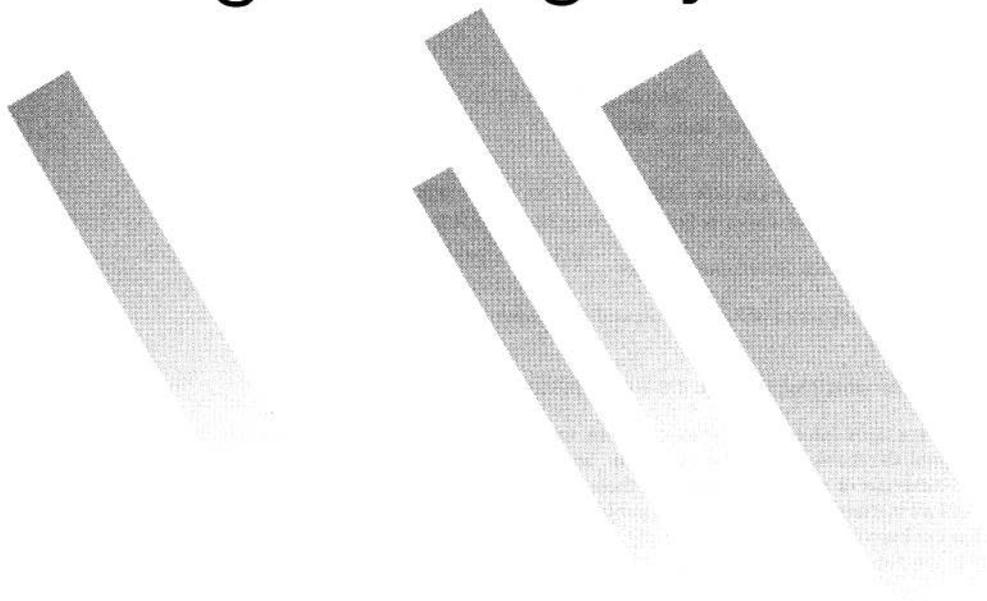


Modeling and Control of Engineering Systems



Clarence W. de Silva



CRC Press
Taylor & Francis Group
Boca Raton London New York

CRC Press is an imprint of the
Taylor & Francis Group, an **informa** business

Contents

Preface	xix
Acknowledgments	xxiii
Author	xxv
Further Reading.....	xxvii
Units and Conversions (Approximate)	xxix
1 Modeling and Control of Engineering Systems.....	1
1.1 Control Engineering	1
1.2 Application Areas	2
1.3 Importance of Modeling	4
1.4 History of Control Engineering	5
1.5 Organization of the Book.....	7
Problems	9
2 Modeling of Dynamic Systems	11
2.1 Dynamic Systems.....	11
2.1.1 Terminology.....	12
2.2 Dynamic Models.....	12
2.2.1 Model Complexity	13
2.2.2 Model Types	13
2.2.3 Types of Analytical Models.....	14
2.2.4 Principle of Superposition	15
2.2.5 Lumped Model of a Distributed System	16
2.2.5.1 Heavy Spring	17
2.2.5.2 Kinetic Energy Equivalence.....	18
2.2.5.3 Natural Frequency Equivalence.....	19
2.3 Lumped Elements and Analogies	20
2.3.1 Across Variables and Through Variables	20
2.3.2 Mechanical Elements.....	20
2.3.2.1 Mass (Inertia) Element.....	21
2.3.2.2 Spring (Stiffness) Element	22
2.3.2.3 Damping (Dissipation) Element.....	23
2.3.3 Electrical Elements.....	23
2.3.3.1 Capacitor Element	24
2.3.3.2 Inductor Element	25
2.3.3.3 Resistor (Dissipation) Element.....	26
2.3.4 Fluid Elements.....	26
2.3.4.1 Fluid Capacitor or Accumulator (<i>A</i> -Type Element)	27
2.3.4.2 Fluid Inertor (<i>T</i> -Type Element)	27
2.3.4.3 Fluid Resistor (<i>D</i> -Type Element).....	27
2.3.4.4 Derivation of Constitutive Equations	27
2.3.5 Thermal Elements.....	32
2.3.5.1 Constitutive Equations	32

