Modeling and Computing Two-settlement Oligopolistic Equilibrium in a Congested Electricity Network

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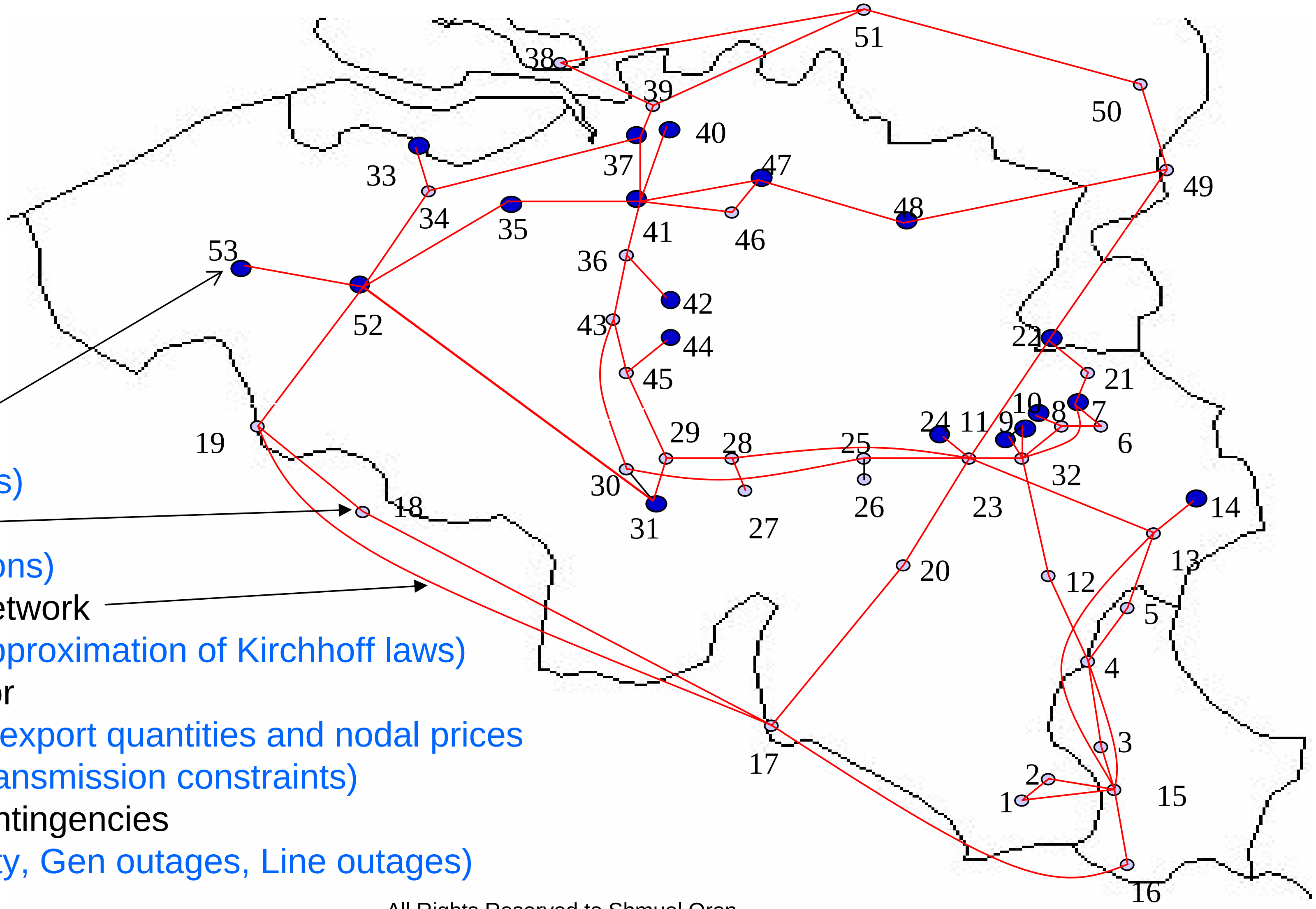
Objectives and Scope

- Generalize the Allaz Vila model to a realistic constrained network setting
  - Explore formulation issues.
  - Does the basic intuition still hold?
  - Explore computational feasibility.
  - Develop special purpose computational tools.
  - Test cases.

