Modeling and Simulation for Automatic Control

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Preface

Modeling and simulation of dynamic processes are very important subjects in control systems design. Most processes that are encountered in practical controller design are very well described in the engineering literature, and it is important that the control engineer is able to take advantage of this information. It is a problem that several books must be used to get the relevant modeling information of a particular process, and it may take a long time to go through all the necessary material. The idea of this book is to supply the control engineer with a sufficient modeling background to design controllers for a wide range of processes. In addition, the book provides a good starting point for going into the specialist literature of different engineering disciplines. In this connection the references indicate where to start. The book also contains more material than what will normally be covered in the lectures of a typical course, so that students may return to the book at a later stage and find additional information about a particular subject. This will be more efficient than to extract the required information from a series of other books. In this sense the book will be of great value for practising control engineers.

The development of new products and systems is often done in a team of experts with different backgrounds. It is hoped that this book will help control engineers to communicate with other experts in this type of team. To achieve this we have been careful to use standard terminology and notation from the different engineering disciplines in question. Here we deliberately break the tradition evident in many books in the control literature where the emphasis is on having a unified formulation specific to automatic control.

The selection of the material is based on the experience of the authors in teaching and research at the Norwegian University of Science and Technology. In addition to this, material has been selected on the basis of extensive industrial activity through research programs between university and industry, and product development in industry. In this activity there has been close cooperation with experts from other disciplines, and this has given useful experience on how to approach different topics, and on how to interact with other specialists.

The style of modeling used in this book is inspired from the field of robotics where modeling is presented in a precise style based on equations. In addition, quite detailed results and optimized algorithms are included in standard textbook in robotics. As a result of this, the development in our book relies on many equations, but it is our experience that this is well appreciated by most students as they do no have to waste time on trying to understand long written descriptions on subject that are easily understood in terms of a series of equations. Moreover, we have experienced that the material presented in this book is suited both for newcomers and for students with prior courses in the topics of the book. In particular we have seen that students with virtually no background in dynamics have been able to master rigid body dynamics after going through the dynamics

chapter of this book. At the same time, students who have taken courses in dynamics also find the material in this book to be useful.

Parts of this book has been taught as a one-semester course at the Norwegian University of Science and Technology. The students are in the third year of their study in electrical engineering with specialization in automatic control, and have taken a basic course in automatic control theory. Standard undergraduate courses in engineering mathematics gives a sufficient background in mathematics.

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