	2.3.5.2 Three Dimensional Conduction	36
	2.3.5.3 Biot Number	37
	2.3.6 Natural Oscillations	37
2.4	Analytical Model Development	39
	2.4.1 Steps of Model Development	40
	2.4.2 I/O Models	40
	2.4.3 State-Space Models	40
	2.4.3.1 State-Space	41
	2.4.3.2 Properties of State Models	41
	2.4.3.3 Linear State Equations	42
	2.4.4 Time-Invariant Systems	45
	2.4.5 Systematic Steps for State Model Development	47
	2.4.6 I/O Models from State-Space Models	50
	Problems	53
3	Model Linearization	63
	3.1 Model Linearization	63
	3.1.1 Linearization about an Operating Point	64
	3.1.2 Function of Two Variables	66
	3.2 Nonlinear State-Space Models	66
	3.2.1 Linearization	67
	3.2.2 Reduction of System Nonlinearities	68
	3.3 Nonlinear Electrical Elements	85
	3.3.1 Capacitor	85
	3.3.2 Inductor	86
	3.3.3 Resistor	86
	3.4 Linearization Using Experimental Operating Curves	87
	3.4.1 Torque-Speed Curves of Motors	87
	3.4.2 Linear Models for Motor Control	88
	Problems	89
4	Linear Graphs	97
	4.1 Variables and Sign Convention	97
	4.1.1 Through Variables and Across Variables	97
	4.1.2 Sign Convention	97
	4.2 Linear Graph Elements	100
	4.2.1 Single-Port Elements	100
	4.2.1.1 Source Elements	101
	4.2.1.2 Effects of Source Elements	102
	4.2.2 Two-Port Elements	102
	4.2.2.1 Transformer	103
	4.2.2.2 Electrical Transformer	104
	4.2.2.3 Gyrator	105
	4.3 Linear Graph Equations	107
	4.3.1 Compatibility (Loop) Equations	107
	4.3.1.1 Sign Convention	107
	4.3.1.2 Number of "Primary" Loops	107
	4.3.2 Continuity (Node) Equations	109
	4.3.3 Series and Parallel Connections	110

4.4	State Models from Linear Graphs.....	110
4.4.1	System Order	111
4.4.2	Sign Convention.....	111
4.4.3	Steps of Obtaining a State Model	112
4.4.4	General Observation.....	112
4.4.5	Topological Result.....	113
4.5	Miscellaneous Examples.....	128
4.5.1	Amplifiers	128
4.5.1.1	Linear Graph Representation.....	129
4.5.2	DC Motor	130
4.5.3	Linear Graphs of Thermal Systems.....	134
4.5.3.1	Model Equations.....	134
	Problems	138
5	Transfer-Function and Frequency-Domain Models	149
5.1	Laplace and Fourier Transforms.....	149
5.1.1	Laplace Transform	150
5.1.2	Laplace Transform of a Derivative	150
5.1.3	Laplace Transform of an Integral	151
5.1.4	Fourier Transform	151
5.2	Transfer-Function.....	152
5.2.1	Transfer-Function Matrix	153
5.3	Frequency Domain Models	159
5.3.1	Frequency Transfer-Function (Frequency Response Function).....	159
5.3.1.1	Response to a Harmonic Input.....	159
5.3.1.2	Magnitude (Gain) and Phase.....	160
5.3.2	Bode Diagram (Bode Plot) and Nyquist Diagram.....	161
5.4	Transfer-Functions of Electro-Mechanical Systems.....	162
5.4.1	Significance of Transfer-Functions in Mechanical Systems	162
5.4.2	Mechanical Transfer-Functions	163
5.4.2.1	Mechanical Impedance and Mobility.....	164
5.4.3	Interconnection Laws	164
5.4.3.1	Interconnection Laws for Mechanical Impedance and Mobility.....	164
5.4.3.2	Interconnection Laws for Electrical Impedance and Admittance.....	164
5.4.3.3	A-Type Transfer-Functions and T-Type Transfer-Functions	165
5.4.4	Transfer-Functions of Basic Elements.....	166
5.4.5	Transmissibility Function.....	170
5.4.5.1	Force Transmissibility.....	170
5.4.5.2	Motion Transmissibility	171
5.4.5.3	Single-Degree-of-Freedom System	171
5.4.5.4	Two-Degree-of-Freedom System.....	173
5.5	Equivalent Circuits and Linear Graph Reduction.....	175
5.5.1	Thevenin's Theorem for Electrical Circuits.....	176
5.5.2	Mechanical Circuit Analysis Using Linear Graphs.....	179
5.5.3	Summary of Thevenin Approach for Mechanical Circuits	188
5.5.3.1	General Steps.....	188

