y^*)\left\{s'(N^*)C^* + \frac{s_0 C^{*2}}{(L(N^*))^2}L'(N^*)\right\}.$$

Remark. It can be seen that the condition (4.1) is automatically satisfied when $s(N)$ and $L(N)$ are independent of N, i.e., when the density of carrier population remains unaffected due to human population related factors.

In the following we show that E_2 is nonlinearly asymptotically stable. It may be pointed out here that a region of attraction for the system (2.1) can be given by the set

$$\begin{aligned} \Omega &= \{Y, N, C) : 0 \le Y \le N \le A/d,\, 0 \le C \le C_m\}\,, \\ C_m &= \tfrac{L(A/d)}{s_0}\{s(A/d) - s_1\}\,, \end{aligned} \tag{4.2}$$

which attracts all solutions initiating in the positive quadrant.

Theorem 4.2. *In addition to assumptions* (2.2) *and* (2.3), *let* $s(N)$ *and* $L(N)$ *satisfy* $0 \le s'(N) \le p$ *and* $0 \le L'(N) \le q$ *for some positive constants* p *and* q *in* Ω, *then* E_2 *is nonlinearly asymptotically stable in* Ω *provided the following inequalities are satisfied*:

$$\alpha\lambda^2 C_m^2 < \frac{2}{3} d\beta^2 Y^{*2}\,, \tag{4.3}$$

$$3\alpha\lambda^2 (N^* - Y^*)^2 L^2(N^*) \left\{ p + \frac{s_0 q C_m}{L_0^2} \right\}^2 < 2ds_0^2\beta^2 Y^{*2}\,. \tag{4.4}$$

It is noted here that in absence of human factors, i.e., $p = q = 0$, the inequality (4.4) is satisfied. This implies that human population related factors, causing the growth of carrier population, have a destabilizing effect on this system.

Proof. We prove the above theorem by using the following positive definite function:

$$V = k_0 \left(Y - Y^* - Y^* \ln \frac{Y}{Y^*} \right) + \frac{k_1}{2}(N - N^*)^2 + K_2 \left(C - C^* - C^* \ln \frac{C}{C^*} \right), \tag{4.5}$$

where $k_0 = \alpha/\beta$, $k_1 = 1$ and k_2 has been chosen later. □

Differentiating (4.5) and using (3.1), we get, after a little simplification,

$$\begin{aligned} \dot{V}^{\&} = {} & \frac{-k_0\lambda C N}{Y Y^*}(Y - Y^*)^2 - \frac{k_0\beta}{3}(Y - Y^*)^2 \\ & - \left[\frac{k_0\beta}{3}(Y - Y^*)^2 + \frac{k_1 d}{2}(N - N^*)^2 - \frac{k_0\lambda C}{Y^*}(Y - Y^*)(N - N^*) \right] \\ & - \left[\frac{k_0\beta}{3}(Y - Y^*)^2 + \frac{k_2 s_0}{2L(N^*)}(C - C^*)^2 - \frac{k_0\lambda}{Y^*}(N^* - Y^*)(Y - Y^*)(C - C^*) \right] \\ & - \left[\frac{k_1 d}{2}(N - N^*)^2 + \frac{k_2 s_0}{2L(N^*)}(C - C^*)^2 \right. \\ & \left. - k_2\{f(N) + s_0 C g(N)\}(N - N^*)(C - C^*) \right], \end{aligned} \tag{4.6}$$

where, $f(N)$ and $g(N)$ are defined as follows:

$$f(N) = \begin{cases} \dfrac{s(N) - s(N^*)}{N - N^*}\,, & N \neq N^* \\ \dfrac{ds}{dN}\,, & N = N^*\,, \end{cases} \tag{4.7}$$

$$g(N) = \begin{cases} \dfrac{L(N) - L(N^*)}{N - N^*} \cdot \dfrac{1}{L(N) \cdot L(N^*)}\,, & N \neq N^* \\ \dfrac{1}{L^2(N^*)} \cdot \dfrac{dL}{dN}\,, & N = N^*\,. \end{cases} \tag{4.8}$$

