

Modeling and Numerical Simulations of Immiscible Compressible Two-Phase Flow in Porous Media by the Concept of Global Pressure

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Abstract A new formulation is presented for the modeling of immiscible compressible two-phase flow in porous media taking into account gravity, capillary effects and heterogeneity. The formulation is intended for the numerical simulation of multi-dimensional flows and is fully equivalent to the original equations, unlike the one introduced in Chavent and Jaffré (1986). The main feature of this formulation is the introduction of a global pressure. The resulting equations are written in a fractional flow formulation and lead to a coupled system which consists of a nonlinear parabolic (the global pressure equation) and a nonlinear diffusion–convection one (the saturation equation) which can be efficiently solved numerically. A finite volume method is used to solve the global pressure equation and the saturation equation for the water and gas phase in the context of gas migration through engineered and geological barriers for a deep repository for radioactive waste. Numerical results for the one-dimensional problem are presented. The accuracy of the fully equivalent fractional flow model is demonstrated through comparison with the simplified model already developed in Chavent and Jaffré (1986).

Keywords Immiscible compressible two-phase flow · Global pressure · Porous media · Water hydrogen ·

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1 Introduction

The understanding and prediction of multiphase flow through porous media is of great importance in various areas of research and industry. Petroleum engineers need to model multiphase and multicomponent flow for production of hydrocarbons from petroleum reservoirs. Hydrologists and soil scientists are concerned with underground water flow in connection with applications to civil and agricultural engineering, and, of course, the design and evaluation of remediation technologies in water quality control rely on the properties of underground fluid flow. More recently, modeling multiphase flow received an increasing attention in connection with the disposal of radioactive waste and sequestration of CO_2 .

In this paper, we focus our attention on the modeling and simulation of immiscible compressible two-phase flow in porous media, in the framework of the geological disposal of radioactive waste. As a matter of fact, one of the solutions envisaged for managing waste produced by nuclear industry is to dispose it in deep geological formations chosen for their ability to prevent and attenuate possible releases of radionuclides in the geosphere. In the frame of designing nuclear waste geological repositories appears a problem of possible two-phase flow of water and gas. Multiple recent studies have established that in such installations important amounts of gases are expected to be produced in particular due to the corrosion of metallic components used in the repository design. The creation and transport of a gas phase is an issue of concern with regard to capability of the engineered and natural barriers to evacuate the gas phase and avoid overpressure, thus preventing mechanical damages. It has become necessary to carefully evaluate those issues while assessing the performance of a geological repository, see for instance OECD/NEA (2006). As mentioned above, the most important source of gas is the corrosion phenomena of metallic components (e.g. steel lines, waste containers). The second source, generally less important depending on the type of waste, is the water radiolysis by radiation issued from nuclear waste. Both processes would produce mainly hydrogen. Furthermore the microbial activity will generate some methane and carbon dioxide and also would transform some hydrogen into methane. Hydrogen is expected to represent more than 90% of the total mass of produced gases. Recently, the Couplex-Gas benchmark (http://www.gdrmommas.org/ex_qualifications.html) was proposed by Andra and MoMAS to improve the simulation of the migration of hydrogen produced by the corrosion of nuclear waste packages in an underground storage. This is a system of two-phase (water–hydrogen) flow in a porous medium. This benchmark generated some interest and engineers encountered difficulties in handling numerical simulations for this model.

Historically, there have been two main approaches to modeling multiphase flow in porous media. The first is based on individual balance equations for each of the fluids, while the second involves manipulation and combination of those balance equations into modified forms, with concomitant introduction of ancillary functions that we will refer to as the fractional flow or global pressure saturation formulation. The notion of global pressure was first introduced by Antontsev et al. (1990); Chavent and Jaffré (1986) and was then revisited by other authors, see for instance Chen and Ewing in TPM (1997). It has been since used in a wide range of engineering specialties related to numerical simulation in hydrology and petroleum reservoir engineering, see for instance Chen et al. (2006) and the references therein. It has been proved that this fractional flow approach is far more efficient than the original two-pressure approach from the computational point of view Chen et al. (2006).

The study of two immiscible incompressible two-phase flow using the feature of global pressure is well known, see for instance Antontsev et al. (1990); Chavent and Jaffré (1986); Chen et al. (2006). This is not the case for two compressible phases except in the case of small capillary pressure so that the densities are assumed to depend on the global pressure, see for instance Chavent and Jaffré (1986); Galusinski and Saad (2008) and the references therein. In this simplified model, it is assumed that the nonlinear functions appearing in the system depend on the global pressure by ignoring the error caused by calculating them for the fluid phase at the global pressure instead of the phase pressure. These assumptions have limited the use of the global pressure formulation in numerical simulations codes. However, comparison with other formulations Chen et al. (2006) show the computational effectiveness of the global pressure when it can be put to work, which may explain the current revival of interest for the concept of global pressure for numerical modeling of multiphase flow in porous media. A fully equivalent global pressure formulation of compressible two phase flows was announced in Amaziane and Jurak (2008) and it was established for three-phase flows in Chavent (2009).

In this paper, we focus our attention on the study of immiscible compressible two-phase flow in porous media taking into account gravity, capillary effects and heterogeneity. The fractional flow formulation employs the saturation of one of the phases and a global pressure as independent variables. The fractional flow approach treats the two-phase flow problem as a total fluid flow of a single mixed fluid, and then describes the individual phases as fractions of the total flow. This approach leads to a less strong coupling between the two coupled equations: the global pressure equation and the saturation equation.

Numerical methods are very sensitive to the choice of form of the governing equation. In the light of the new and continuing developments in numerical methods for the solution of the multiphase flow equations, it is worthwhile revisiting the question of the form of the governing equations and exploring the implications of this equation form for a numerical method based on it.

The aim of this paper is to derive a new fully equivalent fractional flow formulation for immiscible compressible two-phase flow in porous media which can be efficiently solved numerically. We will consider a general case of two compressible fluids but in numerical simulations we restrict our attention to water (incompressible) and gas such as hydrogen (compressible) in the context of gas migration through engineered and geological barriers for a deep repository for radioactive waste.

The global pressure is the primary variable in the new fractional flow formulation and a computation of phase pressures corresponding to a given global pressure requires a solution of a differential equation (see (19)). It follows that an evaluation of the coefficients depending on phase pressures requires more computations in the fully equivalent fractional flow model (28)–(30) than in the original model (1), (2) which could numerically be done by using standard libraries existing in the literature. For this reason, we also develop an approximate fractional flow formulation which is not fully equivalent to the corresponding phase equations but which is more simple to use. We then compare approximate fractional flow formulation to fully equivalent fractional flow formulation by comparing the coefficients in the two formulations and by performing numerical simulations in one-dimensional test problems for both models. Our goal is to recognize situations in which approximate fractional flow formulation can be safely used and to show differences in approximate and fully equivalent formulations when they are significant.

The rest of the paper is organized as follows. In the next section, we review the differential problem describing immiscible compressible two-phase flow in porous media. In Section 3 we derive a new fully equivalent fractional flow formulation. This formulation leads to a coupled system which consists of a nonlinear parabolic (the global pressure) equation and a nonlinear diffusion-convection one (the saturation equation). In Section 4 we recall a simplified fractional flow formulation described in Chavent and Jaffré (1986), and show its deficiency in certain range of pressures and propose a modification that makes the model more applicable. In Section 5 we compare the fully equivalent fractional flow model with the simplified one. Since the two models differ only in the way of calculating coefficients in the partial differential equations, we first compare the coefficients. Then we perform numerical simulations with both models for one-dimensional problems. We will present differences in pressure phases and water saturation. All numerical results are presented for water-gas system with incompressible water and compressible gas. Data are chosen from the Couplex-Gas benchmark. Additional conclusions are drawn in Section 6.