5.6	Block Diagrams and State-Space Models	189
5.6.1	Simulation Block Diagrams.....	191
5.6.2	Principle of Superposition	191
5.6.3	Causality and Physical Realizability	207
	Problems	208
6	Response Analysis and Simulation	217
6.1	Analytical Solution	217
6.1.1	Homogeneous Solution.....	218
6.1.1.1	Repeated Poles	218
6.1.2	Particular Solution.....	219
6.1.3	Impulse Response Function.....	219
6.1.3.1	Convolution Integral.....	221
6.1.4	Stability.....	222
6.2	First-Order Systems	223
6.3	Second-Order Systems	225
6.3.1	Free Response of an Undamped Oscillator	225
6.3.2	Free Response of a Damped Oscillator.....	227
6.3.2.1	Case 1: Underdamped Motion ($\zeta < 1$)	228
6.3.2.2	Case 2: Overdamped Motion ($\zeta > 1$).....	229
6.3.2.3	Case 3: Critically Damped Motion ($\zeta = 1$)	230
6.4	Forced Response of a Damped Oscillator	232
6.4.1	Impulse Response.....	232
6.4.2	The Riddle of Zero ICs	234
6.4.3	Step Response.....	234
6.4.4	Response to Harmonic Excitation	236
6.5	Response Using Laplace Transform	241
6.5.1	Step Response Using Laplace Transforms	242
6.5.2	Incorporation of ICs.....	243
6.5.2.1	Step Response of a First-Order System	243
6.5.2.2	Step Response of a Second-Order System	244
6.6	Determination of ICs for Step Response	245
6.7	Computer Simulation	253
6.7.1	Use of Simulink® in Computer Simulation	254
6.7.1.1	Starting Simulink®	254
6.7.1.2	Basic Elements.....	255
6.7.1.3	Building an Application	255
6.7.1.4	Running a Simulation.....	256
	Problems	260
7	Control System Structure and Performance	271
7.1	Control System Structure.....	271
7.1.2	Feedforward Control.....	272
7.1.2.1	Computed-Input Control.....	274
7.1.3	Terminology.....	276
7.1.4	Programmable Logic Controllers (PLCs)	277
7.1.4.1	PLC Hardware	278
7.1.5	Distributed Control	280
7.1.5.1	A Networked Application.....	281

7.1.6	Hierarchical Control	283
7.2	Control System Performance	285
7.2.1	Performance Specification in Time-Domain	286
7.2.2	Simple Oscillator Model	288
7.3	Control Schemes	291
7.3.1	Feedback Control with PID Action	294
7.4	Steady-State Error and Integral Control	296
7.4.1	Final Value Theorem (FVT)	297
7.4.2	Manual Reset	297
7.4.3	Automatic Reset (Integral Control)	299
7.4.4	Reset Windup	299
7.5	System Type and Error Constants	300
7.5.1	Definition of System Type	301
7.5.2	Error Constants	301
7.5.2.1	Position Error Constant K_p	302
7.5.2.2	Velocity Error Constant K_v	302
7.5.2.3	Acceleration Error Constant K_a	303
7.5.3	System Type as a Robustness Property	305
7.5.4	Performance Specification Using s -Plane	305
7.6	Control System Sensitivity	309
7.6.1	System Sensitivity to Parameter Change	310
	Problems	313
8	Stability and Root Locus Method	329
8.1	Stability	329
8.1.1	Natural Response	329
8.2	Routh–Hurwitz Criterion	331
8.2.1	Routh Array	332
8.2.2	Auxiliary Equation (Zero-Row Problem)	333
8.2.3	Zero Coefficient Problem	334
8.2.4	Relative Stability	335
8.3	Root Locus Method	336
8.3.1	Rules for Plotting Root Locus	338
8.3.1.1	Complex Numbers	338
8.3.1.2	Root Locus Rules	340
8.3.1.3	Explanation of the Rules	341
8.3.2	Steps of Sketching Root Locus	343
8.3.4	Variable Parameter in Root Locus	356
8.4	Stability in the Frequency Domain	358
8.4.1	Response to a Harmonic Input	359
8.4.2	Complex Numbers	360
8.4.3	Resonant Peak and Resonant Frequency	361
8.4.4.1	Damped Simple Oscillator	363
8.4.3.2	Peak Magnitude	365
8.4.4	Half-Power Bandwidth	365
8.4.4.1	Damped Simple Oscillator	365
8.4.5	Marginal Stability	367
8.4.5.1	The (1,0) Condition	367
8.4.6	PM and GM	369

