Modeling and Control Design of Magnetic Levitation System

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***Abstract* — The aim of this paper is to design of control algorithm for the Magnetic levitation system using an exact input – output feedback linearization method. In this paper is given nonlinear simulation model of the Magnetic levitation based on mathematical model of the Magnetic levitation system. Designed control algorithm together with simulation model of the Magnetic levitation is implemented into control structure with purpose of control on steady state defined by a reference trajectory, which is verified in Matlab/Simulink language.**

***Keywords* –- modeling, control design, magnetic levitation, exact linearization method**

I. Introduction

The Magnetic levitation system is an example of nonlinear, open loop unstable system with fast dynamics. For these properties of the Magnetic levitation system, modeling and mainly control design is very difficult. However, Magnetic levitation system has wide application in various industries than high-speed trains, frictionless bearing, etc and therefore this field of research is devoted significant effort in recent years.

Some linear and nonlinear approaches were used to design control algorithm for Magnetic levitation system as a linear state control [1], adaptive control [2] or an exact linearization [3], [4].

In this paper will be presented application of the exact input - output feedback linearization method in design of the control algorithm for simulation model of the Magnetic levitation based on a real model Magnetic levitation CE 152 of Humusoft [7], which is located at the laboratory of Cybernetics at the Department of Cybernetics and Artificial Intelligence at FEI TU Kosice.

The paper is organized as follows. A mathematical model of the Magnetic levitation system is shown in the part two. The third part includes a general description of the exact input – output feedback linearization method, define a relative order of the system and internal resp. zero dynamics of the system [6]. The fourth part contains linearization of the nonlinear model of Magnetic levitation using the exact input – output feedback linearization method and control algorithm design for the model thus defined. Simulation nonlinear model of the Magnetic levitation together with the proposed control algorithm is implemented into control structure with purpose of control on steady state resp. more steady states defined by the reference trajectory, which is then verified in Matlab/Simulink program language.

II. Nonlinear Model of the Magnetic Levitation System

The Magnetic levitation system is shown on Fig. 1. It consists of a Magnetic levitation education model, laboratory card and control computer. The essence of this system is keep levitate the steel ball in the air by using electromagnetic force, which is produced from electric current going through the coil with soft magnetic core.

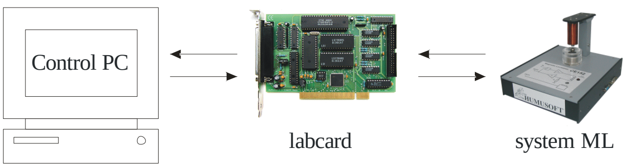

Figure 1. Magnetic levitation system

The mathematical model of the Magnetic levitation system is divided into five subsystems, namely ball and coil, power amplified, position sensor, and A/D and D/A converters.

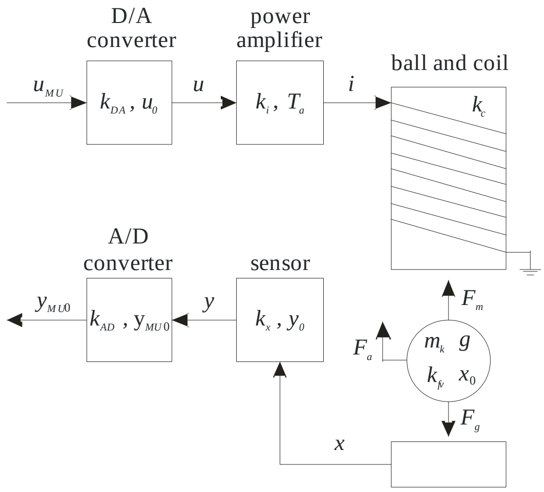

Figure 2. Magnetic levitation model

The mathematical model of the ball and coil subsystem is based on the balance of forces acting on the ball

$$F_a = F_m - F_g \tag{1}$$

where: F_a - acceleration force [N]
F_m - electromagnetic force [N]
F_g - gravity force [N]