2 Governing Equations

The usual equations describing immiscible compressible two-phase flow in a porous medium are given by the mass balance equation and Darcy's law for each of the fluid phases (see, e.g., Bear and Bachmat (1991); Chavent and Jaffré (1986); Chen et al. (2006); Helmig (1997)):

$$\Phi \frac{\partial}{\partial t}(\rho_\alpha S_\alpha) + \operatorname{div}(\rho_\alpha \mathbf{V}_\alpha) = \mathcal{F}_\alpha \quad \text{and} \quad \mathbf{V}_\alpha = -\mathbf{K} \frac{kr_\alpha(S_\alpha)}{\mu_\alpha} (\nabla p_\alpha - \rho_\alpha \mathbf{g}), \quad (1)$$

where Φ and \mathbf{K} are the porosity and the absolute permeability of the porous medium; $\alpha = w$ denotes the wetting phase (e.g. water), $\alpha = g$ indicates the nonwetting phase (e.g. gas), ρ_α , S_α , p_α , \mathbf{V}_α and μ_α are, respectively, the density, (reduced) saturation, pressure, volumetric velocity, and viscosity of the α -phase, \mathcal{F}_α is the source/sink term, kr_α is the relative permeability of the α -phase, and \mathbf{g} is the gravitational, downward-pointing, constant vector. In addition to (1), we also have the customary property for saturations and the capillary pressure function:

$$S_w + S_g = 1, \quad \text{and} \quad p_c(S_w) = p_g - p_w. \quad (2)$$

The primary variables are S_α , p_α , and \mathbf{V}_α . Here we assume that the porosity Φ and the absolute permeability \mathbf{K} are functions of space and viscosities μ_w , μ_g are constant. Finally, we assume that the capillary pressure and relative permeabilities depend upon the saturation solely. For notational simplicity, we neglect their dependence on space. With respect to the mass densities we do not set any particular restrictions, but in numerical examples we will be interested in the special case of water-gas system where ρ_w is a constant and ρ_g is given by the ideal gas law: $\rho_g(p_g) = c_g p_g$, with constant c_g .

For expository convenience, we introduce the phase mobility functions:

$$\lambda_\alpha(S_w) = kr_\alpha(S_w)/\mu_\alpha, \quad \alpha = w, g \quad (3)$$

and the total mobility

$$\lambda(S_w, p_g) = \rho_w(p_w) \lambda_w(S_w) + \rho_g(p_g) \lambda_g(S_w). \quad (4)$$

Then, we define the fractional flow functions:

$$f_\alpha(S_w, p_g) = \rho_\alpha(p_\alpha)\lambda_\alpha(S_w)/\lambda(S_w, p_g), \quad \alpha = w, g, \quad (5)$$

and also the following nonlinear functions:

$$\rho(S_w, p_g) = (\lambda_w(S_w)\rho_w(p_w)^2 + \lambda_g(S_w)\rho_g(p_g)^2)/\lambda(S_w, p_g), \quad (6)$$

$$\alpha(S_w, p_g) = \rho_w(p_w)\rho_g(p_g)\lambda_w(S_w)\lambda_g(S_w)/\lambda(S_w, p_g), \quad (7)$$

and

$$b_g(S_w, p_g) = (\rho_w(p_w) - \rho_g(p_g))\alpha(S_w, p_g), \quad a(S_w, p_g) = -\alpha(S_w, p_g)p'_c(S_w). \quad (8)$$

Note that in the above formulas we have chosen the nonwetting pressure p_g and the wetting phase saturation S_w as independent variables, which is possible due to (2).

The governing equations (1)–(2) are a set of coupled, nonlinear partial differential equations (PDEs). The basic equations can be mathematically manipulated into several alternate forms (see Chen and Ewing in JCP (1997), Chen et al. (2006)) with various choices of primary dependent variables. The choice of equation form and primary solution defined by variables have considerable implications for the mathematical analysis and the numerical method used to solve these equations.

Here we rewrite the equations (1)–(2) by summation of the two equations and introduction of total flux, $\mathbf{Q}_t = \rho_w(p_w)\mathbf{V}_w + \rho_g(p_g)\mathbf{V}_g$, as follows:

$$\begin{aligned} \Phi \frac{\partial}{\partial t} (S_w \rho_w(p_w) + (1 - S_w)\rho_g(p_g)) \\ - \operatorname{div}(\lambda(S_w, p_g)\mathbf{K}[\nabla p_g - f_w(S_w, p_g)\nabla p_c(S_w) - \rho(S_w, p_g)\mathbf{g}]) = \mathcal{F}_w + \mathcal{F}_g, \end{aligned} \quad (9)$$

$$\mathbf{Q}_t = -\lambda(S_w, p_g)\mathbf{K}(\nabla p_g - f_w(S_w, p_g)\nabla p_c(S_w) - \rho(S_w, p_g)\mathbf{g}), \quad (10)$$

$$\Phi \frac{\partial}{\partial t} (\rho_w(p_w)S_w) + \operatorname{div}(f_w(S_w, p_g)\mathbf{Q}_t + b_g(S_w, p_g)\mathbf{K}\mathbf{g}) - \operatorname{div}(a(S_w, p_g)\mathbf{K}\nabla S_w) = \mathcal{F}_w. \quad (11)$$

The governing equation for the saturation (11) is a nonlinear convection-diffusion PDE and the equation for pressure (9) is a nonlinear PDE strongly coupled to the saturation equation through the gradient of capillary pressure and the time derivative term. Our goal is to partially decouple equations (9) and (11) by eliminating capillary pressure gradient in (9).

Remark 1 Phase fluxes can be expressed through the total flux as:

$$\rho_w(p_w)\mathbf{V}_w = f_w(S_w, p_g)\mathbf{Q}_t - a(S_w, p_g)\mathbf{K}\nabla S_w + b_g(S_w, p_g)\mathbf{K}\mathbf{g}, \quad (12)$$

$$\rho_g(p_g)\mathbf{V}_g = f_g(S_w, p_g)\mathbf{Q}_t + a(S_w, p_g)\mathbf{K}\nabla S_w - b_g(S_w, p_g)\mathbf{K}\mathbf{g}. \quad (13)$$

3 A Fully Equivalent Fractional Flow Formulation

We can eliminate the capillary pressure gradient term from (9) by expressing the total flux \mathbf{Q}_t as the Darcy flux of some *mean pressure* p , which leads to

$$\nabla p_g - f_w(S_w, p_g) p_c'(S_w) \nabla S_w = \omega(S_w, p) \nabla p, \quad (14)$$

where the function $\omega(S_w, p)$ is to be determined. To that aim we introduce the following unknown function π such that

$$p_g = \pi(S_w, p), \quad (15)$$

which relates the gas pressure p_g and a new variable p which will be called *the global pressure*. The global pressure is expected to be an intermediate pressure between p_w and p_g . From (14) and (15) we have

$$\nabla p_g = \omega(S_w, p) \nabla p + f_w(S_w, \pi(S_w, p)) p_c'(S_w) \nabla S_w,$$

or

$$\frac{\partial \pi}{\partial S_w}(S_w, p) \nabla S_w + \frac{\partial \pi}{\partial p}(S_w, p) \nabla p = \omega(S_w, p) \nabla p + f_w(S_w, \pi(S_w, p)) p_c'(S_w) \nabla S_w.$$