	8.4.6.1 GM.....	369
	8.4.6.2 PM.....	369
	8.4.7 Bode and Nyquist Plots.....	370
	8.4.8 PM and Damping Ratio Relation.....	372
8.5	Bode Diagram Using Asymptotes.....	373
	8.5.1 Slope-Phase Relationship for Bode Magnitude Curve.....	375
	8.5.1.1 Nonminimum-Phase Systems.....	376
	8.5.2 Ambiguous Cases of GM and PM.....	383
	8.5.3 Destabilizing Effect of Time Delays.....	384
8.6	Nyquist Stability Criterion.....	385
	8.6.1 Nyquist Stability Criterion.....	386
	8.6.2 Loop Poles on the Imaginary Axis.....	387
	8.6.3 Steps for Applying the Nyquist Criterion.....	387
	8.6.4 Relative Stability Specification.....	394
8.7	Nichols Chart.....	394
	8.7.1 Graphical Tools for Closed-Loop Frequency Response.....	394
	8.7.2 <i>M</i> Circles and <i>N</i> Circles.....	395
	8.7.3 Nichols Chart.....	398
	Problems.....	400
9	Controller Design and Tuning.....	409
	9.1 Controller Design and Tuning.....	409
	9.1.1 Design Specifications.....	410
	9.1.2 Time-Domain Design Techniques.....	411
	9.1.3 Frequency-Domain Design Techniques.....	411
	9.2 Conventional Time-Domain Design.....	411
	9.2.1 Proportional Plus Derivative Controller Design.....	412
	9.2.2 Design Equations.....	414
	9.3 Compensator Design in the Frequency Domain.....	414
	9.3.1 Lead Compensation.....	415
	9.3.1.1 Design Steps for a Lead Compensator.....	417
	9.3.2 Lag Compensation.....	420
	9.3.2.1 Design Steps for a Lag Compensator.....	420
	9.3.3 Design Specifications in Compensator Design.....	423
	9.4 Design Using Root Locus.....	427
	9.4.1 Design Steps Using Root Locus.....	427
	9.4.2 Lead Compensation.....	428
	9.4.3 Lag Compensation.....	431
	9.5 Controller Tuning.....	436
	9.5.1 Ziegler–Nichols Tuning.....	436
	9.5.2 Reaction Curve Method.....	437
	9.5.3 Ultimate Response Method.....	439
	Problems.....	441
10	Digital Control.....	447
	10.1 Digital Control.....	447
	10.1.1 Computer Control Systems.....	447
	10.1.2 Components of a Digital Control System.....	448
	10.1.3 Advantages of Digital Control.....	449