Electricity Market Model

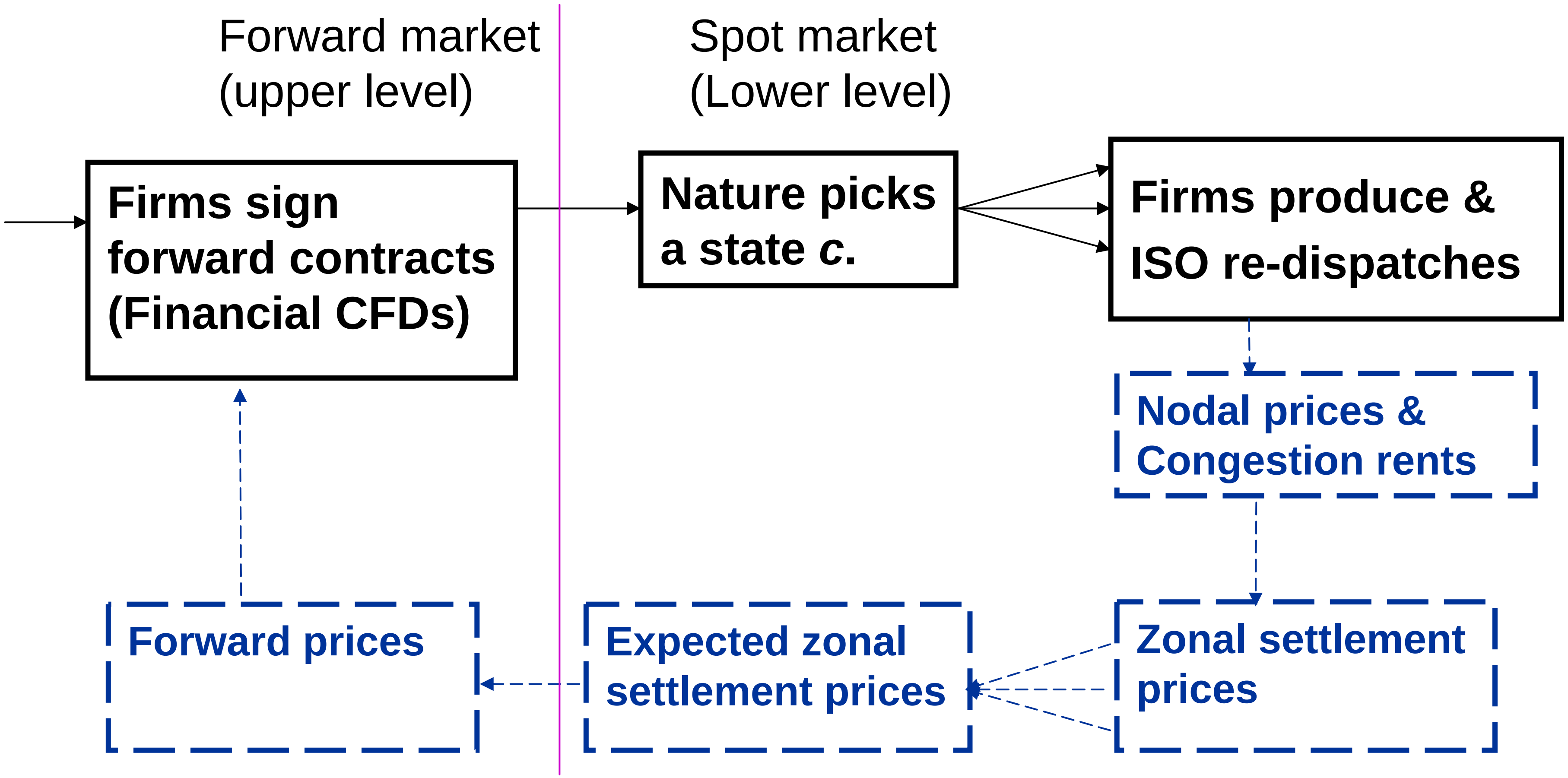


Generators
(Cournot Players)
Load nodes
(Demand functions)
Transmission network
(Lossless DC approximation of Kirchhoff laws)
System Operator
(Set import and export quantities and nodal prices so as to meet transmission constraints)
Probabilistic Contingencies
(Load uncertainty, Gen outages, Line outages)

Two Settlement Market Equilibrium

- Generators enter into forward contracts to supply specified quantities at agreed upon prices (forward markets) and decide in real time (spot market) how much to produce.
- Forward and spot markets may have different granularly of settlement points.
  - Nodal spot market
  - Forward contracts are settled at clusters of nodes (Hubs) based on a weighted average of the nodal spot prices.

Two-settlement Model Structure



Solution Concept: Subgame Perfect Nash equilibrium

The ISO Problem

- Conducts the energy redispatch
- Sets locational prices and transmission charges
- Maximizes social welfare

Import/Export quantity

Local production

$$\max_{r_i^c,\, i\in N} \sum_{i\in N} \left(\int_0^{r_i^c + q_i^c} P_i(\tau_i)\, d\tau_i - C_i(q_i^c) \right)$$

Inverse demand function

subject to:

$$\sum_{i\in N} r_i^c = 0$$

Balancing constraint

$$-K_l^c \le \sum_{i\in N} D_{l,i}^c r_i^c \le K_l^c,\ \ l \in L$$

Flows constraints

KKT conditions for the ISO problem

$$P_i(q_i + r_i) - p - \varphi_i = 0 \quad i \in N$$

$$\varphi_i = \sum_{m \in L} (\lambda_m^+ - \lambda_m^-) D_{m,i}$$

$$\sum_{j \in N} r_j = 0$$

$$0 \le \lambda_l^- \perp \sum_{j \in N} D_{l,j} r_j + K_l \ge 0 \quad l \in L$$

$$0 \le \lambda_l^+ \perp K_l - \sum_{j \in N} D_{l,j} r_j \ge 0 \quad l \in L$$

Modeling Choices

- Cournot generation firms act as multi-Stackelberg leaders anticipating the outcome of the ISO redispatch
- Cournot generation firms and ISO move simultaneously as Nash players taking each other's strategic variables as parameters in their optimization problem
  - The ISO's strategic variables are import/export quantities for each node (Cournot-Cournot)
  - The ISO's strategic variables are the nodal injection charges = nodal premium relative to the slack bus price (Cournot-Bertrand)

