Modeling and Control of Complex Physical Systems

The Port-Hamiltonian Approach

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(Eds.)

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This book is dedicated to all the people that worked in the GEOPLEX project and to all the students and researchers that joined the GEOPLEX public events: their enthusiasm deeply motivated us to prepare this book

Foreword

The story behind this book begins during the International Conference on Robotics and Automation 2001 which took place in Seoul, Korea in May of 2001. During the farewell reception I had a nice talk with Henrik Christensen, founder and scientific director of EURON, the European Robotic Network. Henrik mentioned that during some discussion with the European commission it came out that there was interest for good fundamental projects for the last call of FP5 related to the KA 4, Action Line IV2.1 on Advance Control Systems. At the end of August, after discussions with Arjan van der Schaft also at the University of Twente, it was decided to work on a proposal related to the great potentials of port-Hamiltonian systems initially introduced by Arjan van der Schaft and Bernhard Maschke. The deadline for the Submission was October 17th 2001 and it was at that stage not even clear who would be involved in the consortium. Around the end of September the real writing began and the consortium was formed by the University of Twente (NL) as a coordinator under my responsibility, Control Lab Products (NL) who with the 20-sim modeling and simulation program should have provided the tools implementing the new ideas in the project, Université Claude Bernard Lyon 1 (F) under the leadership of Bernhard Maschke, Universitat Politècnica de Catalunya (SP) under the leadership of Enric Fossas Colet, Supelec (F) under the leadership of Romeo Ortega, Johannes Kepler Universitat Linz (A) under the leadership of Kurt Schlacher, Katholieke Universiteit Leuven (B) under the leadership of Ir. Herman Bruyninckx, l'Università degli Studi di Bologna (I) under the leadership of Claudio Melchiorri and finally the CNRS (F) with Francoise Couenne: a great consortium was born.

During an incredible active period and difficult moments in which I did not believe we were going to make it, many sleepless nights and hard work brought us to a successful submission before the deadline. The project name was *"Geometric Network Modeling and Control of Complex Physical Systems"* with Acronyms GEOPLEX proposed by Herman Bruyninckx on an email dated 7th September 2001.

During the project preparation we decided to have as a deliverable a book which would collect some of the major results of the project. This volume is the final result of this effort. Many people have contributed to this volume and tough decisions have been made in what to include and what to leave out in order to have a volume with didactic value and with a reasonably homogeneous notation.

The official "book deliverable" was a draft and a lot of work still needed to be done in order to get it to a useful publishable volume. The initial editorial efforts where done by Vincent Duindam, Herman Bruyninckx and myself during many meetings and discussions about the structure, the homogeneity and the various possibilities. After the end of the project, due to many obligations, the book has been in stand-by for a while until I kindly asked the help and support of Alessandro Macchelli (Alex) from the University of Bologna who has been also extremely scientifically (and not only) active during the project. Thanks to Alex this major group effort has finally become a real book and I can speak for the all consortium that we are really proud of this result.

The four years of the project beside having been scientifically productive and brought us to many new results, have created a wonderful interpersonal synergy which is still bringing fruits to this beautiful field. The GEOPLEX journey has been a great one and even if not all of you have enjoyed the great atmosphere and scientific discussions, I hope you will enjoy the result of this successful project and wonderful theory on port-Hamiltonian systems.

Enschede (NL), March 2009 *Stefano Stramigioli*

Preface

This preface gives a "bird's eye" view on the *paradigm* of port-Hamiltonian systems [137] for modelling and control of complex dynamical systems, which will be explained in detail in the rest of this book. The mentioned complexity comes, in the first place, from the *scale* of the systems, which is too large to be captured and controlled reliably by the traditional "block-diagram" approaches. This preface explains why this paradigm has a large potential to be successful in tackling some of the big challenges in modern control theory and engineering. Three of the paradigm's major features are:

i) its scalability to very large interconnected multi-physics systems;
ii) its ability for incorporating non-linearities while retaining underlying conservation laws;
iii) its integration of the treatment of both finite-dimensional and infinite-dimensional components.