10.2	Signal Sampling and Control Bandwidth.....	449
10.2.1	Sampling Theorem.....	450
10.2.2	Antialiasing Filter	450
10.2.3	Control Bandwidth.....	451
10.2.4	Bandwidth Design of a Control System	454
10.2.5	Control Cycle Time	455
10.3	Digital Control Using z-Transform	456
10.3.1	z-Transform	457
10.3.2	Difference Equations	458
10.3.3	Discrete Transfer Functions.....	460
10.3.4	Time Delay	461
10.3.5	s-z Mapping	462
10.3.6	Stability of Discrete Models.....	464
10.3.7	Discrete Final Value Theorem (FVT).....	464
10.3.8	Pulse Response Function	466
10.3.8.1	Unit Pulse and Unit Impulse.....	467
10.4	Digital Compensation.....	467
10.4.1	Hold Operation.....	468
10.4.2	Discrete Compensator	469
10.4.3	Direct Synthesis of Digital Compensators.....	473
10.4.4	Causality Requirement.....	473
10.4.5	Stability Analysis Using Bilinear Transformation	474
10.4.6	Computer Implementation.....	475
	Problems	476
11	Advanced Control.....	483
11.1	Modern Control	483
11.2	Time Response.....	484
11.2.1	The Scalar Problem	484
11.2.1.1	Homogeneous Case (Input $u = 0$).....	484
11.2.1.2	Nonhomogeneous (Forced) Case	484
11.2.2	Time Response of a State-Space Model.....	486
11.2.2.1	Case of Constant System Matrix	486
11.2.2.2	Matrix Exponential	486
11.2.2.3	Methods of Computing e^{At}	487
11.2.3	Time Response by Laplace Transform	494
11.2.4	Output Response.....	495
11.2.4.1	Transfer Function Matrix	495
11.2.5	Modal Response	496
11.2.5.1	State Response through Modal Response	497
11.2.5.2	Advantages of Modal Decomposition.....	497
11.2.6	Time-Varying Systems.....	501
11.2.6.1	Properties of the State-Transition Matrix.....	503
11.3	System Stability.....	503
11.3.1	Stability of Linear Systems.....	503
11.3.1.1	General Case of Repeated Eigenvalues	505
11.3.1.2	Generalized Eigenvectors	505
11.3.2	Stability from Modal Response for Repeated Eigenvalues	507
11.3.2.1	Possibilities of Jordan Blocks and Modal Responses	508

11.3.3	Equilibrium	508
11.3.3.1	Bounded-Input Bounded-State (BIBS) Stability	509
11.3.3.2	Bounded-Input Bounded-Output (BIBO) Stability	509
11.3.4	Stability of Linear Systems	509
11.3.4.1	Frobenius' Theorem	509
11.3.4.2	First Method of Lyapunov	510
11.3.5	Second Method (Direct Method) of Lyapunov	513
11.3.5.1	Lyapunov Equation	515
11.4	Controllability and Observability	517
11.4.1	Minimum (Irreducible) Realizations	521
11.4.2	Companion Form and Controllability	525
11.4.3	Implication of Feedback Control	525
11.4.4	State Feedback	526
11.4.5	Stabilizability	527
11.5	Modal Control	528
11.5.1	Controller Design by Pole Placement	529
11.5.2	Pole Placement in the Multiinput Case	534
11.5.3	Procedure of Pole Placement Design	536
11.5.4	Placement of Repeated Poles	538
11.5.5	Placement of Some Closed-Loop Poles at Open-Loop Poles	539
11.5.6	Pole Placement with Output Feedback	542
11.6	Optimal Control	544
11.6.1	Optimization through Calculus of Variations	544
11.6.2	Cost Function having a Function of End State	555
11.6.3	Extension to the Vector Problem	556
11.6.4	General Optimal Control Problem	557
11.6.5	Boundary Conditions	559
11.6.6	Hamiltonian Formulation	559
11.6.7	Pontryagin's Minimum Principle	559
11.7	Linear Quadratic Regulator (LQR)	560
11.7.1	The Euler Equations	560
11.7.2	Boundary Conditions	561
11.7.3	Infinite-Time LQR	562
11.7.4	Control System Design	566
11.8	Other Advanced Control Techniques	569
11.8.1	Nonlinear Feedback Control	569
11.8.2	Adaptive Control	571
11.8.3	Sliding Mode Control	573
11.8.4	Linear Quadratic Gaussian (LQG) Control	574
11.8.5	H_∞ Control	576
11.9	Fuzzy Logic Control	577
11.9.1	Fuzzy Logic	578
11.9.2	Fuzzy Sets and Membership Functions	579
11.9.3	Fuzzy Logic Operations	580
11.9.3.1	Complement (Negation, NOT)	580
11.9.3.2	Union (Disjunction, OR)	580
11.9.3.3	Intersection (Conjunction, AND)	581
11.9.3.4	Implication (If-Then)	582