After substituting relationship for each power in equation (1) and adding a damping force F_{fv} is obtained mathematical model of the ball and coil subsystem, which is described by nonlinear differential equation of second order:

$$\underbrace{m_k \ddot{x}(t)}_{F_a} - \underbrace{k_{fv}\dot{x}(t)}_{F_{fv}} = \underbrace{\frac{i(t)^2 k_c}{(x(t)-x_0)^2}}_{F_m} - \underbrace{m_k g}_{F_g} \qquad (2)$$

where: $i(t)$ - electric current [A]
- $x(t)$ - ball position [m]
- m_k - mass of ball [kg]
- k_c - coil constant [A/V]
- x_0 - coil offset [m]
- g - gravity constant [m/s^{-2}]
- k_{fv} - damping constant [N/m.s]

Position of the ball in the magnetic field is controlled by electric current $i(t)$, which is generated from the power amplified. The power amplified is designed as a source of constant current and is described transfer function F_z:

$$F_z = \frac{I(s)}{U(s)} = \frac{k_i}{T_a s + 1} \qquad (3)$$

where: $I(s)$ - image of electric current $i(t)$
- $U(s)$ - image of input voltage $u(t)$
- k_i - coil and amplified gain [A/V]
- T_a - coil and amplified time constant [s]

An inductive sensor is used to determine the ball position, which is approximated by a linear equation

$$y(t) = k_x x(t) + y_0 \qquad (4)$$

where: $y(t)$ - sensor output voltage [V]
- $x(t)$ - ball position [m]
- k_x - sensor gain [V/m]
- y_0 - sensor offset [V]

The signal incoming from laboratory card resp. from inductive sensor for communication with surrounding is necessary adjusted for further processing and therefore D/A resp. A/D converter is added into mathematical model. The converters can be described by linear equations [7]:

D/A converter : $u(t) = k_{DA} u_{MU}(t) - u_0$ (5)

A/D converter : $y_{MU}(t) = k_{AD} y(t) - y_{MU0}$ (6)

where: $u(t)$ - converter output voltage [V]
- $u_{MU}(t)$ - converter input voltage [MU]
- k_{DA} - converter gain [V/MU]
- u_0 - converter offset [V]
- $y_{MU}(t)$ - converter output voltage [MU]
- $y(t)$ - converter input voltage [V]
- k_{AD} - converter gain [MU/V]
- y_{MU0} - converter offset [MU]

Based on equations (2) to (6), which describe mathematical model of the Magnetic levitation system was programmed simulation scheme of the Magnetic levitation nonlinear model (Fig. 3).

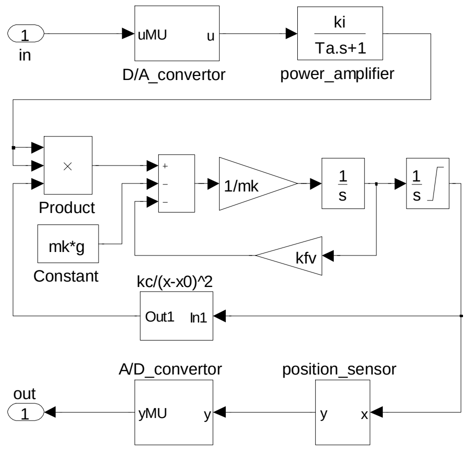

Figure 3. Simulation scheme of Magnetic levitation model

Since the Magnetic levitation system is unstable in open loop, physical analysis of the simulation model was carried in a feedback control structure with defined reference signal y_{ref} and experimental proposed PID parameters [7]. The result graph (Fig. 4) shows the possibility of further use of Magnetic levitation model in the control structure using exact feedback linearization method (Fig. 5).