But also for more traditional control problems, the port-Hamiltonian systems paradigm provides a solid foundation, which suggests new ways to look at control problems and offers powerful tools for analysis and control.

The port-Hamiltonian systems paradigm has, over the last decade, succeeded in matching the "old" framework of port-based network modeling of multi-domain physical systems with the "new" framework of geometric dynamical systems and control theory. It provides a very systematic approach to modelling, analysis and control, via

i) the separation of the network interconnection structure of the system from the constitutive relations of its components;
ii) the emphasis on *power flow* and the ensuing distinction between different kind of variables;
iii) the analysis of the system through the properties of its interconnection structure and the component constitutive relations;
iv) the achievement of *control by interconnection*, by means of *stabilization by Casimir generation and energy shaping* , *energy routing control* (transferring energy between components in the system), and *port and impedance control*.

Some familiarity with ‘geometry’, in particular ‘coordinate-free’ thinking and the identification of physically different types of variables with different mathematical objects, is the price to pay for a complete understanding of the paradigm: while port-Hamiltonian systems may at first sight make things “unnecessarily complicated” for the most simple systems, it can reach much further than traditional paradigms, with not much more than the same set of concepts that are used for these simple systems. This book’s major ambition is to convince its readers that

i) the extra mathematical complexity introduced in port-Hamiltonian systems is the *necessary minimum* to represent the essential inherent properties of large-scale interconnected physical systems;
ii) understanding these mathematical concepts drastically reduces the human effort to master the intellectual “curse of dimensionality” created by tackling complex dynamical systems by only the traditional “block-diagram” control approaches.

Complex dynamical systems

Modern control engineering is continuously challenged to provide modelling, analysis and control for ever more *complex* systems:

- Complexity in the sense that the modern consumer expects to see more ‘intelligent’, better performing, and yet more miniaturized and/or lighter products. Most of the current sensing and actuation components do not scale well towards these meso- and milli-scales; at least, much less than the computational components. Inevitably, this evolution requires the development of sensor/actuator components that are integrated into the mechanical structure of the products. This will most certainly lead to components that cannot be modelled (and produced !) anymore as traditional finite-dimensional, i.e. “lumped parameter”, systems.
- Complexity in the sense that production, logistics and service facilities evolve towards more distributed systems, with more decentrally controlled degrees of freedom, more interactions between various controlled subsystems, less possibilities to define, let alone measure, all relevant variables in the system, etc. Continental-scale power grids are a nice example: electricity is often produced further and further away from the final consumer; new energy sources (e.g., wind, biomass, energy recuperation) require more flexibility and bring higher load and source irregularities in the grid control system. Experience has shown that the traditional “optimized” control of the power grids has quite some problems with robustness against sudden transient effects, such as line breakage.

The traditional control approaches have, up to now, to a very large extent been focusing on “human-scale” systems, where one single control engineer can comprehend the whole system, one centralized controller can do the whole job, and one “Simulink” block diagram suffices to model the whole system dynamics to the required level of detail and to optimize its control to the required level of performance.

This approach, however, seldom scales well in the above-mentioned evolution towards more complex systems. Some of the major scaling problems are:

