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1971

*Document Version:*
Publisher's PDF, also known as Version of record

Link to publication

*Citation for published version (APA):*
Åström, K. J. (1971). *Modeling and Identification of Power System Components*. (Technical Reports TFRT-7014). Department of Automatic Control, Lund Institute of Technology (LTH).

*Total number of authors:*
1

# MODELING AND IDENTIFICATION OF POWER SYSTEM COMPONENTS.

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REPORT 7116(B) OCTOBER 1971
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ABSTRACT

There is a continuing tendency to apply many of the powerful results of modern control theory to control of various industrial processes. Power systems have been indicated as one area where significant progress can be expected. Practically all results of modern control theory require that models of the processes in terms of state equations are available. The need to obtain such models have been a strong motivation for research in the area of modeling and identification. Some progress made in this area is reviewed in this paper. Modeling based on physical equations and plant experiments are discussed and compared. Particular emphasis is given to parameter estimation techniques like the maximum likelihood method which offer a possibility to combine physical a priori knowledge with experimental investigations. The formulation of identification problems is discussed including the choice of criteria and model structures.

The techniques are illustrated by applications to data obtained from measurements on various components of a power system. The examples include an electric generator, a nuclear reactor and a drum boiler. The examples illustrate the potentials and limitations of system identification and modeling techniques when they are applied to real data.

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This work has been supported by the Swedish Board of Technical Development under Contract 71-50/U33.

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## 1. INTRODUCTION

The design of a control system is frequently divided into two steps determination of a mathematical model and design of a control strategy. In fact most of the control theory that has been developed postulates that a model of the system and its environment is available. To use much of the existing the control theory it is therefore necessary to have techniques to determine suitable models for the processes to be controlled.

Before the advent of modern control theory most results were restricted to linear systems, assuming a model specified by a transfer function. The modeling then reduces to the determination of transfer functions. This is conveniently done experimentally by introducing a sinousoidal variation in the input and measuring amplitude and phase relations between the input and the output. Interesting applications of this techniques to power systems were done early. In Sweden, for example, a group at the ASEA company under the direction of Dr. A. Garde, made extensive measurements on power system components for the purpose of designing control system. A typical study was the determination of the transfer function from power input to frequency variations in the swedish power net reported by Oja [20].

A characteristic feature of many significant results in control theory that have been developed over the past 20 years is that they require other models than transfer functions. Typically many of the results of modern control theory assume that the system is described by time domain models like

$$\frac{dx}{dt} = f(x,u,v)$$

$$y = g(x,u,v) \tag{1.1}$$

where u is the input, y the output, x the state variable and v a disturbance.

To apply the results of modern control theory to industrial processes is it therefore necessary to have techniques available to determine models like (1.1) for the different processes. In this paper we will outline some progress towards the solution of this problem. The results which can be applied to many industrial processes are illustrated by computations on data obtained from experiments on power system components. Two approaches, modeling from physical a priori knowledge and modeling based on measurements of inputs and outputs only are discussed in section 2. Possibilities to combine the techniques are also investigated. Parameter estimation methods and formulation of identification problems are covered in section 3. The choice of model structures for linear deterministic and linear stochastic systems is reviewed in sections 4 and 5. Section 6 deals with criteria for identification problems. In sections 7, 8 and 9 the techniques discussed in the previous sections are applied to the modeling of power system components. The results are based on measurements on real processes. The examples include a power generator, a nuclear reactor and a drum boiler.

## 2. MODELING AND IDENTIFICATION

The problem to be considered is thus how to obtain a model like the one given by equation (1.1) for industrial processes. This problem is sometimes called the inverse problem because a solution is given and the problem is to find the equation which has the given solution. Problems of this type do of course arise in many fields, biology, medicine, economy, physics and chemistry. There are certain advantages to the modeling and identification problems originating from the field of automatic control.

- There is a specific purpose to do the modeling. (Design of control strategies.)
- It is frequently fairly easy to do experiments. (Control systems are designed in such a way that control variables can be manipulated and that outputs can be measured.)

Strictly speaking the problem may not be so well defined. Even if the design of a control system is the final goal it is of course often very valuable to have insight and understanding of several of the system properties that do not enter the control design directly. The possibility of making experiments may be severly limited because it may be necessary to experiment under normal operating conditions and large changes in inputs may be prohibitive for safety and economic reasons.

Process models can be obtained from basic physical laws from pure input-output experiments or from a combination. The different approaches have advantages and disadvantages which are briefly discussed below.

### Modeling from physical principles

The required models can in principle be derived from basic physical laws, expressing conservation of mass, momentum and energy, combined with material equations like Boyle's law or Hooke's law. The models obtained in this way have the great advantage of a wide range of validity. Usually they also provide a good insight into the behaviour of the system. The drawbacks of modeling from physical laws is that the required knowledge is not always available. It is frequently time consuming. (Compare the time required to develop Newtonian mechanics.) It is often difficult to make sensible approximations. A typical difficulty is to find good approximations of distributed parameter systems. Experience has shown that the models developed from basic physical principles tend to be very complex. Since the complexity of the model indirectly implies a complex control strategy and conversely, it means that if a system can be successfully controlled by a simple strategy it can probably also be modeled satisfactory by a simple model. In the area of power systems, components like generators, motors, transmission lines and dynamics of hydroelectric stations are well understood in the sense that models can conveniently be derived from physical principles. On the other hand components like thermal boilers and nuclear power stations are not sufficiently well-known in the sense that models suitable for control can be derived from physical principles alone. In thermal boilers we have e.g. the difficulty with the two phase flow and the uncertainty in heat transfer coefficients. Even if the basic neutron kinetics in nuclear power stations is well-known it leads to very complex models if three dimensioned effects are considered. A large portion of a nuclear reactor dynamics also consists of thermal dynamics and hydromechanics.

Input-output modeling

The pure case of input-output modeling consists of the determination of a model from input-output measurements only. This is often called the black box approach. The advantage of this method is that it is usually done fairly quickly. Experience has also shown that it usually leads to fairly simple models. One serious disadvantage is that in most methods it is possible to determine linear models only. This means that the validity of the model is limited. A change in operating conditions, input signals etc. may thus lead to a different model. Another disadvantage is that available a priori knowledge is not used. For example it is almost impossible to exploit a priori knowledge when a transfer function is determined using frequency response methods. However, in some cases the input-output approach may be the only possibility. This may be the case in the characterization of disturbancies, e.g. load variations.

Recognizing the advantages and disadvantages of modeling from physical equations and from input-output experiments alone it seems highly desirable to try to exploit both methods in order to solve the modeling problem. In the next section we will discuss techniques that can be used to do this.

## 3. PARAMETER ESTIMATION

Using available a priori knowledge about the system to be controlled it is frequently possible to arrive at a class of models {M} that can represent the system. The class of models can for example be the class of all stable linear systems having positive impulse responses, all systems described by equations like (1.1) where the functions f and g depend on a parameter, etc.

The problem is to design an experiment on the real system which makes it possible to select one model in the class which is a good representation of the real system. For a control system the most natural way to select a suitable model would be to compare the performances of the control strategies designed on the models when used to control the real system. Since such a selection is very difficult to do simpler ways are often chosen. It is common to select the models by comparing the error between the modelvariables and the corresponding system variables. The comparison is often based on minimization of a lossfunctional for example of the type

$$V(y,y_m) = \int_0^\infty e^2(t)\, dt \qquad (3.1)$$

where y is the system output, $y_m$ the model output and e the error. The error can for example be defined as

$$e = y - y_m \qquad (3.2)$$

or

$$e = Ay - Ay_m \qquad (3.3)$$

where A is some operator. Examples of this are discussed in [4].

Using this formulation the identification problem reduces to an optimization problem. Select the model in the class {M} such that the chosen criterion is as small as possible. It is natural to ask if there is a unique minimum, if there is a natural choice of lossfunctions, how are the results influenced by the choice of lossfunctions. Some of there problems will be covered in the following sections. In particular it will be shown that if auxiliary assumptions are made there are in fact natural lossfunctions and there will be unique minima.

## 4. LINEAR DETERMINISTIC SYSTEMS

Consider the case when the models is given by a state equation like (1.1) where the functions f and g depend on a set of parameters $\alpha_1, \alpha_2, \ldots \alpha_m$ which are considered as components of a vector $\alpha$. Also assume that the functions f and g are linear in x and u and that disturbances v are neglected. We thus have the standard state space description of a linear system

$$\frac{dx}{dt} = \tilde{A}x + \tilde{B}u$$
$$y = Cx + Du \tag{4.1}$$

where the elements of the matrices $\tilde{A}$, $\tilde{B}$, C and D depend on the parameter $\alpha$. It will be shown that some of the problems raised in the previous section are nontrivial even in such simple cases. Let the experiment be arranged in such a way that the input u is constant over sampling intervals of constant length. The values of the state variables, the inputs and the outputs at the sampling intervals are then given by

$$x(t+1) = Ax(t) + Bu(t)$$
$$y(t) = Cx(t) + Du(t) \tag{4.2}$$

where the sampling interval is chosen as the time unit and A, B, C and D are constant matrices whose elements depend on the parameter $\alpha$. The matrices A and B are related to $\tilde{A}$ and $\tilde{B}$ through well-known equations.

Let n, p and r be the dimensions of x, u and y respectively. The descriptions (4.1) and (4.2) of the system then contains

$$N_1 = n^2 + n(r+p) + rp \tag{4.3}$$

parameters. It is well-known that the input-output relation for (4.2) can be characterized by at most

$$N_2 = n(r+p) + rp \tag{4.4}$$

parameters. It is thus clear that in an identification experiment where the input u of (4.1) or (4.2) is perturbed and the output y observed it is possible to determine at most $N_2$ parameters. Since the models (4.1) and (4.2) have $N_1$ coefficients it is clear that it is not possible to determine all coefficients of (4.1) or (4.2) from an identification experiment. We can thus say that the model is not identifiable. The difficulty can be overcome by choosing a canonical structure or by using additional information about the system.

It is well-known that if the coordinates in the state space of (4.2) are changed by the linear transformation

$$z(t) = Tx(t) \tag{4.5}$$

the equations (4.2) transform to

$$\begin{aligned} z(t+1) &= TAT^{-1}z(t) + TBu(t) \\ y(t) &= CT^{-1}z(t) + Du(t) \end{aligned} \tag{4.6}$$

which have the same input-output relation as (4.2). It is now natural to ask if there are transformations T such that the transformed equations (4.6) are characterized by fewer coefficients than (4.2). Many transformations with that property are known. See e.g. [1], [18], [26] . One example is given below.