Generation Firms Anticipate ISO Redispatch

$$\max_{q_i : i \in N_g} \sum_{i \in N_g} \left(P_i(q_i + r_i) q_i - C_i(q_i) \right)$$

subject to:

$$\underline{q}_i \le q_i \le \overline{q}_i \quad i \in N_g$$

$$\boxed{\begin{aligned}
&P_i(q_i + r_i) - p - \varphi_i = 0 \;\; i \in N \\
&\varphi_i = \sum_{m \in L} (\lambda_m^+ - \lambda_m^-) D_{m,i} \\
&\sum_{j \in N} r_j = 0 \\
&0 \le \lambda_l^- \perp \sum_{j \in N} D_{l,j} r_j + K_l \ge 0 \;\; l \in L \\
&0 \le \lambda_l^+ \perp K_l - \sum_{j \in N} D_{l,j} r_j \ge 0 \;\; l \in L
\end{aligned}}$$

Problem with Sequential Move Formulation

- A two settlement model will result in a non-tractable three level optimization problem
- Generation firms in spot market have incentive to induce degeneracy in ISO problem (Induce flow just below constraints to avoid congestion rents)
  - Non-uniqueness
  - Discontinuity in reaction functions
  - Possible non-existence of pure strategy equilibrium

Simultaneous Move Model (generation firms do not account for impact on congestion)

$$\max_{q_i : i \in N_g} \sum_{i \in N_g} \left(P_i(q_i + r_i) q_i - C_i(q_i) \right)$$

subject to:

$$\underline{q}_i \le q_i \le \overline{q}_i \quad i \in N_g$$

$$P_i(q_i + r_i) - p - \varphi_i = 0 \;\; i \in N$$

$$\varphi_i = \sum_{m \in L} (\lambda_m^+ - \lambda_m^-) D_{m,i}$$

$$\sum_{j \in N} r_j = 0$$

$$0 \le \lambda_l^- \perp \sum_{j \in N} D_{l,j} r_j + K_l \ge 0 \;\; l \in L$$

$$0 \le \lambda_l^+ \perp K_l - \sum_{j \in N} D_{l,j} r_j \ge 0 \;\; l \in L$$

Market Equilibrium when ISO's Strategic Variables are Import/Export Quantities $\{r_i\}$

- Mixed Nonlinear Complementarity Problem (NCP)

$$P_i(q_i + r_i) + q_i \frac{\partial P_i(q_i + r_i)}{\partial q_i} - \frac{dC_i(q_i)}{dq_i} + \rho_i^- - \rho_i^+ = 0 \quad i \in N$$

$$0 \le \rho_i^- \perp q_i - \underline{q}_i \ge 0 \quad i \in N$$

$$0 \le \rho_i^+ \perp \overline{q}_i - q_i \ge \quad i \in N$$

$$P_i(q_i + r_i) - p - \varphi_i = 0 \quad i \in N$$

$$\varphi_i = \sum_{m \in L} (\lambda_m^+ - \lambda_m^-) D_{m,i}$$

$$\sum_{j \in N} r_j = 0$$

$$0 \le \lambda_l^- \perp \sum_{j \in N} D_{l,j} r_j + K_l \ge 0 \quad l \in L$$

$$0 \le \lambda_l^+ \perp K_l - \sum_{j \in N} D_{l,j} r_j \ge 0 \quad l \in L$$

- When demand function are linear and supply functions quadratic it can be reduced to an LCP

$$w = t + My, 0 \le w \perp y \ge 0$$

M is symmetric or bisymmetric PSD

Implications

- No effect of multiple ownership (problem is separable so that each unit is priced independently)
- When there is no congestion import/exports variables are selected so as to equalize nodal prices across nodes.
  - The resulting market equilibrium is different than the Cournot equilibrium when nodal demand is aggregated (residual demands at each node retain the slope of the local demand function)

Firms' Optimization when ISO Strategic Variables are the Nodal Price Premiums

$$\max_{q_i : i \in N_g, p} \sum_{i \in N_g} (p + \varphi_i) q_i - \sum_{i \in N_g} C_i(q_i)$$

subject to:

$$q_i \geq \underline{q}_i \quad i \in N_g$$

$$q_i \leq \overline{q}_i \quad i \in N_g$$

$$\sum_{j \in N} q_j = \sum_{j \in N} P_j^{-1}\left(p + \varphi_i\right)$$ Implicit residual demand function

Firms do not see transmission constraints only nodal price premiums

Market equilibrium for simultaneous move model where ISO strategic variables are the locational premiums (Cournot-Bertrand)

$$p + \varphi_i - \beta - \frac{dC_i(q_i)}{dq_i} + \rho_i^- - \rho_i^+ = 0 \quad i \in N_g$$

$$\beta \sum_{j \in N} \frac{dP_j^{-1}\left(p + \varphi_j\right)}{dp} + \sum_{j \in Ng} q_j = 0$$

$$\sum_{j \in N} q_j = \sum_{j \in N} P_j^{-1}\left(p + \varphi_j\right)$$

$$0 \le \rho_i^- \perp q_i - \underline{q}_i \ge 0 \quad i \in N_g$$

$$0 \le \rho_i^+ \perp \overline{q}_i - q_i \ge 0 \quad i \in N_g$$

$$P_i(q_i + r_i) - p + \sum_{m \in L} (\lambda_m^- - \lambda_m^+) D_{m,i} = 0 \quad i \in N$$