- Block diagram modeling of physical systems lacks compositionality: whenever interconnecting a physical system to additional components the block diagram modeling usually needs to be redone completely.
- The presence of fundamental physical properties such as conservation laws and energy balance is not reflected in the block diagram structure, and easily gets out of sight when the scale of the system grows.
- *Block diagram causality:* the large majority of control engineers is only familiar with the "Simulink"-like block diagram approach. Most of them don't even realize that, for modelling and computational simplicity, this approach *imposes* one specific physical and computational causality (i.e., a fixed choice of what is *input* and what is *output*) onto the system model and onto its controller, while the real physical system does not have these causal constraints.
- *Non-linearities:* the more nonlinear components appear in the system, the more difficult it gets to provide stable, efficient and optimized controllers with the traditional linear state space control theory that most engineers are trained in.
- *Integration of finite-dimensional and infinite-dimensional components:* no modelling and control paradigm has yet achieved the breakthrough in this domain.
- *Network size:* when the number of components in the system under control grows, the number of state variables also grows, as well as the communication delays between actuators, plant and sensors, the transmission line effects, etc. This means that a traditional centralized controller will not work anymore.
- *Robustness:* the traditional state space control paradigm has a big focus on optimized control, and the algorithms for designing robust controllers are still (implicitly) targeted at centralized systems. However, all the above-mentioned complexities drastically reduce the robustness of any optimal controller, if it has to work on a real-world complex system.

The port-Hamiltonian systems paradigm

Port-Hamiltonian systems represent a control *paradigm*, in the sense that they provide a set of models, thought patterns, techniques, practices, beliefs, systematic procedures, terminology, notations, symbols, implicit assumptions and contexts, values, performance criteria, ..., shared by a community of scientists, engineers and users in their modelling, analysis, design and control of complex dynamical systems. Being a coherent paradigm by itself does not mean that port-Hamiltonian systems have nothing in common with other control paradigms. The name "port-Hamiltonian" systems, for example, refers to the two major components of the paradigm, which exist for quite some time already:

- *Port:* the modelling approach is *port-based*, more in particular it builds upon the successful multi-domain Bond Graph way of composing complex systems by means of *power*-preserving interconnections.
- *Hamiltonian:* the mathematical framework extends the geometric Hamiltonian formulation of mechanics, by emphasizing the geometry of the state space and the Hamiltonian function (total stored *energy*) as basic concepts for modelling multi-physics systems.

Because of its roots in port-based modeling he port-Hamiltonian systems paradigm extends the geometric Hamiltonian formulation of physics by generalizing the geometry of the classical *phase space* to the geometry of the state space of energy variables which is determined by the (power-conserving) *interconnection structure*. Furthermore, it allows for the incorporation of *energy-dissipating*[1] components, and the presence of open ports modelling interaction with an (unknown) environment or accessible for controller interaction. Port-Hamiltonian systems theory relies on a rather limited amount of concepts from *geometry*. Geometry, in particular geometric linear algebra in the linear case and differential geometry in the nonlinear case, has proven to be a very appropriate mathematical formalism whenever one wants to separate generic, coordinate-free and (hence) intrinsic descriptions of systems from the details and particularities of specific representations, and if one wants to capture the physical characteristics of the variables involved in the mathematical description. Port-Hamiltonian systems shares this sympathy for geometry with, among others, the *geometric nonlinear control theory* paradigm [93, 156], and geometric mechanics [24, 40].

The *systematic procedure* for the modelling and control of complex dynamical systems, as it is beginning to materialize in the port-Hamiltonian systems theory, is as follows:

i) Model the system as energy storing and energy dissipating components, connected via ports to power conserving transmissions and conversions.
ii) Separate the network structure from the constitutive relations of the components.
iii) (Optionally) reduce the order of the system model while respecting the invariant *structure* of the system dynamics.
iv) Identify the *Casimir functions* (conservation laws) in the system, in order to use them in the design of the controller.
v) Control the system by *interconnecting* it to *energy shaping* and/or *damping injection* components, and by adding energy routing controllers.

The first focus of a port-Hamiltonian controller is to achieve a *feasible, stable and robust* control, instead of being driven by performance optimization from the start. As motivated above, this focus is already difficult enough when controlling increasingly complex dynamical systems.

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[1] Of course, from a thermodynamical perspective energy is not dissipated but converted from, say, the mechanical or electrical domain to, e.g., the thermal domain. It would therefore be more appropriate to speak of 'free energy' that is dissipated.

Fig. P.1 The logo of the GEOPLEX project.