Canonical structures

Assume that the system (4.2) is completely controllable and completely observable. Let A be nonsingular and let C have rank r then there exists a transformation T such that the transformed system (4.6) becomes

$$z(t+1) = \begin{bmatrix} a_{11} & a_{12} \dots a_{1n} \\ a_{21} & a_{22} \dots a_{2n} \\ \vdots & \vdots \quad\;\; \vdots \\ a_{r1} & a_{r2} \dots a_{rn} \\ & E & \end{bmatrix} z(t) + \begin{bmatrix} b_{11} & b_{12} \dots b_{1p} \\ b_{21} & b_{22} \dots b_{2p} \\ \vdots & \vdots \quad\;\; \vdots \\ b_{n1} & b_{n2} \dots b_{np} \end{bmatrix} u(t)$$

$$y(t) = \begin{bmatrix} 1 & 0 \dots 0 \dots 0 \\ 0 & 1 \dots 0 \dots 0 \\ \vdots & \vdots \quad \vdots \quad \vdots \\ 0 & 0 \dots 1 \dots 0 \end{bmatrix} z(t) + Du(t) \tag{4.7}$$

where E is a matrix with one nonzero element in each row. (This element can be chosen as 1.)

The proof is straight forward and omitted. There are also dual representations.

The representation (4.7) contains $N_2$ parameters and it is thus a minimum parameter representation. There are, however, two difficulties with this representation. The matrix E reflects the way in which the state variables are coupled to the output. In general there might be

$$N_3 = \binom{n-1}{r-1} \tag{4.8}$$

different matrices E.

Hence if there is no a priori knowledge about the manner in which the outputs are coupled to the state there are $\left(N_3\right)$ different models of the type (4.7). In practice this means that in order to fit a general linear model (4.7) to experimental data it is necessary to determine the best values of the parameters $a_{ij}$, $b_{ij}$ and $d_{ij}$ for all $N_3$ possible E matrices. If $r = 1$ then $N_3 = 1$ and there are no difficulties since there is only one alternative. The case $r = 1$ implies that the system has only one output. Completely observable and completely controllable linear systems with one output thus have only one structural parameter namely the order of the system. For such systems it is thus possible to obtain unique canonical representations. To identify linear systems with one output it is thus sufficient to chose a minimal parameter structure and determine the parameters of models with successively increasing order. For true multivariable system the problem is much more difficult, since for each model order n there are $N_3$ models with different internal couplings. For each order this means that if no structural information is available a priori it is necessary to consider $N_3$ cases for each order of the model. Since $N_3$ is a fairly large number even for moderate values of r and n the investigation of all possible internal couplings is a significant burden.

There are many canonical structures similar to (4.7). They will, however, all suffer from the same difficulty i.e. unless the internal couplings are known there are many different models.

If it is attempted to bypass the difficulties by identifying a multivariable system as an interconnection of single output systems an other difficulty is encountered. Due to uncertainties poles of the single output systems originating from the same mode of the multivariable system are easily estimated as beeing different. This means that the model obtained will be of too high order due to false modes and that the internal couplings of the multivariable system will also be represented incorrectly.

It is thus crucial to exploit a priori information in the identification of a multivariable system.

There is also another difficulty with the representation (4.7). Due to its peculiar structure it can be shown to be very sensitive to parameter variations. Examples are given in [15].

Using a priori knowledge

Having found some difficulties associated with the choice of canonical structures, we will now consider some problems associated with the use of a priori physical knowledge. Having some knowledge about the physics of the process it may be possible to impose some conditions on the elements of the matrices $\tilde{A}$, $\tilde{B}$, C and D of (4.1) or A,B,C,D (4.2). An example is given below.

Example 4.1

Consider a system described by the equations (4.1) where

$$\tilde{A} = \begin{bmatrix} \alpha_1 & 0 & \alpha_6 & \alpha_{10} & \alpha_{15} \\ \alpha_2 & 0 & \alpha_7 & \alpha_{11} & \alpha_{16} \\ \alpha_3 & 0 & \alpha_8 & \alpha_{12} & \alpha_{17} \\ \alpha_4 & 0 & 0 & \alpha_{13} & 0 \\ \alpha_5 & 0 & \alpha_9 & \alpha_{14} & \alpha_{18} \end{bmatrix}$$

$$\tilde{B} = \begin{bmatrix} 0 & \alpha_{20} & \alpha_{24} \\ 0 & \alpha_{21} & \alpha_{25} \\ 0 & \alpha_{22} & \alpha_{26} \\ \alpha_{19} & 0 & 0 \\ 0 & \alpha_{23} & \alpha_{27} \end{bmatrix}$$

$$C = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \end{bmatrix}$$

$$D = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

This is in fact a simplified model of a drumboiler. See Eklund [9]. The statevariables and the inputs have the following meaning

$x_1$ = drum pressure
$x_2$ = drum level
$x_3$ = drum liquid mean temperature
$x_4$ = riser tube mean temperature
$x_5$ = mean value of steam to water ratio in drum and risers

$u_1$ = fuel flow
$u_2$ = feedwater flow
$u_3$ = steam flow.

The fact that certain elements of the matrices $\tilde{A}$, $\tilde{B}$, C and D are zero is obtained from physical considerations, [9].

If it is attempted to fit a model having the structure given in example 4.1 it is of course of interest to know if all the parameters $\alpha_1, \alpha_2, \ldots \alpha_{27}$ can be determined from input-output experiments. The existence of a neat criterion to decide this is still an open problem. Algorithms which shows that the parameters can be determined locally are available, see [6]. In the particular example we have $n = 5$, $r = 2$, $p = 3$. Also notice that the matrix D vanishes identically. The input-output relation can be characterized by 25 parameters. In the particular example we thus find that the 27 parameters of the model can not be determined from an input-output experiment.

## 5. LINEAR STOCHASTIC SYSTEMS

When solving control problems, the characteristics of disturbances are frequently as important as the process dynamics. A significant contribution in modern control theory has also been to model disturbances as stochastic processes and to exploit the theory of stochastic processes to obtain control strategies that take into account certain characteristics of the disturbances. See e.g. [3]. The following model is frequently used to represent a control system subject to random disturbances

$$\begin{aligned} x(t+1) &= Ax(t) + Bu(t) + v(t) \\ y(t) &= Cx(t) + Du(t) + e(t) \end{aligned} \qquad (5.1)$$

In this model $\{v(t),\ t = 0, \pm 1, \pm 2, \ldots\}$ and $\{e(t),\ t = 0, \pm 1, \pm 2, \ldots\}$ are sequences of independent equally distributed random vectors with zero mean values and given covariances

$$\begin{aligned} \text{cov}\left[v(t), v(t)\right] &= R_1 \\ \text{cov}\left[e(t), e(t)\right] &= R_2 \\ \text{cov}\left[v(t), e(t)\right] &= R_{12} \end{aligned} \qquad (5.2)$$

The model (5.1) is fairly general. It can e.g. be used to represent linear finite dimensional linear systems whose disturbances are weakly stationary random processes with rational spectral densities.

The problem of determining the parameters of the model (5.1) will now be discussed. If all the elements of the matrices A, B, C, D, $R_1$, $R_{12}$ and $R_2$ are considered as parameters the model contains

$$N_4 = n(1.5n + p + 2r + 0.5) + r(p + 0.5r + 0.5) \qquad (5.3)$$

parameters. Apart from the difficulties associated with the determination of the parameters of the deterministic part of the system i.e. A, B, C and D it can be shown that all elements of the matrices $R_1$, $R_{12}$ and $R_2$ can not be determined from the data of an input-output experiment.

Assume for example that the matrix $R_2$ is positive definite then the model given by (5.1) is equivalent to the model

(5.4)

$$\hat{x}(t+1) = A\hat{x}(t) + Bu(t) + K\varepsilon(t)$$
$$y(t) = C\hat{x}(t) + Du(t) + \varepsilon(t), \tag{5.4}$$

where $\{\varepsilon(t), t = 0, \pm 1, \pm 2, \ldots\}$ is a sequence of independent equally distributed random vectors with zero mean values and covariances R. The models are equivalent in the sense that the input-output relations are the same and that the stochastic properties of the output are also the same, [3].

The proof of the statement follows from the Kalman-Bucy filtering theorem. The assumption that $R_2$ is positive definite can be relaxed. The model (5.4) is not unique. There are in general many matrices K for which the stochastic properties are the same. The representation is unique, however, if it is required that the matrix A - KC has all eigenvalues inside the unit circle.

The model (5.4) contains

$$N_5 = n(n + 2r + p) + r(0.5r + p + 0.5) \tag{5.5}$$

parameters. It has several interesting properties. The state variable $\hat{x}$ of (5.4) can be interpreted as the best linear estimate of the state x of (5.1) based on observed outputs. The quantities $\{\varepsilon(t)\}$ are the innovations associated with the stochastic process $\{y(t), t = 0, \pm 1, \pm 2, \ldots\}$ of (5.1) or (5.4). The matrix K can be interpreted as the steady state gain of the Kalman filter associated with the model (5.1). The Kalman filter for estimating the state of (5.1) is thus given by

$$\hat{x}(t+1) = A\hat{x}(t) + Bu(t) + K\left[y(t) - C\hat{x}(t) - Du(t)\right] \tag{5.6}$$

This equation is trivially obtained by eliminating $\varepsilon$ in (5.4).

Hence if the purpose of the identification is to design a predictor or a regulator based on linear stochastic control theory the model (5.4) has several advantages over the model given by (5.1). If the model (5.1) is determined it is necessary to solve a Riccati equation in order to obtain the gain of the Kalman filter. It also turns out that the algorithms for the identification problem are simpler for the model structure (5.4). Notice, however, that even with (5.4) there are the difficulties with ambiguities discussed previously for deterministic systems.

In the special case of systems with one output the redundancy in the general model can be reduced by first transforming the state variables so that $TAT^{-1}$ becomes a matrix on companion form and then use the transformation given by Theorem 2. We will then obtain the following model

$$z(t+1)=\begin{bmatrix} -a_1 & 1 & 0\dots0 \\ -a_2 & 0 & 1\dots0 \\ \vdots & \vdots & \vdots \\ -a_{n-1} & 0 & 0\dots1 \\ -a_n & 0 & 0\dots0 \end{bmatrix} z(t) + \begin{bmatrix} b_{11} & \dots b_{1p} \\ b_{21} & \dots b_{2p} \\ \vdots & \vdots \\ b_{n-11} & \dots b_{n-1,p} \\ b_{n1} & \dots b_{np} \end{bmatrix} u(t) + \begin{bmatrix} k_1 \\ k_2 \\ \vdots \\ k_{n-1} \\ k_n \end{bmatrix} \varepsilon(t)$$

$$y(t) = \begin{bmatrix} 1 & 0 & 0\dots0 \end{bmatrix} z(t) + \varepsilon(t) \tag{5.7}$$

By eliminating the state variable z the following input-output relation is obtained.

$$\begin{aligned} y(t)+a_1y(t-1)+\dots+a_ny(t-n) &= b_{11}u_1(t-1)+\dots+b_{n1}u_1(t-n)+\dots \\ &+b_{1p}u_p(t-1)+\dots+b_{np}u_p(t-n)+\varepsilon(t)+c_1\varepsilon(t-1)+\dots+c_n\varepsilon(t-n) \end{aligned} \tag{5.8}$$

where

$$c_i = a_i + k_i , \qquad i = 1, 2, \dots n \tag{5.9}$$

The model (5.8) is thus a canonical representation of a linear stochastic system with one output and several inputs.

