

# Modeling and control of induction motors\*

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## 1 General introduction

Induction motors constitute a theoretically interesting and practically important class of nonlinear systems. They are described by a fifth order nonlinear differential equation with two inputs and only three state variables available for measurement. The control task is further complicated by the fact that induction motors are subject to unknown (load) disturbances and the parameters are highly uncertain. We are faced then with the challenging problem of controlling a highly nonlinear system, with unknown time varying parameters, where the regulated output, besides being not measurable, is perturbed by an unknown additive signal.

Existing solutions to this problem, in particular the *de facto* industry standard field-oriented control (FOC), were not theoretically well understood. Consequently, no guidelines were available for the designer, which had to rely on trial-and-error and intuition for commissioning and high performance applications. These compelling factors, together with the recent development of powerful theoretical tools for analysis and synthesis of nonlinear systems, motivated some control researchers to tackle this problem.

The main purpose of this minicourse is to review some of the main developments on the field, with particular emphasis on applications of passivity and flatness ideas. The course starts by presenting the physical model of the motor adopt-

ing an innovative perspective that underscores the control aspects—in lieu of the classical electrical engineering viewpoint. We then review, again from a control theory perspective, the well-known FOC. Connections between this classical technique and passivity ideas have been revealed in the literature, in particular it has been shown that passivity-based control schemes exactly reduce to FOC under some simplifying modeling assumptions. After reviewing these developments we present the recent developments which rely on the property of flatness of the motor.

## 2 Modeling

### 2.1 Introduction

The designers of controls for electrical alternating current (AC) machines frequently use a “natural” method: they try to write “direct models”, that is “physical models” which must be easily “inverted”. Often, in the control algorithms, they make various types of “compensations”: additive or multiplicative compensations. Furthermore, they implement “internal loops” to assure security on variables like currents. In this section, we will present the case of the induction motor (IM) and present how the flux-oriented control (FOC) is deduced from physical criteria and how the various loops directly are deduced from classical input-output linearization by state feedback.

### 2.2 Physical modelling

The induction machine considered here has a three-phase stator and a three-phase rotor (see Fig. 1). We adopt the classical assumptions: linearity of the materials (no saturations),

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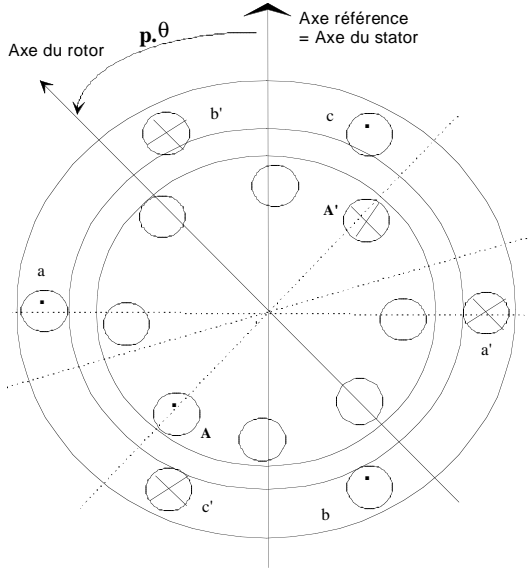


Figure 1: Transverse section of the induction motor

sinusoidal distribution of the field in the air-gap, balanced structure. Vectors relative to stator variables are denoted

$$\mathbf{x}_{abcs} = \begin{pmatrix} x_{as} \\ x_{bs} \\ x_{cs} \end{pmatrix} \triangleq \mathbf{x}_{abc} = \begin{pmatrix} x_a \\ x_b \\ x_c \end{pmatrix}$$

and vectors relative to rotor variables are denoted

$$\mathbf{x}_{abcr} = \begin{pmatrix} x_{ar} \\ x_{br} \\ x_{cr} \end{pmatrix} \triangleq \mathbf{x}_{ABC} = \begin{pmatrix} x_A \\ x_B \\ x_C \end{pmatrix}$$

Fluxes, currents and voltage are (respectively) denoted:  $\psi_{abc}$ ,  $\psi_{ABC}$ ,  $i_{abc}$ ,  $i_{ABC}$ ,  $v_{abc}$ ,  $v_{ABC}$ . The fundamental physical equations of the machine are the relations between fluxes and currents:

$$\psi_{abc} = \mathbf{l}_s i_{abc} + \mathbf{m}_{sr}(\theta) i_{ABC} \quad (1a)$$

$$\psi_{ABC} = \mathbf{m}_{rs}(\theta) i_{abc} + \mathbf{l}_r i_{ABC} \quad (1b)$$

with:

$$\mathbf{l}_s = \begin{pmatrix} l_s & m_s & m_s \\ m_s & m_s & m_s \\ m_s & m_s & l_s \end{pmatrix}, \quad \mathbf{l}_r = \begin{pmatrix} l_r & m_r & m_r \\ m_r & l_r & m_r \\ m_r & m_r & l_r \end{pmatrix}$$

$$\mathbf{m}_{sr}(\theta) = M_o \begin{pmatrix} \cos(n_p \theta) & \cos(n_p \theta + \gamma) & \cos(n_p \theta - \gamma) \\ \cos(n_p \theta - \gamma) & \cos(n_p \theta) & \cos(n_p \theta + \gamma) \\ \cos(n_p \theta + \gamma) & \cos(n_p \theta - \gamma) & \cos(n_p \theta) \end{pmatrix}$$

where  $n_p$  is the number of pair of poles and  $\theta$  is the mechanical position of the rotor and  $\gamma = \frac{2\pi}{3}$ ; the other parameters (inductances) are constant. The second systems of equations are the voltages equations:

$$v_{abc} = R_s i_{abc} + \frac{d\psi_{abc}}{dt} \quad (2a)$$

$$v_{ABC} = R_r i_{ABC} + \frac{d\psi_{ABC}}{dt} \quad (2b)$$

**Note:** In practical cases of squirrel-cage motor, the rotor voltages are zero:  $v_{ABC} = (0 \ 0 \ 0)^t$ . The final equation is given by the expression of the electromagnetic torque:

$$\tau_{em} = i_{abc}^t \frac{\partial \mathbf{m}_{sr}(\theta)}{\partial \theta} i_{ABC} \quad (3)$$

### 2.3 The criteria for the flux oriented control (FOC)

The designers have chosen the two following criteria to have a good control of the IM:

- Controlling the electromagnetic torque  $\tau_{em}$ .
- Controlling the magnitude of the rotor flux  $\psi_{ABC}$  :  $\psi_r$ .

The problem has now two aspects:

- First, what are the currents needed to impose the torque and the magnitude of the rotor flux? That is: how to “inverse” equations (1) and (3) which are “algebraic equations”. If we observe (1) and (3), we see that we have 6 unknown variables (the currents  $i_{abc}$  and  $i_{ABC}$ ) and only 2 equations given by  $\tau_{em}$  and  $\psi_r$ .
- Second: what are the voltages, which can create the appropriate currents? That is: how to inverse (2), which are “differential equations”. Furthermore, we must protect the motor against too large magnitudes of stator currents.

### 2.4 Algebraic properties of the coupling matrix $\mathbf{m}_{sr}(\theta)$

The most important term in equations (1) and (3) is the coupling matrix  $\mathbf{m}_{sr}(\theta)$  which describes the electromechanical conversion. We must detail some of its algebraic properties. Its eigenvalues are:  $0$ ,  $\frac{3M_o}{2} e^{j n_p \theta}$  and  $\frac{3M_o}{2} e^{-j n_p \theta}$ . It is possible to diagonalize this matrix with these eigenvalues. But for convenience, we make two choices: first, we eliminate the terms relative to the “zero-sequence” component (associated to the eigenvalue equal to zero) because these terms are almost always zero and do not participate to the energy conversion; and, second, we prefer to use a “real” transformation; for this we observe that if we define the “real rotation matrix”:

$$P(\xi) = \begin{pmatrix} \cos(\xi) & -\sin(\xi) \\ \sin(\xi) & \cos(\xi) \end{pmatrix} \quad (4)$$

and the “Clarke sub-matrix”:

$$C_{32} = \begin{pmatrix} 1 & -\frac{1}{2} & -\frac{1}{2} \\ 0 & +\frac{\sqrt{3}}{2} & -\frac{\sqrt{3}}{2} \end{pmatrix}^t$$

furthermore, also for convenience, we prefer use “normalized” matrices for later transformations and we define the “Concordia sub-matrix” by:  $T_{32} = \sqrt{\frac{2}{3}} C_{32}$ , and we have:

$$T_{32}^t T_{32} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \triangleq I_2 \text{ (i.e., } T_{32} \text{ is an orthogonal matrix). Thus, we have the following “factorization” (which has analogue properties to diagonalization):}$$

$$\mathbf{m}_{sr} = M T_{32} P(n_p \theta) T_{32}^t \quad (5)$$

where  $M = \frac{3}{2} M_o$ . Furthermore,  $T_{32}$  diagonalizes the matrices like  $\mathbf{l}_s$  and  $\mathbf{l}_r$ :  $\mathbf{l}_s T_{32} = L_s T_{32}$  and  $\mathbf{l}_r T_{32} = L_r T_{32}$ , with:  $L_s = l_s - m_s$  and  $L_r = l_r - m_r$ . Thus the fundamental equations (1) and (3) can be rewritten as following:

$$\psi_{abc} = L_s i_{abc} + M T_{32} P(n_p \theta) T_{32}^t i_{ABC} \quad (6a)$$

$$\psi_{ABC} = M T_{32} P(-n_p \theta) T_{32}^t i_{abc} + L_r i_{ABC} \quad (6b)$$

$$\tau_{em} = n_p M i_{abc}^t T_{32} P(n_p \theta + \frac{\pi}{2}) T_{32}^t i_{ABC} \quad (7)$$