The GEOPLEX project

The last few years, significant progress has been made in the port-Hamiltonian systems paradigm, to a large extent thanks to the concerted efforts of the GEOPLEX project [1], whose logo is displayed in Fig. P.1. Some of the major evolutions are: the port-Hamiltonian approach has excellent results in the systematic separation (conceptually, as well as in the mathematical representation) of the *interconnection structure* and the *dynamical properties of interconnected system*; finite-dimensional systems and infinite-dimensional systems can be described with unified concepts and mathematical representations, and one is nearing the above-mentioned breakthrough towards unified *control* of both domains; the *energy shaping* and *damping injection*, as well as the *energy routing*, control approaches begin to mature and show their advantages for the construction of safe and predictable controllers for complex systems.

GEOPLEX does not only have theoretical developments, but also real-world experimental verifications of the port-Hamiltonian systems approach: walking robots, teleoperated and haptic devices, piezo-controlled beams and plates, chemical engineering of reaction processes, electrical grids with sources and flywheels, etc..

Port-based modelling

Port-based modelling as in the Bond Graph formalism [165] models a physical system as the *interconnection* of (possibly a large amount) of components from a rather small set of dynamic elements: energy storage, energy dissipation, energy transportation or energy conversion. Each element interacts with the system via a *port*, that consists of a couple of “dual” *effort* and *flow* quantities, whose product gives the power flow in and out of the component. For example, force and velocity for a mechanical system, or current and voltage for an electric network. The network allows (loss-less) power exchange between all components and describes the power flows within the system and between the system and the environment. Some advantages of the Bond Graph approach are:

- It focuses on *energy* (in all its instantiations) as the fundamental physical concept to appropriately model the real world.

- It is *multi-domain*: the same concepts and mathematical representations are used for mechanical, electrical, hydraulic, pneumatic, thermo-dynamical, ..., components.
- It is *multi-scale*: components can be decomposed hierarchically in smaller interconnected components.
- Its models are *acausal*: each component contains only "constitutive relationships" which describe the dynamic relations between the port variables, without imposing which ones are inputs and which ones are output.

The Bond Graph approach has proven to be very successful in the modelling of complex systems, at least in the lumped parameter domain.

Differential geometry for systematic structuring

The above-mentioned system model networks, derived from port-based network modelling, can be mathematically described and analysed by means of the concept of a *Dirac structure* [54, 59, 183], at least for "lumped parameter" (finite-dimensional) systems. A Dirac structure can be regarded as a generalization of the well-known Kirchhoff laws of electrical circuit theory. It separates the (power conserving) *network topology* ("interconnection") from the (power storing or dissipating) dynamics of the components; both together provide a complete model of the dynamical system under study. The Dirac structure allows to bring *all* possible system models, however complex, into the same mathematical form, strongly facilitating a highly systematic treatment.

This systematic treatment has been extended to infinite-dimensional systems, for which the Dirac structure is generalized into the *Stokes-Dirac structure*, by incorporating Stokes theorem applied to the underlying conservation laws. Again, the same concepts are being reused.

Control by interconnection

The port-Hamiltonian systems paradigm uses the system's *interconnection structure* and its *Hamiltonian* (i.e., its total energy) as the primary vehicles for modelling and control. If one wants to steer the system to one of its stable equilibrium states, it is easy to do so: the Hamiltonian of the system assumes its minimum at this state, so, by introducing dissipation in the controller ("damping injection"), the energy in the system decreases until the minimum of energy, or, equivalently, the desired equilibrium configuration is reached.

However, the natural equilibrium states of the system seldom correspond to the desired system state. So, "energy shaping" is necessary, i.e., one has to add a control component to the system network, in such a way that the desired state in one way or another corresponds to a stable equilibrium of the new system. In summary, the port-Hamiltonian approach to set-point control is “control by interconnection”: the controller components that one adds to the plant network are the same kind of components as the ones that make up the model of the plant itself. In doing so, the first emphasis is on getting the controlled system robustly stable, possibly giving up on “optimal performance” of some of the subsystems.