Since p and S_w are independent variables we must have

$$\frac{\partial \pi}{\partial S_w}(S_w, p) = f_w(S_w, \pi(S_w, p)) p_c'(S_w) \quad (16)$$

$$\frac{\partial \pi}{\partial p}(S_w, p) = \omega(S_w, p). \quad (17)$$

We will integrate (16) to obtain π , and use (17) as a definition for ω . If we assume that the capillary pressure is equal to zero at $S_w = 1$ then we set $\pi(1, p) = p$, and we can rewrite the differential equation (16) in a form of the following integral equation:

$$\pi(S_w, p) = p + \int_1^{S_w} f_w(s, \pi(s, p)) p_c'(s) ds, \quad (18)$$

which gives us

$$p \leq \pi(S_w, p) \leq p + p_c(S_w),$$

and therefore $p_w \leq p \leq p_g$. The integral equation (18) can be rewritten in a form of the Cauchy problem for an ordinary differential equation as follows:

$$\begin{cases} \frac{d\pi(S, p)}{dS} = \frac{\rho_w(\pi(S, p) - p_c(S)) \lambda_w(S) p_c'(S)}{\rho_w(\pi(S, p) - p_c(S)) \lambda_w(S) + \rho_g(\pi(S, p)) \lambda_g(S)}, & S < 1 \\ \pi(1, p) = p. \end{cases} \quad (19)$$

Problem (19) can be given a form easier to solve by introducing the capillary pressure as an independent variable. Since the capillary pressure is invertible, $u = p_c(S_w)$ can be written as $S_w = S_w(u)$; any function of the water saturation $f(S_w)$ can be replaced

by the corresponding function of capillary pressure $\hat{f}(u) = f(S_w(u))$. Therefore, we introduce the function $\hat{\pi}(u, p)$ as a solution of the Cauchy problem (p is a parameter),

$$\begin{cases} \frac{d\hat{\pi}(u, p)}{du} = \frac{\rho_w(\hat{\pi}(u, p) - u)\hat{\lambda}_w(u)}{\rho_w(\hat{\pi}(u, p) - u)\hat{\lambda}_w(u) + \rho_g(\hat{\pi}(u, p))\hat{\lambda}_g(u)}, & u > 0 \\ \hat{\pi}(0, p) = p. \end{cases} \quad (20)$$

For problem (20) it is easy to see that it has a global solution, and then a solution of (19) is given by a simple change of variables as

$$\pi(S_w, p) = \hat{\pi}(p_c(S_w), p).$$

Having found the function π we can give a formula for ω based on the equation (17). First we introduce a new notation for the coefficients which now depend on the global pressure p instead of the phase pressures p_g and p_w . All these functions will carry the superscript n :

$$\rho_w^n(S_w, p) = \rho_w(\pi(S_w, p) - p_c(S_w)), \quad \rho_g^n(S_w, p) = \rho_g(\pi(S_w, p)), \quad (21)$$

$$\lambda^n(S_w, p) = \rho_w^n(S_w, p)\lambda_w(S_w) + \rho_g^n(S_w, p)\lambda_g(S_w), \quad (22)$$

$$f_w^n(S_w, p) = \frac{\rho_w^n(S_w, p)\lambda_w(S_w)}{\lambda^n(S_w, p)}, \quad f_g^n(S_w, p) = \frac{\rho_g^n(S_w, p)\lambda_g(S_w)}{\lambda^n(S_w, p)} \quad (23)$$

$$\rho^n(S_w, p) = \rho(S_w, \pi(S_w, p)), \quad a^n(S_w, p) = a(S_w, \pi(S_w, p)), \quad (24)$$

$$b_g^n(S_w, p) = b_g(S_w, \pi(S_w, p)). \quad (25)$$

Note that the coefficients (21)–(25) are obtained from (3)–(8) by replacing p_g by $\pi(S_w, p)$ and p_w by $\pi(S_w, p) - p_c(S_w)$. Fluid compressibilities are defined as:

$$\nu_w^n(S, p) = \frac{\rho_w'(\pi(S, p) - p_c(S))}{\rho_w(\pi(S, p) - p_c(S))}, \quad \nu_g^n(S, p) = \frac{\rho_g'(\pi(S, p))}{\rho_g(\pi(S, p))}. \quad (26)$$

From (16) and (17) it follows that $\omega(S_w, p)$ satisfies a linear ordinary differential equation

$$\frac{\partial \omega}{\partial S_w}(S_w, p) = \partial_\pi f_w(S_w, \pi(S_w, p))p_c'(S_w)\omega(S_w, p),$$

(p being a parameter) which has a solution

$$\omega(S_w, p) = \exp\left(\int_{S_w}^1 (\nu_g^n(s, p) - \nu_w^n(s, p)) \frac{\rho_w^n(s, p)\rho_g^n(s, p)\lambda_w(s)\lambda_g(s)p_c'(s)}{(\rho_w^n(s, p)\lambda_w(s) + \rho_g^n(s, p)\lambda_g(s))^2} ds\right), \quad (27)$$

where we have taken $\omega(1, p) = 1$, as a consequence of $\pi(1, p) = p$. From (27) it is evident that ω is strictly positive function and it is less than one if the nonwetting phase compressibility is greater than the wetting phase compressibility.

Finally, replacing p_g by $\pi(S_w, p)$ in equations (9)–(11), and using (14), we obtain the following system of equations:

$$\Phi \frac{\partial}{\partial t}(S_w \rho_w^n(S_w, p) + \rho_g^n(S_w, p)(1 - S_w)) \quad (28)$$

$$- \operatorname{div}\left(\lambda^n(S_w, p)\mathbf{K}(\omega(S_w, p)\nabla p - \rho^n(S_w, p)\mathbf{g})\right) = \mathcal{F}_w + \mathcal{F}_g,$$

$$\mathbf{Q}_t = -\lambda^n(S_w, p)\mathbf{K}(\omega(S_w, p)\nabla p - \rho^n(S_w, p)\mathbf{g}), \quad (29)$$

$$\Phi \frac{\partial}{\partial t}(S_w \rho_w^n(S_w, p)) + \operatorname{div}(f_w^n(S_w, p)\mathbf{Q}_t + b_g^n(S_w, p)\mathbf{K}\mathbf{g}) = \operatorname{div}(a^n(S_w, p)\mathbf{K}\nabla S_w) + \mathcal{F}_w. \quad (30)$$

The system (28)–(30) is expressed in the variables S_w and p . The phase pressures p_g and p_w are given as smooth functions of S_w and p through $p_g = \pi(S_w, p)$ and $p_w = \pi(S_w, p) - p_c(S_w)$. Since the derivative $\partial\pi(S_w, p)/\partial p = \omega(S_w, p)$ is strictly positive, we can find the global pressure p in the form $p = \eta_g(S_w, p_g)$, with some smooth function η_g , allowing us to conclude that the flow equations (28)–(30) are fully equivalent to equations (9)–(11), and therefore to (1), (2).

Remark 2 Using the global pressure we can write the total flow \mathbf{Q}_t in the form of Darcy-Muskat law. The global pressure can be then interpreted as a mixture pressure where the two phases are considered as mixture constituents (see Wang (1997)). Let us also note that the sum of “phase energies” can be decomposed as

$$\begin{aligned} & \rho_w(p_w)\lambda_w(S_w)\mathbf{K}\nabla p_w \cdot \nabla p_w + \rho_g(p_g)\lambda_g(S_w)\mathbf{K}\nabla p_g \cdot \nabla p_g \\ & = \lambda^n(S_w, p)\omega(S_w, p)^2\mathbf{K}\nabla p \cdot \nabla p + \alpha^n(S_w, p)\mathbf{K}\nabla p_c(S_w) \cdot \nabla p_c(S_w). \end{aligned} \quad (31)$$

The equation (31) shows again physical relevance of the global pressure.

Remark 3 In development of (19) and (20) we have assumed that the capillary pressure is equal to zero for $S_w = 1$. However, a common situation is also to have $p_c(S_w = 1) = p_0$, where $p_0 > 0$ is an entry pressure. In that case wetting and nonwetting relative permeabilities, for capillary pressures in the interval $(0, p_0)$, are naturally defined as one and zero respectively, and problem (20) is again well defined. Consequently, $\pi(S_w, p) = \hat{\pi}(p_c(S_w), p)$ gives $\pi(1, p) = p + p_0$, that is, at $S_w = 1$ the global pressure is equal to the wetting phase pressure. One could equally introduce other definitions of the global pressure attaining at $S_w = 1$ any value between p_w and p_g . They would all differ by an additive constant.