$$\sum_{j \in N} r_j = 0$$

$$0 \le \lambda_l^- \perp \sum_{j \in N} D_{l,j} r_j + K_l \ge 0 \quad l \in L$$

$$0 \le \lambda_l^+ \perp K_l - \sum_{j \in N} D_{l,j} r_j \ge 0 \quad l \in L$$

- Can be reduced to an LCP when demand functions are linear and cost functions are quadratic

$$w = t + My, \quad 0 \le w \perp y \ge 0$$

M is a bisymmetric PSD matrix

Implications

- Ownership structure affects results
- When there is no congestion nodal price premiums go to zero and market equilibrium is identical to the single node oligopoly solution.
- The market equilibrium will reflect an oligopoly solution even when the market is separated (strategically decoupled) due to a thin line (zero capacity) or permanently congested line (fixed imports/exports)

Firms' programs with forward contracts for the Cournot-Bertrand Case

Forward quantity

$$\max_{q_i^c : i \in N_g,\, p^c} \sum_{i \in N_g} [(p^c + \varphi_i^c) \cdot q_i^c - C_i(q_i^c)] + \sum_{z \in Z} (F_z - u_z^c) \cdot x_{g,z}$$

Forward settlement price

Forward contract price

subject to:

$$q_i^c \geq \underline{q}_i^c \quad i \in N_g$$

$$q_i^c \leq \overline{q}_i^c \quad i \in N_g$$

$$\sum_{j \in N} q_j^c = \sum_{j \in N} (P_j^c)^{-1} \left(p^c + \varphi_j^c \right)$$

$$u_z^c = \sum_{j : z(j) = z} \left(p^c + \varphi_j^c \right) \delta_j \quad z \in Z$$

Forward Market Decisions

EPEC Formulation – Each firm solves an MPEC

$$\max_{x_{g,z}:z\in Z} \quad \sum_{z\in Z} F_z x_{g,z} + \sum_{c\in C} \Pr(c)\left(\sum_{i\in N_g}\left(\left(p^c - \sum_{m\in L}(\lambda_{m-}^c - \lambda_{m+}^c)D_{m,i}^c\right)q_i^c - C_i\left(q_i^c\right)\right) - \sum_{z\in Z} u_z^c x_{g,z}\right)$$

subject to:

$$F_z = \sum_{c\in C} \Pr(c) u_z^c \qquad z\in Z$$

$$u_z^c = \sum_{j:z(j)=z} \left(p + \varphi_i\right)\delta_i \qquad z\in Z$$

and

Complementarity conditions characterizing spot market

The EPEC problem structure

- The MPECs for each firm

$$
\boxed{\begin{array}{l}
\min \quad f_g(x_g, y, w, x_{-g}) \\
s.t.\ x_g \in X_g \\
w = t + A^g x_g + A^{-g} x_{-g} + My,\ 0 \le w \perp y \ge 0
\end{array}}
$$

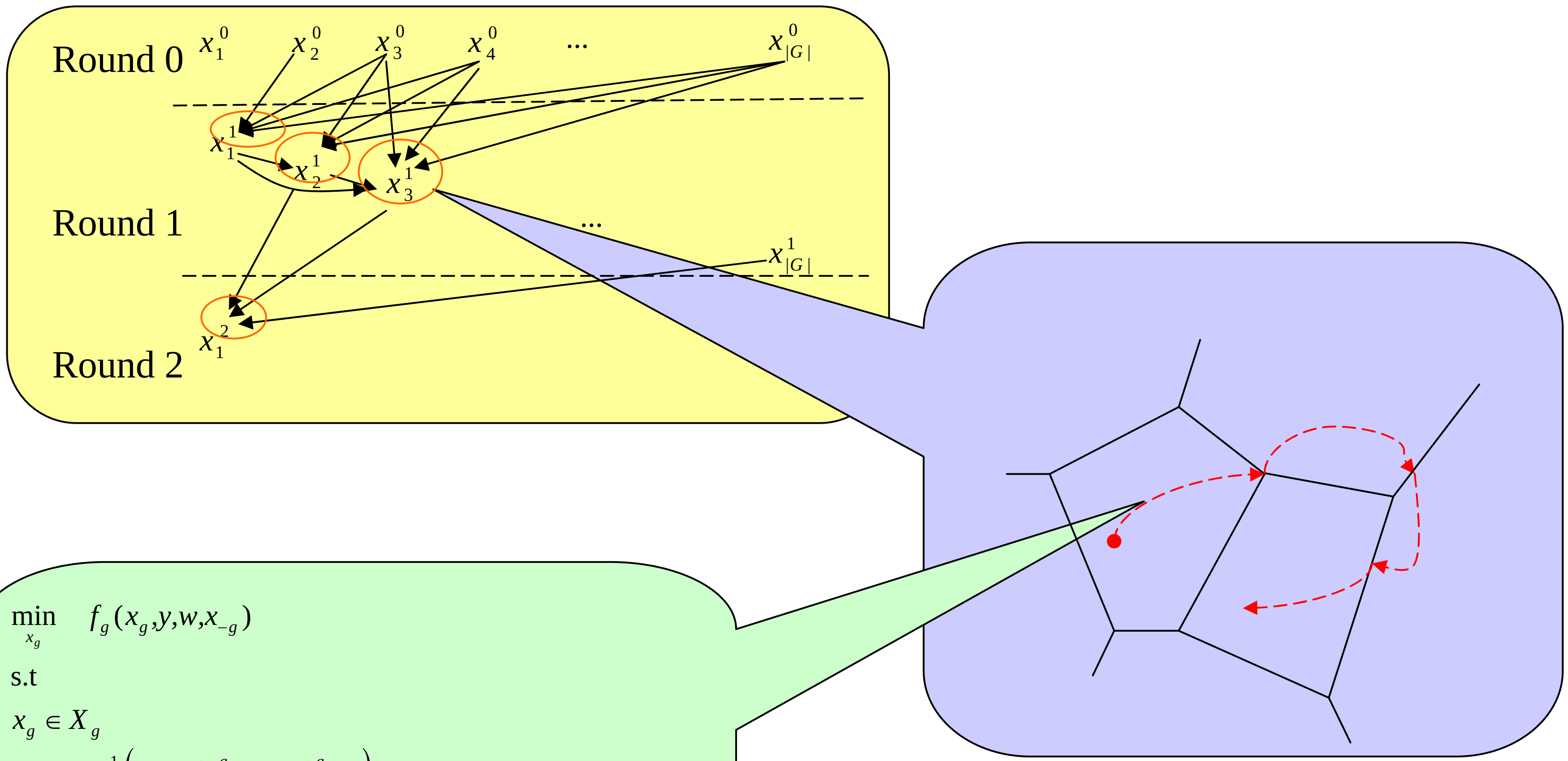
x_g: decision variable, y,w: state variables, x_{-g}: parameters