How is this energy-shaping achieved? Clearly, the Hamiltonian of the control components may be chosen arbitrarily, but do not directly influence the shape of the Hamiltonian with respect to the state variables of the original system. One way to achieve energy-shaping is to add controller components in such a way that conserved quantities (*Casimirs*) are enforced involving the state variables of the original system and the controller states. Interestingly enough, these conserved quantities are determined by the Dirac structure of the interconnected system (original system *plus* controller system), thus leading to the problem of how to ‘shape’ or manipulate this Dirac structure by the interconnection with a controller Dirac structure. The total closed-loop energy function is then shaped by combining the Hamiltonian of the original system with the Casimirs and suitably chosen Hamiltonians for the controller systems. This procedure is systematic, but not explicit (that is, it still requires insight from the control designer), and it is not guaranteed to be applicable in all situations. Another (but very much related !) approach to stabilization, which in principle offers more options, is the Interconnection-Damping-Assignment methodology, where the energy function, the interconnection structure, as well as the damping structure, are directly modified by state feedback

Software support

The GEOPLEX project consists of some academic research institutions, plus one company: Controllab Products, which develops and markets the 20-sim [51] simulation and control software. 20-sim has Bond Graph modelling tools under the hood, but can also provide more user-friendly (because domain-specific) iconic diagrams; it has extensive algorithmic support for causality determination, for solving the differential equations governing a particular model (including controller), and for transforming the control design into code for embedded control computers. In addition, the software is being extended with some of the differential-geometric concepts and tools that are developed by the academic GEOPLEX partners.

Who should read this book?

The target public of this book consists of “traditional” control engineers, confronted with complex, multi-domain control problems, and graduate students in *Systems and Control*. This book can extend their knowledge and understanding of advanced modelling, analysis and control methods using the port-Hamiltonian systems paradigm, because port-Hamiltonian systems bring the systems' *inherent structure* to the surface *explicitly*. This is an advantage because every additional structure that the engineer knows about its systems helps *to improve their analysis and control*.

- *Better insight* into complex systems, via the explicit distinction between the *inherent, coordinate-free physical properties* of the systems, and the *artificial, non-physical "properties"* that often show up in the specific *coordinate-based* algorithms that are used to implement analysis and control;
- The *acausal* description of components is very appropriate for the modelling, analysis and control of *open systems*, i.e., systems that can be interconnected to other open systems;
- The port-Hamiltonian approach can interconnect finite-dimensional and infinite-dimensional systems;
- *Scalability*: port-Hamiltonian structure is preserved by interconnection of multiple components modeled as port-Hamiltonian systems, also when components come from different domains;
- *"Re-use"* of the same theory for finite- and infinite-dimensional systems;
- More *modular, structured, and re-usable software framework*, leading to more user-friendly and more reliable modelling, simulation and design software tools;
- More structure means more "constraints" that make the "solution search space" smaller, hence *leading to potentially more efficient and more precise algorithms*.

Outline of the book

This book aims at presenting a unified framework for modelling, analysis, simulation and control of *complex* dynamical systems based on the port-Hamiltonian formalism. Background and concepts of a port-based approach to integrated modelling and simulation of physical systems and their controllers are illustrated in **Chapter 1**. These important notions are the conceptual motivation from a physical point of view of what is elaborated mathematically and applied to particular cases in the remaining chapters. In fact, in **Chapter 2**, it is shown how the representation of a lumped-parameter physical system as a bond graph naturally leads to a dynamical system endowed with a geometric structure, called port-Hamiltonian system. The notion of Dirac structure is here introduced as the key mathematical concept to unify the description of complex interactions in physical systems. Moreover, it is shown how the port-Hamiltonian structure is related, among others, to the classical Hamiltonian structure of physical systems. Furthermore, different representations of port-Hamiltonian systems are discussed, as well as the ways to navigate between them, and tools for analysis are introduced.