In numerical simulations based on the system (28)–(30), we need to compute the coefficients (21)–(27) by integrating the equation (20) for different initial values of the global pressure p . From a practical point of view one can solve (20) approximately for certain values of initial data and then use an interpolation procedure to extend these values to the whole range of interest. Necessary calculations can be done in a preprocessing phase, without penalizing the flow simulation.

We end up this section by the following remarks.

Remark 4 The definition of the global pressure, introduced in this section by (15) and (19), is not affected by possible variations in absolute permeability and porosity and it is therefore applicable to heterogeneous media. If the porous medium is composed of different rock types, having different set of saturation functions in different subdomains, such as relative permeabilities and capillary pressure functions, then the global pressure must be defined in each subdomain separately. At the boundaries of the subdomains the global pressure will be generally discontinuous and continuity of phase pressures will introduce compatibility condition on the global pressure, namely that the function $\pi(S, p)$ must be continuous across the subdomain boundaries. In this respect the global pressure variable in multiple rock-type domains is similar to the saturation variable.

Remark 5 The main advantage of the new fractional flow formulation (28)–(30) over other equivalent formulations obtained by simple manipulations from the original equations is that the coupling between the two PDEs is much less strong. This was obtained by elimination of the gradient of the capillary pressure from equation (9). Furthermore the form of the PDEs of the system from the new fractional flow formulation (28)–(30) is more adapted for the mathematical and numerical analysis.

4 A Simplified Fractional Flow Formulation

In the existing literature the global pressure is always introduced in compressible two and three phase models by means of an approximation. More precisely, it is assumed that one can ignore the error caused by calculating the phase density ρ_α at the global pressure p instead of the phase pressure p_α . This assumption is introduced in Chavent and Jaffré (1986) and widely used in petroleum engineering applications (see, e.g., Chen and Ewing in JCP (1997); Chen et al. (2006)), but it cannot be satisfied for all the existing immiscible compressible two-phase flow. We will describe shortly this simplified global pressure formulation in the case of the two-phase flow and then compare it to the new formulation introduced in the previous section.

We return now to the equation (14) and we will solve it by using the assumption that the phase pressure in the water fractional flow function can be replaced by the global pressure p . Therefore, (14) is transformed to:

$$\nabla p_g - f_w(S_w, p)p_c'(S_w)\nabla S_w = \omega(S_w, p)\nabla p,$$

which can be satisfied by

$$p_g = p - \gamma(S_w, p), \quad \gamma(S_w, p) = - \int_0^{p_c(S_w)} \hat{f}_w(u, p) du, \quad (32)$$

where, as before, $\hat{f}_w(u, p) = f_w(S_w, p)$ for $u = p_c(S_w)$. From (32) it follows

$$\nabla p = \nabla p_g + \frac{\partial}{\partial p}\gamma(S_w, p)\nabla p - f_w(S_w, p)p_c'(S_w)\nabla S_w,$$

which means that

$$\omega(S_w, p) = 1 - \frac{\partial}{\partial p}\gamma(S_w, p). \quad (33)$$

The total flux is now expressed in a form of the Darcy law:

$$\mathbf{Q}_t = -\lambda(S_w, p)\mathbf{K}(\omega(S_w, p)\nabla p - \rho(S_w, p)\mathbf{g}). \quad (34)$$

The system (9)–(11), written in the unknowns p and S_w , now takes the form:

$$\Phi \frac{\partial}{\partial t}(S_w \rho_w(p) + (1 - S_w)\rho_g(p)) \quad (35)$$

$$- \operatorname{div}(\lambda(S_w, p)\mathbf{K}[\omega(S_w, p)\nabla p - \rho(S_w, p)\mathbf{g}]) = \mathcal{F}_w + \mathcal{F}_g,$$

$$\mathbf{Q}_t = -\lambda(S_w, p)\mathbf{K}(\omega(S_w, p)\nabla p - \rho(S_w, p)\mathbf{g}), \quad (36)$$

$$\Phi \frac{\partial}{\partial t}(S_w \rho_w(p)) + \operatorname{div}(f_w(S_w, p)\mathbf{Q}_t + \mathbf{K}\mathbf{g}b_g(S_w, p)) = \operatorname{div}(\mathbf{K}a(S_w, p)\nabla S_w) + \mathcal{F}_w, \quad (37)$$

where, according to our initial assumption, we have systematically approximated $\rho_g(p_g)$ by $\rho_g(p)$ and $\rho_w(p_w)$ by $\rho_w(p)$. The coefficients in (35)–(37) are, therefore, given by

$$\lambda(S_w, p) = \rho_w(p)\lambda_w(S_w) + \rho_g(p)\lambda_g(S_w). \quad (38)$$

$$f_\alpha(S_w, p) = \rho_\alpha(p)\lambda_\alpha(S_w)/\lambda(S_w, p), \quad \alpha = w, g, \quad (39)$$

$$\rho(S_w, p) = (\lambda_w(S_w)\rho_w(p)^2 + \lambda_g(S_w)\rho_g(p)^2)/\lambda(S_w, p), \quad (40)$$

$$\alpha(S_w, p) = \rho_w(p)\rho_g(p)\lambda_w(S_w)\lambda_g(S_w)/\lambda(S_w, p), \quad (41)$$

$$b_g(S_w, p) = (\rho_w(p) - \rho_g(p))\alpha(S_w, p), \quad (42)$$

$$a(S_w, p) = -\alpha(S_w, p)p_c'(S_w), \quad (43)$$

and (33). It is clear that the systems (35)–(37) and (28)–(30) are different only in the way that the coefficients are calculated.

Remark 6 Note that (32) defines p_g as a function of S_w and p in a form $p_g = \pi(S_w, p) = p - \gamma(S_w, p)$, but for fixed p_g and S_w , (32) is a nonlinear equation in p and its solvability is to be demonstrated if we want to have an invertible change of variables. From (32) it also follows that the new global pressure p is between the two phase pressures:

$$p_w \leq p \leq p_g.$$

Remark 7 The simplified definition of the global pressure (32) is also applicable in the case when $p_c(S_w = 1) = p_0 > 0$, assuming that the relative permeabilities are defined as in Remark 3, giving $p = p_w$ at $S_w = 1$.

Let us now turn to the question of well-posedness of the simplified global formulation. The system (35)–(37) is physically relevant only if the function ω , introduced by (33), is strictly positive since ω is a certain correction factor of the total mobility that takes into account that the gas pressure is now a nonlinear function of the global pressure. This function can be written as

$$\begin{aligned} \omega(S_w, p) &= 1 + \int_0^{p_c(S_w)} \frac{\partial}{\partial p} \hat{f}_w(u, p) du \\ &= 1 - \int_0^{p_c(S_w)} \frac{\hat{\lambda}_w(u) \hat{\lambda}_g(u)}{(\hat{\lambda}_w(u) + M(p) \hat{\lambda}_g(u))^2} du M'(p), \end{aligned} \quad (44)$$

where we set

$$M(p) = \frac{\rho_g(p)}{\rho_w(p)}, \quad M'(p) = M(p) \left(\frac{\rho'_g(p)}{\rho_g(p)} - \frac{\rho'_w(p)}{\rho_w(p)} \right). \quad (45)$$

We see that the condition $\omega > 0$ is critical only in the case when the nonwetting phase compressibility is greater than the wetting phase compressibility, which is usually the case. In that case we have to impose

$$\int_0^\infty \frac{\hat{\lambda}_w(u) \hat{\lambda}_g(u)}{(\hat{\lambda}_w(u) + M(p) \hat{\lambda}_g(u))^2} du M'(p) < 1, \quad (46)$$

for all pressures p in the range of interest. Under the condition (46) we can show that the change of variables $(S_w, p) \rightarrow (S_w, p_g)$ is invertible.