## 2.5 Concordia transformation

If we observe equations (6) and (7) we observe that we do not have six unknown variables, but only four one, which are given by:  $T_{32}^t i_{abc}$  and  $T_{32}^t i_{ABC}$ . Then it is useful to define the ‘‘Concordia transformation’’ applied to all electric variables (voltages  $v$ , fluxes  $\psi$ , currents  $i$ ) by:

$$x_{\alpha\beta s} \triangleq \begin{pmatrix} x_{\alpha s} \\ x_{\beta s} \end{pmatrix} = T_{32}^t x_{abc} \quad (8a)$$

$$x_{\alpha\beta r} \triangleq \begin{pmatrix} x_{\alpha r} \\ x_{\beta r} \end{pmatrix} = T_{32}^t x_{ABC} \quad (8b)$$

Then the equations of fluxes and torque (6) and (7) can be rewritten:

$$\begin{pmatrix} \psi_{\alpha s} \\ \psi_{\beta s} \end{pmatrix} = L_s \begin{pmatrix} i_{\alpha s} \\ i_{\beta s} \end{pmatrix} + M P(+n_p \theta) \begin{pmatrix} i_{\alpha r} \\ i_{\beta r} \end{pmatrix} \quad (9a)$$

$$\begin{pmatrix} \psi_{\alpha r} \\ \psi_{\beta r} \end{pmatrix} = M P(-n_p \theta) \begin{pmatrix} i_{\alpha s} \\ i_{\beta s} \end{pmatrix} + L_r \begin{pmatrix} i_{\alpha r} \\ i_{\beta r} \end{pmatrix} \quad (9b)$$

$$\tau_{em} = n_p M i_{\alpha\beta s}^t P(n_p \theta + \frac{\pi}{2}) i_{\alpha\beta r} \quad (10)$$

Furthermore, we have for the voltages:

$$v_{\alpha\beta s} = R_s i_{\alpha\beta s} + \frac{d\psi_{\alpha\beta s}}{dt} \quad (11a)$$

$$v_{\alpha\beta r} = R_r i_{\alpha\beta r} + \frac{d\psi_{\alpha\beta r}}{dt} \quad (11b)$$

## 2.6 Choice of the ‘‘useful variables’’

Equations (9) and (10) show that we have now 4 unknown variables to determine the components of the stator and rotor currents and only 2 equations relative to the torque  $\tau_{em}$  and to the magnitude of the rotor flux  $\psi_r$ . For convenience, we choose to use, not exactly the rotor flux, but the so-called ‘‘magnetizing current’’  $i_{\mu r}$ . This new variable, its magnitude  $i_{\mu}$  and its polar angle  $\xi_r$  are defined by a Cartesian to polar transformation:

$$\begin{aligned} \begin{pmatrix} \psi_{\alpha r} \\ \psi_{\beta r} \end{pmatrix} &\triangleq M i_{\mu\alpha\beta r} = M \begin{pmatrix} i_{\mu\alpha r} \\ i_{\mu\beta r} \end{pmatrix} \\ &= M \begin{pmatrix} i_{\mu r} \cos(\xi_r) \\ i_{\mu r} \sin(\xi_r) \end{pmatrix} \\ &= M i_{\mu r} P(\xi_r) \begin{pmatrix} 1 \\ 0 \end{pmatrix} \end{aligned} \quad (12)$$

Then it appears that it will be natural to choose the following ‘‘useful variables’’ (physical signification of the future ‘‘state variables’’, which will be defined in a later section): the stator currents, which are measurable, and the magnetizing current (magnitude and polar angle). We can rewrite the other variables (torque, stator fluxes, rotor currents) with the help of these two vector variables:

$$\tau_{em} = n_p L_m i_{\mu s} i_{\alpha\beta s}^t P(n_p \theta + \xi_r + \frac{\pi}{2}) \begin{pmatrix} 1 \\ 0 \end{pmatrix} \quad (13)$$

$$\psi_{\alpha\beta s} = N_1 i_{\alpha\beta s} + L_m i_{\mu r} P(n_p \theta + \xi_r) \begin{pmatrix} 1 \\ 0 \end{pmatrix} \quad (14)$$

$$\begin{pmatrix} i_{\alpha r} \\ i_{\beta r} \end{pmatrix} = \frac{M}{L_r} \left( \begin{pmatrix} i_{\mu\alpha r} \\ i_{\mu\beta r} \end{pmatrix} - P(-n_p \theta) \begin{pmatrix} i_{\alpha s} \\ i_{\beta s} \end{pmatrix} \right) \quad (15)$$

with the definition of new parameters:

$$\sigma = 1 - \frac{M^2}{L_s L_r} \quad (16a)$$

$$N_1 = \sigma L_s = \left( L_s - \frac{M^2}{L_r} \right) \quad (16b)$$

$$L_m = (1 - \sigma) L_s = \frac{M^2}{L_r} \quad (16c)$$

## 2.7 Park transformation

If we examine the equation (13), we observe that it will be much more simpler to write it, if we make the following transformation for the stator variables (voltages, currents, fluxes):

$$P(-\xi_s) \begin{pmatrix} x_{\alpha s} \\ x_{\beta s} \end{pmatrix} = \begin{pmatrix} x_d \\ x_q \end{pmatrix} \quad (17a)$$

$$P(-\xi_r) \begin{pmatrix} x_{\alpha r} \\ x_{\beta r} \end{pmatrix} = \begin{pmatrix} x_D \\ x_Q \end{pmatrix} \quad (17b)$$

$$\xi_s = \xi_r + n_p \theta \quad (17c)$$

this leads to the following torque equation, which has the most simple form:

$$\tau_{em} = n_p L_m i_{\mu r} i_q \quad (18)$$

This transformation is, in fact, a rotation of the axes and the  $d$ -axe is given by the direction of the rotor flux. It is known as ‘‘Park transformation’’.

## 2.8 State variables and state equations

The most practical state variables are: first, the stator currents in Cartesian representation  $i_d$  and  $i_q$  (see (17a)); and second, the magnetizing current in polar form (12). The state equations, which determine these state variables, are, first for the equations deduced from stator voltage:

$$\frac{di_d}{dt} = \frac{1}{N_1} (v_d - e_d) \quad (19a)$$

$$\frac{di_q}{dt} = \frac{1}{N_1} (v_q - e_q) \quad (19b)$$

with the following definitions for the “back-electromotive forces”  $e_d$  and  $e_q$ :

$$e_d = R_s i_d + \frac{L_m}{T_2} (i_d - i_{\mu r}) - \left( n_p \omega + \frac{1}{T_2} \frac{i_q}{i_{\mu r}} \right) N_1 i_q \quad (20a)$$

$$e_q = R_s i_q + \left( n_p \omega + \frac{1}{T_2} \frac{i_q}{i_{\mu r}} \right) (N_1 i_d + L_m i_{\mu r}) \quad (20b)$$

and, second, for the equations deduced from the rotor variables:

$$\frac{di_{\mu r}}{dt} = \frac{1}{T_2} (i_d - i_{\mu r}) \quad (21a)$$

$$\frac{d\xi_r}{dt} = \frac{1}{T_2} \frac{i_q}{i_{\mu r}} \quad (21b)$$

### 3 Field oriented control

#### 3.1 Estimator

If we want to impose a dynamic with the help of a state feedback, it is necessary to determine the three variables which are not measurable:  $\xi_s, \xi_r, i_{\mu r}$  with the help of the measurable variables: that is stator currents,  $i_{\alpha\beta s} = T_{32}^t i_{abc}$ , and mechanical variables:  $\omega$  and  $\theta$ . The equations of the estimator are deduced from the equations (21a) and (21b):

$$\hat{i}_{dq} = P(-\hat{\xi}_s) T_{32}^t i_{abc} \quad (22a)$$

$$\frac{d\hat{i}_{\mu r}}{dt} = \frac{1}{\hat{T}_2} (\hat{i}_d - \hat{i}_{\mu r}) \quad (22b)$$

$$\frac{d\hat{\xi}_r}{dt} = \frac{1}{\hat{T}_2} \frac{\hat{i}_q}{\hat{i}_{\mu r}} \quad (22c)$$

$$\hat{\xi}_s = n_p \theta + \hat{\xi}_r \quad (22d)$$

$$\hat{\tau}_{em} = n_p L_m \hat{i}_{\mu s} \hat{i}_q \quad (22e)$$

The symbol  $\hat{\cdot}$  denotes estimated variables and parameters.

#### 3.2 Closed loop control and introduction of physical constraints

For the design of the controllers, we have to solve two problems:

1. We want to impose the dynamic and the steady state behaviors of the two variables of interest: the torque  $\tau_{em}$  and the amplitude of the magnetizing current:  $i_{\mu r}$ . For this we will use an input-output linearization by state-feedback using the differential equations:

$$\frac{d\tau_{em}}{dt} = n_p L_m \left( \frac{di_{\mu r}}{dt} i_q + \frac{di_q}{dt} i_{\mu s} \right) \quad (23a)$$

$$N_1 \frac{di_{\mu r}}{dt} + N_1 T_2 \frac{d^2 i_{\mu r}}{dt^2} = v_d - e_d \quad (23b)$$

$$i_{\mu r} + T_2 \frac{di_{\mu r}}{dt} = i_d \quad (23c)$$

And we choose arbitrarily the following dynamic models, which have the lowest orders physically realisable

(1<sup>st</sup> for the torque, 2<sup>nd</sup> for the magnetizing current):

$$\begin{aligned} \frac{d\tau_{em}}{dt} &= \frac{1}{\tau_c} ((\tau_{em})_{ref} - \tau_{em}) \\ &= \frac{1}{\tau_c} n_p L_m i_{\mu r} (I_{q_{ref}} - i_q) \end{aligned} \quad (24a)$$

$$i_{\mu r} + \frac{2\xi}{\omega_n} \frac{di_{\mu r}}{dt} + \frac{1}{\omega_n^2} \frac{d^2 i_{\mu r}}{dt^2} = i_{\mu r_{ref}} \quad (24b)$$

