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Modeling and Identification of Rate Dependent Hysteresis in Piezoelectric Actuated Nano-Stage: A Gray Box Neural Network Based Approach

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ABSTRACT A modeling and parameter identification method for rate dependent hysteresis of piezoelectric actuated nano-stage is presented in this work. A system level quasi-static hysteresis model is employed to construct a neural network. To better describe the rate dependent behavior of hysteresis in piezoelectric actuated stage, a Nonlinear AutoRegressive Moving Average with eXogenous input (NARMAX) based dynamic model is incorporated with the quasi-static hysteresis model, where the weights of specifically designed neural network corresponds to the model parameters. To handle the multivalued problem of hysteresis, generalized input gradient is proposed to convert multivalued mapping of hysteresis into one-to-one mapping. The parameters of the nonlinear rate dependent hysteresis in piezoelectric actuated stage is identified by neural network training, taking advantage of their universal function approximation capabilities. The proposed scheme is also compared with conventional black box and particle swarm optimization identification based methods, simulation and experimental results demonstrate significant performance improvement with an error of 20.77nm for proposed method whereas 96.56nm and 31.46nm for black box and particle swarm optimization respectively.

INDEX TERMS Extended input space, neural network, model identification, piezoelectric actuator.

I. INTRODUCTION

Piezoelectric (PZT) actuators have received a great deal of attention over the last few decades in nano-positioning applications due to their high speed, stiffness, and fast response [1]. PZT actuators are widely used in applications such as micro/nano positioners, micro grippers, robotic systems [2], [3], positioning of microscope stage [4] etc. But the presence of nonlinear rate dependent hysteresis in PZT's output response, due to their ferromagnetic nature, deteriorates their positioning accuracy. Thus the precise modelling of PZT's output response is very necessary in order to increase its positioning accuracy.

Different models have been presented in the literature to describe the hysteresis phenomenon. For PZT actuators, phenomenological models are widely used, including the Preisach [5]–[8], Maxwell-slip [9], [10], Prandtl-Ishlinskii (PI) [11]–[13], and Krasnosel'skii-Pokrovskii (KP) [14]. Hysteresis compensation is complicated in these models due to the difficulty they present in deriving the inverse hysteresis model. On the other hand, physiological models, such as the Bouc-Wen [15], [16] and Duhem [17] models, require too many parameters to be identified [18], thereby making the identification process a difficult task. Recently, a System Level Model (SLM) was presented in [19], where hysteresis behavior is described through a single function that links the derivatives of the output and input quantities. As such, a separate expression is not required for describing the increasing and decreasing curves of hysteresis. However, the curve-fitting-based parameter identification approach presented in [19] limits its performance.

Several works have been reported in the past on parameter identification of hysteresis models for PZT actuators. Parameter identification using recursive least mean squares based

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algorithms is the prominent model identification technique presented in literature [20]–[23]. Parameter identification of NARMAX model by using least-squares support vector machine is presented in [23]. In [22] a fuzzy least square support vector machine technique which can overcome the slow convergence problem of least-squares support vector machine is presented. An adaptive least squares support vector regression based method is proposed in [21], where the particle swarm optimization is used to optimize the hyper-parameters. The support vector regressions based methods usually suffer from slow training speed also hyperparameters in such algorithms substantially affect the accuracy of regression. The swarm intelligence optimization based method has also gain much importance for model identification recently, due to their advantages of fast convergence and efficient global optimization. Reference [24] presented an improved partial swarm optimization (PSO) method to identify non-linear dynamic hysteresis model. PSO and broad learning system are used to identify nonlinear dynamical systems in [25]. Generally, the optimization techniques suffer from trapping within local optima which deteriorates the accuracy of identified model.

Due to their universal approximation properties, intelligent learning algorithms like neural networks have also received attention recently for the modelling and identification of non-linear systems [26]. To predict the response of nonlinear hysteric system a deep neural network based method is proposed in [27]. NARMAX model based on the Pi-sigma fuzzy neural network to describe the hysteresis in piezo-electric actuated stage is presented in [28]. However, in most the of present literature, the black box neural network model approximation approach is adopted, where parameters of neural network have no mathematical meaning. Since hysteresis corresponds to a multivalued mapping function, it is not possible to directly apply neural networks for the identification of PZT models [29]. Generally gradient of hysteresis output w.r.t. input gradient is used to transform the multivalued problem into a one-to-one mapping problem [30]. But hysteresis in piezoelectric actuators is a rate-dependent non-smooth non-linearity, thus this gradient of hysteresis output w.r.t. input gradient does not exist at non-smooth/extrema points.

This paper explores a feasibility of parameter identification of the system level model [19] by transforming it into a custom designed gray box neural network based model. Unlike black box identification scheme, the weights of neural network in proposed scheme corresponds to the model parameters which have clear mathematical meaning. In order to deal with the multivalued nature of hysteresis a generalized input gradient based mapping scheme is proposed which can extract the moving tendency of hysteresis. An extended input space was created using generalized input gradient to realize the one-to-one mapping between input and output, which is later used by custom design neural network for the identification of model parameters. Finally, a NARMAX model based dynamic model is added to the overall model to describe the rate dependent properties of the PZT actuator. Series