11.9.4	Compositional Rule of Inference	583
11.9.5	Extensions to Fuzzy Decision Making.....	584
11.9.6	Basics of Fuzzy Control	585
11.9.7	Fuzzy Control Surface	589
	Problems	593
12	Control System Instrumentation.....	603
12.1	Control System Instrumentation.....	603
12.2	Component Interconnection	605
12.2.1	Cascade Connection of Devices	605
12.2.2	Impedance Matching Amplifiers.....	607
12.2.3	Operational Amplifier	607
12.2.3.1	Use of Feedback in Op-Amps.....	609
12.2.4	Instrumentation Amplifiers.....	609
12.2.4.1	Differential Amplifier.....	609
12.3	Motion Sensors	610
12.3.1	Linear-Variable Differential Transformer (LVDT).....	611
12.3.2	Signal Conditioning	612
12.3.3	DC Tachometer	613
12.3.3.1	Electronic Commutation	614
12.3.4	Piezoelectric Accelerometer.....	614
12.3.4.1	Charge Amplifier	616
12.3.5	Digital Transducers.....	616
12.3.6	Shaft Encoders	617
12.3.7	Optical Encoder	618
12.4	Stepper Motors.....	619
12.4.1	Stepper Motor Classification.....	620
12.4.2	Driver and Controller	620
12.4.3	Stepper Motor Selection	623
12.4.3.1	Torque Characteristics and Terminology	623
12.4.3.2	Stepper Motor Selection Process.....	625
12.5	dc Motors	630
12.5.1	Rotor and Stator.....	632
12.5.2	Commutation	633
12.5.3	Brushless dc Motors.....	633
12.5.4	DC Motor Equations	634
12.5.4.1	Steady-State Characteristics.....	635
12.5.5	Experimental Model for dc Motor	637
12.5.5.1	Electrical Damping Constant	637
12.5.5.2	Linearized Experimental Model.....	637
12.5.6	Control of dc Motors	638
12.5.6.1	Armature Control.....	639
12.5.6.2	Motor Time Constants.....	640
12.5.6.3	Field Control.....	641
12.5.7	Feedback Control of dc Motors	642
12.5.7.1	Velocity Feedback Control	643
12.5.7.2	Position Plus Velocity Feedback Control	643
12.5.7.3	Position Feedback with PID Control	644
12.5.8	Motor Driver	644

12.5.8.1	Interface Board.....	645
12.5.8.2	Drive Unit.....	646
12.5.9	dc Motor Selection.....	648
12.5.9.1	Motor Data and Specifications	648
12.5.9.2	Selection Considerations	649
12.5.9.3	Motor Sizing Procedure	650
12.5.9.4	Inertia Matching.....	651
12.5.9.5	Drive Amplifier Selection.....	651
12.5.10	Summary of Motor Selection.....	652
12.6	Control Experiments Using LabVIEW®.....	654
12.6.1	Experiment 1: Tank Level Display	654
12.6.1.1	Procedure	654
12.6.1.2	Creating the Front Panel	654
12.6.1.3	Creating the Block Diagram	656
12.6.1.4	Calibrating the VI.....	658
12.6.1.5	Finding the Resistances of the Process Valves.....	659
12.6.2	Experiment 2: Process Control Using LabVIEW®	660
12.6.2.1	Two-Tank System.....	660
12.6.2.2	Description of the Front Panel.....	661
12.6.2.3	ON/OFF Control Algorithm.....	662
12.6.2.4	Proportional Control Algorithm	662
12.6.2.5	Single-Tank Process Control.....	662
Problems	666
Appendix A: Transform Techniques.....		677
A.1	Laplace Transform.....	677
A.1.1	Laplace Transforms of Some Common Functions.....	678
A.1.1.1	Laplace Transform of a Constant	679
A.1.1.2	Laplace Transform of the Exponential	679
A.1.1.3	Laplace Transform of Sine and Cosine.....	679
A.1.1.4	Laplace Transform of a Derivative.....	681
A.1.2	Table of Laplace Transforms	682
A.2	Response Analysis.....	682
A.3	Transfer Function	689
A.4	Fourier Transform	691
A.4.1	Frequency-Response Function (Frequency Transfer Function).....	692
A.5	The s -Plane	693
A.5.1	An Interpretation of Laplace and Fourier Transforms	693
A.5.2	Application in Circuit Analysis.....	693
Appendix B: Software Tools		695
B.1	Simulink®.....	695
B.2	MATLAB®	695
B.2.1	Computations.....	695
B.2.2	Arithmetic	696
B.2.3	Arrays.....	697
B.2.4	Relational and Logical Operations	698
B.2.5	Linear Algebra.....	698
B.2.6	M-Files.....	699