Port and port-Hamiltonian concepts are the basis of the detailed examples of modelling in several domains illustrated in **Chapter 3**. Here, it is shown how port-Hamiltonian systems can be fruitfully used for the structured modelling of electromechanical systems, robotic mechanisms and chemical systems. As far as the chemical domain is concerned, expressions of the models representing momentum, heat and mass transfer as well as chemical reactions within homogeneous fluids are reported in the port-based formalism. Furthermore, some insights are also given concerning the constitutive equations and models allowing to calculate transport and thermodynamic properties.

These last concepts serve as a starting point for the generalization of the port-Hamiltonian description of lumped parameter systems towards the distributed parameter ones. This is accomplished in **Chapter 4** by extending the definition of Dirac structure. In fact, it is shown how the Dirac structure and the port-Hamiltonian formulation arise from the description of distributed parameter systems as systems of conservation laws. In case of systems of two conservation laws, which describe two physical domains in reversible interaction, they may be formulated as port-Hamiltonian systems defined on a canonical interconnection structure, called canonical Stokes-Dirac structure. Several examples of physical systems are provided in order to illustrate the power of the proposed approach.

In the remaining chapters, it is shown how the port-Hamiltonian formulation offers powerful methods for control of complex multi-physics systems. In **Chapter 5**, a number of approaches to exploit the model structure of port-Hamiltonian systems for control purposes is illustrated. Formulating physical systems as port-Hamiltonian systems naturally leads to the consideration of *impedance* control problems, where the *behavior* of the system at the interaction port is sought to be shaped by the addition of a controller system. As an application of this strategy of *control by interconnection* within the port-Hamiltonian setting, the problem of stabilization of a desired equilibrium by shaping the Hamiltonian into a Lyapunov function for this equilibrium is considered. The mathematical formalism of port-Hamiltonian systems provides various useful techniques, ranging from Casimir functions, Lyapunov function generation, shaping of the Dirac structure by composition, and the possibility to combine finite-dimensional and infinite-dimensional systems.

In this respect, the control problem of distributed parameter port-Hamiltonian systems is discussed in **Chapter 6**. This chapter aims at extending some of the well-established control techniques developed for finite dimensional port-Hamiltonian systems illustrated in Chapter 5 to the infinite dimensional case. First result concerns the control by damping injection, which is applied to the boundary and distributed control of the Timoshenko beam. Then, the control by interconnection and energy shaping via Casimir generation is also discussed, giving particular emphasis to the stabilization of *mixed finite and infinite dimensional* port Hamiltonian system and to the dynamical control of a Timoshenko beam.

March 2009 *Herman Bruyninckx*, *Arjan J. van der Schaft*

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It is difficult to write acknowledgments of a book because undoubtedly there will be people which will not be specifically mentioned which directly or indirectly have contributed to the realization of this work.

I would like to acknowledge first the two persons which can be considered the inventors of the line of research presented in this volume which are Arjan van der Schaft and Bernhard Maschke who beside being two incredibly inspiring scientists, are wonderful people to work with and to have informal discussions or a nice dinner at the end of a working day.

Furthermore, the people which I would like to specifically mention for the birth and support of the project are Henrik Christensen who suggested to work on such a proposal, as also mentioned in the foreword, Alkis Konstantellos, the EC project officer who has always supported the project with great enthusiasm and Alberto Isidori and Andreas Kugi, the reviewers of the project which have always given useful feedback during the formal project reviews and elsewhere.

On the editorial side I would like to stress the incredible work that especially Vincent Duindam and Alessandro Macchelli have done without whom we would still be waiting for this volume.

But of course, the people which are really the architects of this book are the authors reported in each chapter but not only, because the result of each chapter is a consequence of many discussions which took place among all members of the consortium; thank you GEOPLEX consortium! The project has been a wonderful journey and this book is the result of a unique scientific collaboration.

Enschede (NL), March 2009 *Stefano Stramigioli*

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