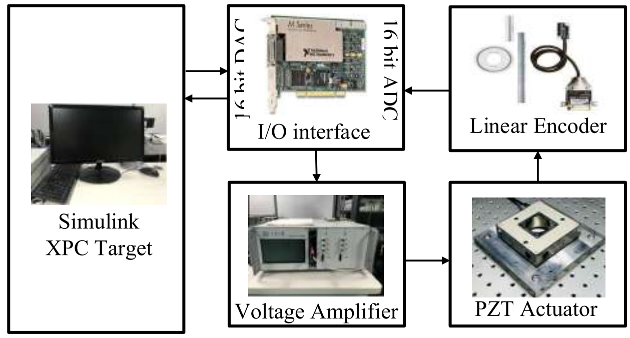

FIGURE 1. The experimental setup.

of experiments were conducted to validate the performance proposed modeling and identification scheme. The overall contributions of this paper are, modeling of quasi static hysteresis model into a gray box neural network, development of generalized input gradient model to convert multivalued hysteresis into one-to-one mapping and finally introduction NARMAX based dynamic model to describe the rate dependent characteristics of pizo-driven nano stage.

The remainder of this paper is organized as follows. Section II states the rate dependent hysteresis phenomenon in PZT actuators. Concept of generalized input gradient is described in Section III. Neural networks based modelling approach is shown in section IV. Results of the experiments with the proposed method are presented in Section V. Finally, Section VI concludes the article.

II. EXPERIMENTAL SETUP AND HYSTERESIS PHENOMENON IN PIEZOELECTRIC ACTUATED NANO-STAGE

This section illustrates the rate-dependent hysteresis phenomenon in piezoelectric actuated motion stage using an experimental example. Figure 1 shows the experimental setup to excite the PZT actuated motion stage and measure its output response. Input signal, generated by Matlab Simulink/xPc Target, is given to high bandwidth voltage amplifier by 16bit resolution DAC interface of NI-6259 for amplification. The amplified voltage signal from high-bandwidth voltage amplifier is fed to the X-Y PZT actuated nano stage. The PZT actuated nano stage is excited in X-direction to record input-output data of single DOF. MicroE systems Mercury II 6000 series linear encoder, with 1.2nm resolution and maximum speed of 61mm/sec, is employed to get the displacement output data from actuator. Displacement data from linear encoder is recorded via of NI-6259 data acquisition card at a sampling rate of 20kHz. The motion system is mounted on an air-floatation platform to reduce external disturbances.

The piezo-actuated nano-motion stage, with characteristic natural frequency of 512Hz, is excited in single degree of freedom with multiple input voltage signals, with amplitude of 100V but different frequencies (i.e., 10hz, 25hz, 50hz and 100hz), to observe the output displacement. The input signal used here is $\sin(2\pi ft) + \rho$, where ρ is the dc offset to keep input amplitude positive. Output displacement curve is

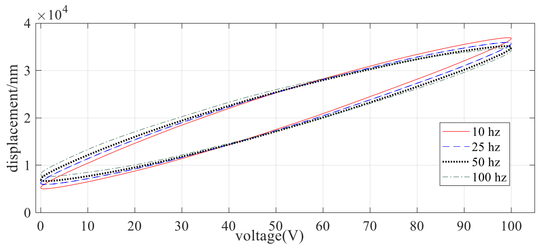

FIGURE 2. Rate-dependent hysteresis in piezoelectric actuated nano-stage.

plotted against input excitation(voltage) is shown in Figure 2. It is observed that with the increase of frequency, the width of output (displacement) to the input(voltage) curve increases. A clockwise twist can also be observed in input output curve with the increase in frequency.

Figure 2 shows that, hysteresis in PZT actuated nano-stage is a multivalued mapping phenomena which is rate-dependent in nature. Whereas the neural network based identification method requires a one-to-one mapping between output and input. Thus neural network based method cannot be directly applied to identify the hysteresis functions. The following section deals with the mapping of multivalued hysteresis into one-to-one mapping.

III. GENERALIZED INPUT GRADIENT

Hysteresis is non-smooth multivalued mapping nonlinearity which makes it difficult to directly apply conventional model identification techniques. The neural networks based identification method requires a continuous relation with one-to-one mapping between input and output data. Usually a hysteresis output gradient with respect to input, is introduced in input space to handle multivalued problem of hysteresis [30]. However, at extrema points this so called gradient of hysteresis with respect to input does not exist. To deal with this problem, a generalized input gradient based method is proposed in this section.

Since hysteresis is a locally Lipschitz function, the input gradient $\acute{f}[u(t)]$ can be obtained in smooth segments:

$$\acute{f}[u(t)] = \frac{u(t+\Delta t) - u(t)}{\Delta t} \tag{1}$$

where $u(t)$ locally Lipschitz continuous input function such that $u(t) : R^n \times R \to R$, $|\Delta t| < \gamma$ is a small change in time and $\gamma \to 0$ is an arbitrary small positive number.