2. We have to limit the variations of the stator currents for security during large movements. Thus we want to have a control structure, which contains internal loops on the stator currents. Algebraic computations give the following results for the control law:

- (a) In the “ $q$ -axis”, we control the torque and the magnitude of the component  $i_q$  of the stator current:

$$v_q = v_{q1} + v_{q2} \quad (25a)$$

$$v_{q1} = k_q (I_{q_{ref}} - \hat{i}_q) \quad (25b)$$

$$\text{with } k_q = \frac{\hat{N}_1}{\tau_c}$$

$$v_{q2} = e_q - \frac{\hat{N}_1}{\hat{T}_2} \frac{\hat{i}_q}{\hat{i}_{\mu r}} (\hat{i}_d - i_{\mu r}) \quad (25c)$$

- (b) In the “ $d$ -axis” we control the rotor flux  $\psi_r = M i_{\mu r}$  and the magnitude of the component  $i_d$  of the stator current: first, the two loops structure is given by:

$$I_{d_{ref}} = k_\mu (I_{\mu r_{ref}} - i_{\mu r}) + I_{d_{ref2}} \quad (26a)$$

$$v_d = k_d (I_{d_{ref}} - i_d) + v_{d2} \quad (26b)$$

and the coefficients of the proportional controllers and the additive compensations are given by:

$$k_d = \frac{N_1}{T_2} (2\xi \omega_n T_2 - 1), v_{d2} = e_d \quad (27a)$$

$$k_\mu = \frac{(\omega_n T_2)^2}{2\xi \omega_n T_2 - 1}, I_{d_{ref2}} = i_{\mu r} \quad (27b)$$

We observe that this structure use proportional controllers like  $k_q, k_\mu, k_d$  and “additive compensations” like  $v_{q2}, v_{d2}$  and  $I_{d_{ref2}}$ .

#### 3.3 Examples of transients

Fig. 2 and Fig. 3 show transients, which prove that this approach gives a complete “inversion” of the dynamical of the model. Fig. 2 shows three responses: First, the “magnetization”, that is, the response of the stator flux to a step reference (dynamic of 2nd order, given by (24b)). Fig. 3 shows that the  $i_d$  component of the stator current reaches the authorized maximum value (denoted  $i_{d_{sat}}$ ), but without overshoot: that is a protection effect due the internal loop. After the magnetization we see the response of the torque to steps reference: the dynamic is 1st order, given by (24a). The speed

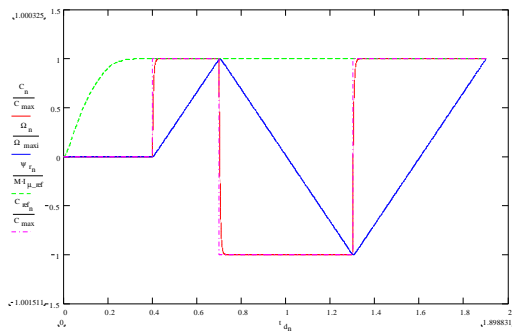


Figure 2: Flux, torque and speed responses.

$\omega$  has quasi-linear responses (the torque is well controlled). We observe that the decoupling between the two axes is perfect: the magnitude of the flux remains constant when the torque has large variations. The “inversion” of the model is completed.

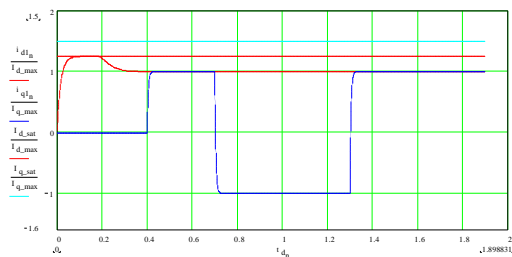


Figure 3: Responses of  $i_d$  and  $i_q$ .

### 3.4 Conclusion

This brief approach of the control of induction machines has two aims. First, to prove that the classical Field Oriented Control (FOC) is the logical solution to the problem of controlling simultaneously the two outputs (torque and magnitude of the rotor flux); this shows that the Park transformation in the referential of the rotor flux is necessary to resolve simply the algebraic equations which describe the physical systems. Second, the necessity of imposing simultaneously the steady state and dynamic behavior of the two outputs leads to control in the two-axis  $d$  and  $q$ ; we can limit (for protection) the magnitudes of the two components of the stator currents by the help of internal loops, which can realize the input-output linearization of the induction machine. This proves the ability of determination of the complete “inverse model” of the machine.

## 4 Passivity-based control

### 4.1 Background on passivity-based control

The term passivity-based control (PBC) was first introduced in [18] to define a controller design methodology which achieves stabilization by rendering *passive*<sup>1</sup> a suitably defined map. This idea has been very successful to control physical systems, in particular those described by Euler-Lagrange equations of motion, which as thoroughly detailed in [19], includes mechanical, electrical and electromechanical applications. PBC has its roots in the work of Takegaki and Arimoto [20] on state-feedback regulation of fully actuated robot manipulators. PBC design proceeds along two basic stages. First, an *energy shaping* stage where we modify the total energy of the system in such a way that the new energy function has a strict local minimum in the desired behavior. Second, a *damping injection* stage where we now modify the dissipation function to ensure asymptotic stability.

There are several important advantages of PBC which, to a large extent, explain its practical success:

1. The action of the controller has a clear physical interpretation as an interconnection of the system with its environment. In particular, stabilization can be understood in terms of energy balance between them.
2. Passive systems are robust *vis a vis* uncertain parameters and unmodeled dynamics. For instance, in mechanical systems the passive outputs are generalized velocities, thus unmodeled effects like friction (which is a dissipative action) cannot destroy stability.
3. In some cases it is possible to attach a clear physical interpretation to the controller tuning parameters. For instance, as dampers or springs. This property can hardly be overestimated in engineering applications where commissioning of the controller for a robust behavior is an issue of prime importance, (alas! largely neglected in control theory).
4. Since PBC has evolved from considerations of physical properties like energy conservation and passivity -which should be contrasted with feedback linearization that results from purely mathematical considerations- the design is consistent with the physical constraints, without cancelling dynamics or introducing controller singularities.

In this part of the minicourse we show how PBC can be applied for speed (or position) regulation of induction motors. To provide the reader with a suitable range of references on PBC of electrical machines, and at the same time give some perspective on this approach, we present in the next section

<sup>1</sup>We recall here that a passive system is one where the stored energy cannot exceed the energy supplied to it from its environment, the difference being dissipated.

a brief historical review of these developments. In Subsection 4.3, we take off from the model of the motor presented in Section 2, to exhibit two important properties of the machine: its passivity and “invertibility”. In Subsection 4.4 we derive a PBC, which assuming measurement only of stator currents and rotor speed (or position), yields a globally convergent behavior. We also prove that, if the stator dynamics is assumed sufficiently fast (like in the so-called current-fed machines), then our PBC can be considerably simplified and exactly reduces to the *de facto* industry standard field-oriented control introduced before in Section 3.

## 4.2 A historical review

To give some historical perspective to the developments of PBC as applied to electrical machines, we briefly summarize them here in a chronological order. These results are presented in full detail in [19].

- In [21] the controller design method used in robot motion control to solve the output tracking problem for a class of underactuated Euler-Lagrange systems, was extended to torque regulation of the induction motor, with all internal states bounded. There were no controller singularities, but exact model knowledge and full state measurement had to be assumed. It was also indicated how to follow sinusoidally varying torque references. A model representation in a  $dq$ -frame of reference was used, and this model became the basis for later designs.

- The previous design was extended to a globally stable controller for torque regulation without measurements of rotor variables in [22]. This globally defined and globally stable interlaced design of controller and observer, was *the first such result reported in the control literature* [23]. Exact model knowledge was assumed, but it was indicated how to compensate for unknown rotor resistance and load torque, unfortunately under the assumption of full state measurement. The torque reference was restricted to be below a certain upper limit depending on motor and controller parameters, but again it was shown how this could be avoided in the case of full state measurement. This is a foundational paper for PBC that first illustrated the application of this technique for electromechanical systems.

- Torque regulation with a globally defined and stable controller without measurements of rotor variables, was extended to torque tracking with adaptation of unknown linearly parameterized load torque in [24].

- In [25] PBCs were extended to include the important case of rotor flux norm regulation without rotor variable measurements. The coordinate independent properties of this approach were also rigorously explained. It follows that PBC can be derived in any frame of reference chosen for model representation, hence clarifying some erroneous claims made in [26]

- Recently, a new approach to the induction motor control problem was presented in [27], where it was shown that global torque tracking and rotor flux norm regulation could be done without flux measurement or estimation. This was

accomplished by the fundamental observation that the mechanical part of the induction motor dynamics defines a passive feedback around the electrical subsystem, which is also passive. Hence, instead of shaping the energy of the total system as in previous designs, the control goal could be achieved by shaping only the energy of the electrical subsystem, with the mechanical subsystem as a passive disturbance. It was also shown in this paper how to extend the controller for speed tracking with adaptation of a constant load torque. Drawbacks of this scheme are that it is open loop in speed, and that the convergence rate of the speed tracking error is bounded from below by the mechanical time constant, relying on a positive damping of the mechanical system. However, this paper gave a first rigorous solution to the longstanding problem of avoiding rotor flux estimates in induction motor control, still with global stability results, but unfortunately under the assumption of known parameters.

- The problems with the convergence rate and the speed controller were solved in [28]. In this paper mechanical damping was injected into the closed loop by use of linear filtering of the speed tracking error, giving a globally stable observer-less speed (or position) tracking controller with flux regulation. That paper established also the fundamental fact that our PBC reduces to the well-known field-oriented control scheme for current-fed machines. This “downward compatibility” of PBC with current engineering practice is a remarkable feature whose importance can hardly be overestimated. On one hand, it provides a solid system-theoretic foundation to popular control strategies which enhances their understanding and paves the way for subsequent improvements. On the other hand, viewing the new controllers as “upgrades” of the existing ones, it facilitates the transfer of these developments to practitioners.