Lemma 1 *For given $p_g > 0$ and $0 < S_w \leq 1$, assume that the condition (46) is satisfied for $p \in (p_w, p_g)$, with $p_w = p_g - p_c(S_w)$. Then, the global pressure p is well defined by the equation (32).*

Proof For $S_w = 1$ we obviously have $p = p_w = p_g$. For $S_w \in (0, 1)$ we define a function

$$\Phi_{S_w}(p) = p - p_g + \int_0^{p_c(S_w)} \frac{\rho_w(p) \hat{\lambda}_w(u)}{\rho_w(p) \hat{\lambda}_w(u) + \rho_g(p) \hat{\lambda}_g(u)} du.$$

The global pressure p is defined by $\Phi_{S_w}(p) = 0$. Note that $\Phi'_{S_w}(p) = \omega(S_w, p)$, and by (46) we have $\Phi'_{S_w}(p) > 0$ for all $p \in (p_w, p_g)$. It easily follows that $\Phi_{S_w}(p_g) > 0$, and $\Phi_{S_w}(p_w) < 0$, so that Φ_{S_w} must have a unique zero in interval (p_w, p_g) . \square

Note that the condition (46) is not always satisfied in the whole range of the global pressure p . If we take, for example, incompressible wetting phase and the ideal gas law $\rho_g(p) = c_g p$ for a nonwetting phase, then (46) reduces to

$$\int_0^\infty \frac{c_g \rho_w \hat{\lambda}_w(u) \hat{\lambda}_g(u)}{(\rho_w \hat{\lambda}_w(u) + c_g p \hat{\lambda}_g(u))^2} du < 1,$$

which will generally be true only for p sufficiently large. Therefore, the simplified fractional flow model is not well defined if the field pressure in the porous domain is not sufficiently large.

In order to correct this deficiency of the simplified fractional flow model we will redefine the function ω . The formula (44) was obtained as a consequence of calculation of mass densities ρ_w and ρ_g in the global pressure instead of the appropriate phase pressure. We can make this kind of approximation directly in the formula (27) for ω in a fully equivalent fractional flow model, leading to

$$\begin{aligned} \omega(S_w, p) &= \exp \left(\int_{S_w}^1 (\nu_g(p) - \nu_w(p)) \frac{\rho_w(p) \rho_g(p) \lambda_w(s) \lambda_g(s) p'_c(s)}{(\rho_w(p) \lambda_w(s) + \rho_g(p) \lambda_g(s))^2} ds \right) \\ &= \exp \left(- \int_0^{p_c(S_w)} M'(p) \frac{\hat{\lambda}_w(u) \hat{\lambda}_g(u)}{(\hat{\lambda}_w(u) + M(p) \hat{\lambda}_g(u))^2} du \right). \end{aligned} \quad (47)$$

A benefit of the formula (47), in contrast to (44), is its strict positivity. It is clear that the formula (44) gives only the first two terms in Taylor's expansion for the exponential function in (47) and these remarks allow us to conclude that (47) is a more consistent approximation than (44). Consequently, in numerical simulations with the simplified fractional flow model, we will use ω given by (47).

Remark 8 There is another way of introducing a global pressure in compressible two-phase flow based on approximate calculation of mass densities. Namely, one can use the global pressure definition from incompressible case Chavent and Jaffré (1986):

$$p = p_g - \int_{S_w}^1 \frac{\lambda_w(s)}{\lambda_w(s) + \lambda_g(s)} p'_c(s) ds. \quad (48)$$

This change of variables permits to eliminate the saturation gradient from *the total velocity*, $\mathbf{V}_t = \mathbf{V}_w + \mathbf{V}_g$, but it will not eliminate it from the pressure equation (9), except in the case $\rho_w(p) = \rho_g(p)$. In that particular case, the global pressure (48) is the same as the global pressure in the simplified fractional flow formulation presented in this section. We will not consider further formulation based on (48), noting only that the existence of its weak solution was recently proved in Galusinski and Saad (2008, 2009).

5 Comparison of Fully Equivalent and Simplified Formulations

Note that the simplified assumption introduced in Section 4 leads to a fractional flow model in which the coefficients are calculated from the mass densities, the relative permeabilities and the capillary pressure, without solving a large number of the Cauchy problems for ordinary differential equation as in (28)–(30). This makes the simplified

model (35)–(37) interesting and sets a question of error introduced by replacing systematically the phase pressures by the global pressure in the calculations of the mass densities. We will address this question by comparing the coefficients in the two models and by performing 1D numerical simulations using these two models. In what follows we will denote differential equations (28)–(30), with coefficients given by (19) and (21)–(27), as *the new model*, and differential equations (35)–(37), with coefficients given by (38)–(43) and (47), as *the simplified model*.

5.1 Comparison of the Coefficients

The difference in the coefficients of the two models is introduced by replacing the fluid phase pressures $p_g = \hat{\pi}(u, p)$ and $p_w = \hat{\pi}(u, p) - u$ by the global pressure p . We will estimate differences $p_g - p$ and $p - p_w$ using the fact that $p_w \leq p \leq p_g$ and that the mass density is a non decreasing function of the corresponding phase pressure. Then we have

$$\frac{\rho_g(p_g)}{\rho_w(p_w)} \geq \frac{\rho_g(p)}{\rho_w(p)} = M(p), \quad (49)$$

where $M(p)$ is introduced in (45). From (18) and (5) we have

$$0 \leq \pi(S_w, p) - p = \int_0^{p_c(S_w)} \frac{\hat{\lambda}_w(u)}{\hat{\lambda}_w(u) + (\rho_g(p_g)/\rho_w(p_w))\hat{\lambda}_g(u)} du,$$

from which, using (49) we get

$$0 \leq \pi(S_w, p) - p \leq \int_0^{p_c(S_w)} \frac{\hat{\lambda}_w(u)}{\hat{\lambda}_w(u) + M(p)\hat{\lambda}_g(u)} du. \quad (50)$$

Note also that in general, the relative permeability functions depend on a dimensionless variable of the form $v = u/p_c^0$, where u is the capillary pressure and p_c^0 is some characteristic capillary pressure value. In that case we can write, by a simple change of variables,

$$0 \leq \pi(S_w, p) - p \leq p_c^0 \int_0^{+\infty} \frac{\hat{\lambda}_w(v)}{\hat{\lambda}_w(v) + M(p)\hat{\lambda}_g(v)} dv, \quad (51)$$

where the integral on the right hand side is independent of the strength of the capillary pressure.

Difference between the wetting fluid pressure and the global pressure can be calculated from (18) and (5) as

$$p - p_w = \int_0^{p_c(S_w)} \frac{\hat{\lambda}_g(u)}{(\rho_w(p_w)/\rho_g(p_g))\hat{\lambda}_w(u) + \hat{\lambda}_g(u)} du,$$

and therefore, using again the estimate (49), it follows

$$\int_0^{p_c(S_w)} \frac{M(p)\hat{\lambda}_g(u)}{\hat{\lambda}_w(u) + M(p)\hat{\lambda}_g(u)} du \leq p - p_w \leq p_c(S_w). \quad (52)$$

From the estimates (51) and (52) we can draw several conclusions. In order to simplify discussion we consider only the most common case where the nonwetting phase is more compressible than the wetting phase, i.e. the case in which the function $M(p)$ is increasing.