B.3	Control Systems Toolbox	700
B.3.1	Compensator Design Example	700
B.3.1.1	Building the System Model	700
B.3.1.2	Importing Model into SISO Design Tool	700
B.3.1.3	Adding Lead and Lag Compensators	703
B.3.2	PID Control with Ziegler–Nichols Tuning	703
B.3.2.1	Proportional Control	703
B.3.2.2	PI Control	705
B.3.2.3	PID Control	707
B.3.3	Root Locus Design Example	708
B.3.4	MATLAB® Modern Control Examples	708
B.3.4.1	Pole Placement of a Third Order Plant	708
B.3.4.2	Linear Quadratic Regulator (LQR) for a Third Order Plant	711
B.3.4.3	Pole Placement of an Inverted Pendulum on Mobile Carriage	712
B.3.4.4	LQG Controller for an Inverted Pendulum Mounted with Mobile Carriage	715
B.4	Fuzzy Logic Toolbox	716
B.4.1	Graphical Editors	716
B.4.2	Command Line Driven FIS Design	717
B.4.3	Practical Stand-Alone Implementation in C	717
B.5	LabVIEW®	719
B.5.1	Introduction	719
B.5.2	Some Key Concepts	719
B.5.3	Working with LabVIEW®	719
B.5.3.1	Front Panel	720
B.5.3.2	Block Diagrams	721
B.5.3.3	Tools Palette	722
B.5.3.4	Controls Palette	723
B.5.3.5	Functions Palette	723
B.6	LabVIEW® Sound and Vibration Tools	724
B.6.1	Sound and Vibration Toolkit	724
B.6.2	Signal Acquisition and Simulation	725
B.6.2.1	Integration	725
B.6.2.2	Vibration-Level Measurements	725
B.6.2.3	Frequency Analysis	726
B.6.2.4	Transient Analysis	726

Appendix C: Review of Linear Algebra	729
C.1 Vectors and Matrices	729
C.2 Vector–Matrix Algebra	731
C.2.1 Matrix Addition and Subtraction	732
C.2.2 Null Matrix	733
C.2.3 Matrix Multiplication	733
C.2.4 Identity Matrix	734
C.3 Matrix Inverse	734

C.3.1	Matrix Transpose	735
C.3.2	Trace of a Matrix.....	736
C.3.3	Determinant of a Matrix	736
C.3.4	Adjoint of a Matrix.....	737
C.3.5	Inverse of a Matrix	738
C.4	Vector Spaces.....	739
C.4.1	Field (F)	739
C.4.2	Vector Space (L).....	739
C.4.3	Subspace S of L	740
C.4.4	Linear Dependence	740
C.4.5	Bases and Dimension of a Vector Space.....	740
C.4.6	Inner Product.....	741
C.4.7	Norm	741
C.4.8	Gram–Schmidt Orthogonalization.....	742
C.4.9	Modified Gram–Schmidt Procedure	742
C.5	Determinants	742
C.5.1	Properties of Determinant of a Matrix.....	743
C.5.2	Rank of a Matrix.....	743
C.6	System of Linear Equations	743
C.7	Quadratic Forms.....	744
C.8	Matrix Eigenvalue Problem	745
C.8.1	Characteristic Polynomial.....	745
C.8.2	Characteristic Equation	745
C.8.3	Eigenvalues	745
C.8.4	Eigenvectors	745
C.9	Matrix Transformations.....	745
C.9.1	Similarity Transformation	745
C.9.2	Orthogonal Transformation.....	746
C.10	Matrix Exponential	746
C.10.1	Computation of Matrix Exponential	747
Index		749