- In [29] it was shown how the controller could be extended from regulation to tracking of rotor flux norm, an important result for power efficient operation of induction motor drives.

- The results cited above are specific to the induction motor, and it was of interest to see if these results could be extended to other types of electric machines. An answer to this question was given in the seminal paper [30], where it is shown that passivity-based controllers can be designed for a large class of electric machines, including synchronous-, stepper- and reluctance motors.

- In [31] the first globally stable *discrete-time* induction motor PBC was presented. The controller was based on the *exact* discrete-time model of a current-fed induction motor.

- Experimental results from the application of passivity-based controllers, have been presented in several publications, e.g. [32, 30, 33, 34, 35].

## 4.3 Control properties of the induction motor model

In this section we establish some properties of the model - input-output and geometrical- that will be instrumental for further developments.

### 4.3.1 Model

In Section 2.5 we have derived the standard two phase  $\alpha\beta$ -model of an  $n_p$  pole pair squirrel-cage induction motor with uniform air-gap. For convenience in the sequel, set

$$\begin{aligned}\psi &= [\psi_s^\top, \psi_r^\top]^\top \\ i &= [i_s^\top, i_r^\top]^\top\end{aligned}$$

Thus, the relation between the flux and the currents reads:

$$\psi = L(\theta)i \quad (28)$$

where  $L(\theta) = L^\top(\theta) > 0$  is the  $4 \times 4$  inductance matrix of the windings defined as

$$L(\theta) = \begin{bmatrix} L_s I_2 & M e^{\mathcal{J} n_p \theta} \\ M e^{-\mathcal{J} n_p \theta} & L_r I_2 \end{bmatrix}$$

where we have defined the (skew-symmetric) rotation matrices

$$\mathcal{J} = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} = -\mathcal{J}^\top$$

and

$$e^{\mathcal{J} n_p \theta} = P(n_p \theta) \quad (\text{see (4)})$$

The electrical dynamics are defined by the voltage balance equation

$$\dot{\psi} + Ri = Nu \quad (29)$$

where we have introduced the matrices

$$R = \begin{bmatrix} R_s I_2 & 0 \\ 0 & R_r I_2 \end{bmatrix}, \quad N = \begin{bmatrix} I_2 \\ 0 \end{bmatrix}$$

with  $R_s, R_r > 0$  stator and rotor resistances. Particularly useful for further developments is the following relationship between rotor fluxes and rotor currents

$$\dot{\psi}_r + R_r i_r = 0 \quad (30)$$

The model is completed computing the torque of electrical origin as

$$\tau_{em} = \frac{1}{2} i^\top \frac{\partial L(\theta)}{\partial \theta} i = -\frac{1}{2} \psi^\top \frac{\partial L^{-1}(\theta)}{\partial \theta} \psi \quad (31)$$

and replacing it in the mechanical dynamics

$$J\ddot{\theta} = \tau_{em} - \tau_L \quad (32)$$

where  $J > 0$  is the rotational inertia of the rotor, and we have introduced a term of load torque  $\tau_L$ , which we will assume constant but unknown. For simplicity, we have neglected the effect of friction but, as shown in [19], this can be easily accommodated into our analysis.

**Remark** As pointed out above, we established in [25] that PBC is coordinate independent. That is, it can be derived in any frame of reference chosen for model representation. For instance, in [27] the  $ab$ -model was used, while the developments of [22] and [24] relied on the  $dq$ -model. Finally, the work of [30] was carried-out in the original  $\alpha\beta$  frame.

**Input-output properties.** The *cornerstone* of the passivity-based design philosophy is to reveal the passivity property of the system, and identify -as a by-product- its *workless forces*. This is easily established from the systems total energy, which for the induction motor is given as

$$\mathcal{H}(\psi, \dot{\theta}, \theta) = \underbrace{\frac{1}{2} \psi^\top L^{-1}(\theta) \psi}_{\mathcal{H}_e} + \underbrace{\frac{1}{2} \mathcal{J} \dot{\theta}^2}_{\mathcal{H}_m}$$

where  $\mathcal{H}_e(i, \theta)$ ,  $\mathcal{H}_m(\dot{\theta})$  denote the electrical energy and the mechanical kinetic co-energy, respectively. We have neglected the capacitive effects in the windings of the motor, and considered a rigid shaft, hence, the potential energy of the motor is zero.

The rate of change of the energy (*the systems work*) is given by

$$\dot{\mathcal{H}} = i_s^\top u - \dot{\theta} \tau_L - i_s^\top R i_s$$

From the integration of the equation above we obtain the energy-balance

$$\begin{aligned} \underbrace{\mathcal{H}(t) - \mathcal{H}(0)}_{\text{stored energy}} &= - \underbrace{\int_0^t i^\top(s) R i(s) ds}_{\text{dissipated}} + \underbrace{\int_0^t [i_s^\top(s) u(s) - \dot{\theta}(s) \tau_L] ds}_{\text{supplied - extacted}} \quad (33) \end{aligned}$$

which proves that the mapping  $[u^\top, -\tau_L]^\top \mapsto [i_s^\top, \dot{\theta}]^\top$  is passive, with storage function  $\mathcal{H}$ .

Furthermore, as shown in [27], the motor model can be decomposed as the feedback interconnection of two passive operators with storage functions  $\mathcal{H}_e$  and  $\mathcal{H}_m$ , respectively. These passivity properties, and their corresponding storage functions, are the basis for two different PBCs studied in [19].

**Geometric properties.** We now exhibit an ‘‘invertibility’’ property of the induction motor model which is essential for obtaining an explicit expression of the PBC.

From (31) and (28) we see that the torque can be written as

$$\tau_{em} = n_p M i_s^\top \mathcal{J} e^{\mathcal{J} n_p \theta} i_r \quad (34)$$

where the fact that  $\mathcal{J}$  and  $e^{\mathcal{J} n_p \theta}$  commute ( $\mathcal{J} e^{\mathcal{J} n_p \theta} = e^{\mathcal{J} n_p \theta} \mathcal{J}$ ), and the skew-symmetry of  $\mathcal{J}$  ( $\mathcal{J}^\top = -\mathcal{J} \rightarrow x^\top \mathcal{J} x = 0, \forall x \in \mathcal{R}^2$ ) has been used. Now, solving (30) for  $i_r$  and substituting it into (34) gives

$$\tau_{em} = n_p \frac{M}{L_r} i_s^\top \mathcal{J} e^{\mathcal{J} n_p \theta} \psi_r \quad (35)$$

Finally, (28) can be solved for  $i_s$  as

$$i_s = \frac{1}{M} e^{\mathcal{J} n_p \theta} (\psi_r - L_r i_r)$$

and then substituted into (35) to give

$$\tau_{em} = \frac{n_p}{R_r} \dot{\psi}_r^\top \mathcal{J} \psi_r \quad (36)$$

where (30) has been used again. This is a key expression that allows us to “invert” the systems dynamics, that is, explicitly “solve” this equation as

$$\dot{\psi}_r = \frac{\tau_{em} R_r}{\|\psi_r\| n_p} \mathcal{J} \psi_r \quad (37)$$

where  $\|\cdot\|$  is the Euclidean norm.

The two equations above will be instrumental in the next Section for the derivation of the PBC. In [30], where we study the model of the generalized rotating machine, we assume that the machine is Blondel-Park transformable to ensure this “invertibility” property. The underlying fundamental assumption for the machine to be Blondel-Park transformable, is that the windings are sinusoidally distributed, giving a sinusoidal air-gap magnetomotive force and sinusoidally varying elements in the inductance matrix  $L(\theta)$ . For a practical machine, this means that the magnetomotive force can be suitably approximated with the first harmonic in a Fourier approximation. Examples of machines in which higher order harmonics must be taken into account, are the square wave brushless DC motors, and machines with significant saliency in the air gap. For this class of machines the application of PBC is still an open issue.

The equation (37) also shows that the zero dynamics of the motor with outputs  $\tau_{em}$  and  $\|\psi_r\|$  are periodic. This fact becomes clearer if we evaluate the angular speed of the rotor flux vector relative the rotor fixed frame (*the slip speed*) as

$$\begin{aligned} \dot{\rho} &= \frac{d}{dt} \arctan\left(\frac{\psi_{r2}}{\psi_{r1}}\right) = \frac{1}{1 + \left(\frac{\psi_{r2}}{\psi_{r1}}\right)^2} \frac{\dot{\psi}_{r2}\psi_{r1} - \psi_{r2}\dot{\psi}_{r1}}{\psi_{r1}^2} \\ &= \frac{1}{\|\psi_r\|^2} \dot{\psi}_r^\top \mathcal{J} \psi_r \\ &= \frac{R_r}{n_p \|\psi_r\|^2} \tau_{em} \end{aligned} \quad (38)$$

From this equation we that if  $\tau_{em}$  and  $\|\psi_r\|$  are fixed to constant values, the rotor flux rotates at a constant speed. This expression also shows that torque can be controlled by controlling rotor flux norm and slip speed -as is well known in the drives community.

#### 4.4 Nested-loop passivity-based control

It is shown in [19] that for electromechanical systems the PBC approach can be applied in at least *two different ways*, leading to different controllers. In the first, more direct form, a PBC is designed for the whole electromechanical system using as storage function the total energy of the full system. This is the way PBCs are typically defined for mechanical and electrical systems and is usually referred as *PBC with total energy shaping*.

Another route stems from the application of a passive subsystems decomposition to the electromechanical system.