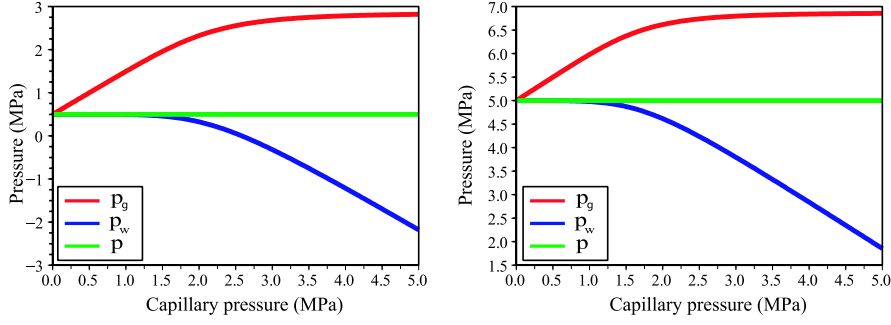


Fig. 1 Phase pressures $p_g = \hat{\pi}(u, p)$ and $p_w = \hat{\pi}(u, p) - u$, for two fixed global pressures $p = 0.5$ and $p = 5$ MPa, as functions of capillary pressure u .

1. The global pressure will be uniformly close to the gas pressure if the characteristic capillary pressure p_c^0 is small. From (51) we also see that the difference between the global and the gas pressure is smaller for higher global pressures.
2. From (50) and (52) we see that for S_w close to one, the global pressure is closer to the wetting phase pressure than to the nonwetting one. But, for small S_w , the difference $p - p_w$ can be arbitrary large.
3. From preceding conclusions we see that the approximation errors are larger when the two fluids are equally compressible, and capillary pressure is large. There is also an influence of the pressure p that affects differently $p_g - p$ and $p - p_w$: when p increases difference $p_g - p$ decreases and $p - p_w$ increases.

For numerical tests, we take a set of data from the Couplex-Gas benchmark Couplex-Gaz (2006), with *incompressible wetting phase* (water) and the ideal gas law $\rho_g(p_g) = c_g p_g$ for the nonwetting phase (hydrogen) and a set of van Genuchten's saturation-functions.

The choice of incompressible wetting phase makes the approximation of coefficients independent of the difference $p - p_w$. Error introduced by replacing p_g by p in density calculation depends now only on the characteristic capillary pressure p_c^0 and the global pressure p .

The fluid characteristics are given in Table 1, where n and P_e are van Genuchten's parameters.

μ_w Pa s	μ_g Pa s	ρ_w kg/m ³	c_g kg/(m ³ MPa)	n -	P_e MPa
$0.86 \cdot 10^{-3}$	$9 \cdot 10^{-6}$	996.5	0.808	2	2

Table 1 Fluid properties

Van Genuchten's capillary pressure is given by

$$p_c(S) = P_e \left(S^{-1/m} - 1 \right)^{1/n},$$

and the relative permeabilities are

$$kr_w(S) = \sqrt{S} \left[1 - \left(1 - S^{1/m} \right)^m \right]^2, \quad kr_g(S) = \sqrt{1-S} \left[1 - S^{1/m} \right]^{2m}.$$

In the above formulas we have $m = 1 - 1/n$ and we assume that water and gas residual saturation are equal to zero.

In Figure 1 we show $p_g = \hat{\pi}(u, p)$ and $p_w = \hat{\pi}(u, p) - u$ for a two fixed global pressures, $p = 0.5$ and 5 MPa. These figures show that the global pressure is more close to the wetting phase pressure p_w for small to intermediate capillary pressure values but the difference $p - p_w$ grows unbounded when capillary pressure augments. The difference $p_g - p$ tends to a constant when the capillary pressure grows.

In Figures 2 and 3 we compare the coefficients in new and simplified models: b_g and b_g^n ($= \mathbf{b_g}$ on the figure), a and a^n , λ and λ^n ($= \mathbf{tot_mob}$ on the figure), f_w and f_w^n ($= \mathbf{f_w}$ on the figure), and functions ω ($= \mathbf{omega}$ on the figure) given by formulas (27) and (47). The comparisons are given at the global pressure of $p = 0.1$ MPa in Figure 2 and at $p = 5$ MPa in Figure 3. All functions are presented as functions of wetting fluid saturation, while the global pressure plays a role of a parameter.

These figures confirm that the difference in the coefficients diminishes when the global pressure augments, which is a consequence of the fact that ρ_w is constant and the approximation error depends only on $p_g - p$. Similarly it could be demonstrated that the difference in the coefficients diminishes when the capillary pressure diminishes.

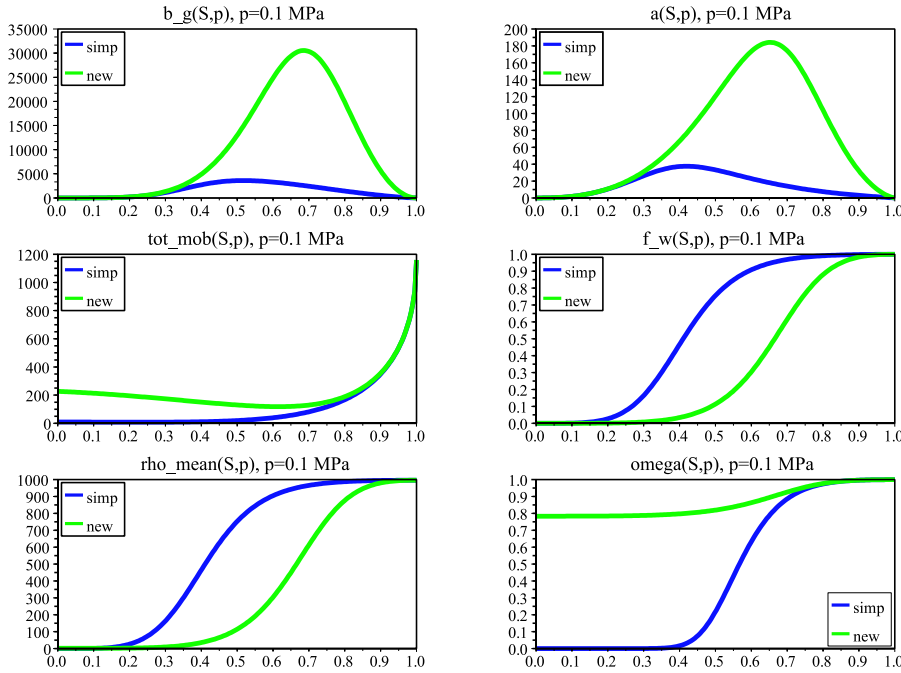


Fig. 2 Comparison of the coefficients in new and simplified global pressure models at the global pressure of 0.1 MPa. The coefficients are presented as functions of the wetting phase saturation S with fixed global pressure p .

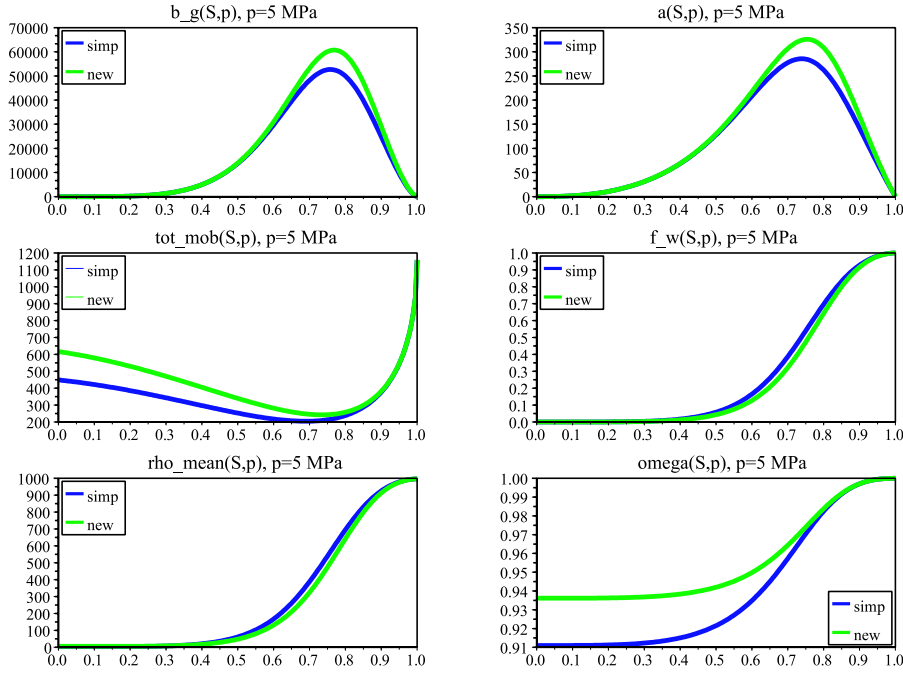


Fig. 3 Comparison of the coefficients in new and simplified models at the global pressure of 5 MPa. The coefficients are presented as functions of the wetting phase saturation S with fixed global pressure p .