The model (5.8) can be written in a slightly more compact form if the polynomials

$$\begin{aligned} A(z) &= z^n + a_1z^{n-1} + \dots + a_n \\ B(z) &= b_1z^{n-1} + \dots + b_n \\ C(z) &= z^n + c_1z^{n-1} + \dots + c_n \end{aligned} \tag{5.10}$$

and the shift operator q defined by

$$qx(t) = x(t+1) \tag{5.11}$$

are introduced. The model (5.8) then becomes

$$A(q)y(t) = B(q)u(t) + C(q)\varepsilon(t) \tag{5.12}$$

## 6. CRITERIA

Having chosen a model whose parameters can be determined from input-output data it remains to find the parameters of the model such that the model fits the experimental data. The crucial problem is then to find suitable criteria. Having the control application in mind it would be natural to evaluate the model on the basis of the performance of the control strategies designed from them. This is generally very difficult to do and it is thus necessary to use other criteria. The criteria can be chosen ad hoc for example like (3.1).

If statistical assumptions are made the parameters can also be determined using statistical parameter estimation techniques. This leads frequently to an optimization problem with a given criterion. It is thus often possible to give statistical interpretations to many criteria.

### Maximum likelihood estimates

Assume that the process is governed by the model (5.4) where $\{\varepsilon(t)\}$ is a sequence of independent gaussian random variables, the parameters can be determined using the method of maximum likelihood. It can be shown that the likelihood function for estimating the model parameters is given by

$$-\ln L = \frac{1}{2}\sum_{t=1}^{N} \varepsilon^T(t)\, R^{-1}\varepsilon(t) + \frac{N}{2}\ln \det R + \text{const.} \qquad (6.1)$$

where $N$ is the record length and $R$ the covariance of $\varepsilon(t)$. The identification problem then reduces to the problem of minimizing the function (6.1) with respect to the unknown parameters. The matrix $R$ is frequently not known a priori. This means that it is necessary to minimize (6.1) with respect to $R$ also. Notice that this can be done analytically. We have

$$\min_R \left[\frac{1}{2}\sum_{t=1}^{N} \varepsilon^T(t)\, R^{-1}\varepsilon(t) + \frac{N}{2}\ln \det R\right]$$

$$= \frac{rN}{2} + \frac{N}{2}\ln \det \frac{1}{N}\sum_{t=1}^{N} \varepsilon(t)\varepsilon^T(t) \qquad (6.2)$$

The minimum is assumed for

$$R = R_0 = \frac{1}{N} \sum_{t=1}^{N} \varepsilon(t)\, \varepsilon^T(t) \tag{6.3}$$

This follows from the identity

$$\sum_{i=1}^{N} x_i^T \left( \sum_{j=1}^{N} x_j x_j^T \right)^{-1} x_i = n \tag{6.4}$$

where n is the dimension of the vectors $x_i$.

The maximum likelihood identification thus reduces to the minimization of the lossfunction

$$V = \det \sum_{t=1}^{N} \varepsilon(t)\, \varepsilon^T(t) \tag{6.5}$$

and conversely an identification problem with the criterion (6.6) can be interpreted as a parameter estimation problem. An estimate of the covariance R is then given by (6.3).

In the special case of systems with one output, $\varepsilon$ is a scalar and the lossfunction reduces to

$$V = \sum_{t=1}^{N} \varepsilon^2(t) \tag{6.6}$$

Under the restrictive assumption that the data was actually generated by a model (5.4) where $\{\varepsilon(t)\}$ is a sequence of independent gaussian random variables it is possible to pose and answer several statistical problems. With mild additional assumptions it can be shown that

- the estimates converge to the true parameters with probability one as the record length N increases (Consistency),
- for large N there are no other estimation procedures that give estimates with smaller variations (Asymptotic efficiency),
- an estimate of the covariance of the estimate $\hat{\alpha}$ is given by

  $$\operatorname{cov}\left[\alpha, \alpha\right] = \frac{2V}{N} V_{\alpha\alpha}^{-1} \tag{6.7}$$

  (Asymptotic normality)

The precise statements of these results are given in [2] for the single output case and in [7] and [27] for the multivariable case.

Many other statistical problems can also be posed. For example the determination of the order of a model can be approached as a statistical problem. Let $V_n$ denote the lossfunction obtained for a model with n parameters. The function $V_n$ decreases with increasing n. The problem is to decide if the decrease in V is significant or not. For systems with one output the testquantity

$$F_{n_1,n_2} = \frac{V_{n_1} - V_{n_2}}{V_{n_2}} \cdot \frac{N - n_2}{n_2 - n_1} \qquad n_2 > n_1 \tag{6.8}$$

where N is the number of observations of the output, can be shown to be asymptotically F-distributed. The quantities $F_{n_1,n_2}$ can thus be used to test if the lossfunction is reduced significantly when the number of parameters in the model is increased from $n_1$ to $n_2$.

Notice that it can be tested if the residuals are gaussian and uncorrelated. It is, however, virtually impossible to assert that the data was actually generated by a model (5.4) with specific parameters. In practice this is of course never true since the model (5.4) is only an approximation of a complex process. The results obtained by using the statistical theories thus must be handled by great care when applied to real data. Notice that when the methods are tried on simulated data it is always possible to assert that the assumptions required by the statistical theory are fulfilled!

Other interpretations

The algorithm obtained by minimizing the criterion (6.5) can be given a physical interpretation even in the case when no statistical assumptions are made. The equation (5.4) can be rewritten as

$$\begin{aligned} \hat{x}(t+1) &= A\hat{x}(t) + Bu(t) + K\left[y(t) - C\hat{x}(t) - Du(t)\right] \\ \varepsilon(t) &= y(t) - C\hat{x}(t) - Du(t) \end{aligned} \tag{6.9}$$

The quantity $C\hat{x}(t) + Du(t)$ can be interpreted as a one step prediction of y(t) based on y(t-1), y(t-2), ... The quantity $\varepsilon(t)$ can thus be interpreted as the difference between the value of the actual measured process output at time t-1. To minimize the lossfunction (6.5) thus means that the parameters of the model are changed in such a way that the error in predicting the output one step ahead is as small as possible according to the criterion (6.5). In the case of single output systems the criterion (6.6) is simply the mean squares prediction error.

## 7. POWER GENERATOR DYNAMICS

In this section the techniques described in the previous sections will be applied to power generator modeling. The results of this section are based on [17] .

The analysis is based on experiments made by Dr. Stanton on a 50 MW turboalternator [24], [25]. The experiment consisted in recording the variations in the terminal voltage V, the active and reactive components of the armature current $I_r$ and $I_q$, respectively and the angular velocity $\omega$ during normal operation. The instrumentation used is described in [23]. This report also contains a description of the experimental procedure and the difficulties with this type of experiment. Several experiments were performed by Stanton. In this example we will base computations on an experiment performed with the governor blocked since we are interested in open loop dynamics. The record length was 600 seconds the sampling interval was 0.5 seconds for the angular velocity and 0.125 seconds for the other variables.

In this example the dynamics relating angular velocity to electric torque $M = VI_r/\omega$ will be considered. A plot of these two variables are shown in the upper part of Fig. 7.1. The generator is frequently described by the simplified model

$$J \frac{d\omega}{dt} + D\omega = M_m - \frac{V \cdot I_r}{\omega} \tag{7.1}$$

where $\omega$ is the angular velocity of the rotor, J the moment of inertia of the rotor, D the damping coefficient, V the terminal voltage, $I_r$ the active component of the armature current and $M_m$ the mechanical torque. There are also more elaborate descriptions leading to models of higher order.

Since it is not known a priori that a model like (7.1) is compatible with the data we could first attempt to fit general linear models with the canonical structure (5.8) and different orders n. This will give an indication of the complexity required to describe the data. The lossfunction obtained from a maximum likelihood identification of models of different order are shown in Table 7.1. To save computer time the computations are based on 1000 input-output pairs only. Since the sampling interval of the angular velocity is 0.5 seconds and the sampling interval of the electric power is 0.125 seconds a compatible data set is created by interpolating the angular velocity. The initial state of the model (5.8) is also determined.