Generalized input gradient can be defined as:

$$f[u(t)] = \begin{cases} \tilde{f}[u(t)], & \text{extrema} \\ \acute{f}[u(t)], & \text{else} \end{cases} \tag{2}$$

from [30] the input gradient at extrema points, $\tilde{f}[u(t)]$ can be defined as:

$$\tilde{f}[u(t)] \triangleq \overline{co}\{\lim \acute{f}[u(t_i)] \,|t_i \to t, u(t_i) \to u(t), \times u(t_i) \notin \Omega_V \cup N\} \tag{3}$$

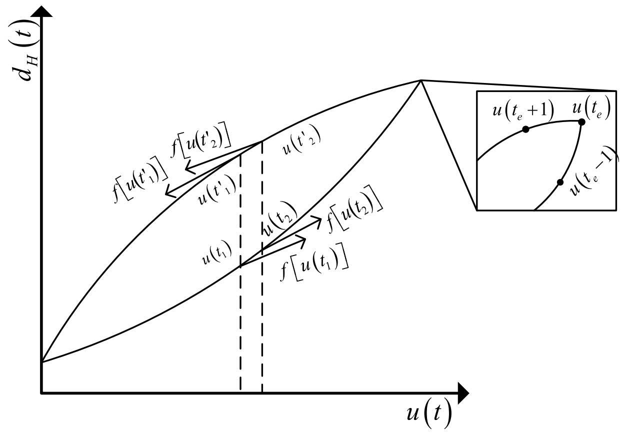

FIGURE 3. Multivalued nature of hysteresis.

where N represents an arbitrary set of zeros, Ω_V is the set containing Lebesgue measure zero where $\acute{f}[u(t)]$ does not exist and $\overline{co}$ is the convex closure.

From (3) it is clear that at extrema points the generalized input gradient is bounded. The generalized input gradient at extrema points, from the properties of rate-dependent hysteresis, can be expressed as:

$$\tilde{f}[u(t)] = \acute{f}[u(t-\zeta)] \tag{4}$$

where $\zeta \to 0$ is a small positive number. Since at turning points the conventional input gradient from (1) does not exists, $\tilde{f}[u(t)]$ can be utilized as the input gradient to extract the change tendency of hysteresis.

However, it is well established that neural networks based identification can only be applied to a continuous system with one-to-one mapping[31]–[34]. In the following it is shown that $f[u(t)]$ is a continuous mapping function.

Lemma 1: For time instants $t_1 \& t_2 \in t$, where $t_1 \neq t_2$, $f[u(t)]$ is a continuous mapping function such that if $u(t_2) - u(t_1) \to 0$ then $f[u(t_2)] - f[u(t_1)] \to 0$, where $u(t) \in M(R^+)$ for time "t", $M(R^+)$ is the set of continuous function of R^+ and $R^+ = \{t|t \geq 0\}$.

Proof: Consider t_1 and t_2 are not extrema points, $t_2 - t_1 \to 0$ and $t_2 > t_1$, also consider rising curve case of hysteresis loop from Figure 3 i.e. $u(t_2) > u(t_1)$. For input $u(t_1)$ at time t_1:

$$f[u(t_1)] = \frac{u(t_1+\Delta t) - u(t_1)}{\Delta t} \tag{5}$$

where $\Delta t \to 0$ is a small change in time.

$$f[u(t_2)] - f[u(t_1)] = \frac{[u(t_2+\Delta t) - u(t_1+\Delta t)] - [u(t_2) - u(t_1)]}{\Delta t} \tag{6}$$

For any periodic continuous input $u(t)$, if $u(t_2) - u(t_1) \to 0$ then $u(t_2+\Delta t) - u(t_1+\Delta t) \to 0$. From (6) if $u(t_2) - u(t_1) \to 0$ then $f[u(t_2)] - f[u(t_1)] \to 0$. Same can be proven for descending curve of hysteresis loop i.e. that if $u(t'_1) - u(t'_2) \to 0$ then $f[u(t'_1)] - f[u(t'_2)] \to 0$ shown in Figure 3. Since from (4) $\tilde{f}[u(t)] \subset \acute{f}[u(t)]$, hence it is clear that $f[u(t)]$ is continuous mapping for a periodic continues input $u(t)$.

Theorem 1: For hysteresis space defined as $R^2 : U \times F$, where $u(t) \in U$ is the input and $f[u(t)] \in F$ is the mapping defined in (2), $d_H(t)$ is the output of rate dependent hysteresis, and input $u(t)$ satisfies the condition in Lemma 1 then there exist a continuous one-to-one mapping $\Gamma : R^2 \rightarrow R$ such that:

$$d_H(t) = \Gamma(f[u(t)], \quad u(t)) \tag{7}$$

Proof: Taking the contradiction approach to prove theorem 1, consider a time instant $(t+\Delta t)$ and $(t+\Delta t) > t$ also $(t+\Delta t) - t = \delta$ where $\delta \rightarrow 0$ is an arbitrary small positive number. Consider input $u(t_1)$ and $u(\acute{t}_1)$ are the inputs at time instants t_1 and $\acute{t}_1$ respectively as shown in Figure 3 where:

$$u(t_1) = u(\acute{t}_1), \quad t_1 \neq \acute{t}_1 \tag{8}$$

for time instants t_1 and $\acute{t}_1$ are not extrema points, proposed genitalized input gradient is given as:

$$f[u(t_1)] = \frac{u(t_1+\Delta t) - u(t_1)}{\Delta t} \tag{9}$$

$$f[u(\acute{t}_1)] = \frac{u(\acute{t}_1+\Delta t) - u(\acute{t}_1)}{\Delta t} \tag{10}$$

let's assume for $u(t_1) = u(\acute{t}_1)$:

$$f[u(t_1)] = f[u(\acute{t}_1)] \tag{11}$$

from (9) and (10):

$$u(t_1) - u(t_1+\Delta t) = u(\acute{t}_1) - u(\acute{t}_1+\Delta t) \tag{12}$$

from (12):

$$u(t_1+\Delta t) = u(\acute{t}_1+\Delta t) \tag{13}$$

from Figure 3, it is clear that (13) contradicts with the property of hysteresis which means that assumption made in (11) does not hold. Hence:

$$(f[u(t_1)], u(t_1)) \neq (f[u(\acute{t}_1)], u(\acute{t}_1)). \tag{14}$$

Thus there exist a one-to-one mapping such that $d_H(t) = \Gamma(f[u(t)], u(t))$, that can be used with conventional identification methods, such as neural networks, for the identification multivalued hysteresis.