- The EPEC

$$
\begin{array}{llll}
\min \quad f_1(x_1, y, w, x_{-1}) & \min \quad f_2(x_2, y, w, x_{-2}) & \min \quad f_3(x_3, y, w, x_{-3}) & \cdots \\
\boxed{s.t.\ x_1 \in X_1} & \boxed{s.t.\ x_2 \in X_2} & \boxed{s.t.\ x_3 \in X_3} &
\end{array}
$$

$$
\boxed{w = t + \sum_g A^g x_g + My,\ 0 \le w \perp y \ge 0}
$$

The EPEC Algorithm



$$\min_{x_g} \quad f_g(x_g, y, w, x_{-g})$$

s.t

$$x_g \in X_g$$

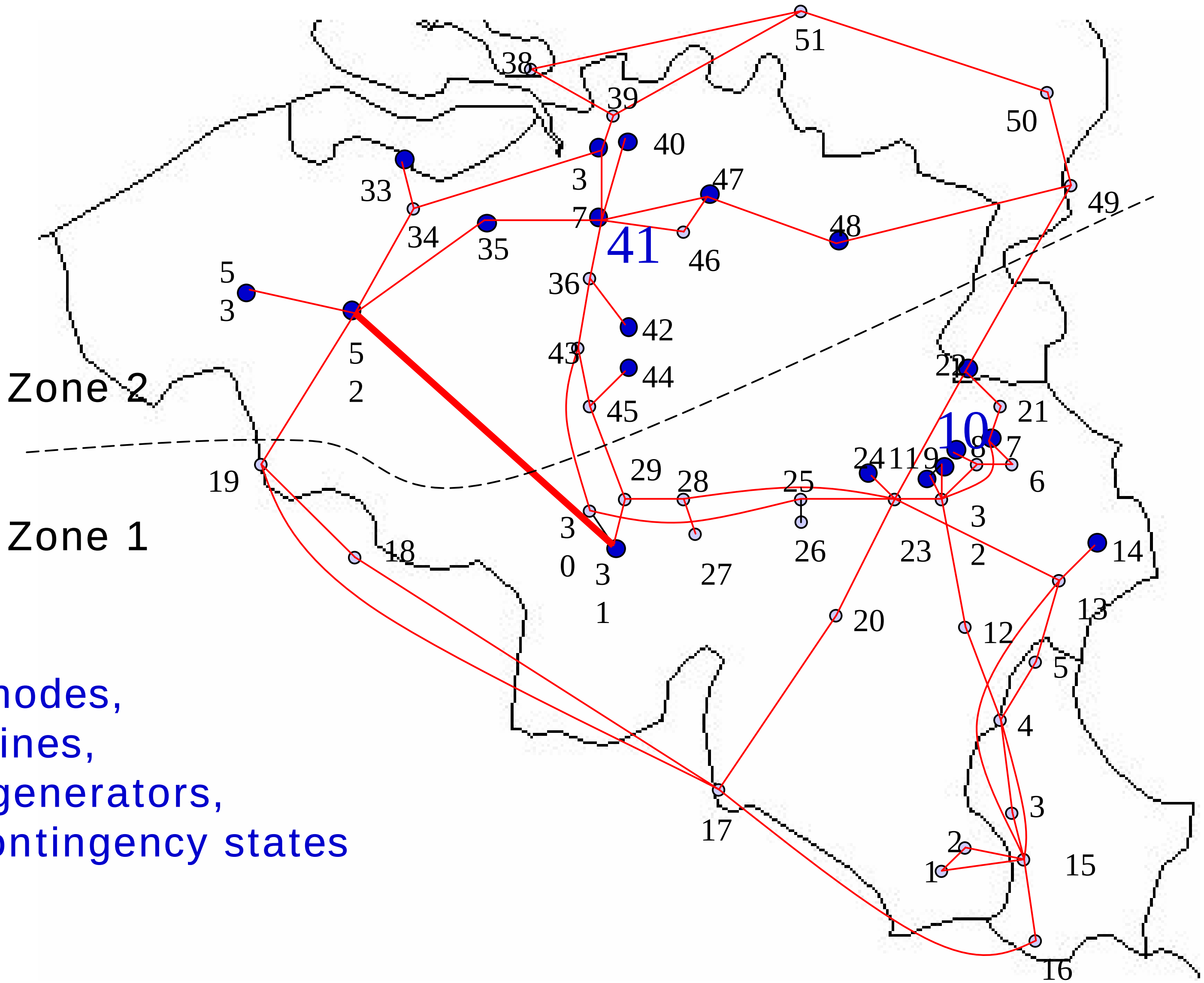
$$y_\alpha = M_{\alpha\alpha}^{-1}\left(a_\alpha + A_{\alpha\bullet}^{-g}\bar{x}_{-g} + A_{\alpha\bullet}^{g} x_g\right)$$