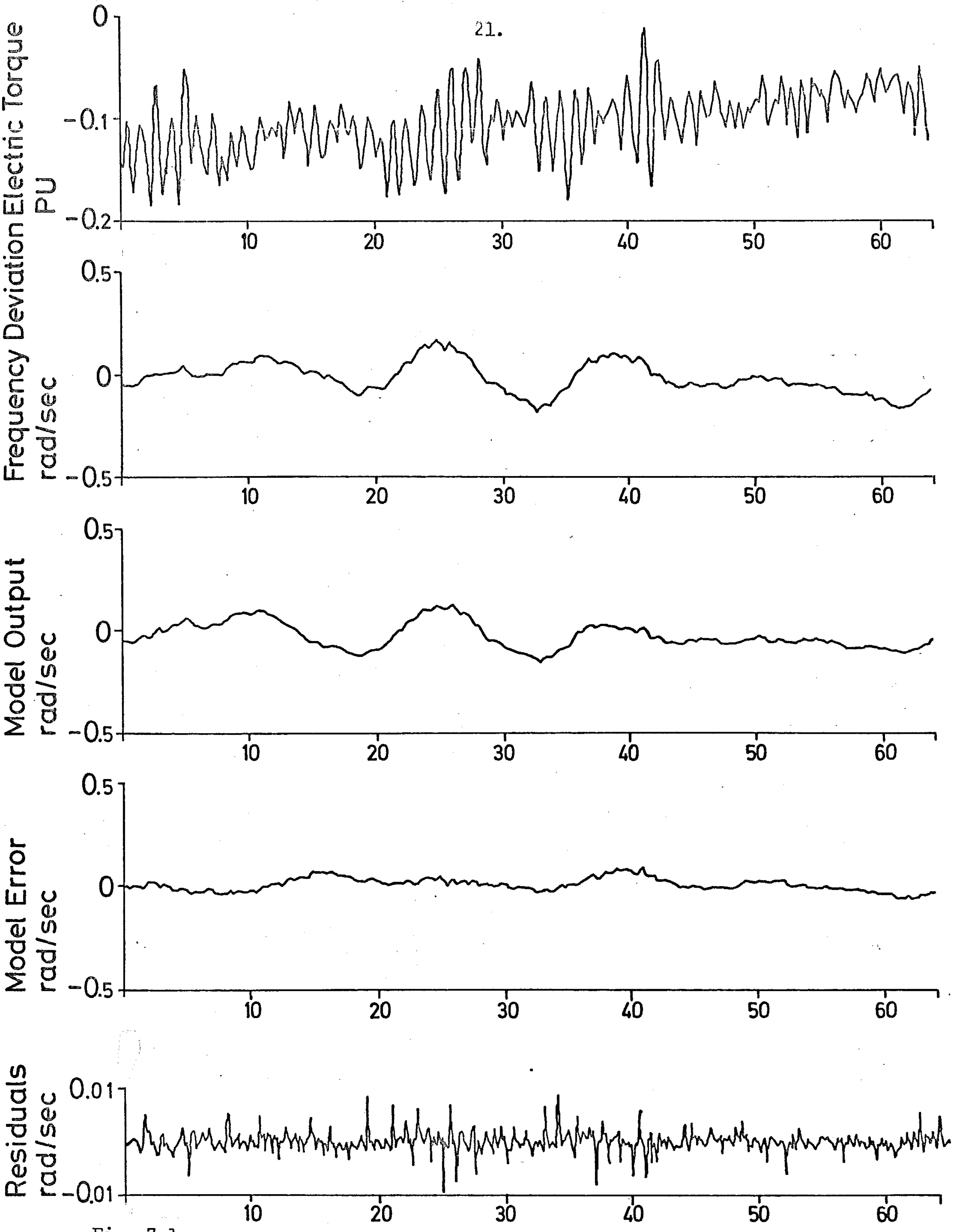


Fig. 7.1

Illustrates the results of identification of power generator dynamics. The upper most curves shows the measured input (electric torque $\delta M = I_r V/\omega$) and the measured output (angular velocity $\omega$). The model output, computed from the fifth order model with coefficients given in Table 7.1 and disturbances neglected, the model error and the residuals $\{\varepsilon(t)\}$ are also shown in the diagram.

Table 7.1. Minimal values of the lossfunction for maximum likelihood identification of models of different order relating angular velocity $\omega$ to electric torque (IrV/N) for the electric generator. The computations are based on 1000 data points.

| $n$ | $V_n$ | $F_{n,n+1}$ |
|---|---|---|
| 1 | 0.01442 | 126 |
| 2 | 0.009562 | 26 |
| 3 | 0.008560 | 73 |
| 4 | 0.006656 | 17 |
| 5 | 0.006230 | 2.0 |
| 6 | 0.006180 | |

In Table 7.1 the testquantities (6.8) for testing the order of the system are also evaluated. Assuming that all assumptions required for the order test are fulfilled a straight forward application of the order test would thus indicate that a fifth order model is appropriate. The coefficients of the fifth order model are shown in Table 7.2. The accuracy estimates are obtained from (6.7). The results obtained by performing the identification on a longer data set N = 2404 data are also shown in Table 7.1. A comparison of the results obtained for N = 1000 and N = 2400 shows that it is not unreasonable to assume that the system is time invariant. A comparison of the accuracy estimates also indicates that they decrease as 1/ N as can be expected from the theory. Notice in Table 7.2 that there is a significant difference between the relative accuracies of the parameters $b_i$ and the relative accuracy of $a_i$ and $c_i$. The results are quite typical to those obtained in other cases when no perturbations in the input are used.

Table 7.2. Coefficients of fifth order model relating angular velocity $\omega$ to electric power for electric generator.

| | | |
|---|---|---|
| N | 1.000 | 2.400 |
| $a_1$ | -2.628±0.031 | -2.581±0.024 |
| $a_2$ | 2.290±0.074 | 2.204±0.049 |
| $a_3$ | -0.715±0.082 | -0.640±0.048 |
| $a_4$ | 0.078±0.067 | 0.024±0.044 |
| $a_5$ | -0.025±0.026 | -0.004±0.019 |
| $b_1$ | -0.835±0.092 | -0.836±0.061 |
| $b_2$ | 1.120±0.244 | 1.016±0.016 |
| $b_3$ | -0.805±0.308 | -0.529±0.197 |
| $b_4$ | 0.599±0.250 | 0.198±0.162 |
| $b_5$ | -0.080±0.100 | 0.150±0.075 |
| $c_1$ | -0.899±0.036 | -0.854±0.027 |
| $c_2$ | -0.005±0.039 | -0.011±0.024 |
| $c_3$ | -0.013±0.036 | -0.027±0.024 |
| $c_4$ | -0.624±0.034 | -0.617±0.021 |
| $c_5$ | 0.579±0.029 | 0.541±0.021 |
| $\lambda$ | 0.0025 | 0.0025 |

The results of the identification are illustrated in Fig. 7.1 which shows the measured inputs and outputs, the model output, the difference between the measured output and the model output. The model output is computed from the model (5.8) with coefficients given in the first column of Table 7.2 and disturbances $\{e(t)\}$ neglected. The model output thus explains how much of the actual output that can be explained from the input. Fig. 7.1 reveals that only about half of the observed output can be related to the input. The signal to noise ratio is thus fairly low. The situation is quite typical for data obtained during normal operation with no extra perturbations introduced. Also notice from table 7.2 that with a record of length 125 seconds it is possible to get reasonable parameter estimates of the parameters $a_i$ and $c_i$.

The residuals can be interpreted as the one step prediction errors obtained from a predictor determined from the model (5.8). The residuals thus shows how well the output can be predicted one step ahead. The standard deviation of the residuals is $\lambda$ = 0.0025 which means that it is possible to predict the angular velocity one sampling interval ahead with a standard deviation of 0.0025 rad/sec using the model obtained.

If the identification procedure should have a nice statistical interpretation the residulas should be independent stochastic variables. In Fig. 7.2 the covariance of the residuals is shown. This diagram indicates that the residuals at least are uncorrelated. The residuals are, however, not normally distributed as is seen by the diagram of cumulative frequencies in Fig. 7.3. This means for example that the results of the order test can be questioned in a case like this.

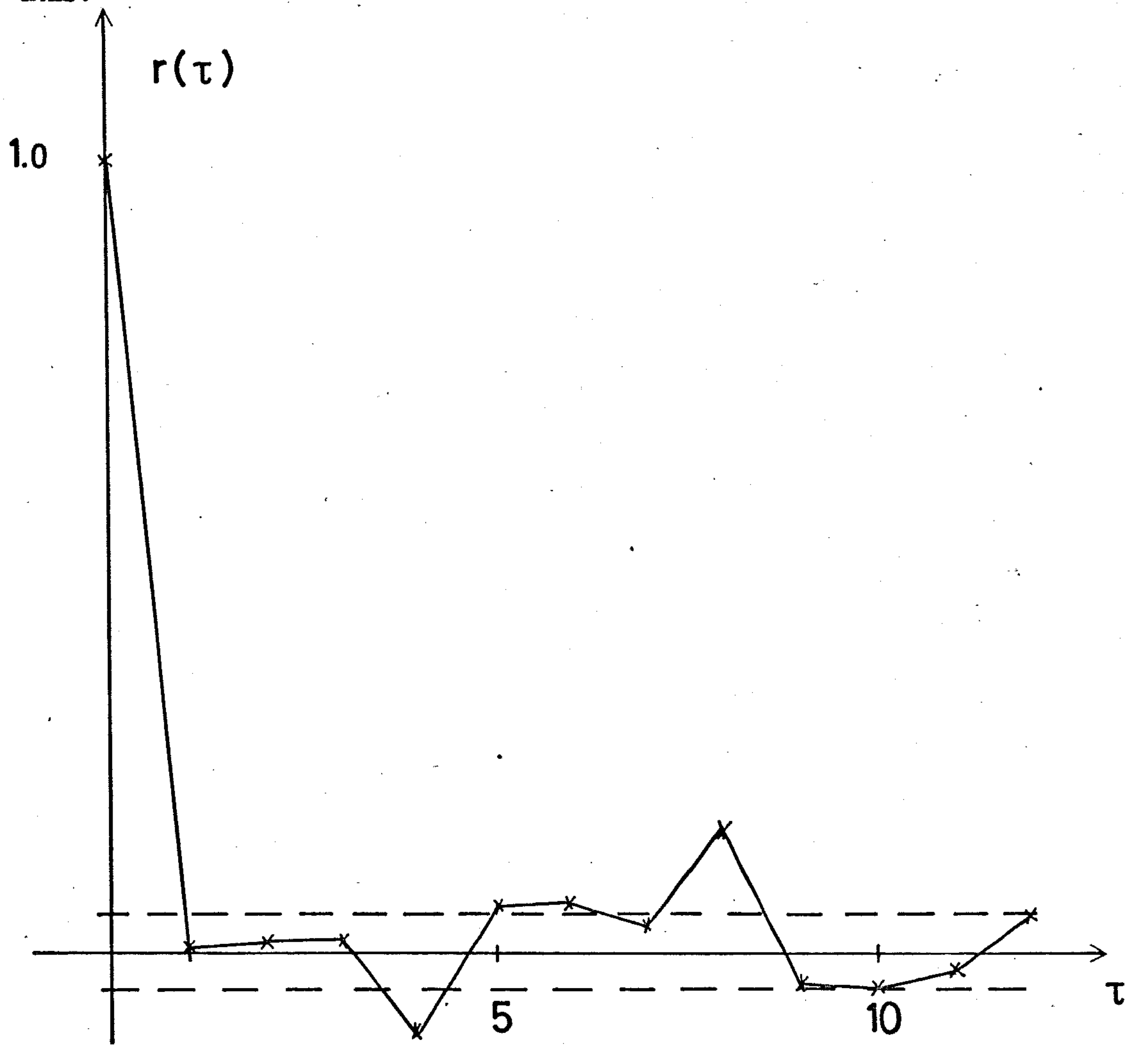


Fig. 7.2

Sample covariance function for the residuals of the model (5.8) with parameters according to the first column of Table 7.2.