IV. NEURAL NETWORK MODEL OF HYSTERESIS

The hysteresis behavior of piezoelectric actuated stage is a rate dependent phenomenon i.e. the overall output of the actuator not only depends on the amplitude of input voltage but also on input frequency. The output displacement due to amplitude of input voltage is the quasi static behavior whereas the change in output hysteresis with the change in input frequency is the dynamic behavior of piezoelectric actuated stage. The overall displacement output $y(t)$ of the actuator is sum of its static and dynamic response. Here the system level model [19] is used to describe the quasi static behavior of piezoelectric actuated nano-stage and NARMAX model based dynamic model is proposed to describe its rate dependent properties. Quasi static and dynamic sub-models of piezoelectric actuated nano-stage are presented in Section IV-A and B respectively.

A. NEURAL NETWORK QUASI STATIC SUB-MODEL

1) QUASI STATIC RELATION BETWEEN INPUT AND OUTPUT

For a piezoelectric actuated stage in quasi static state relation between input voltag $u(t) : R^n \times R \rightarrow R$ and output displacement $d_H(t)$ [19] is given as:

$$d_H(t) = \int d(u(t)) \frac{du(t)}{dt} dt \tag{15}$$

$$d(u(t)) = d_o + g\left(u(t) - V_{shift}(t)\right) \tag{16}$$

where d_o and $d(u(t))$ are the initial conditions and pre-exponential coefficient, $u(t)$ is the input voltage, V_c is the voltage constant. The exponential model fit $g\left(u(t) - V_{shift}\right)$ is given by:

$$g\left(u(t) - V_{shift}\right) = \Delta d\left(1 - \gamma e^{-\frac{\left|u(t)-V_{shift}(t)\right|}{V_c}}\right) \tag{17}$$

where $\Delta d, \gamma$ and V_c are the loop shaping parameters, $V_{shift}(t) \subset u(t)$ is the value of input voltage at last zero value of input derivative:

$$V_{Shift}(t) = \sum_{k=0}^{\infty} u(t) \prod\left(\frac{t-t_k}{t_{k+1}-t_k} - \frac{1}{2}\right) \tag{18}$$

where $t_k \in t$ is the time instant when input voltage derivative attains its last zero value, such that

$$\forall \tau \in IR : \frac{du(\tau)}{dt} = 0 \Rightarrow \exists k \in IN : \tau = t_k, \quad t_k < t_{k+1} \tag{19}$$

and rectangular function $\Pi(x)$ is defined as:

$$\prod(x) = \begin{cases} 0 & if \ |x| > \frac{1}{2} \\ 1 & if \ |x| \leq \frac{1}{2} \end{cases} \tag{20}$$

2) CONSTRUCTION OF NEURAL NETWORK BASED ON QUASI STATIC MODEL

This section demonstrates the approximation of quasi static sub-model presented in previous section by converting it into a custom design neural network. Displacement of PZT actuator can be given as [19]

$$d_H(t) = d_o + \Delta d\left(1 - \gamma e^{\frac{-\left|u(t)-V_{shift}(t)\right|}{V_c}}\right) u(t) \tag{21}$$

$\Delta d, \gamma$, and V_c are the parameters to be identified. Considering the universal approximation properties of neural network, the generalized input gradient is introduced in (21) to convert multi valued mapping of hysteresis into one to one mapping. Rewriting (21) in discrete form:

$$d_H(k+1) = d_o + \Delta d_o T\left(1 - \gamma_o e^{\frac{-\left|u(k)-V_{shift}(k)\right|}{V_{co}}}\right) \times u(k) + h_s f[u(k)] \tag{22}$$

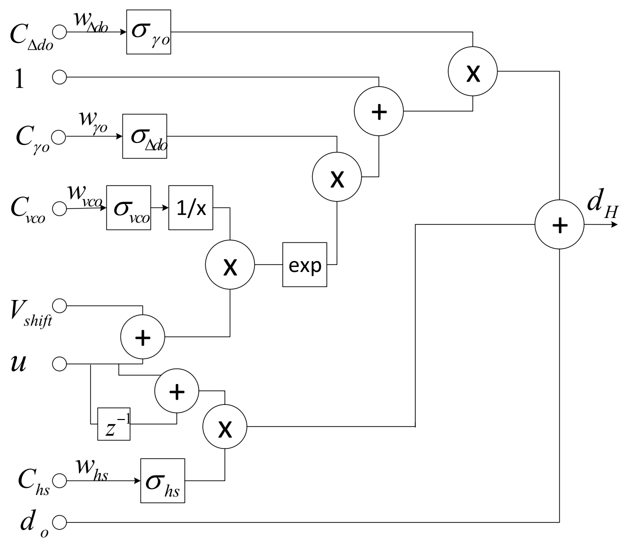

FIGURE 4. Static sub-model neural network structure.

where T is the sampling time, $\Delta d_o, \gamma_o$ and V_{co} are the model parameter with the one-to-one mapping introduced in the model and h_s is hysteresis mapping coefficient. The topological architecture of proposed neural network can be designed by utilizing the interconnections and relationships among the variables in (22). Figure 4 shows the structure of neural network where symbols $\oplus$ and $\otimes$ represent addition and multiplication node respectively. $\sigma_{\Delta d_o}, \sigma_{\gamma_o}, \sigma_{hs}$ and σ_{Vco} are the activation functions of the training nodes of neural network for the identification of the parameters h_s, Δd_o, γ_o and V_{co}. Specially designed inputs $C_{\Delta d_o}, C_{\gamma_o}, C_{hs}$ and C_{Vco} are used to train the weights $w_{\Delta d_o}, w_{\gamma_o}, w_{hs}$ and $w_{V_{co}}$ [18].