Namely, we show that (under some reasonable assumptions) we can decompose the system into its electrical and mechanical dynamics, where the latter can be treated as a “passive disturbance”. We design then a PBC *for the electrical subsystem* using as storage function only the electrical part of the systems total energy. An outer-loop controller (which can also be a PBC, but here is a simple pole-placement) is then added to regulate the mechanical dynamics. The so-designed controller will be called *nested-loop PBC*. There are at least three motivations for this approach: firstly, using this feedback-decomposition leads to simpler controllers, which in general do not require observers. Secondly, typically there is a time-scale separation between the electrical and the mechanical dynamics. Finally, since the nested-loop configuration is the prevailing structure in practical applications, we can in some important cases establish a clear connection between our PBC and current practice.

Although for both controllers we can prove global asymptotic speed/position tracking, for the sake of brevity we present here only the torque tracking version of the nested-loop PBC.

##### 4.4.1 Controller structure

In this section we solve the speed-position tracking problem adopting a nested-loop (i.e. cascaded) scheme, where  $\mathcal{C}_{il}$  is an inner-loop torque tracking PBC, and  $\mathcal{C}_{ol}$  is an outer-loop speed controller, which generates the desired torque<sup>2</sup>  $\tau_{emd}$ . We will show in this section that  $\mathcal{C}_{ol}$  may be taken as an LTI system that asymptotically stabilizes the mechanical dynamics. The main technical obstacle for its design stems from the fact that  $\mathcal{C}_{il}$  requires the knowledge of  $\dot{\tau}_{emd}$ , and this in its turn implies measurement of acceleration. To overcome this obstacle we proceed, as done in [19] for the robotics problem, and replace the acceleration by its approximate differentiation, while preserving the global stabilization property. (In simple applications, of course,  $\mathcal{C}_{ol}$  is just a PI around speed error. We go here through these additional complications to provide a complete proof of stability.)

A very interesting property of the resulting scheme, which is further elaborated in Subsection 4.4.4, is that if the inverter can be modeled as a current source and the desired speed and rotor flux norm are constant, the controller exactly reduces to the well known indirect field-oriented control, hence providing a solid theoretical foundation to this popular control strategy.

##### 4.4.2 Torque tracking PBC

**Implicit and explicit forms.** In this subsection we derive a torque tracking PBC from the perspective of systems “inversion”. Towards this end, using (28), we rewrite (29) and (31) as

$$\dot{\psi} + RL^{-1}(\theta)\psi = Nu \quad (39)$$

<sup>2</sup>We will adopt throughout the following notation convention. If a signal is explicitly given as an external reference we denote it  $(\cdot)_*$ . If instead it is generated by the controller, we use  $(\cdot)_d$ .



$$\tau_{em} = \frac{n_p}{R_r} \dot{\psi}_r^\top \mathcal{J} \psi_r \quad (40)$$

where, for ease of reference, we have repeated (36).

Typically the PBC is a ‘‘copy’’ of the electrical dynamics of the motor (39), (40) with an additional *damping injection* term that improves the transient performance. To simplify the presentation we will omit the damping injection here, and refer the reader to [19]. Thus, we define the PBC in an *implicit* form as

$$Nu = \dot{\psi}_d + RL^{-1}(\theta)\psi_d \quad (41)$$

$$\tau_{em*} = \frac{n_p}{R_r} \dot{\psi}_{rd}^\top \mathcal{J} \psi_{rd} \quad (42)$$

where  $\tau_{em*}$  is the torque reference, and  $\psi_d = [\psi_{sd}^\top, \psi_{rd}^\top]^\top$  define desired values for the fluxes.

An *explicit* realization of the PBC above is obtained by ‘‘inversion’’ of (42) as

$$\dot{\psi}_{rd} = \frac{1}{\beta_*^2(t)} \left( \frac{R_r}{n_p} \tau_{em*} \mathcal{J} + \dot{\beta}_*(t) \beta_*(t) I_2 \right) \psi_{rd}$$

where  $\psi_{rd}(0) = [\beta_*(0), 0^\top]$ , and  $\beta_*(t)$  is the (time-varying) reference for  $\|\psi_r\|$ . The latter equation can actually be solved as

$$\psi_{rd} = e^{\mathcal{J}\rho_d} \begin{bmatrix} \beta_*(t) \\ 0 \end{bmatrix} \quad (43)$$

$$\dot{\rho}_d = \frac{R_r}{n_p \beta_*^2(t)} \tau_{em*}, \quad \rho_d(0) = 0 \quad (44)$$

The description of the controller is completed by replacement of  $\psi_{rd}$  and  $\dot{\psi}_{rd}$  in the last two equations of (41) to get  $\dot{\psi}_{sd}$ . After differentiation we get  $\dot{\psi}_{sd}$  which can be replaced in the first two equations of (41) to get<sup>3</sup>

$$u = \dot{\psi}_{sd} + \begin{bmatrix} I_2 & 0 \end{bmatrix} R_s L^{-1}(\theta) \psi_d$$

the expression above we can see a difficulty for the implementation of the nested-loop scheme. Namely that the control law depends on  $\dot{\psi}_{sd}$ , which in its turn will depend on  $\tau_{em*}$ . On the other hand, the signal  $\tau_{em*}$  will now be generated by an outer-loop controller  $\mathcal{C}_{ol}$ , which will generally depend on  $\dot{\theta}$ . We will see in the Proposition 4.4.3 how to overcome this obstacle by the use of a linear filter.

**Stability.** Let us now analyze the *stability* of the closed-loop. The error equation for the fluxes is obtained from (39) and (41) as

$$\dot{\tilde{\psi}} + RL^{-1}(\theta)\tilde{\psi} = 0$$

where  $\tilde{\psi} \triangleq \psi - \psi_d$  are the flux errors. Global convergence can be easily established considering the storage function<sup>4</sup>

$$H_\psi = \frac{1}{2} \tilde{\psi}^\top R^{-1} \tilde{\psi} \geq 0$$

<sup>3</sup>An explicit state space description is given in Proposition 4.4.3.

<sup>4</sup>This function was used in [36] to give an ‘‘implicit observer’’ interpretation of the PBC controller.

whose derivative satisfies

$$\dot{H}_\psi = -\tilde{\psi}^\top L^{-1}(\theta)\tilde{\psi} \leq -\alpha H_\psi$$

for some  $\alpha > 0$ . Hence,  $\tilde{\psi} \rightarrow 0$  exponentially fast.

To illustrate the second difficulty in the stability analysis of the nested-loop scheme, let us turn our attention to the torque tracking error  $\tau_{em} \triangleq \tau_{em} - \tau_{em*}$ . After some simple operations from (40) and (42) we get

$$\tau_{em} = \frac{n_p}{R_r} \left\{ \tilde{\psi}_r^\top \mathcal{J} \tilde{\psi}_r + \dot{\psi}_r^\top \mathcal{J} \psi_{rd} + \dot{\psi}_{rd} \mathcal{J} \tilde{\psi}_r \right\}$$

We have shown above that  $\tilde{\psi} \rightarrow 0$  (exp.), consequently also  $\dot{\tilde{\psi}} \rightarrow 0$ . Also,  $\psi_{rd}$  is bounded by construction, see (43). Unfortunately, we cannot prove that  $\dot{\psi}_{rd}$  is bounded, unless  $\tau_{em*}$  is bounded. In position-speed control  $\tau_{em*}$  is not *a priori* bounded, since it will be generated by  $\mathcal{C}_{ol}$ . Therefore,  $\mathcal{C}_{ol}$  must be chosen with care and a new argument should be invoked to complete the proof. Proposition 4.4.3 below shows that  $\mathcal{C}_{ol}$  can be taken as a linear filter.

**Connection with system inversion.** Before closing this section, we will view the PBC from the standard systems inversion perspective of nonlinear geometric control [37]. The purpose of the exercise is to show that if we complete in a suitable manner the ‘‘output’’ vector, then the standard inversion algorithm, -as applied to the *reference signals*-, will give us the PBC controller above. This fact is important for at least two reasons, first because the inversion algorithm is a general *systematic* procedure, while the procedure used above is somewhat *ad hoc*. Second, it provides a clear connection with feedback linearization where the same inversion algorithm is invoked, but now applied to the *output signals*.

To this end, we propose to complete the ‘‘output’’ vector for the system (39) as  $y = [\tau_{em}, y_2]^\top$ , where

$$y_2 \triangleq -n_p \psi_r^\top \begin{bmatrix} 0 & I_2 \end{bmatrix} L^{-1}(\theta) \psi = \frac{n_p}{R_r} \dot{\psi}_r^\top \psi_r$$

Recall that  $\tau_{em}$  can be written in the form (40).

Following the inversion algorithm we evaluate the decoupling matrix by taking the time derivative of  $y$

$$\dot{y} = \frac{n_p}{R_r} \begin{bmatrix} \dot{\psi}_r^\top \mathcal{J} \\ \dot{\psi}_r^\top \end{bmatrix} (RL^{-1}(\theta))_{21} u + m(\psi) \triangleq G(\psi_r, \theta) u + m(\psi)$$

where  $m(\psi)$  is some function of  $\psi$ . It can be shown that

$$(RL^{-1}(\theta))_{21} = \frac{R_r M}{L_s L_r \sigma} e^{\mathcal{J} n_p \theta}$$

which is globally invertible. On the other hand

$$\begin{bmatrix} \dot{\psi}_r^\top \mathcal{J} \\ \dot{\psi}_r^\top \end{bmatrix}^{-1} = \frac{1}{\|\dot{\psi}_r\|^2} \begin{bmatrix} -\mathcal{J} \psi_r & \psi_r \end{bmatrix}$$

Consequently, the decoupling matrix  $G(\psi_r, \theta)$  is nonsingular everywhere except when  $\|\psi_r\| = 0$ . This implies that the

system (39) with output  $y$  has relative degree  $\{1, 1\}$ . (This should be contrasted with the signal  $\|\psi_r\|$ , which has relative degree 2.)