5.2 Numerical Simulations

Now we compare the two models by means of a numerical simulation in one space dimension, with gravity being neglected. The porous domain is $L = 100$ meters long with homogeneous permeability K of 1 mD and porosity $\Phi = 0.1$. Fluid properties are given in Table 1. Discretization of the systems (28)–(30) and (35)–(37) is performed by a vertex centered finite volume method, see, e.g., Afif and Amaziane (2002), with a fully implicit time stepping. Now we give some details of the numerical scheme which applies to the simplified and the fully equivalent models since the difference is only in the differential equation coefficients.

Let us give a short description of the numerical scheme used for the simulations. As usual, let (t_k) , $k = 0, 1, \dots, N_T$ be a partition of the time interval $[0, T]$ with a time step $\Delta t := t_{k+1} - t_k$, and let (x_i) , $i = 0, 1, \dots, N_x$ be a partition of the spatial domain $I = [0, L]$ with $x_i = i\Delta x$. We denote by $x_{i+1/2}$ the center of $I_{i+1/2} := [x_i, x_{i+1}]$, $x_{1/2} := 0$, $x_{N_x+1/2} := L$, $\Delta x = x_{i+1/2} - x_{i-1/2}$, and the control volume $I_i = [x_{i-1/2}, x_{i+1/2}]$, $i = 0, 1, \dots, N_x$. Let p_i^k [resp. S_i^k] be an approximation of $p(x_i, t_k)$ [resp. $S(x_i, t_k)$] which will be defined precisely in the sequel. For simplicity in notation, we will only present the scheme for the simplified model. Integrating the system (35)–(37) over the set $I_i \times [t_k, t_{k+1}]$, we obtain the following finite volume scheme: for $k = 0, 1, \dots, N_T - 1$ and $i = 0, 1, \dots, N_x - 1$

$$\Phi \Delta x \frac{M_i^{k+1} - M_i^k}{\Delta t} = R_{p,i}^{k+1}, \quad (53)$$

$$\Phi \Delta x \frac{N_i^{k+1} - N_i^k}{\Delta t} = R_{S,i}^{k+1}, \quad (54)$$

where $M := M(S, p) = \rho_w(p_w)S + \rho_g(p_g)(1 - S)$ and $N := N(S, p) = \rho_w(p_w)S$, and where p_w and p_g are calculated from p and S by using formulas developed in Section 3 or, simply replacing them by global pressure p .

The term $R_{p,i}^{k+1}$ is given by

$$R_{p,i}^{k+1} = (\lambda\omega)_{i+1/2}^{k+1} K^*(p_{i+1}^{k+1} - p_i^{k+1}) - (\lambda\omega)_{i-1/2}^{k+1} K^*(p_i^{k+1} - p_{i-1}^{k+1}) + \Delta x (\mathcal{F}_{w,i}^{k+1} + \mathcal{F}_{g,i}^{k+1})$$

where $K^* = K/\Delta x$, and the functions ω and λ are calculated at $x_{i+1/2}$ as arithmetic means; $\mathcal{F}_{w,i}^{k+1}$, $\mathcal{F}_{g,i}^{k+1}$ are the water and gas source terms. The term $R_{S,i}^{k+1}$ is given by

$$\begin{aligned} R_{S,i}^{k+1} &= \alpha_{i+1/2}^{k+1} K^*(\beta(S_{i+1}^{k+1}) - \beta(S_i^{k+1})) - \alpha_{i-1/2}^{k+1} K^*(\beta(S_i^{k+1}) - \beta(S_{i-1}^{k+1})) \\ &\quad - Q_{i+1/2}^{k+1} f_{w,i+1/2}^{up,k+1} + Q_{i-1/2}^{k+1} f_{w,i-1/2}^{up,k+1} + \mathcal{F}_{w,i}^{k+1} \end{aligned}$$

where $\alpha(S, p) = \rho_w(p)\rho_g(p)/\lambda(S, p)$ and

$$\beta(S) = - \int_S^1 \lambda_w(s)\lambda_g(s)p'_c(s) ds.$$

The upwind value $f_{w,i+1/2}^{up,k+1}$ is determined in the usual way (see, e.g., Afif and Amaziane (2002)) with respect to the total flux $Q_{i+1/2}^{k+1}$, which is discretized as

$$Q_{i+1/2}^{k+1} = -(\lambda\omega)_{i+1/2}^{k+1} K^*(p_{i+1}^{k+1} - p_i^{k+1}).$$

Equations (53) and (54) are complemented by incorporating the boundary conditions, then the obtained nonlinear system is solved by the Newton method.

We compare the global pressure in new and simplified models denoted by p^{new} and p^{simp} , the phase pressures in both models, denoted by p_α^{new} and p_α^{simp} , for $\alpha = g$ (gas) and $\alpha = w$ (water), and water saturation in both models, denoted by S_w^{new} and S_w^{simp} .

In the following we present three different simulations corresponding to gas injection, water injection and gas source term.

Simulation 1 In this simulation, the gas is injected on the left end of the porous medium initially saturated by water. As in other simulations we use Dirichlet boundary conditions for the global pressure and the Dirichlet condition for water saturation on injection boundary completed by the Neumann condition on the output end of the domain. Therefore, we solve (28)–(30) and (35)–(37) with the following boundary conditions:

$$S_w(0, t) = 0.4, \quad p(0, t) = 2.0, \quad p(100, t) = 0.1, \quad \frac{\partial}{\partial x} S_w(100, t) = 0,$$

and the initial conditions:

$$S_w(x, 0) = 1.0, \quad p(x, 0) = 0.1.$$

The initial pressure is chosen to be low in order to maximize the difference in the coefficients of the two models. The source terms are equal to zero: $\mathcal{F}_w = \mathcal{F}_g = 0$. The results of this simulation are shown in Figure 5 for the time instance of 45 days.

Simulation 2 In this simulation pure water is injected in the porous domain filled with 30 % of gas. We solve (28)–(30) and (35)–(37) with the following boundary conditions:

$$S_w(0, t) = 1.0, \quad p(0, t) = 4.0, \quad p(100, t) = 0.5, \quad \frac{\partial}{\partial x} S_w(100, t) = 0,$$

and the initial conditions:

$$S_w(x, 0) = 0.7, \quad p(x, 0) = 0.5.$$

The source terms are equal to zero: $\mathcal{F}_w = \mathcal{F}_g = 0$. The results of this simulation are shown in Figure 6 for the time instance of 45 days.

Simulation 3 In this simulation we have added a constant source term \mathcal{F}_g on the interval [47, 53] with intensity of 0.01 kg/day; the water source is equal to zero: $\mathcal{F}_w = 0$. The porous medium is initially saturated by water at pressure of 0.1 MPa and boundary conditions, consistent with the initial conditions, are of Dirichlet type. The results of this simulation are shown in Figure 7 for the time instance of 12 days.