Adjustment of weights during training process can set an optimized value of model parameters. Activation function for the identification of parameters are defined as:

$$\sigma_{\Delta d_o}(x) = e^{a_{\Delta d_o} x} \quad (23)$$

$$\sigma_{\gamma_o}(x) = e^{a_{\gamma_o} x} \quad (24)$$

$$\sigma_{hs}(x) = e^{a_{hs} x} \quad (25)$$

$$\sigma_{Vco}(x) = e^{a_{Vco} x} \quad (26)$$

activation function parameters $a_{\Delta d_o}, a_{\gamma_o}, a_{hs}$ and a_{Vco} values are the stepping stair height in order to achieve optimized vales of neural network parameters.

Quasi static sub-model is trained by using a low frequency data set according to the rules defined in Figure 4. Levenberg-Marquardt training algorithm is used to train neural network. Weight updating during training of static sub-model is done by following:

$$\Delta w_{st}(k) = \frac{J_{st}^T e_{st}(k)}{(J_{st}^T J_{st} - \mu I)} \quad (27)$$

wher

$$e_{st}(k) = y_L(k) - d_H(k) \quad (28)$$

$$J_{st} = \left[\left[\frac{\partial e_{st}(k)}{\partial h_s} \right] \left[\frac{\partial e_{st}(k)}{\partial \Delta d_o} \right] \left[\frac{\partial e_{st}(k)}{\partial \gamma_o} \right] \left[\frac{\partial e_{st}(k)}{\partial V_{co}} \right] \right] \quad (29)$$

where $e_{st}(k)$ is the error matrix, y_L is the output from training data set, $d_H(k)$ is the output of static sub-model. Neural network is custom designed such that the error function $e_{st}(k)$ can only be tuned by changing weights $w_{\Delta d_o}, w_{\gamma_o}, w_{hs}, w_{Vco}$. This implies that most of the entities in J_{st}^T will be zero because they are not effecting the training process, thus the overall training time will be less than the conventional black box neural network identification technique.

B. NARMAX DYNAMIC SUB MODEL

In order to describe the rate dependent behavior of PZT actuator, a dynamic sub-model is introduced. Since linear model cannot approximate the dynamical properties of piezoelectric actuated stage properly, thus NARMAX model based dynamic sub-model is proposed here. Output y_D of NARMAX based dynamic sub-model is defined as:

$$y_D(t) = \Gamma\left(u(t), u(t-1) \ldots u\left(t-n_f\right), \ldots y(t-1), y(t-2) \ldots y\left(t-n_y\right)\right) \quad (30)$$

$$y_D(t) = \Gamma\left(x_1, x_2, \ldots x_p\right) \quad (31)$$

where $u(t)$ is the input signal, $n_f > 0, n_y > 0$ are the input and output lags, $p = n_f + n_y$, x_p represents the entities of vector $y_D(t)$ and Γ is the mapping between entities of NARMAX model inputs. (30) & (31) can be written in discrete form as:

$$y_D(k) = \Gamma\left(u(k-1), u(k-2) \ldots u\left(k-n_f\right), y(k-1), y(k-2) \ldots y\left(k-n_y\right)\right) \quad (32)$$

$$y_D(k) = \Gamma\left(x_1, x_2, \ldots x_p\right) \quad (33)$$

The corresponding NARMAX model can be described as:

$$y_D(k) = \sum_{i_1=1}^{p} \theta_{i_1} x_{i_1} + \sum_{i_1=1}^{p} \sum_{i_2=1}^{p} \theta_{i_1 i_2} x_{i_1 i_2} \cdots + \sum_{i_1=1}^{p} \cdots \sum_{i_q=1}^{p} \theta_{i_1 \ldots i_q} x_{i_1 \ldots i_q} \quad (34)$$

where q is the order and θ^T is the vector comprising coefficients of NARMAX model. Dynamic model of PZT actuator can be approximated accurately as two order system [17]. (34) can be modified in two order form as:

$$y_D(k) = \sum_{i=1}^{p} \theta_i x_i + \sum_{i=1}^{p} \sum_{j=1}^{p} \theta_{ij} x_{ij} \quad (35)$$

the parameter matrices θ_i and θ_{ij} can be defined as:

$$\theta_i^T = \left[\theta_1, \theta_2, \quad \ldots, \quad \theta_p\right] \quad (36)$$

$$\theta_{ij} = \begin{bmatrix} \theta_{11} & \cdots & \theta_{1p} \\ \vdots & \ddots & \vdots \\ \theta_{p1} & \cdots & \theta_{pp} \end{bmatrix} \quad (37)$$

$$x = \left[x_1, \quad x_2 \quad \ldots, \quad x_p\right] \quad (38)$$

Figure 5 shows the neural network approximation of the NARMAX model. The activation functions, σ_{θ_i} and $\sigma_{\theta_{ij}}$ are