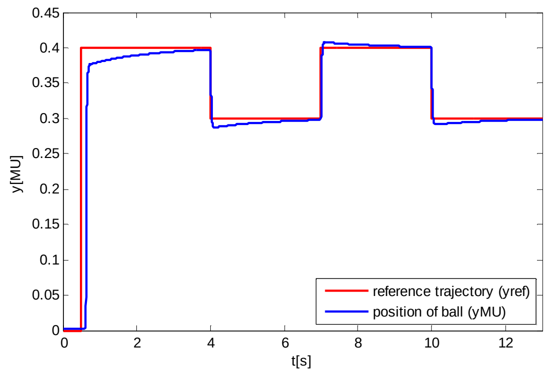

Figure 4. Physical analysis of the Magnetic levitation simulation model

III. METHODS OF SYNTHESIS OF NONLINEAR SYSTEM

The basic task of the synthesis of a nonlinear systems is to propose control algorithm, which ensures that the nonlinear system will be behave according to desired goals of control, either stabilization of system or system tracking trajectory.

Many nonlinear systems can be controlled by classical PID controllers, which ensure desired quality of regulation. However, there are systems with large range or with nonlinearities, which can not be linearized and for such dynamic systems is not sufficient PID control algorithm. In such cases can be used the nonlinear synthesis methods, which are directly for nonlinear systems such as Gain – Scheduling method, control design by Lyapunov function, exact linearization method and etc. The analysis and synthesis of the nonlinear systems requires knowledge of mathematical disciplines such as differential geometry, etc., and also because these methods have limited use in practice, but with development of computer technology grows use of these methods.

In this paper is given procedure of linearization using exact input – output feedback linearization method for nonlinear model of the Magnetic levitation.

A. *Exact input-output feedback linearization method*

The exact input – output feedback linearization method is one of the structure methods. This method based on the idea to compensate nonlinearities in the system by adding nonlinear compensators thus that the resulting system will be behave as linear to respect to the new input $v(t)$ and output $y(t)$.

The assumption for use of the exact input – output feedback linearization method is nonlinear systems written in state space representation

$$\begin{aligned} \dot{x}(t) &= f(x,t) + g(x,t) * u(t) \\ y(t) &= h(x,t) \end{aligned} \qquad (7)$$

where $x \in R^n$ is state vector, $u(t)$ is control input, $y(t)$ is system output, $f(x,t)$, $g(x,t)$ and $h(x,t)$ smooth nonlinear functions.

Principle of the exact input – output feedback linearization method is based on repeatedly derivative of the output $y(t)$ until a dependence on the input $u(t)$. Number of derivation indicates a relative order of system r. For better overview, further will not write dependence of variables on the time t.

Linearization of the nonlinear system (7) using exact input – output feedback linearization method in steps:

1. calculation of first derivation :

$$\dot{y} = \frac{\partial h}{\partial x} f + \frac{\partial h}{\partial x} gu = L_f h(x) + L_g h(x) u$$

2. if valid $L_g h(x) u \neq 0$ then after substitution $\dot{y} = v$ is possible to determine the resulting input transformation

$$u = \frac{1}{L_g h(x)} (-L_f h(x) + v)$$

3. but if valid $L_g h(x) u = 0$ then continues derivative of the output

$$\ddot{y} = L_f{}^2 h(x) + L_g L_f h(x) u$$

4. if valid $L_g h(x) u = 0$ then continues derivative of the output y
5. derivative of the output continues until will not valid $L_g L_f{}^{r-1} h(x) u \neq 0$ the after substitution $y^r = v$ the resulting input transformation is equal

$$u = \frac{1}{L_g L_f{}^{r-1} h(x)} (-L_f{}^r h(x) + v) \qquad (8)$$

6. to determine state transformation $z = T(x)$

$$z = T(x) = [h(x); L_f h(x), ..., L_f{}^{r-1} h(x)] \qquad (9)$$

7. nonlinear system (7) can be transformed into linear form

$$\dot{z} = \begin{bmatrix} 0 & 1 & ... & 0 \\ 0 & 0 & 1 & 0 \\ ... & ... & ... & ... \\ 0 & 0 & ... & 0 \end{bmatrix} z + \begin{bmatrix} 0 \\ 0 \\ ... \\ 1 \end{bmatrix} v \qquad (10)$$