A feedback linearizing controller is chosen as

$$u = G(\psi_r, \theta)^{-1}[\dot{y}_* - m(\psi) + v]$$

where  $v$  is an additional stabilizing controller. On the other hand, the control signal for the PBC derived above can be obtained by “evaluating the inversion for the reference signals”, that is

$$u = G(\psi_{rd}, \theta)^{-1}[\dot{y}_* - m(\psi_d)] + u_{di}$$

where  $u_{di}$  is the damping injection term and  $\psi_{rd}$  is obtained from (43) and (44).

Roughly speaking, we can summarize the discussion above as follows: While input-output linearization implements a *right inverse* of the system  $\Sigma(\psi, \theta)$ , that is

$$u_{FL} = \Sigma^{-1}(\psi, \theta)(y_* + v),$$

the PBC implements a *left inverse*

$$u_{PBC} = \Sigma^{-1}(\psi_d, \theta)y_* + u_{di}$$

Notice that, except for the damping injection, the PBC is open-loop in  $\psi$ . However, the loop is closed with  $\theta$ .

#### 4.4.3 Speed tracking PBC

**Main result.** A globally stable speed tracking PBC is presented in the proposition below, whose proof may be found in [19].

**Proposition 4.4.3** The nonlinear dynamic output feedback nested-loop controller

$$u = \underbrace{L_s \dot{i}_{sd} + M e^{\mathcal{J} n_p \theta} \dot{i}_{rd} + n_p M \mathcal{J} e^{\mathcal{J} n_p \theta} \dot{\theta} i_{rd} + R_s i_{sd}}_{\dot{\psi}_{sd}} \quad (45)$$

with

$$i_d = \begin{bmatrix} \frac{1}{M} \left[ \left(1 + \frac{L_r \dot{\beta}_*}{R_r \beta_*}\right) I_2 + \frac{L_r}{n_p \beta_*^2} \tau_{emd} \mathcal{J} \right] e^{\mathcal{J} n_p \theta} \psi_{rd} \\ - \left( \frac{\tau_{emd}}{n_p \beta_*^2} \mathcal{J} + \frac{\dot{\beta}_*}{R_r \beta_*} I_2 \right) \psi_{rd} \end{bmatrix} \quad (46)$$

where

$$\tilde{i} = \begin{bmatrix} \tilde{i}_s \\ \tilde{i}_r \end{bmatrix} = \begin{bmatrix} i_s - i_{sd} \\ i_r - i_{rd} \end{bmatrix}$$

$$\tau_{emd} = J \ddot{\theta} * - z + \tau_L$$

and controller state equations

$$\dot{\psi}_{rd} = \left( \frac{R_r}{n_p \beta_*^2} \tau_{emd} \mathcal{J} + \frac{\dot{\beta}_*}{\beta_*} I_2 \right) \psi_{rd} \quad (47)$$

$$\dot{z} = -az + b\dot{\theta} \quad (48)$$

with  $\tilde{\theta} \triangleq \theta - \theta_*$  and  $a, b > 0$ , provides a solution to the speed and rotor flux norm tracking problem. That

is, when placed in closed-loop with (28),(29), (31) and (32) ensures

$$\lim_{t \rightarrow \infty} \tilde{\theta} = 0, \lim_{t \rightarrow \infty} \|\psi_r\| - \beta_*(t) = 0$$

for all initial conditions and with all internal signals uniformly bounded.

**Extensions.** • [Position control.] It is easy to see that choosing the desired torque in the controller above as

$$\tau_{emd} = J \ddot{\theta}_* - z - f\tilde{\theta} + \tau_L \quad (49)$$

yields global asymptotic *position* tracking for all positive values of  $a, b, f$ . The proof of global asymptotic rotor flux norm and position tracking follows *verbatim* from the proof of the main result above.

• [Adaptation of load torque.] We can extend the result in Proposition 4.4.3 to the case of unknown but linearly parameterized load

$$\tau_L = \eta^\top \phi(\theta, \dot{\theta})$$

where  $\eta \in \mathbb{R}^q$  is a vector of unknown constant parameters, and  $\phi(\theta, \dot{\theta})$  is a measurable regressor.

• [Integral action in stator currents] It is common in applications to add an integral loop around the stator current errors to the input voltages. The experimental evidence presented [19] shows that this indeed robustifies the PBC by compensating for unmodeled dynamics. It is interesting to note that the global tracking result above still holds for this case.

#### 4.4.4 PBC is downward compatible with FOC

Under assumptions that can be satisfied in many practical applications, the stator currents can be taken as control inputs for the induction motor. In other words, in some applications the inverter can be modeled as an ideal current source. In this subsection we will prove that, under this condition, the nested-loop PBC for voltage-fed machines derived above considerably simplifies and actually *reduces* to the well-known indirect FOC.

For current-fed machines the control signal is actually the stator currents. Hence, they can be set equal to  $i_{sd}$ , the first row of (46), and we do not need to calculate  $u$  as in (45). Consequently  $\tau_{emd}$ , which was required because of the presence of the term  $\dot{i}_d$ , is no longer needed for the implementation. Hence, we can remove the filtered speed error and replace it directly by the speed error. In this way the controller reduces to

$$M i_s = \left[ \left(1 + T_r \frac{\dot{\beta}_*}{\beta_*}\right) I_2 + \frac{L_r}{n_p \beta_*^2} \tau_{emd} \mathcal{J} \right] e^{\mathcal{J} n_p \theta} \psi_{rd} \quad (50)$$

$$\dot{\psi}_{rd} = \left( \frac{R_r}{n_p \beta_*^2} \tau_{emd} \mathcal{J} + \frac{\dot{\beta}_*}{\beta_*} \right) \psi_{rd} \quad (51)$$

$$\tau_{emd} = J \dot{\theta}_* - a\dot{\theta} + \tau_{emL} \quad (52)$$

Now, as pointed out above the controller states in (51) can be exactly integrated as

$$\psi_{rd} = \beta_* \begin{bmatrix} \cos(\rho_d) \\ \sin(\rho_d) \end{bmatrix} = e^{\mathcal{J} \rho_d} \begin{bmatrix} \beta_* \\ 0 \end{bmatrix}$$

where  $\rho_d$  is the solution of

$$\dot{\rho}_d = \frac{R_r}{n_p \beta_*^2} \tau_{\text{emd}}, \quad \rho_d(0) = 0 \quad (53)$$

By use of these expressions in (50) we get

$$i_s = \frac{1}{M} e^{\mathcal{J}(n_p \theta + \rho_d)} \begin{bmatrix} \beta_* + T_r \dot{\beta}_* \\ \frac{L_r}{n_p \beta_*} \tau_{\text{emd}} \end{bmatrix} \quad (54)$$

On the other hand, assuming that the desired speed is constant ( $\ddot{\theta}_* = 0$ ), and replacing the exact load torque cancellation by an integral action we get from (52)

$$\tau_{\text{emd}} = -(a + \frac{K_I}{p}) \dot{\theta}, \quad K_I > 0 \quad (55)$$

That is, a PI action around the speed error. The last three equations are exactly the indirect FOC.

**Remark.** The observer-based feedback-linearizing controller of [26]<sup>5</sup> may be viewed as a variation of direct FOC where, in order to achieve a linear system in closed loop, some terms are added to the control law to cancel the motor nonlinearities. The same idea had been already advanced in [38] for the case of the full motor dynamics, see also [39] where some adaptation terms are added to the basic scheme of [38]. Although the objective of feedback linearization is quite luring, the resulting schemes suffer from serious drawbacks, from both theoretical and practical viewpoints. First, they invariably require the explicit implementation of an observer. This, besides increasing the computational burden, makes the stability analysis extremely difficult. For instance, it is well known that for nonlinear systems the certainty equivalence principle fails. Also, it is widely recognized that at this stage, our understanding of nonlinear observers is quite rudimentary. It suffices to say that in spite of many years of research, to the best of our knowledge, a complete stability analysis for exact linearization schemes in the full motor model case is conspicuous by its absence. However, it should be pointed out that for the simplified model of current-fed machines this problem is elegantly solved in [26]. Second, since these schemes are based on nonlinearity cancellations, it is expected that potential instability due to parameter mismatch will arise. One such instability mechanism, which appears even in the state feedback case, was identified in [32] this work, and observed in the experiments.

## 5 Flatness-based control

### 5.1 Structural properties of the model

#### 5.1.1 Complex form of the model

For the sake of simplicity we prefer to work with a complex model instead of the real one given by (9)–(10)–(11). To simplify the proof of the flatness [40], it is useful to consider

<sup>5</sup>Called in that paper observer-based adaptive controller.

some variables in the frame rotating at the speed  $n_p \omega$ ; it is the natural frame to consider variables of the rotor. In order to distinguish the value of a given variable whether it is referenced in the fixed frame or in the rotating frame, one put a  $\tilde{\cdot}$  over the variable referenced in the rotating frame. Without the  $\tilde{\cdot}$ , a variable is supposed to be referenced in the fixed frame, i.e. the natural frame for considering the variable of the stator.

Set:

$$\underline{\psi}_s = \psi_{\alpha s} + \mathcal{J} \psi_{\beta s} \quad (56a)$$

$$\underline{\tilde{\psi}}_r = \psi_{\alpha r} + \mathcal{J} \psi_{\beta r} \quad (56b)$$

$$\underline{\dot{i}}_s = i_{\alpha s} + \mathcal{J} i_{\beta s} \quad (56c)$$

$$\underline{\tilde{i}}_r = i_{\alpha r} + \mathcal{J} i_{\beta r} \quad (56d)$$

$$\underline{u}_s = u_{\alpha s} + \mathcal{J} u_{\beta s} \quad (56e)$$

where every variable  $x_{\alpha s}$ ,  $x_{\beta s}$ ,  $x_{\alpha r}$  or  $x_{\beta r}$  is defined in (8). Therefore, equations (9a) and (9b) respectively read:

$$\underline{\dot{\psi}}_s = L_s \underline{\dot{i}}_s + M e^{\mathcal{J} n_p \theta} \underline{\tilde{i}}_r \quad (57a)$$

$$\underline{\dot{\tilde{\psi}}}_r = M e^{-\mathcal{J} n_p \theta} \underline{\dot{i}}_s + L_r \underline{\tilde{i}}_r \quad (57b)$$

Equations (11) become:

$$\frac{d}{dt}(\underline{\psi}_s) + R_s \underline{i}_s = \underline{u}_s \quad (58a)$$

$$\frac{d}{dt}(\underline{\tilde{\psi}}_r) + R_r \underline{\tilde{i}}_r = 0 \quad (58b)$$

The advantage to consider complex variables will clearly appear in the sequel; it reduce the number of equations to be considered, moreover, change of frames are simply done by multiplying complex variables by an appropriate complex exponential. We thus have  $\underline{\tilde{\psi}}_r = \underline{\psi}_r e^{-\mathcal{J} n_p \theta}$ ,  $\underline{\tilde{i}}_s = \underline{i}_s e^{-\mathcal{J} n_p \theta}$  and  $\underline{\tilde{i}}_r = \underline{i}_r e^{\mathcal{J} n_p \theta}$ .