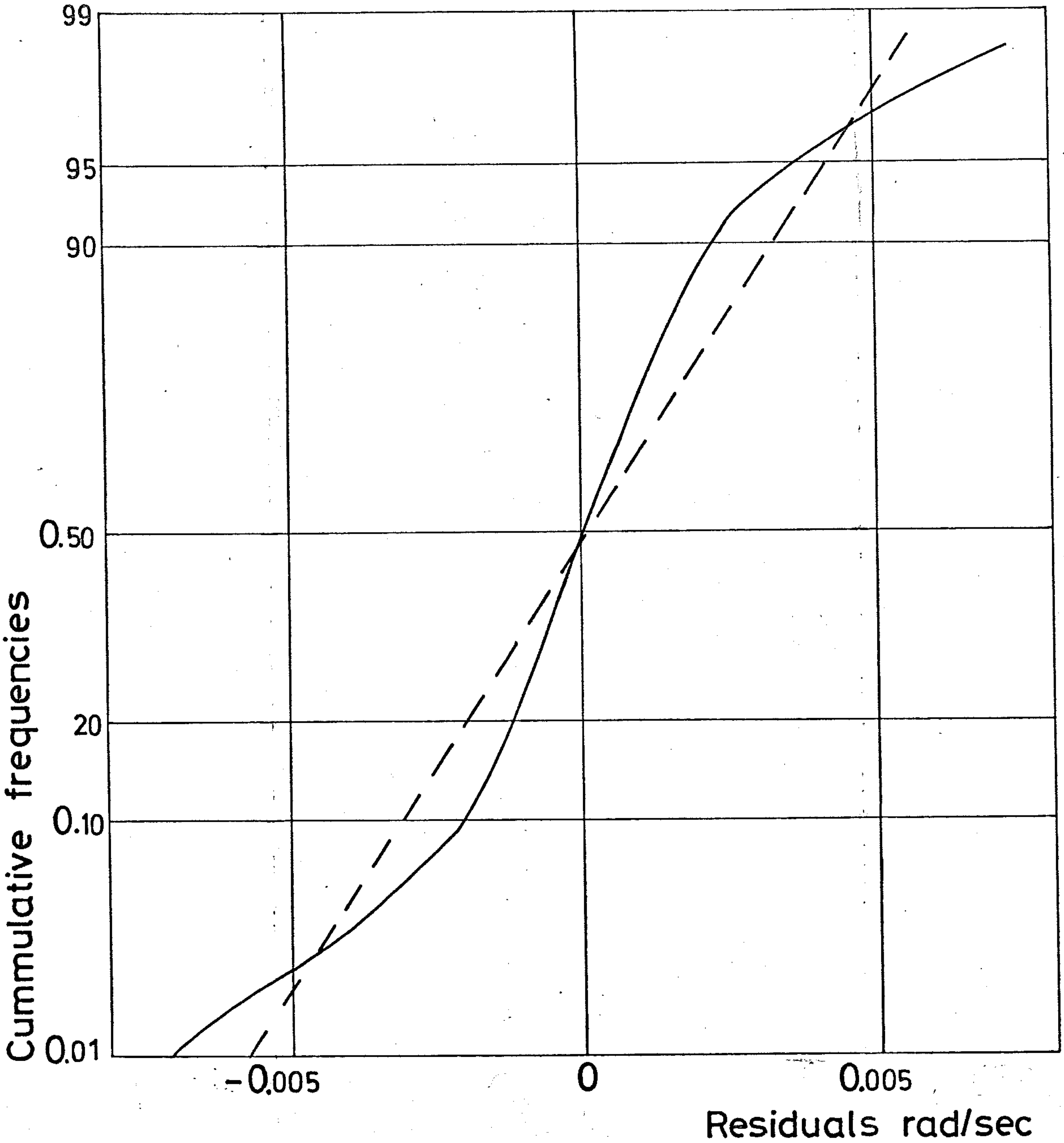


Fig. 7.3

Cumulative frequencies of residuals of the model (5.8). The scales of the diagram are such that a normal distribution corresponds to a straight line.

The results obtained clearly indicates that the simple first order model (7.1) is not compatible with the data.

## 8. NUCLEAR REACTOR DYNAMICS

The experiments were done by AB Atomenergi on The ÅGESTA reactor, located in a suburb of Stockholm. A schematic picture of the reactor is shown in Fig. 8.1. The results of this section are based on [13].

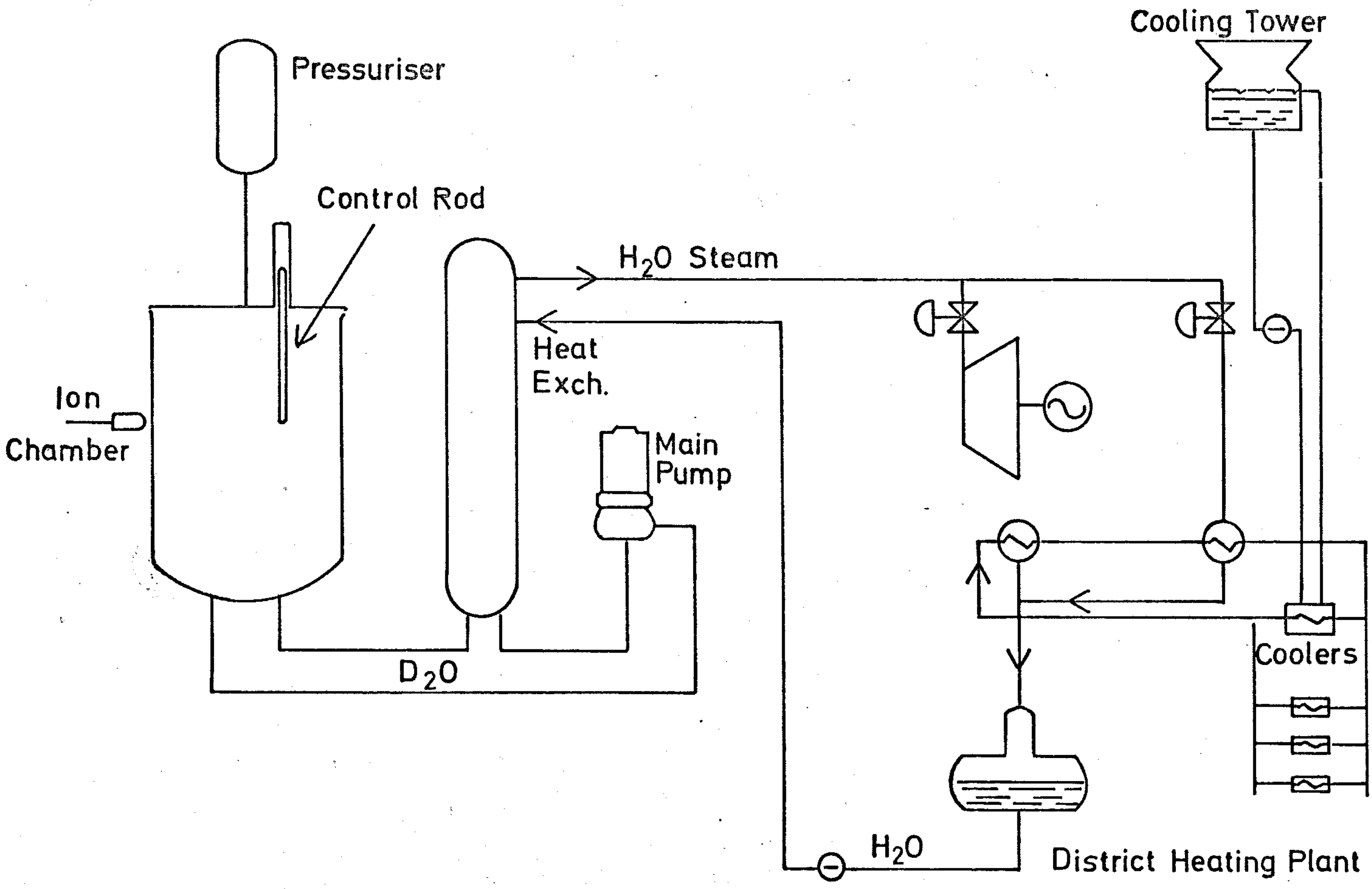


Fig. 8.1

Schematic diagram of the Ågesta nuclear reactor.

In the experiments the control rod position was perturbed and the nuclear power was measured. The input was chosen as a PRBS sequence with period 127. The shortest pulse length was 20 seconds and the sampling interval was 5 seconds. The input-output signals from the experiment are shown in Fig. 8.2.

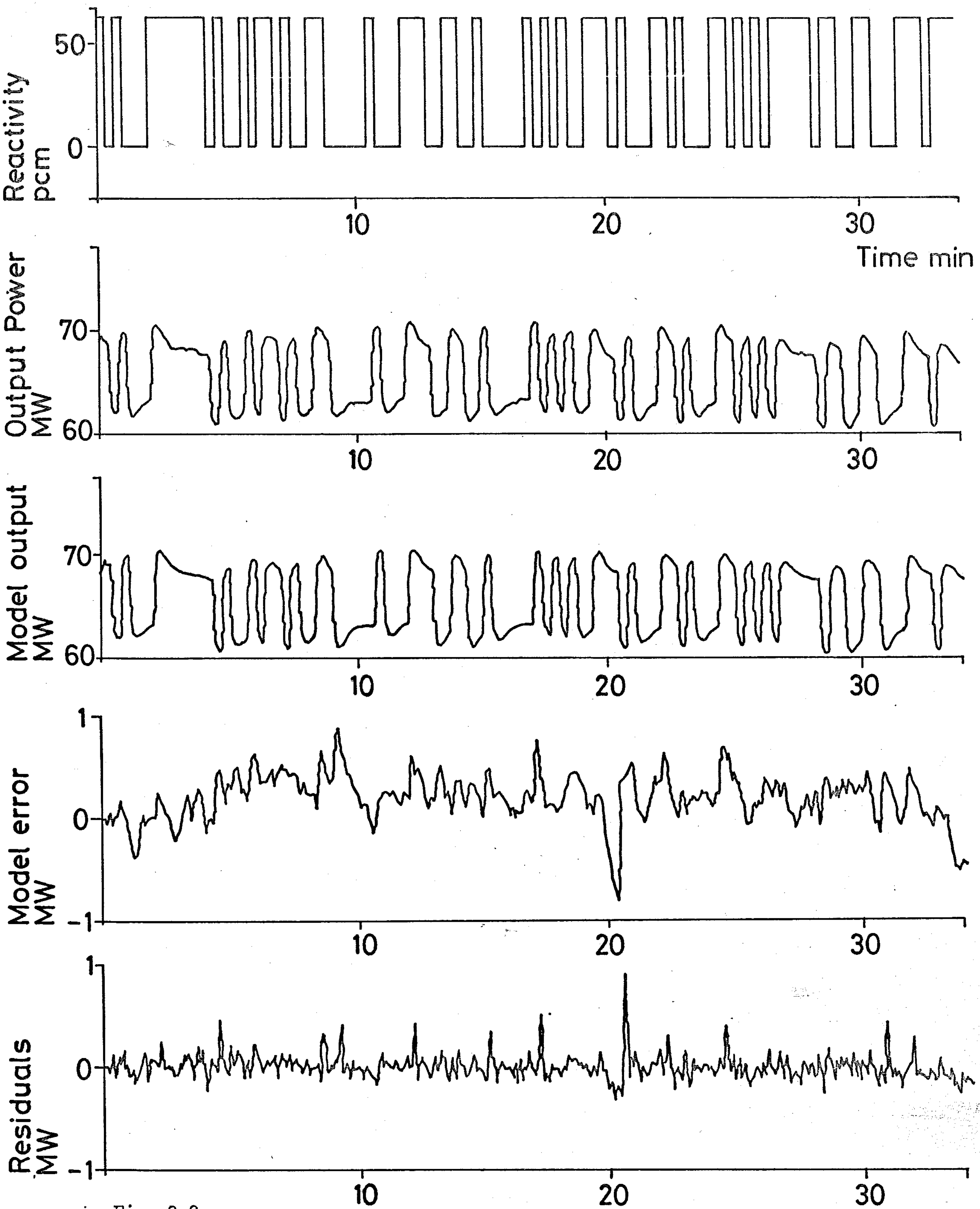


Fig. 8.2

Illustrates the identification of nuclear reactor dynamics. The figure shows the measured input (control rod position), the measured output (nuclear power), the model output, the model error and the residuals.