8. for linear system (10) is necessary to propose feedback control law by linear method of synthesis, to ensure the desired behavior of the system in case of change of the reference trajectory or for compensation disturbance.
9. input transformation (8), state transformation (9) and a control algorithm (step 8) are implemented into control structure (Fig. 5)

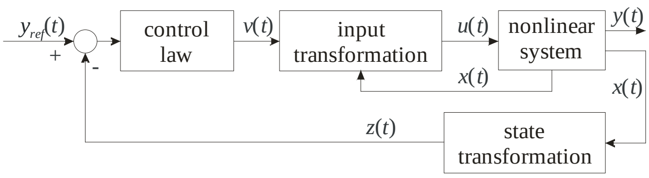

Figure 5. control structure using exact feedback linearization method

When exact input – output feedback linearization method is used then can be case, when relative order of nonlinear system $r < n$. In this case, state variables are divided into two vectors $\xi = [z_1, z_2, ..., z_r]^T$ and $\eta = [z_{r+1}, z_{r+2}, ..., z_n]^T$. For state variables of the vector ξ proceed as in case when $r = n$. Variables from vector η are chosen so as to be mutually independent. Thereafter, control is proposed, that linear dynamic is dictated only output and first r state variables. The other state variables do not contain input of the system u, so they are not controlled, but require that they are limited and do not exceed specified limits.

In this case, the total dynamics of the system can be divided in external controlled part (state variables of the vector ξ) and internal unobserved and uncontrolled part (state variable of the vector η) also known as the internal dynamic of the system. The internal dynamics of the system is dependent on the external dynamic of the system and also can be depend on the input signal. From theory is known, to determine the internal dynamics of the system is very difficult, but designed control for external dynamics is applicable only when the internal dynamics of the system is stable, otherwise may happen to unwanted events such as oscillation of the system. Therefore, the new term was introduced so-called zero dynamics of the system, which simplifies of the investigation of stability of the system. The zero dynamics means the state of the internal dynamics of the system, when input signal into the system causes that output of the system is equal zero during whole process. The examination of the may draw some conclusions for the stability of the internal dynamics of the system, which can be sufficient in some cases for to determining the resulting stability of the internal dynamics of the system. [7]

The problem of the internal dynamics occurs in the control design using this method for the Swinging crane system [8] or Ball&Beam system [9].

IV. DESIGN OF NONLINEAR CONTROL ALGORITHM

This part is given linearization of the nonlinear model of the Magnetic levitation using exact input – output feedback linearization method and control algorithm design for model thus obtained. For better overview, further will not write dependence of variables on the time *t*. The first step is to rewrite equations (2) to (6), which describe the nonlinear system of the Magnetic levitation, to state space form (7). By defining the state vector $x = (x_1, x_2, x_3) = (x, \dot x, i)$, input $u = u_{MU}$ and output $y = y_{MU}$ state space form of the Magnetic levitation has following form:

$$\begin{bmatrix}\dot x_1\\ \dot x_2\\ \dot x_3\end{bmatrix} = \begin{bmatrix} x_2 \\ A\dfrac{x_3^2}{(x_1-x_0)^2} - g + Bx_2 \\ -\dfrac{x_3}{T_a}\end{bmatrix} + \begin{bmatrix}0\\0\\C\end{bmatrix}u \qquad (11)$$

$$y = Dx_1$$

where $A = \dfrac{k_c}{m_k}$, $B = \dfrac{k_{fv}}{m_k}$, $C = \dfrac{k_i k_{da}}{T_a}$, $D = k_{ad} k_x$.