The expression of the electromagnetic torque (10) in this notation is:

$$\tau_{\text{em}} = \frac{n_p M}{J L_r} \Im m \left( \underline{\dot{i}}_s \underline{\tilde{\psi}}_r^* \right) \quad (59)$$

#### 5.1.2 Flatness of the model

The flatness of the model of the induction motor has been established in [41]. We here recall the proof in the present notations. Set  $\rho = |\underline{\tilde{\psi}}_r|$  and define  $\delta$  the angle such that  $\underline{\tilde{\psi}}_r = \rho e^{\mathcal{J} \delta}$ ; this is the angle of the rotor flux w.r.t. a fixed frame. Set  $\alpha = \delta - n_p \theta$ .

A flat output of the induction motor is  $y = (\theta, \alpha)$ . As usual, this flat output has a physical meaning which will simplify the control design:  $\theta$  (or its first derivative  $\omega$ ) is the variable to-be-controlled,  $\alpha$  is the angle of the rotor flux w.r.t. a frame rotating at speed  $n_p \omega$  (recall that  $n_p \omega$  is called the synchronous speed). Notice also that  $\dot{\alpha} = \dot{\delta} - n_p \omega$  is the *slip speed* (usually only defined on constant speed operations).

We thus have  $\underline{\tilde{\psi}}_r = \underline{\psi}_r e^{-\mathcal{J} n_p \theta} = \rho e^{\mathcal{J} \alpha}$ . It is useful to begin to express the electromagnetic torque produced by the motor

in term of  $\rho$  and  $\alpha$ : Eq. (57b) and (58b) lead to:

$$\tilde{i}_s = \underline{i}_s e^{-jn_p\theta} = \frac{1}{M}(\tilde{\psi}_r - L_r \tilde{i}_r) \quad (60)$$

$$\tilde{i}_r = -\frac{1}{R_r} \frac{d}{dt}(\tilde{\psi}_r) \quad (61)$$

Thus,

$$\begin{aligned} \underline{i}_s \underline{\psi}_r^* &= \tilde{i}_s e^{-jn_p\theta} \left( \tilde{\psi}_r e^{-jn_p\theta} \right)^* \\ &= \tilde{i}_s \tilde{\psi}_r^* \\ &= \frac{1}{M} \left( \tilde{\psi}_r + \frac{L_r}{R_r} \frac{d}{dt}(\tilde{\psi}_r) \right) \tilde{\psi}_r^* \\ &= \frac{1}{M} \left( |\tilde{\psi}_r|^2 + \frac{L_r}{R_r} \frac{d}{dt}(\tilde{\psi}_r) \tilde{\psi}_r^* \right) \end{aligned}$$

And finally

$$\begin{aligned} \tau_{em} &= \frac{n_p}{J R_r} \Im \left( \frac{d}{dt}(\tilde{\psi}_r) \tilde{\psi}_r^* \right) \\ &= \frac{n_p}{J R_r} \rho^2 \dot{\alpha} \end{aligned}$$

So, the mechanical equation of the induction motor becomes

$$\dot{\omega} = \frac{n_p}{J R_r} \rho^2 \dot{\alpha} - \frac{f}{J} \omega - \frac{1}{J} \tau_L \quad (62)$$

**Hypothesis:** The torque load is an unknown function of time but it can depend on  $\theta$  or its derivatives, but not on the other variables ( $\underline{i}_s, \underline{i}_r, \underline{\psi}_r, \dots$ ).

It is obvious that

$$\omega = \dot{\theta} \quad (63a)$$

$$\delta = n_p \theta + \alpha \quad (63b)$$

From (62),  $\rho$  satisfies

$$\rho = \sqrt{\frac{R_r(J\ddot{\theta} + f\dot{\theta} + \tau_L)}{n_p \dot{\alpha}}} = a(\dot{\theta}, \ddot{\theta}, \dot{\alpha}, \tau_L) \quad (63c)$$

which is a function of the flat output and its two first derivatives. So

$$\tilde{\psi}_r = \rho e^{j\alpha} = b(\dot{\theta}, \ddot{\theta}, \alpha, \dot{\alpha}, \tau_L) \quad (63d)$$

Continuing the calculations using successively (61), (60), (57b), and (58a) one obtains:

$$\tilde{i}_r = -\frac{1}{R_r} \frac{d}{dt}(\tilde{\psi}_r) = c(\ddot{\theta}, \theta^{(3)}, \dot{\alpha}, \ddot{\alpha}, \tau_L, \dot{\tau}_L) \quad (63e)$$

$$\begin{aligned} \underline{i}_s &= \frac{e^{jn_p\theta}}{M} \left( \tilde{\psi}_r - L_r \tilde{i}_r \right) \\ &= d(\theta, \dots, \theta^{(3)}, \alpha, \dots, \ddot{\alpha}, \tau_L, \dot{\tau}_L) \end{aligned} \quad (63f)$$

$$\begin{aligned} \underline{\psi}_s &= L_s \underline{i}_s + M \underline{i}_r \\ &= e(\theta, \dots, \theta^{(3)}, \alpha, \dots, \ddot{\alpha}, \tau_L, \dot{\tau}_L) \end{aligned} \quad (63g)$$

$$\begin{aligned} \underline{u}_s &= R_s \underline{i}_s + \frac{d}{dt}(\underline{\psi}_s) \\ &= f(\theta, \dots, \theta^{(4)}, \alpha, \dots, \alpha^{(3)}, \tau_L, \dots, \dot{\tau}_L) \end{aligned} \quad (63h)$$

So  $y = (\theta, \delta)$  is a flat output of the induction motor.

### 5.1.3 Stationary operation

The state variables form of the model with the complex notation written in the stator frame:

$$\dot{\theta} = \omega \quad (64a)$$

$$\dot{\omega} = \frac{n_p M}{J L_r} \Im \left( \underline{i}_s \underline{\psi}_r^* \right) - \frac{f}{J} \omega - \frac{1}{J} \tau_L \quad (64b)$$

$$\frac{d}{dt}(\underline{\psi}_r) = \left( -\frac{1}{T_r} + j n_p \omega \right) \underline{\psi}_r + \frac{M}{T_r} \underline{i}_s \quad (64c)$$

$$\begin{aligned} \frac{d}{dt}(\underline{i}_s) &= \frac{M}{\sigma L_s L_r} \left( \frac{1}{T_r} - j n_p \omega \right) \underline{\psi}_r \\ &\quad - \frac{1}{\sigma L_s} \left( R_s + \frac{M^2 R_r}{L_r^2} \right) \underline{i}_s + \frac{1}{\sigma L_s} \underline{u}_s \end{aligned} \quad (64d)$$

However, the most useful frame to study stationary operations of the motors is certainly the frame of the flux — usually called the “ $d$ - $q$  frame”. Denotes as  $x^{dq}$  the value of the variable  $x$  in this frame, i.e.  $x^{dq} = x e^{-j\delta} = \tilde{x} e^{-j\alpha}$ . In this frame, the complex model (64), where (64b) is replaced by (62), reads:

$$\dot{\omega} = \frac{n_p}{J R_r} \rho^2 \dot{\alpha} - \frac{f}{J} \omega - \frac{1}{J} \tau_L \quad (65a)$$

$$\dot{\rho} + j \dot{\alpha} \rho = -\frac{1}{T_r} \rho + \frac{M}{T_r} \underline{i}_s^{dq} \quad (65b)$$

$$\begin{aligned} \frac{d}{dt}(\underline{i}_s^{dq}) &= \frac{M}{\sigma L_s L_r} \left( \frac{1}{T_r} - j n_p \omega \right) \underline{\psi}_r^{dq} \\ &\quad - \left( a + j \dot{\delta} \right) \underline{i}_s^{dq} + \frac{1}{\sigma L_s} \underline{u}_s^{dq} \end{aligned} \quad (65c)$$

with  $a = \frac{1}{\sigma L_s} \left( R_s + \frac{M^2 R_r}{L_r^2} \right)$ . Notice that as  $\rho$  and  $\alpha$  are real variables, equation (65b) can be splitted into:

$$\dot{\rho} = -\frac{1}{T_r} \rho + \frac{M}{T_r} \Re(\underline{i}_s^{dq})$$

$$\dot{\alpha} = \frac{M}{T_r \rho} \Im(\underline{i}_s^{dq})$$

A stationary operation at constant speed  $\omega_o$  with constant load  $\tau_L = \tau_{L_o}$  is obtained when  $\theta = \omega_o t + \theta_o$ ,  $\alpha = \alpha_1 t + \alpha_o$  where  $\alpha_1$  and  $\alpha_o$  are constant (i.e., the slip speed is constant). In this case  $\dot{\delta} = n_p \omega_o + \alpha_1 = \delta_1$  is constant.