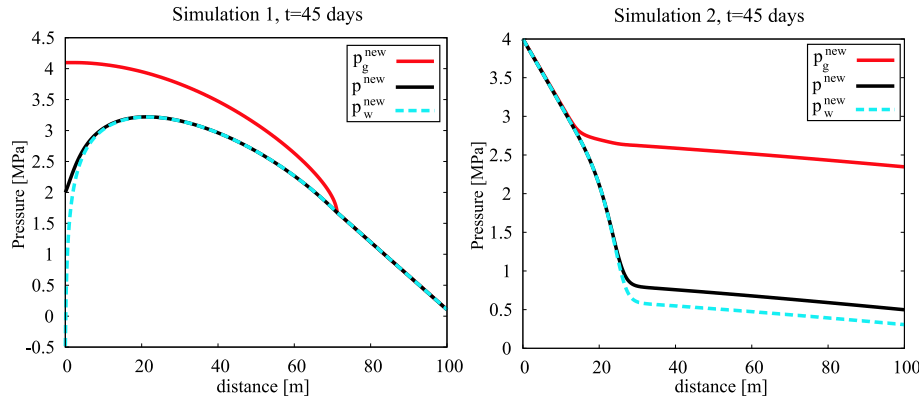


Fig. 4 Global and phase pressures in Simulation 1 (left figure) and Simulation 2 (right figure) given by the fully equivalent fractional flow model.

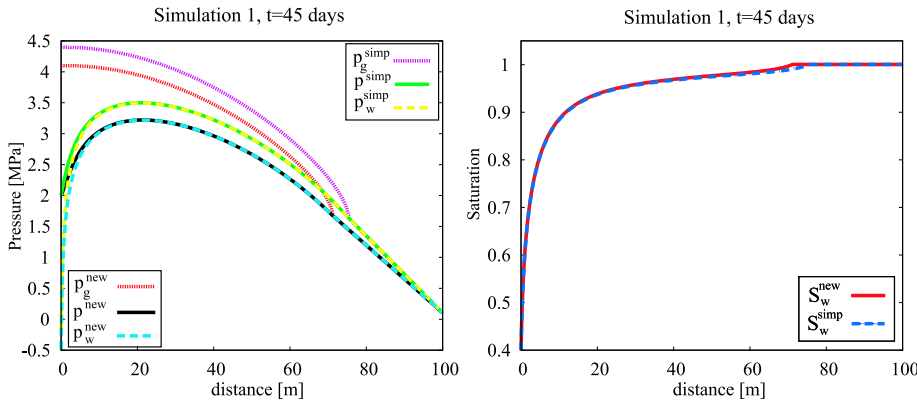


Fig. 5 Simulation 1: Comparison of the global and phase pressures and saturations produced by new and simplified models.

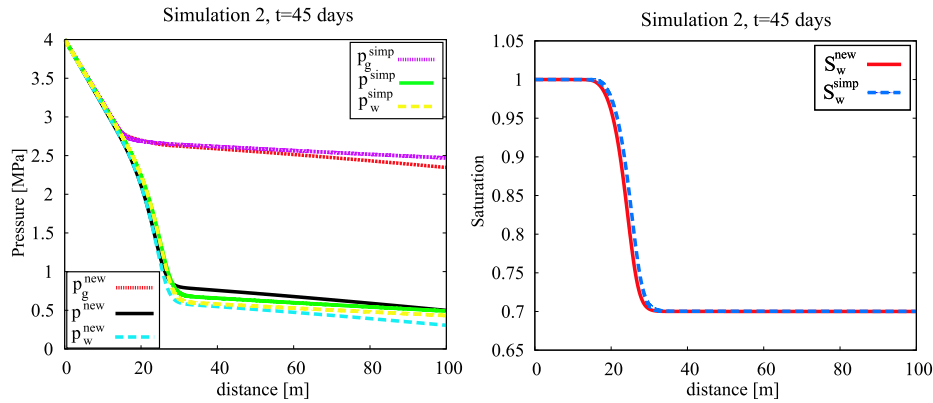


Fig. 6 Simulation 2: Comparison of the global and phase pressures and saturations produced by new and simplified models.

First, in Figure 4, we show the global and the phase pressures in the first two simulations at $t = 45$ days. These figures show that in given saturation range the global pressure is more close to the wetting phase pressure and that the difference to the nonwetting phase pressure can be quite large. Moreover, Simulation 1 shows clearly that the global pressure is a variable with more smoothness than the phase pressures.

In Figure 5 we show the results of the simulation 1 at $t = 45$ days performed by new and simplified models. On the left figure we give a comparison of the global and phase pressures in new model (p^{new} , p_w^{new} and p_g^{new}) and in simplified model (p^{simp} , p_w^{simp} and p_g^{simp}). On the right figure we see water saturations S_w^{new} and S_w^{simp} . Generally, there is very small difference in water saturations, but rather large difference in global pressures and, accordingly, in the gas pressure. Relative difference in total mass of the gas in new and simplified models, at this time instance, is 9.12 %.

In Figure 6 we show the results of the simulation 2 at $t = 45$ days performed by new and simplified models. On the left figure we give a comparison of the global and phase pressures in new model (p^{new} , p_w^{new} and p_g^{new}) and in simplified model (p^{simp} , p_w^{simp} and p_g^{simp}). On the right figure we see water saturations S_w^{new} and S_w^{simp} . In this simulation we have a front propagating from the left to the right end of the domain and we see that the difference in propagation speeds in new and simplified simulations is negligible. Relative difference in the mass of the gas in new and simplified models is of 2 %, at this particular time instance. Smaller difference between the two simulations is a consequence of higher pressures and smaller range of saturations in the simulation.

Finally in Figure 7 we present the results obtained by the two models in the simulation 3. Again, we have relatively small difference in water saturation but larger difference in pressures, giving, at this time instance of 12 days, relative difference in total mass of gas of 9.58 %. As the water saturation in this simulation is close to one, we see that the global pressure in both models are indistinguishable from the corresponding wetting phase pressures.

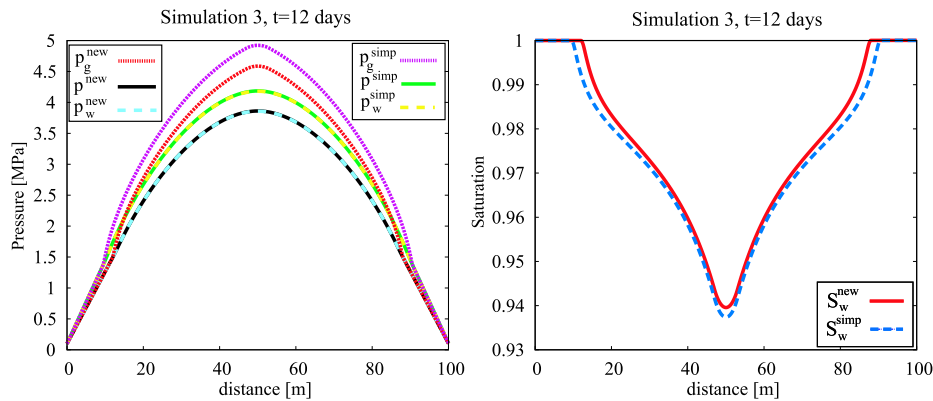


Fig. 7 Simulation 3: Comparison of the global and phase pressures and saturations produced by new and simplified models.

6 Conclusion

We have developed a new immiscible compressible two-phase flow model (28)–(30) based on the concept of the global pressure which is fully equivalent to the original phase equation formulation (1), (2). This model is compared to simplified fractional flow model presented in Section 4. By comparing the coefficients in the two models we show that simplification based on replacing the phase pressures by the global pressure in calculations of mass densities can safely be used in applications with high mean field pressure and relatively small capillary pressure if wetting phase is not strongly compressible, as for example in oil-gas systems. In hydro-geological applications, where capillary pressures may be elevated with respect to mean field pressure, this approximation can introduce unacceptably large errors, especially in predicting total mass of nonwetting phase. These conclusions are verified by means of numerical simulations in a simple homogeneous one-dimensional test case, with incompressible wetting phase. We have shown that although saturations produced by the two models are very close, the difference in pressures may be significant. Our future work will be concentrated on the extension of the numerical results described in this paper to the two-dimensional problem.

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