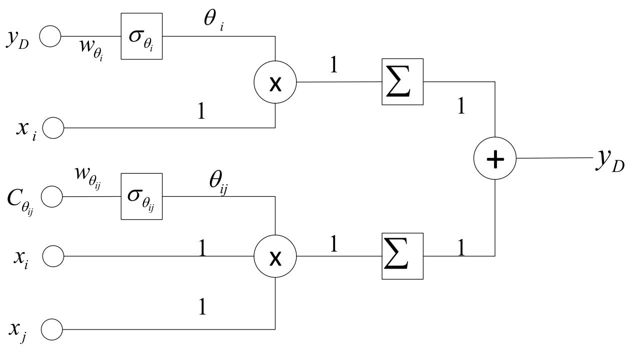

FIGURE 5. Dynamic sub-model neural network structure.

given as:

$$\sigma_{\theta_i}(x) = \frac{1}{1+e^{w_{\theta_i}x}} \tag{39}$$

$$\sigma_{\theta_{ij}}(x) = \frac{1}{1+e^{w_{\theta_{ij}}x}} \tag{40}$$

training rule for dynamic sub-model is given as:

$$e_D(k) = y_H - (d_H(k) + y_D(k)) \tag{41}$$

$e_D(k)$ is the error matrix for dynamic sub-model, y_H is the output of high frequency training set. For Jacobean J_D, parameters for optimized dynamic sub-model can be calculated by

$$\Delta w_D(k) = \frac{J_D^T e_D(k)}{(J_D^T J_D - \mu I)} \tag{42}$$

$$J_D = \left[\left[\frac{\partial e_D(k)}{\partial \theta_i}\right]\left[\frac{\partial e_D(k)}{\partial \theta_{ij}}\right]\right] \tag{43}$$

V. EXPERIMENTAL RESULTS AND DISCUESSION

A. IDENTIFICATION OF HYSTERESIS IN PIEZOELECTRIC ACTUATED NANO-STAGE

This section illustrates the identification of quasi static sub-model. First the training data was obtained by exciting piezoelectric actuated nano-stage by appropriate input signal. For the identification of quasi static model, arousing the actuator dynamics, was avoided by selecting a very low frequency signal as shown in Figure 6(a). Piezoelectric actuated nano-stage was excited at input frequency as low as 0.5Hz with an amplitude of 100V, and input/output data was recorded. Initial condition, d_o, was calculated from a low voltage dc input signal where piezoelectric actuated nano-stage show almost no hysteresis i.e. 5VDC. After required training data was obtained next step was initialization and training of neural network. Training goal was set to 1e-12 mean square error. Train data partition was set to (75:15:15) for training, validation and testing respectively. Max epoch was set to 1000. Levenberg-Marquardt algorithm was used for training purposes and best performance was obtained at 03 epochs. Optimal parameter values can be calculated by evaluating the values of weights and activation function of the neurons of trained neural network as $h_s = 1967$, $\Delta d_o = 0.4783$, $\gamma_o = -370.47$ and $V_{co} = 24629$ with the identified parameters hysteresis nonlinearity in piezoelectric actuated nano-stage can be predicted. Figure 6(b) and 6(c) shows the measured and predicted results of piezoelectric actuated nano-stage for output displacement and hysteresis respectively. Error between measured and predicted results is shown in Figure 6(d) where maximum peak to valley error is 20.77nm. The results show that predicted output curve is very much in consistent with the experimentally obtained data.

Note that, grey box neural network based model identification is computationally inexpensive, i.e. it takes very few epochs to train. This unique feature makes it suitable for real-time reference tracking applications using predictive control. The quasi static system level model can be dynamically linearized on each control cycle of predictive control by updating its parameters using grey box neural network to achieve fast tracking control of nonlinear plants using linear predictive controller [26].

B. IDENTIFICATION OF DYNAMICS OF PIEZOELECTRIC ACTUATED NANO-STAGE

Dynamic model was identified after obtaining training data at different input frequencies. Neural network based NARMAX model, shown in Figure 5 was trained by Levenberg-Marquardt algorithm according to the rules in (42) & (43). Overall system performances (both static and dynamic models) was tested at input excitations of different frequencies i.e. 5Hz,10Hz and 25Hz as shown in Figure 7(a), 8(a) and 9(a) respectively. Figure 7(b), 8(b) and 9(b) shows the experimentally obtained output curve along with the predicted output from the proposed method. It is clear that predicted output at different frequencies is following the experimental data with high precision. The maximum error ranges from 24.62 nm $\sim$ 30.47nm for different frequencies as shown in Figure 7(d), 8(d) and 9(d). Predicted and measured hysteresis responses are shown in Figure 7(c), 8(c) and 9(c). These experimental results show that the proposed model identification method can predict the output dynamics of piezoelectric actuated nano-stage with high accuracy.