Then by using the assumptions of the theorem and the mean value theorem, we have,

$$|f(N)| \leq p \quad \text{and} \quad |g(N)| \leq \frac{q}{L_0^2}\,. \tag{4.9}$$

Now choosing k_2 such that

$$\frac{3\alpha\lambda^2(N^* - Y^*)^2 L(N^*)}{2s_0\beta^2 Y^{*2}} < k_2 < \frac{s_0 d}{L(N^*)\{p + \frac{s_0 q C_m}{L_0^2}\}^2}\,,$$

it can be seen that $\dot{V}^{\&}$ is negative definite under the conditions (4.3) and (4.4) as stated above. Hence the result.

The above theorems imply that under certain conditions if the density of carrier population increases, then the number of infectives in human population increases leading to increased spread of the carrier dependent infectious disease. These theorems also suggest that such an infectious disease becomes more endemic due to immigration.

5. SIRS Model with Constant Immigration

In this section, an SIRS model governing a carrier dependent infectious disease with constant immigration of human population is proposed and analyzed. In this case population density $N(t)$ is divided into three classes: the susceptibles $X(t)$, the infectives $Y(t)$ and the removed class $Z(t)$. The equation governing the density $C(t)$ of the carrier population is assumed to be the same as that of SIS model discussed earlier.

Keeping in view of the above and the considerations in Sec. 2, a mathematical model in this case is proposed as follows:

$$\begin{aligned} \dot{X}^{\&} &= A - \beta XY - \lambda XC + \nu_1 Z - dX\,, \\ \dot{Y}^{\&} &= \beta XY + \lambda XC - (\nu + \alpha + d)Y\,, \\ \dot{Z}^{\&} &= \nu Y - \nu_1 Z - dZ\,, \\ \dot{N}^{\&} &= A - dN - \alpha Y\,, \end{aligned} \tag{5.1}$$

$$\dot{C}^{\&} = s(N)C - \frac{s_0 C^2}{L(N)} - s_1 C\,,$$

$$X + Y + Z = N\,,$$

$$X(0) = X_0 > 0,\ Y(0) = Y_0 \geq 0,\ Z(0) = Z_0 > 0,\ \text{and } C(0) = C_0 \geq 0\,.$$

In model (5.1), ν is the recovery rate constant, i.e., the rate at which individuals recover and transfer to the removed class from the infective class; ν_1 is the rate coefficient at which individuals in the removed class again become susceptibles. All other parameters are the same as defined in the case of SIS model.

6. Equilibrium Analysis

To analyze the model (5.1) we consider the following reduced system (since $X + Y + Z = N$),

$$\begin{aligned}
\dot{Y}^{\&} &= \beta(N - Y - Z)Y + \lambda(N - Y - Z)C - (\nu + \alpha + d)Y\,,\\
\dot{Z}^{\&} &= \nu Y - (\nu_1 + d)Z\,,\\
\dot{N}^{\&} &= A - dN - \alpha Y\,,\\
\dot{C}^{\&} &= s(N)C - \frac{s_0 C^2}{L(N)} - s_1 C\,.
\end{aligned} \tag{6.1}$$

Now we give the results of equilibrium analysis in the following theorem:

Theorem 6.1. *There exist following three equilibrium points of the system* (6.1),

(i) $P_0 = (0, 0, A/d, 0)$.

(ii) $P_1 = (Y_1, Z_1, N_1, 0)$ *which exists if* $\beta A - d(\nu + \alpha + d) > 0$, *where*

$$Y_1 = \frac{\beta N_1 - (\nu + \alpha + d)}{\beta(1 + \frac{\nu}{\nu_1 + d})}\,, \quad Z_1 = \frac{\nu}{\nu_1 + d} \cdot Y_1\,, \quad N_1 = \frac{A - \alpha Y_1}{d}\,.$$

(iii) $P_2 = (\hat{Y}, \hat{Z}, \hat{N}, \hat{C})$.