$$w_{\bar{\alpha}} = -M_{\bar{\alpha}\alpha} M_{\alpha\alpha}^{-1}\left(a_\alpha + A_{\alpha\bullet}^{-g}\bar{x}_{-g} + A_{\alpha\bullet}^{g} x_g\right) + \left(a_{\bar{\alpha}} + A_{\bar{\alpha}\bullet}^{-g}\bar{x}_{-g} + A_{\bar{\alpha}\bullet}^{g} x_g\right)$$

$$w_\alpha = 0$$

$$y_{\bar{\alpha}} = 0$$

Stylized Belgian System



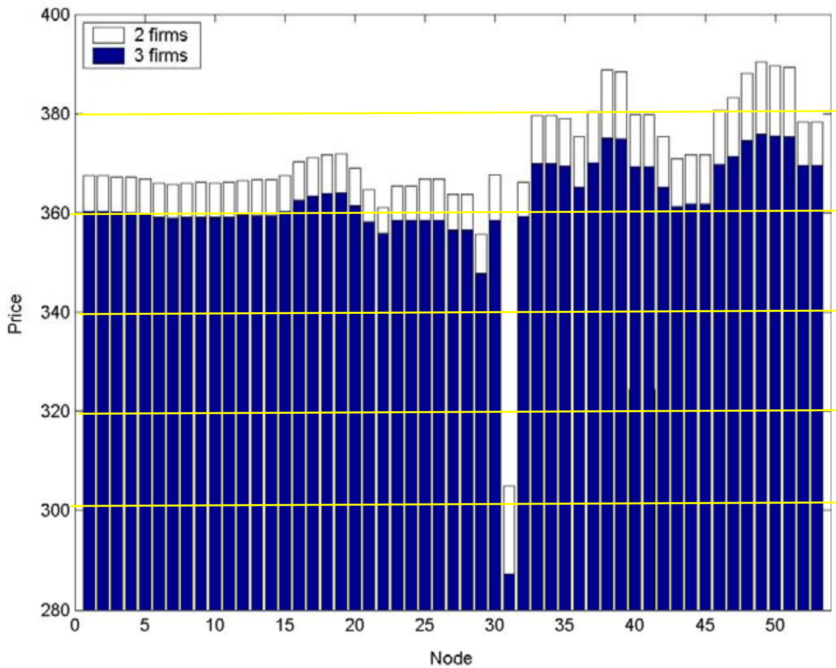
53 nodes,
71 lines,
19 generators,
6 contingency states

Contingency states

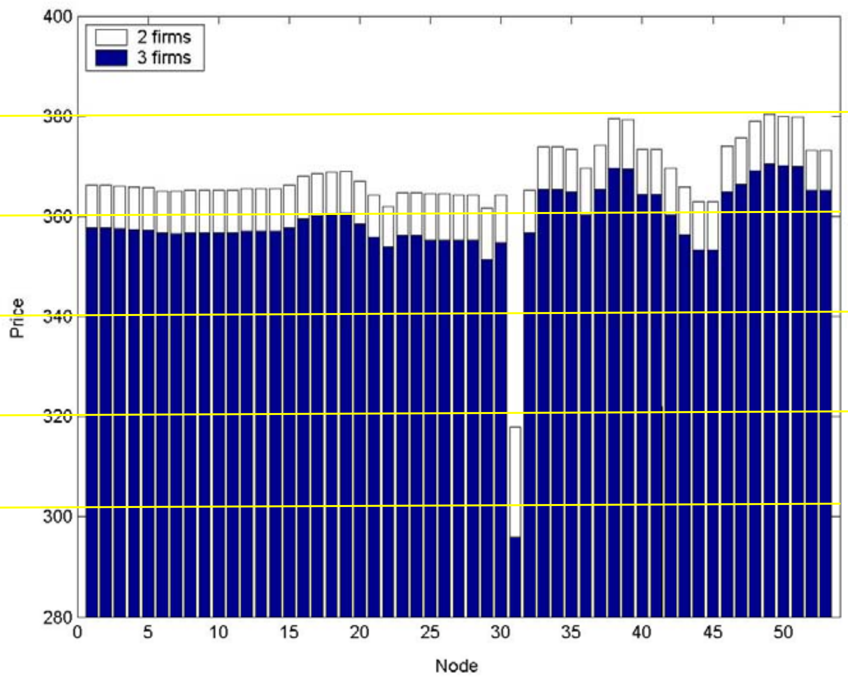
State	Prob	Description
1	.2	On-peak state: All demands are on the peak.
2	.5	Normal state: Demands are at shoulder.
3	.03	Shoulder demands with line breakdown: Line [31,52] goes down.
4	.03	Shoulder demands with generation outage: Plant at node 10 goes down.
5	.04	Shoulder demands with generation outage: Plant at node 41 goes down.
6	.2	Off-peak state: All demands are off-peak.