To compare the performance proposed modeling scheme with existing black box neural network based modeling method, the input/output data set obtained from piezoelectric actuated nano-stage in section V-A was used to train black box neural network. Back propagation training technique with Levenberg–Marquardt training algorithm was used. Table 1 shows comparison between performance of proposed method and different variants of black box identification method, i.e. 17, 25 and 35 neurons. Maximum error comparison between proposed scheme and black box identification method is shown in Figure 10. The proposed method outperforms the conventional black box identification scheme, with maximum error of 20.77nm as compared with best performance of black box model at 35 neurons, i.e. 96.56nm. Which shows the proposed method has obtained more accurate modeling results with less training time as compared to existing black box identification scheme. Furthermore, the proposed

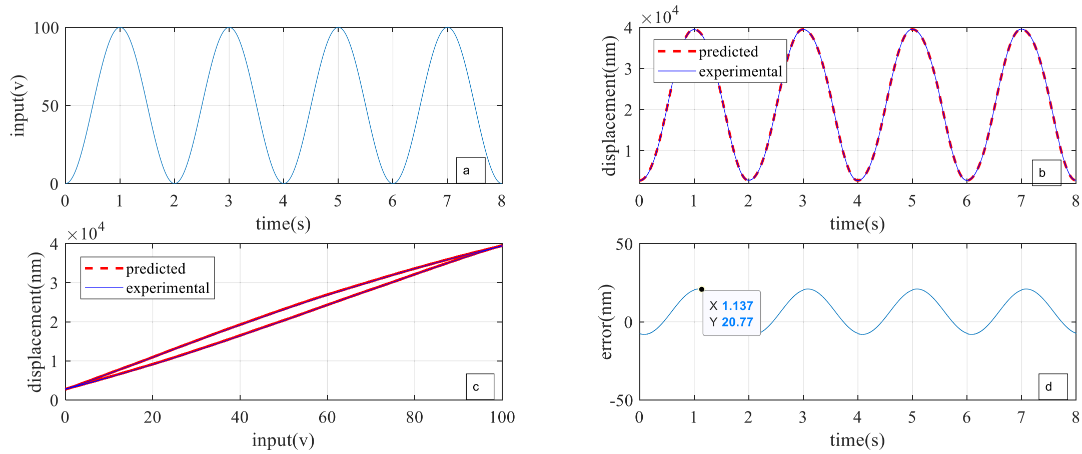

FIGURE 6. Static model identification a) input signal, b) predicted and measured output values, c) predicted and measured static hysteresis, d) error.

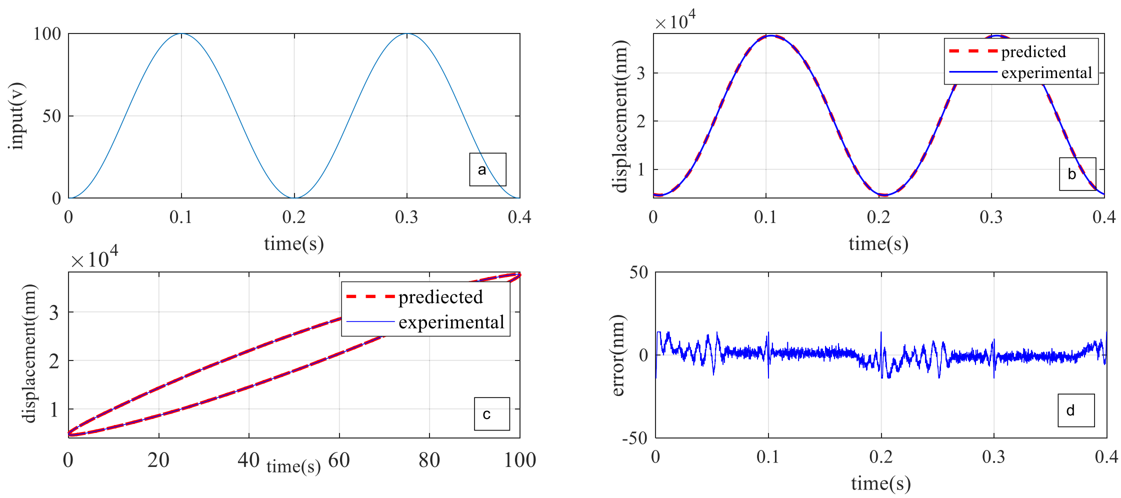

FIGURE 7. Over all model response at 5Hz a) input signal b) output displacement c) hysteresis output d) error.

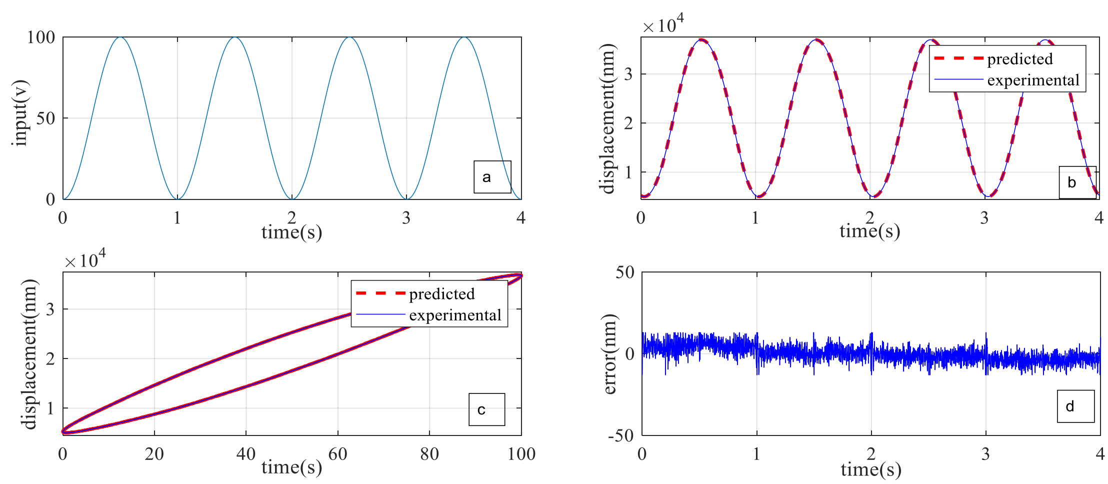

FIGURE 8. Over all model response at 10Hz a) input signal b) output displacement c) hysteresis output d) error.