Where $\hat{Y}, \hat{Z}, \hat{N}$ *and* $\hat{C})$ *are given by the following set of equations*:

$$\beta Y^2 + \{(\nu + \alpha + d) - \beta(N - Z) + \lambda C\}Y - \lambda(N - Z)C = 0\,,$$

$$Z = \frac{\nu Y}{\nu_1 + d}\,,$$

$$Y = \frac{A - dN}{\alpha}\,,$$

$$C = \frac{\{s(N) - s_1\}L(N)}{s_0},\ s_0 > s_1\,.$$

The existence of P_2 can be proved using the same method as employed in the proof of E_2 (see Theorem 3.1).

It can be seen in this case also that $\frac{dY}{dC} > 0$, hence, the infective population density increases as the carrier population density increases.

7. Stability Analysis

The local stability results of these equilibria are stated in the following theorem:

Theorem 7.1. *The equilibria P_0 and P_1 are locally unstable and the equilibrium P_2 is locally asymptotically stable provided*

$$b_3 b_2 - b_1 > 0 \quad \text{and} \quad b_1(b_3 b_2 - b_1) - b_0 b_3^2 > 0\,,$$

where b_0, b_1, b_2, b_3 are defined as follows:

$$b_3 = (2d + \nu_1) + \beta\hat{Y} + \frac{\lambda(\hat{N} - \hat{Z})\hat{C}}{\hat{Y}} + \frac{s_0\hat{C}}{L(\hat{N})}\,,$$

$$b_2 = \nu(\beta\hat{Y} + \lambda\hat{C}) + d(d + \nu_1) + (2d + \nu_1)\left\{\beta\hat{Y} + \frac{\lambda(\hat{N} - \hat{Z})\hat{C}}{\hat{Y}} + \frac{s_0\hat{C}}{L(\hat{N})}\right\}$$

$$+ \frac{s_0\hat{C}}{L(\hat{N})}\left(\beta\hat{Y} + \frac{\lambda(\hat{N} - \hat{Z})\hat{C}}{\hat{Y}}\right) + \alpha(\beta\hat{Y} + \lambda\hat{C})\,,$$

$$b_1 = \nu(\beta\hat{Y} + \lambda\hat{C})\left(d + \frac{s_0\hat{C}}{L(\hat{N})}\right) + d(d + \nu_1)\left(\beta\hat{Y} + \frac{\lambda(\hat{N} - \hat{Z})\hat{C}}{\hat{Y}} + \frac{s_0\hat{C}}{L(\hat{N})}\right)$$

$$+ (2d + \nu_1)\frac{s_0\hat{C}}{L(\hat{N})}\left(\beta\hat{Y} + \frac{\lambda(\hat{N} - \hat{Z})\hat{C}}{\hat{Y}}\right) + \alpha(\beta\hat{Y} + \lambda\hat{C})\left(d + \nu_1 + \frac{s_0\hat{C}}{L(\hat{N})}\right)$$

$$+ \alpha\lambda(\hat{N} - \hat{Y} - \hat{Z})\left\{s'(\hat{N})\hat{C} + \frac{s_0\hat{C}^2}{(L(\hat{N}))^2}L'(\hat{N})\right\}\,,$$

$$b_0 = \nu(\beta\hat{Y} + \lambda\hat{C})\frac{s_0 d\hat{C}}{L(\hat{N})} + \frac{d(d + \nu_1)s_0\hat{C}}{L(\hat{N})}\left(\beta\hat{Y} + \frac{\lambda(\hat{N} - \hat{Z})\hat{C}}{\hat{Y}}\right)$$

$$+ \alpha(\beta\hat{Y} + \lambda\hat{C})\frac{(d + \nu_1)s_0\hat{C}}{L(\hat{N})} + (d + \nu_1)\alpha\lambda(\hat{N} - \hat{Y} - \hat{Z})$$

$$\cdot\left\{s'(\hat{N})\hat{C} + \frac{s_0\hat{C}^2}{(L(\hat{N}))^2}L'(\hat{N})\right\}\,.$$

It may be noted here that first of these conditions is automatically satisfied if $s(N)$ and $L(N)$ are independent of N.