As an initial attempt a maximum likelihood identification is carried out using models (5.8) of different orders. The values of the lossfunctions obtained shown in Table 8.1.

Table 8.1. Minimal values of the lossfunction for maximum likelihood identification of models of different order relating nuclear power to control rod position. The computations are based on 1126 input-output pairs.

| n | $V_n$ | $F_{n,n+1}$ |
|---|---|---|
| 1 | 203.344 | 2502 |
| 2 | 26.378 | 218 |
| 3 | 16.642 | 61.2 |
| 4 | 14.284 | 1.2 |
| 5 | 14.238 | 0.2 |
| 6 | 14.229 | |

A straight forward application of the order test indicates that the model has to be of at least fourth order. At a risk level of 5% the testlimit is 3 and the decrease in the lossfunction obtained when going from a fourth order system to a fifth order system thus is not significant. The coefficients of the fourth order model is shown in Table 8.2.

The results of the identification are illustrated in Fig. 8.2. It is clear from this diagram that a major part of the observed output is caused by the input. The magnitude of the model error never exceeds 0.8 MW. The residuals have a standard deviation of 0.113 MW which means that the output can be predicted 5 seconds ahead with a standard deviation of 0.113 MW. The large model error and the large residual at time 20.2 min. was traced to a malfunction of the control rod servo.

Table 8.2

Parameters of a fourth order model relating nuclear power to control rod position. The estimates are based on 1125 input-output pairs with a sampling interval of 5 seconds.

| | |
|---|---|
| $a_1$ | -2.442±0.023 |
| $a_2$ | 2.096±0.047 |
| $a_3$ | -0.679±0.029 |
| $a_4$ | 0.032±0.004 |
| $b_1$ | 0.1013±0.0002 |
| $b_2$ | -0.2377±0.0024 |
| $b_3$ | 0.1884±0.0045 |
| $b_4$ | -0.517±0.0023 |
| $c_1$ | -1.479±0.038 |
| $c_2$ | 0.502±0.061 |
| $c_3$ | 0.122±0.055 |
| $c_4$ | -0.072±0.030 |
| $\lambda$ | 0.113 |

By comparing the results given in Table 8.2 with those in Table 7.2 we find that the relative accuracy of the parameters associated with the nuclear reactor dynamics are significantly higher than those associated with the dynamics of the electric generator. Compare in particular the coefficients $b_i$. This is due to the fact that perturbation signals were used in the reactor experiment while no perturbations were introduced in the generator experiment. Similar observations have been made in many other situations.

# 9. DRUM BOILER DYNAMICS

In this section applications to drum boiler modeling will be discussed. The results are based on 10 and 5 . The experiments were performed on the P16-G16 powerplant of the Öresundsverket of Sydkraft AB in Malmö, Sweden. The unit consists of a Steinmüller drum boiler and a Stal-Laval turbine. It has a power of 160 MW. A schematic picture of the boiler is shown in Fig. 9.1.

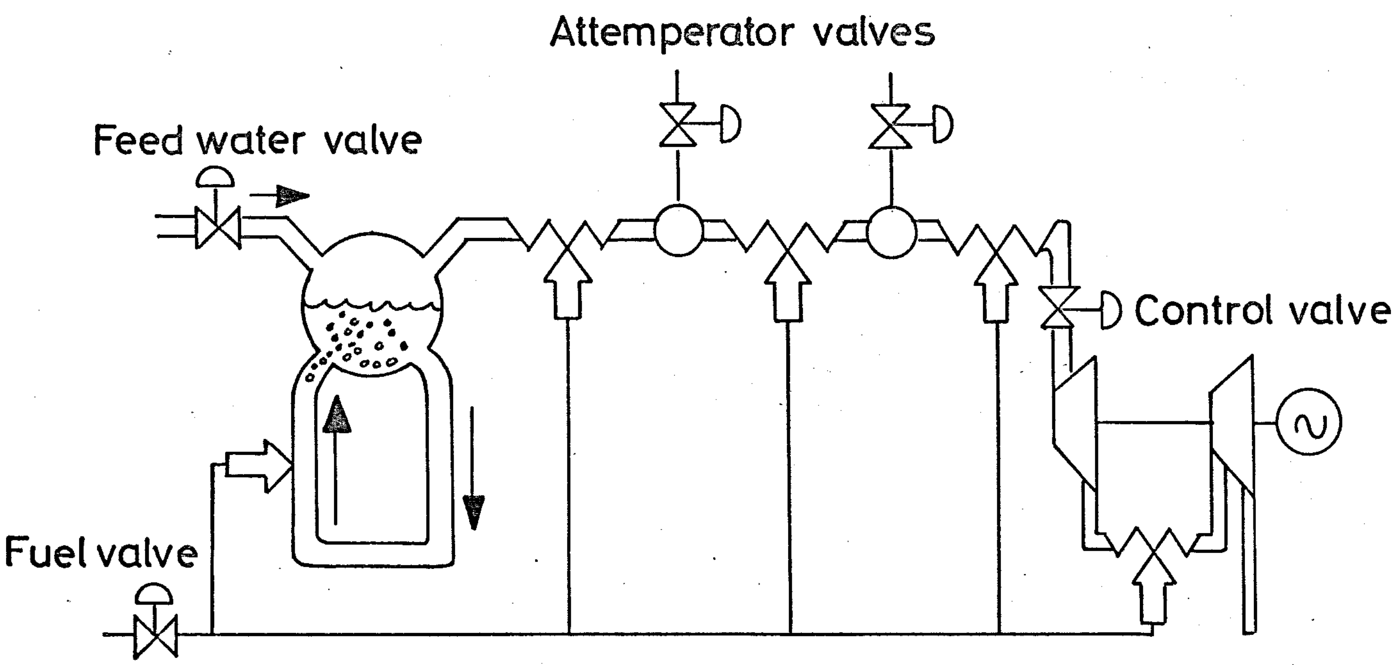


Fig. 9.1.

Schematic diagram of the boiler-turbine unit.

In the experiment the input variables fuel flow, feedwater flow, and control value position were perturbed and the relevant processvariables were recorded. Since the open loop dynamics was of major interest all major regulators except the fuel-air regulator were removed.

For safety reasons only one process variable at a time was changed. The determination of linear multivariable models was done based on superposition of the results of several experiments.

Many experiments were performed both for the purpose of determining linear steady state models and for the purpose of determining nonlinear models. A detailed discussion of the measurements including identification and modeling are given by Eklund [10].

In this example the modeling of the drumboiler only will be discussed. The inputs are taken as fuel flow, feedwater flow and steam flow. The output variables are chosen as drum level and drum pressure. A set of experimental data is shown in Fig. 9.2. The sampling interval was chosen as 10 seconds.

Linear models

Maximum likelihood identification of single output models for drumpressure and drumlevels leads to models of orders 3 and 2. It is thus not unreasonable that a model like the one given in Example 4.1 is compatible with the data. The model given in Example 4.1 was obtained from physical considerations. Since it contains 27 parameters while at most 25 parameters can be determined from input-output data some parameters must be fixed. There are disturbances in the data which are not white measurement noise. It is also necessary to choose the model structure (5.4) which allows for a more general noise model. The 10 elements of K-matrix thus also must be included as parameters. Since a model (5.4) was determined from physical considerations initial estimates of all parameters except those of the K-matrix were available. Several attempts were made to identify different parameter sets. A typical result obtained is listed in Table 9.1. Notice the significant reduction in the value of the lossfunction. The model obtained with the parameters of Table 9.1 is compared with the measurements in Fig. 9.2.

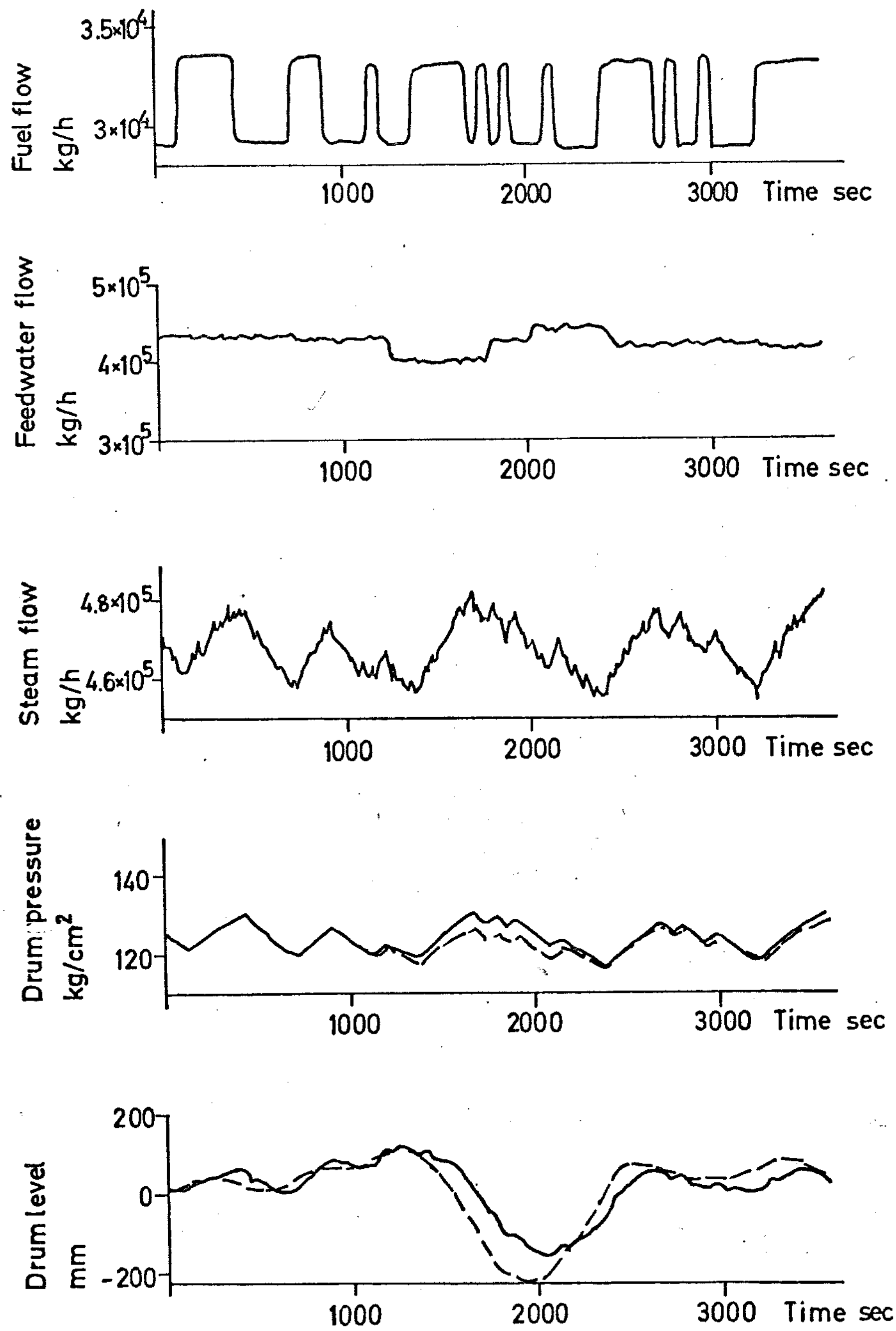


Fig. 9.2

Illustrates results of linear modeling of a drum boiler. The full lines are the measured data and the dashed lines are the model outputs.