Impact of Forward Contracting on Spot Prices (in normal state)

Single Settlement



Two Settlement



Firm's Forward Commitments

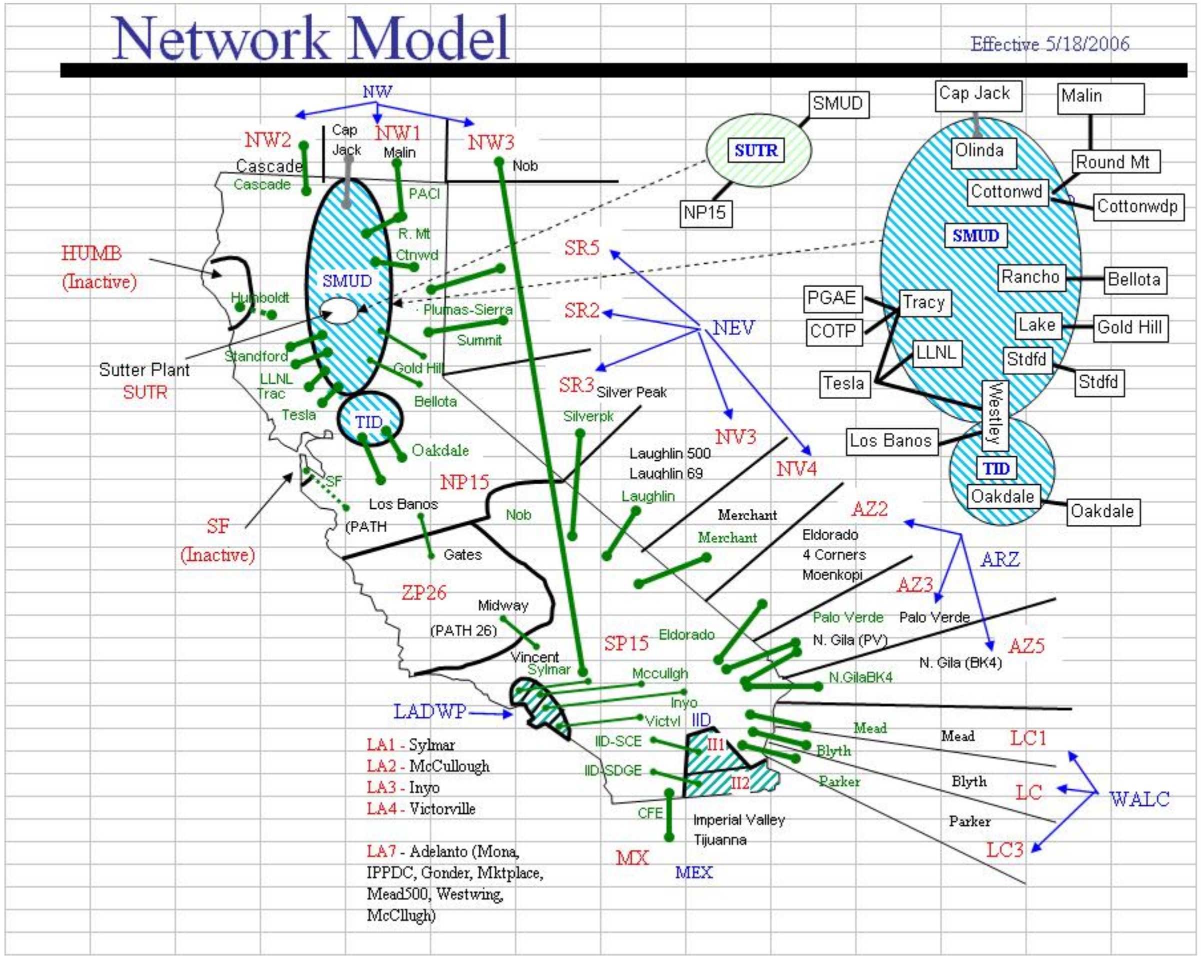
2 Firms

Outer iteration	Firm 1	Firm 2
0	0.000000	0.000000
1	-513.063752	575.219726
2	-331.223467	1546.721883
3	-545.254227	1747.692181
4	-552.287608	1747.692181
5	-552.287608	1747.692181