The next step is derivative of the system output *y* until a dependence on the input *u*:

$$y = Dx_1$$

$$\dot y = Dx_2$$

$$\ddot y = D\left(A\frac{x_3^2}{(x_1-x_0)^2} - g + Bx_2\right)$$

$$\dddot y = \underbrace{\frac{2ADx_2x_3^2}{(x_0-x_1)^3} + BD\left(bx_2 - g + \frac{ax_3^2}{(x_0-x_1)^2}\right) - \frac{2ADx_3^2}{T_a(x_0-x_1)^2}}_{\alpha}\dots$$

$$\dots + \underbrace{\frac{2ADx_3C}{(x_0-x_1)^2}}_{\beta}u$$

After substitution $\dddot y = v$, the resulting input transformation has following form

$$u = \frac{1}{\beta}(-\alpha + v) \qquad (12)$$

and corresponding state transformation $z = T(x)$ can be determined as

$$z = \begin{bmatrix} z_1\\ z_2\\ z_3\end{bmatrix} = \begin{bmatrix} y\\ \dot y\\ \ddot y\end{bmatrix}. \qquad (13)$$

After application of the input transformation *u* (12) and state transformation *z* (13) is possible transformed the nonlinear system (11) to following linear form

$$\dot z = \begin{bmatrix}0 & 1 & 0\\ 0 & 0 & 1\\ 0 & 0 & 0\end{bmatrix} z + \begin{bmatrix}0\\0\\1\end{bmatrix} v . \qquad (14)$$

Based on matrix of the state space (14), suitable chosen roots $p = [p_1, p_2, p_3]$ and using pole – placement method was designed state feedback control law in the form

$$v = -Kz + Ny_{ref} \qquad (15)$$

where: K - vector of gains

N - feedforward gain [5].

The resulting input transformation (12) and state transformation (13) together with control law (15) are implemented into programmed simulation scheme for control of the nonlinear model of the Magnetic levitation (Fig. 6).

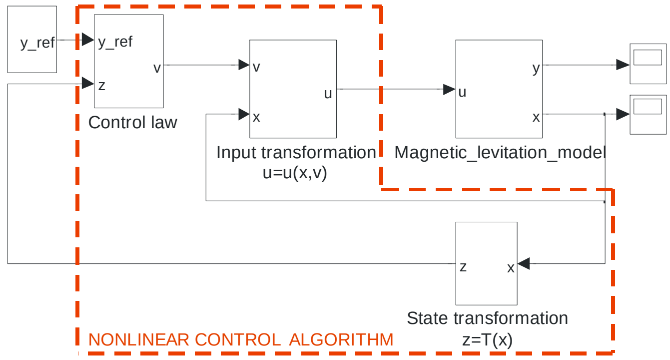

Figure 6. Simulation scheme for control of the nonlinear model of the Magnetic levitation using exact input – output feedback linearization method

The resulting graph of tracking reference trajectory, which is represented the step change between steady state, when using proposed control algorithm and exact input – output feedback linearization method is in Fig. 7.

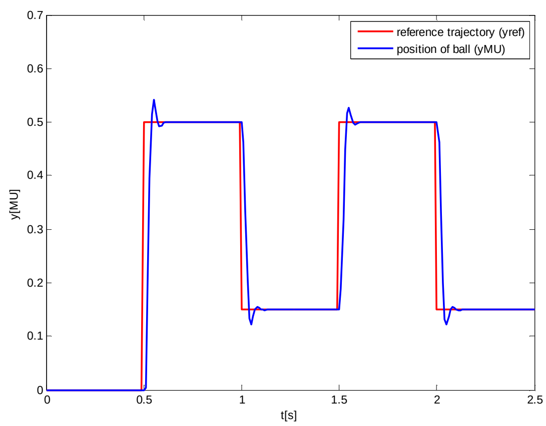

Figure 7. Magnetic levitation system response to track the reference trajectory

Designed control algorithm was applied to the nonlinear model of the Magnetic levitation with all of the limits and this allows its use for real model Magnetic levitation.

V. CONCLUSION

In this paper was presented control algorithm design for nonlinear simulation model of the Magnetic levitation using the exact input – output feedback linearization method and pole – placement method. The proposed control algorithm together with simulation model was implemented into control structure and verified in Matlab/Simulink program language. The resulting graph shows, that output of the model tracks step change of the reference trajectory and therefore can be considered, this approach is suitable for solution problem of control for Magnetic levitation system. The obtained knowledge from field of methods of synthesis of nonlinear systems will be used for control design for real model of the Magnetic levitation and also for to support of teaching in the Optimal and Nonlinear system subject.

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