As a consequence, (65a) implies that  $\rho$  is constant  $\rho = \rho_o$  and, thus,  $\underline{\psi}_r^{dq} = \rho_o$ . Equation (65b) implies that  $\underline{i}_s^{dq} = \underline{i}_{s_o}^{dq}$  is constant, and, finally, with (65c)  $\underline{u}_s^{dq} = \underline{u}_{s_o}^{dq}$  is constant.

In conclusion,  $\underline{\psi}_r$ ,  $\underline{i}_s$  and  $\underline{u}_s$  are periodical functions of time with pulsation  $\delta_1 = n_p \omega_o + \alpha_1$ . To run at constant speed with constant load, the induction motor has to be fed by sinusoidal voltages.

Notice that usually the stationary operation is analyzed by imposing  $\underline{u}_s$  is sinusoidal under constant load and deducing that all electric and magnetic quantities are periodic and finally the speed is constant. Here, with the flatness properties we are able to make the reverse analysis, i.e. beginning with the “to-be-controlled” variable and deducing the control.

### 5.2 Trajectory generation

One has obtain (see (63a–63h)) the expressions of every variables of the systems in terms of the flat output components

and the disturbance  $\tau_L$ . These expression allow in particular to calculate the control  $\underline{u}_s$ , at least when  $\tau_L = 0$  or for a known mean value  $\tau_{L_0}$  of  $\tau_L$ .

As  $\theta^{(4)}$  and  $\alpha^{(3)}$  appear in the expression (63h) of the control  $\underline{u}_s$ , the induction motor can achieve trajectories such that  $t \mapsto \theta$  is everywhere 4-times left- and right-differentiable and  $t \mapsto \alpha$  is everywhere 3-times left- and right-differentiable. The choice of the reference trajectories of  $\theta$  and  $\alpha$  are made in order to respect the constraints on all the variable of the system.

For the first component  $\theta$  of the flat output, the choice of the trajectory is often clear with respect to the control objective whether it is a position or speed control. This correspond to a known function of time  $t \mapsto \theta^d$  on a given interval of time  $[t_i, t_f]$ .

For the second component  $\alpha$  of the flat output, the choice is not so obvious because the value of  $\alpha$  do not correspond to a clear control objective. However, this variable gives a degree a freedom in order to achieve a complementary control objective. For example, it is possible to minimize the copper losses in the stator at every constant speed with a choice such that  $\dot{\alpha} = \frac{1}{T_r}$  (see [42]).

Between two time intervals on which  $\omega$  is constant,  $t \mapsto \alpha^d$  can be chosen as a function of  $\omega$ . For example, we refer to [42] for a detailed planning of the reference trajectories of  $\theta$  and  $\alpha$  in order to start the motor from rest to nominal speed without singularity at starting ( $\underline{u}_s$  remains bounded everywhere).

### 5.3 Stabilization around desired trajectories

We present here a feedback which is designed on the stationary operation of the system. This is possible because of the good separation of the time scales as seen before. This leads to a control scheme which do not necessitate the implementation of a flux observer.

As  $\underline{\psi}_r = \underline{\psi}_r e^{-jn_p\theta}$ ,  $\underline{\dot{\psi}}_r = \underline{\dot{\psi}}_r e^{-jn_p\theta}$  and from (58b) it is possible to write:

$$\begin{aligned} \frac{d}{dt}(\underline{\psi}_r e^{-jn_p\theta}) + R_r \underline{\dot{\psi}}_r e^{jn_p\theta} &= 0 \\ \frac{d}{dt}(\underline{\psi}_r) e^{-jn_p\theta} - jn_p\omega \underline{\psi}_r e^{jn_p\theta} + R_r \underline{\dot{\psi}}_r e^{jn_p\theta} &= 0 \\ \frac{d}{dt}(\underline{\psi}_r) - jn_p\omega \underline{\psi}_r + R_r \underline{\dot{\psi}}_r &= 0 \end{aligned} \quad (67)$$

which is the expression of the electrical equation of the rotor in the fixed frame.

The stationary mode of equations (58a) and (67) are respectively:

$$j\delta \underline{\psi}_s + R_s \underline{\dot{\psi}}_s = \underline{u}_s \quad (68a)$$

$$j(\delta - n_p\omega) \underline{\psi}_r + R_r \underline{\dot{\psi}}_r = 0 \quad (68b)$$

As,

$$\begin{aligned} \underline{\psi}_s &= L_s \underline{\dot{\psi}}_s + M \underline{\dot{\psi}}_r \\ \underline{\psi}_r &= M \underline{\dot{\psi}}_s + L_r \underline{\dot{\psi}}_r \end{aligned}$$

one has

$$\underline{\dot{\psi}}_r = \frac{1}{L_r} (\underline{\psi}_r - M \underline{\dot{\psi}}_s) \quad (69a)$$

$$\underline{\psi}_s = L_s \underline{\dot{\psi}}_s + \frac{M}{L_r} (\underline{\psi}_r - M \underline{\dot{\psi}}_s) \quad (69b)$$

Equations (68) and (69) lead to:

$$\begin{aligned} (R_s + j\sigma\delta L_s) \underline{\dot{\psi}}_s + j\delta \frac{M}{L_s} \underline{\psi}_r &= \underline{u}_s \\ -M \frac{R_r}{L_r} \underline{\dot{\psi}}_s + \left( \frac{R_r}{L_r} + j(\delta - n_p\omega) \right) \underline{\psi}_s &= 0 \end{aligned}$$

Thus,

$$\underline{\dot{\psi}}_s = Z_s(\omega, \delta) \underline{u}_s \quad (70a)$$

$$\underline{\psi}_r = Z_r(\omega, \delta) \underline{u}_s \quad (70b)$$

where

$$Z_s(\omega, \delta) = \frac{\frac{R_r}{L_r} + j(\delta - n_p\omega)}{Z}$$

$$Z_r(\omega, \delta) = \frac{M \frac{R_r}{L_r}}{Z}$$

with  $Z = (R_s + j\sigma\delta L_s) \left( \frac{R_r}{L_r} + j(\delta - n_p\omega) \right) + M \frac{R_r}{L_r} j\delta \frac{M}{L_s}$ .

This leads to the control scheme

$$|\underline{u}_s|^2 = \frac{JL_r}{n_p M \Im(Z_r^*(\omega, \delta^d) Z_s(\omega, \delta^d))} (\dot{\omega}^d - \kappa(\omega - \omega^d))$$

where  $\omega^d = \dot{\theta}^d$  is the reference trajectory of the angular speed and  $\delta^d = \alpha^d - n_p\omega^d$  is the reference speed of the slip speed. This control do not necessitate a flux observer.

### 5.4 Example of experiment

We end up this section whit the presentation of few experimental results of a flatness-based control scheme implemented on the experimental setup of *GDR Automatique*, IRCyN, Nantes.

The first experiment (Fig. 4) consists in accelerating the motor, starting from rest to its nominal speed. We observe a good tracking (it is almost always difficult to distinguish the experimental trajectory from the reference one) and a small overshoot.

Fig. 5 show the braking of the motor from nominal speed to rest.

Finally, Fig. 6 illustrates the inversion of rotating direction at very low speed, i.e. from  $-2\pi$  rad/s to  $2\pi$  rad/s.

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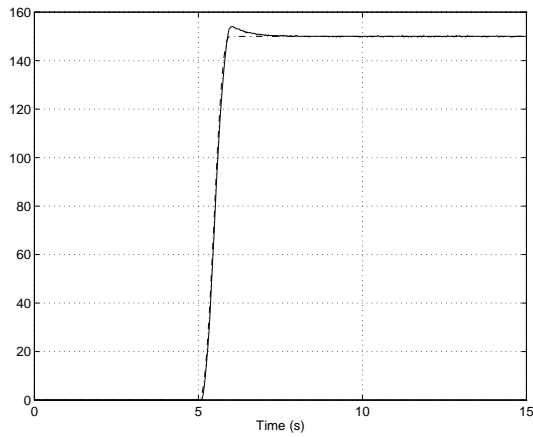


Figure 4: Mechanical speed  $w$  (rd/s) : '-' and Reference trajectory : '-.-' —  $T_{5\%} = 0.84$  s

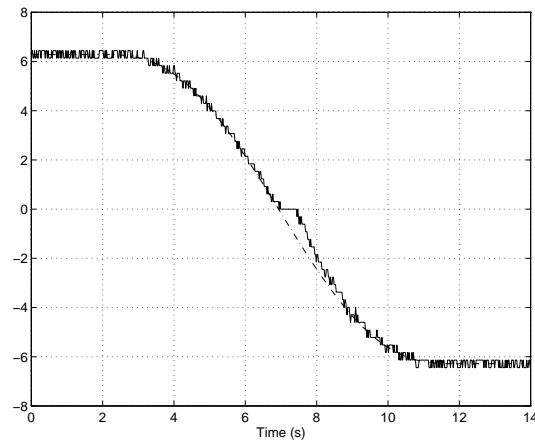


Figure 6: Mechanical speed  $w$  (rd/s) : '-' and Reference trajectory : '-.-'

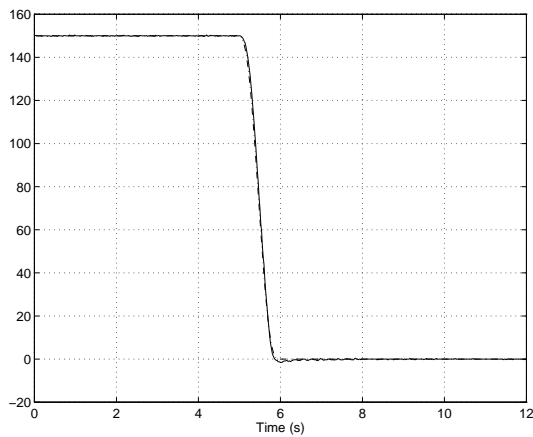


Figure 5: Mechanical speed  $w$  (rd/s) : '-' and Reference trajectory : '-.-' —  $T_{5\%} = 0.76$  s

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