Now we give the non-linear stability result of the nontrivial equilibrium $P_2(\hat{Y}, \hat{Z}, \hat{N}, \hat{C})$, for which we state the following lemma without proof:

Lemma 7.1. *The set*

$$W = \{(Y, Z, N, C) : 0 \le Y \le N \le A/d,\ 0 \le Z \le N \le A/d,\ C \le C_m\},$$

$$C_m = \frac{L(A/d)}{s_0}(s(A/d) - s_1),$$

attracts all solutions initiating in the positive orthant.

Theorem 7.2. *In addition to assumptions* (2.2) *and* (2.3), *let* $s(N)$ *and* $L(N)$ *satisfy in* W : $0 \le s'(N) \le p$ *and* $0 \le L'(N) \le q$ *for some positive constants* p *and* q *in* W, *then* P_2 *is non-linearly asymptotically stable provided the following inequalities hold*:

$$\alpha\lambda^2 C_m^2 < \frac{2}{3} d\beta^2 \hat{Y}^2, \tag{7.1}$$

$$3\alpha\lambda^2(\hat{N} - \hat{Y} - \hat{Z})^2 L^2(\hat{N}) \left\{p + \frac{s_0 q C_m}{L_0^2}\right\}^2 < 2s_0^2\beta^2 d\hat{Y}^2, \tag{7.2}$$

$$\nu\lambda^2 C_m^2 < \frac{4}{3}(\nu_1 + d)\beta^2\hat{Y}^2. \tag{7.3}$$

We note here that due to the presence of removed class, an additional condition (7.3) is required for the non-linear stability implying that it has a destabilizing effect.

We can prove the above theorem by using the following positive definite function and using the same procedure as that of Theorem 4.2:

$$V = k_0\left(Y - \hat{Y} - \hat{Y}\ln\frac{Y}{\hat{Y}}\right) + \frac{k}{2}(Z - \hat{Z})^2 + \frac{k_1}{2}(N - \hat{N})^2 + k_2\left(C - \hat{C} - \hat{C}\ln\frac{C}{\hat{C}}\right), \tag{7.4}$$

where $k_0 = \alpha/\beta$, $k = \alpha/\nu$, $k_1 = 1$ and k_2 is chosen such that

$$\frac{3\alpha\lambda^2(\hat{N} - \hat{Y} - \hat{Z})^2 L(\hat{N})}{2s_0\beta^2\hat{Y}^2} < k_2 < \frac{s_0 d}{L(\hat{N})\{p + \frac{s_0 q C_m}{L_0^2}\}^2}.$$

In this case also the above two theorems imply similar results as in the case of SIS model. In particular they show that the spread of an infectious disease increases as the density of the carrier population, caused by natural environmental as well as human population related factors, increases and the disease becomes more endemic due to immigration. It also suggests that environment and human population related factors have destabilizing effect on the system. The removed class in the model implies that the recovery of infective population is delayed and its effect on the system, in presence of carrier population, is destabilizing.

8. Conclusions

In this paper, we have proposed and analyzed SIS and SIRS models with immigration for carrier dependent infectious diseases by considering that the density of carrier population increases in the habitat by environmental and human population related factors. It has been assumed that the density of the carrier population is governed by a general logistic model, the growth rate per capita and the modified carrying capacity of which increase as the human population density increases. These models have been analyzed by using stability theory of differential equations. It has been shown that, the number of infectives increases as the density of carrier population increases due to environmental and human population related factors leading to fast spread of such infectious diseases. It has also been shown that these infectious diseases become more endemic due to immigration. It has been pointed out that the environmental and human population related factors as well as the presence of removed class have destabilizing effects on the corresponding system.

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