gray box neural network based identification method is compared with conventional PSO based identification. The parameters of quasi static model were identified by PSO, the output curve generated is shown in Figure 10. PSO was

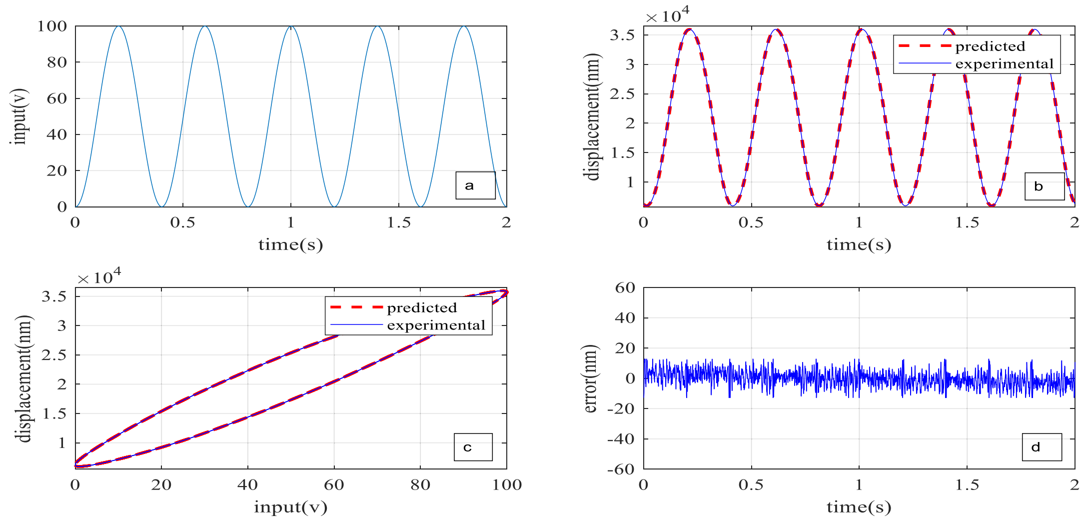

FIGURE 9. Over all model response at 25Hz a) input signal b) output displacement c) hysteresis output d) error.

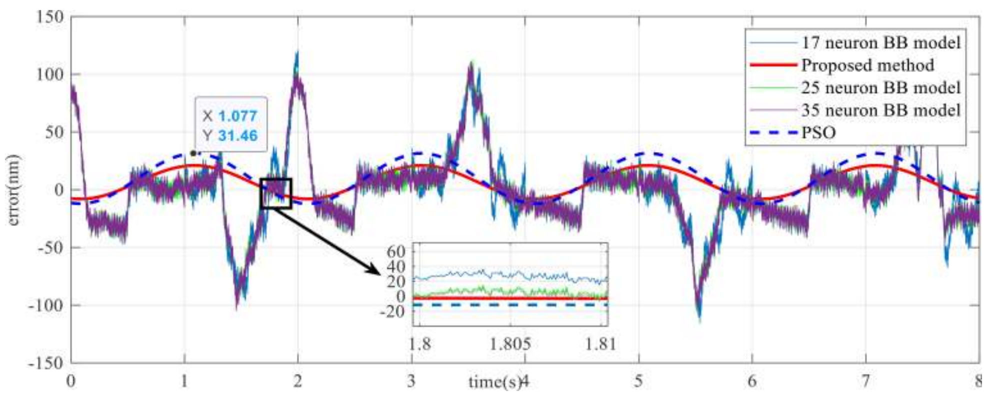

FIGURE 10. Comparison of proposed method with the existing model identification techniques.

TABLE 1. Comparison between proposed and existing model identification techniques.

Method	Training time	Maximum error(nm)
Proposed Method	03 epoch	20.77
17 neuron Black Box method	1796 epoch	120.03
25 neuron Black Box method	622 epoch	103.2
35 neuron Black Box method	578 epoch	96.56
PSO	500 iteration	31.46

run for 500 iterations, from Table 1 it is clear that proposed gray box identification method can identify model parameters in less time with more precision.

VI. CONCLUSION

This paper presented a scheme for the modeling and identification of rate-dependent hysteresis in piezoelectric actuated nano-stage. First step was to convert the System level quasi static hysteresis model into a custom design neural network. To handle the multi-valued problem of hysteresis, generalized input gradient was introduced in the input space to extract the moving tendency of hysteresis. Finally, neural network based NARMAX model was introduced to describe the rate-dependent performance of the PZT. Main features of proposed approach are, the entire model was composed of gray box neural network, where weights of neural network correspond to the unknown model parameters, unlike conventional black box identification it has clear mathematical meaning. Moreover, proposed generalized input gradient based mapping is more effective on non-smooth extrema points than the conventional gradient of hysteresis output with respect to input method. The simulation and experiment validation results demonstrate that the proposed scheme can identify the rate dependent hysteresis more precisely with maximum error of 20.77nm than the black box (96.56nm) and PSO (31.46nm) based model identification methods.

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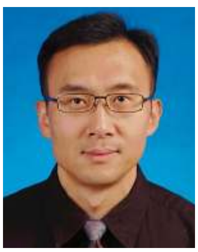

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