3 Firms

Outer iteration	Firm 1	Firm 2	Firm 3
0	0.000000	0.000000	0.000000
1	6739.889190	-16.249658	-288.471837
2	6739.889190	246.601419	-103.536223
3	6851.687937	556.357457	71.319790
4	7001.487699	849.405693	154.719273
5	7154.268773	1001.093059	149.846951
6	7237.416442	1006.167745	149.619740
7	7239.775870	1006.342137	149.611431
8	7239.859233	1006.348165	149.611140
9	7239.862110	1006.348374	149.611129
10	7239.862110	1006.348382	149.611130
11	7239.862110	1006.348382	149.611130

The California Network



Test Case – WECC Light Summer 2005

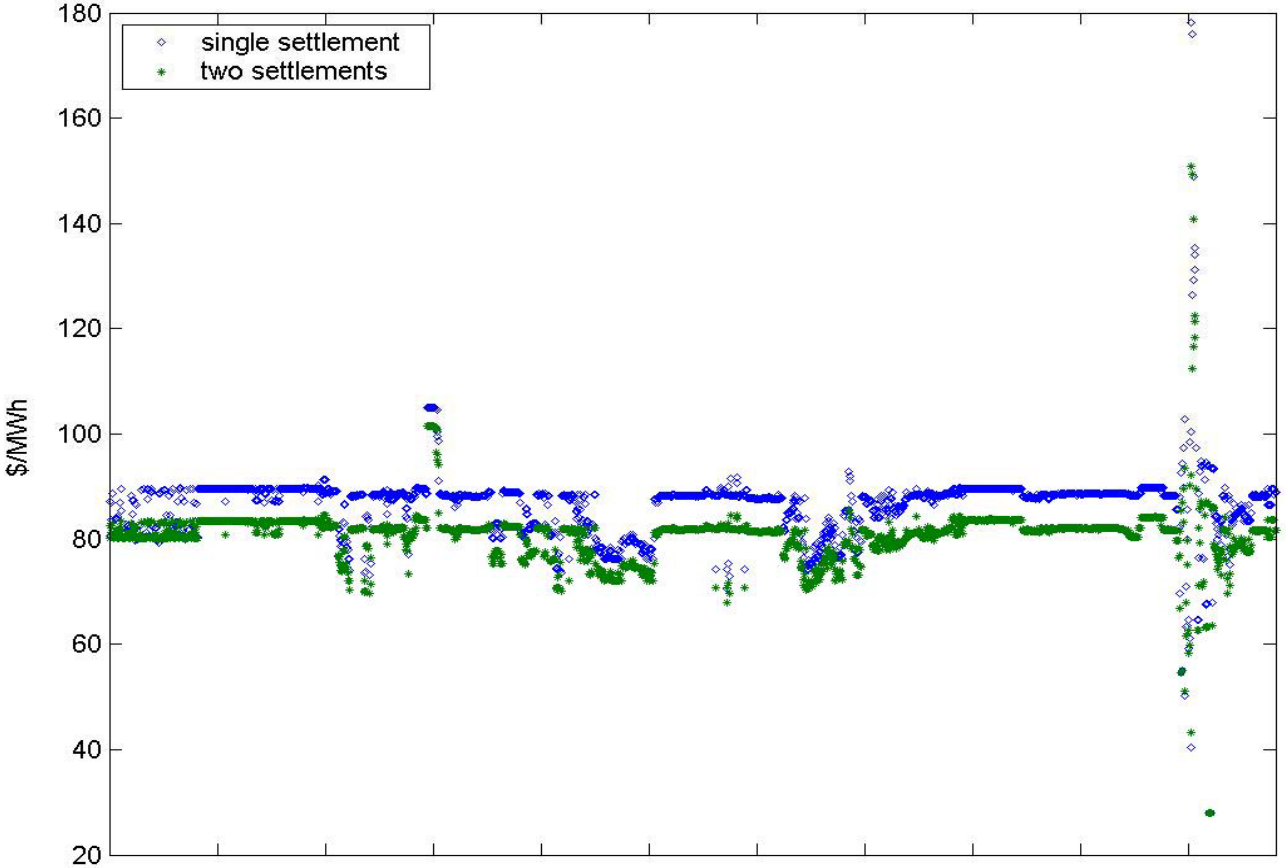
Total number of buses	2161
Number of generation buses	401
Number of consumption buses	1205
Number braches/transformers	3398
Number of firms	16 (9 strategic players)
Total demand	22700 MW

Spot and forward trading

	Total installed capacity (MW)*	Spot output under two settlements (MW)	Forward contracts (MW)
Southern California Edison	22407	4887.1	444.2
SDG&E	3205	2132.2	0
WAPA – SNR	825.8	825.8	825.8
Bureau of Reclamation (PG&E)	1439	1208.6	0
PG&E customer owned facilities	16720	4770.6	508
Department of Water Resources	914.3	914.3	914.3
Sacramento Utility District	2119.5	1893.4	98.8
PG&E	7921	3320.2	76.1
Northern California Power Agency	633.4	633.4	633.4
Total	**56185**	**20847.3**	**3500.5**

- Small units (with capacities less than 10MW) are ignored.
- The total number of iterations is 79 (stopped with a relative error of 1e-5).

Nodal Prices



	Single settlement	Two settlement
Range of nodal prices($/MWh)	**27.9 ~ 179.6**	**28.0 ~ 117.4**
% Change in nodal prices		**-56.5 ~ 29.9**
Average nodal prices	**87.9**	**65.6**

Questions?