Table 9.1

Initial guesses and parameter estimates obtained when fitting a model with the structure of example 4.1 to drum boiler measurements.

| | Initial | Identified |
|---|---|---|
| $V$ | $0.2567\cdot10^{17}$ | $0.3850\cdot10^{5}$ |
| $\alpha_1$ | -0.02835 | -0.02052 |
| $\alpha_3$ | 0.01382 | 0.00896 |
| $\alpha_5$ | $-0.1341\cdot10^{-3}$ | $-0.0738\cdot10^{-3}$ |
| $\alpha_{10}$ | 0.0406 | 0.1678 |
| $\alpha_{13}$ | -0.0454 | -0.3295 |
| $\alpha_{16}$ | -0.2266 | -0.3622 |
| $\alpha_{19}$ | $0.1162\cdot10^{-4}$ | $0.1446\cdot10^{-4}$ |
| $\alpha_{24}$ | -0.01575 | -0.00496 |
| $k_{11}$ | 0. | 1.271 |
| $k_{12}$ | 0. | $-0.1041\cdot10^{-2}$ |
| $k_{21}$ | 0. | 0.01027 |
| $k_{22}$ | 0. | $0.1279\cdot10^{-2}$ |
| $k_{31}$ | 0. | -0.1214 |
| $k_{32}$ | 0. | -0.06576 |
| $k_{41}$ | 0. | 0.5432 |
| $k_{42}$ | 0. | 0.01398 |
| $k_{51}$ | 0. | $-0.7702\cdot10^{-3}$ |
| $k_{52}$ | 0. | $0.3585\cdot10^{-4}$ |

The estimated covariance matrix is

$$\hat{R} = \begin{bmatrix} 0.0240 & 0.016 \\ 0.016 & 12.4 \end{bmatrix}$$

This means that the drumpressure can be predicted 10 seconds ahead with a standard deviation of 0.16 bar and the drumlevel with a standard deviation of 3.5 mm.

Nonlinear models

For power system analysis it is highly desirable to have fairly simple models available which gives the gross behaviour of the different system components.

Based on the studies of linear models like the one just discussed a good deal of insight into the behaviour of boiler turbine units were obtained. It was found that the gross behaviour of a boiler turbine unit could be described by the equations

$$\frac{dp}{dt} = \alpha \left[-f(p,u_2) + g(u_1,u_3)\right] \qquad P = f(p,u_2) \tag{9.1}$$

where p is the drumpressure, P is the output power, $u_1$ is the fuel flow, $u_2$ is the control value setting and $u_3$ is the feedwater flow. A model of the structure (9.1) can also be derived from an energy balance if several (crude) approximations are done. Such a derivation will also give the structure of the functions f and g. Details of this are given in [5]. It is shown that a possibility is

$$f(p,u_2) = \alpha_4 \left[u_2 p^{5/8} - \alpha_5\right] \tag{9.2}$$

$$\alpha g(u_1,u_3) = \alpha_2 u_2 - \alpha_3 u_3 \tag{9.3}$$

$$\alpha = \alpha_1/\alpha_4 \tag{9.4}$$

The boiler-turbine unit can thus be represented by a nonlinear model with five parameters $\alpha_1$, $\alpha_2$, ..., $\alpha_5$. Determining these parameters from several experiments covering a wide operating range the following values are obtained for the unit P16-G16.

$\alpha_1 = 0.00305$
$\alpha_2 = 0.02$
$\alpha_3 = 4.4\cdot10^{-2}$
$\alpha_4 = 11.45$
$\alpha_5 = 8.2$

The performance of the model at two different operating conditions are illustrated in Fig. 9.3 and Fig. 9.4. Notice in particular that the model agrees well with experiments over a wide operating range.

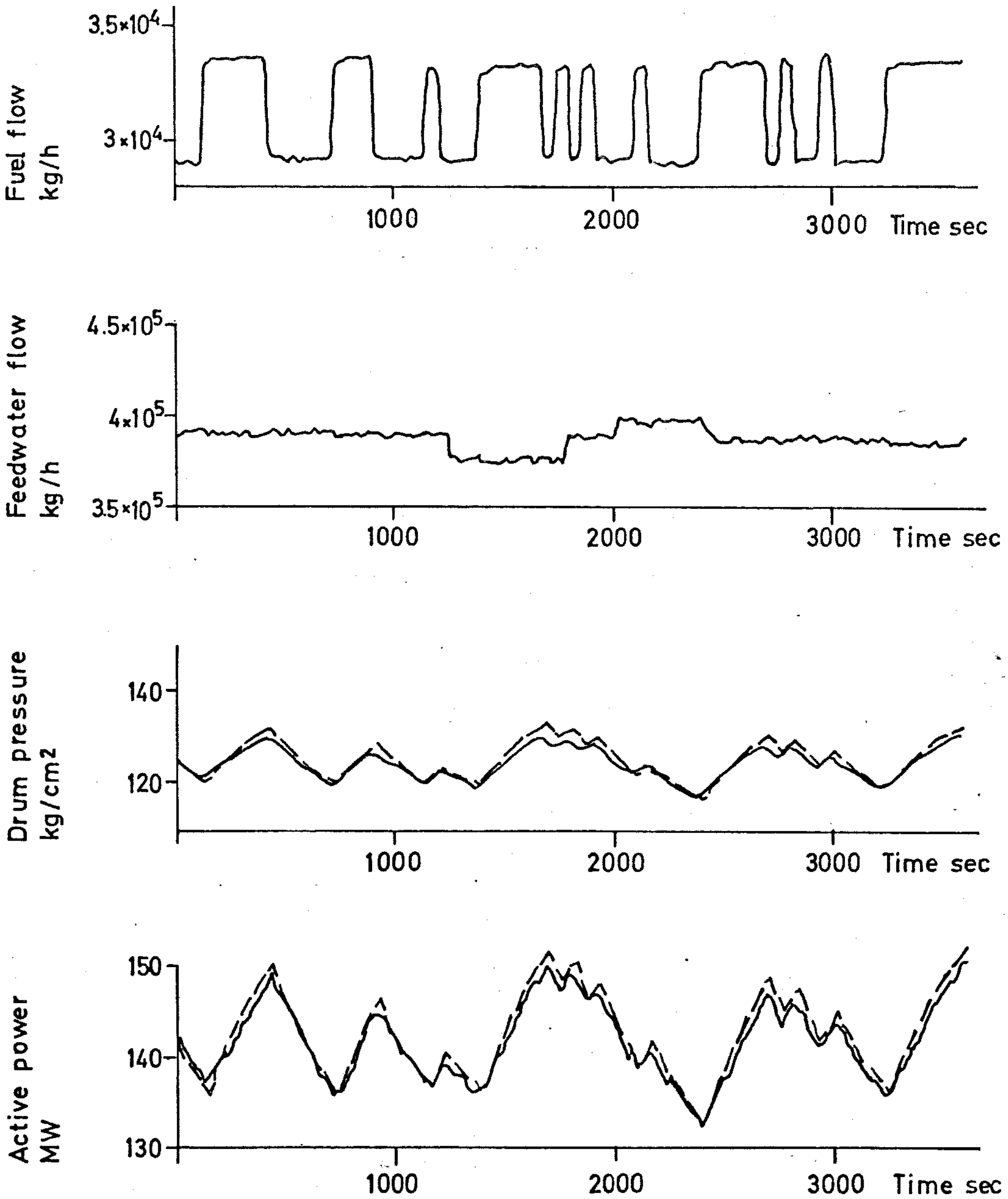


Fig. 9.3

Comparison of measured boiler-turbine (solid) and the responses of the non-linear model (9.1)(dashed).

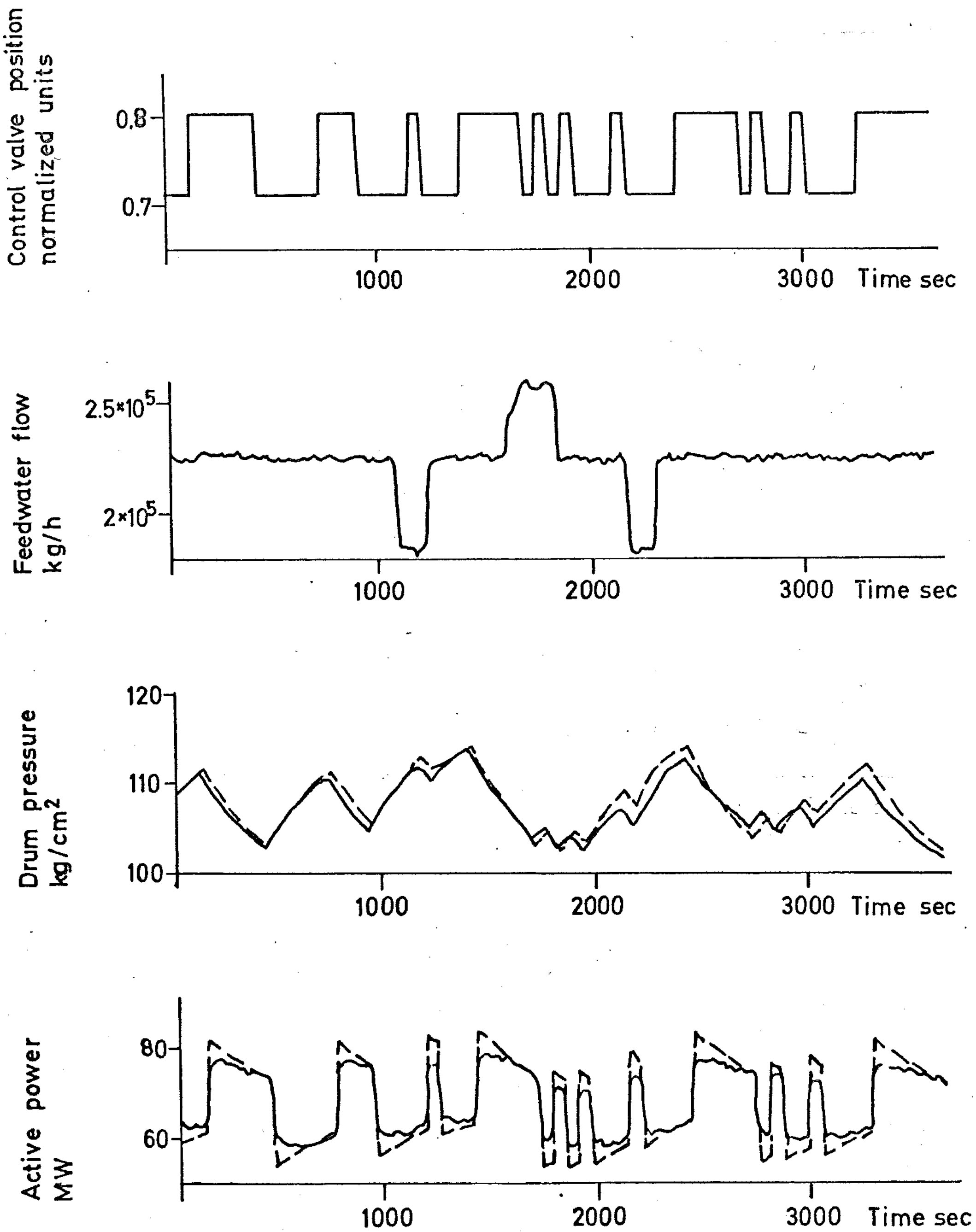


Fig. 9.4

Comparison of measured boiler-turbine (solid) and the responses of the non-linear model (9.1) (dashed).

## 10. ACKNOWLEDGEMENTS

It is my pleasure to acknowledge my gratitude to several persons and institutions. My research in the field of system identification has been partially supported by the Swedish Board for Technical Development (contract 71-50/U33). Data from the power generator measurement were kindly provided by Dr. K.N. Stanton. The results of experiments on the nuclear reactor were provided by AB Atomenergi, Stockholm, Sweden. The experiments on the thermal powerplant were done by Dr. K. Eklund as part of his thesis in collaboration with Sydkraft AB, Malmö, Sweden. The computations reported have been performed by my collaborators at the Lund University Dr. K. Eklund and Mr. S.Lindahl. Special thanks are due to I. Gustavsson with whom I have enjoyed extensive discussions on applications of system identification over many years.

## 11. NOTES AND REFERENCES

System modeling and identification is discussed in many recent articles. Many references are given in the recent survey articles [4] and [19] and in the book [22]. There have also been two IFAC symposia in Prague in 1967 and 1970 entirely devoted to this field. The use of maximum likelihood techniques are discussed in [2], [7], [8], [11], [16], [27]. Canonical structures for linear multivariable systems are treated in [1], [18], [26]. A more detailed discussion of the nuclear reactor identification is given in [13]. The same techniques have also been applied to the modeling of the Halden reactor in Norway. See [21]. The section on boiler modeling is based on [5] and [10] which includes detailed analyses of many measurements.

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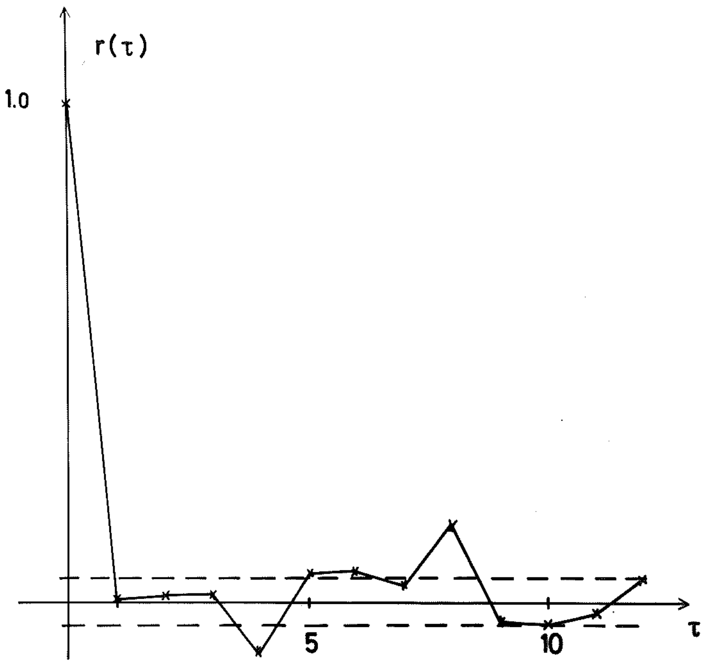


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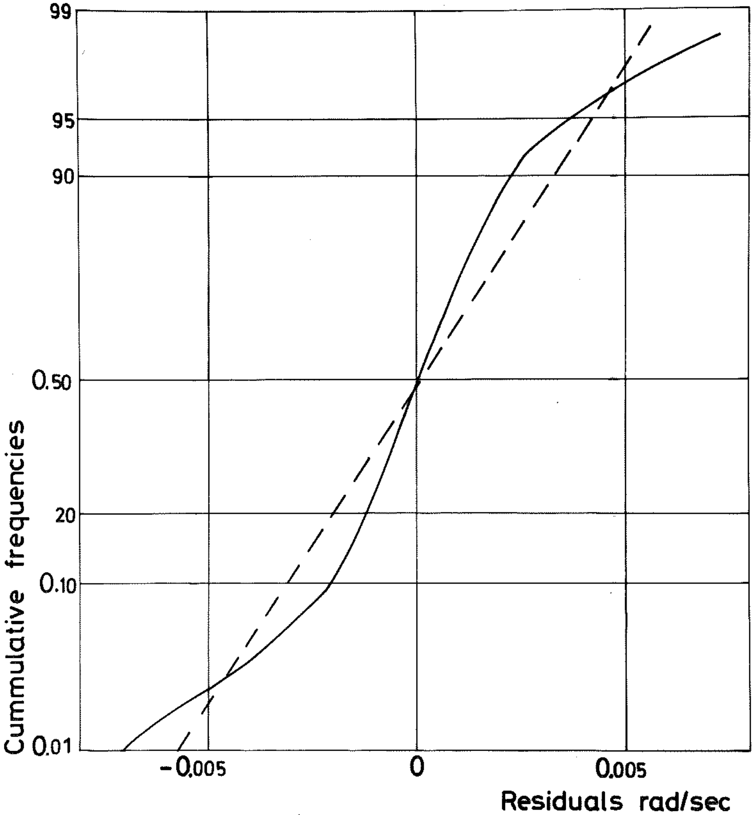


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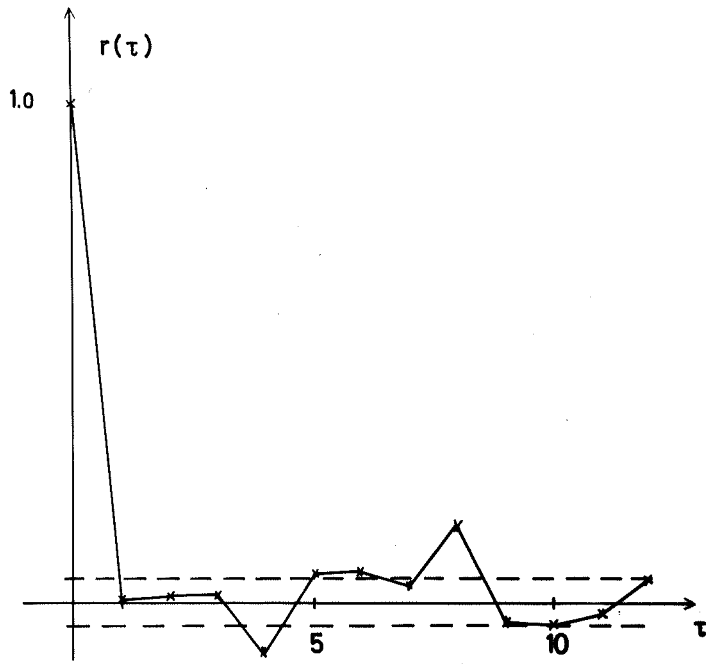


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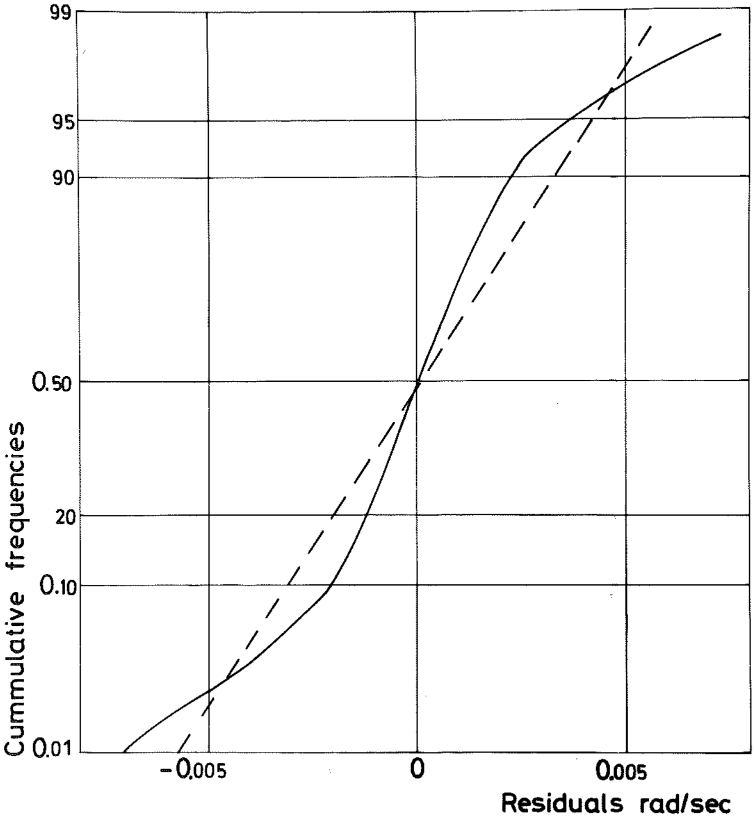


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