Modeling Driver Behavior as Sequential Risk-Taking Task

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Acceleration models are at the core of operational driving behaviors and include car-following models that capture interactions between a lead and following vehicles. The main assumption in these models is that the behavior of the following vehicle (e.g., change in acceleration) is related directly to a stimulus observed or perceived by the driver, defined relative to the lead vehicle (e.g., difference in speeds or headways). An important aspect missing from previous formulations pertains to the stochastic character of the cognitive processes used by drivers, such as perception, judgment, and execution while driving. A car-following model that reflects the psychological and cognitive aspects of the phenomenon and captures risk-taking behavior under uncertainty is explored and evaluated. In this model, Tversky and Kahneman's prospect theory provides a theoretical and operational basis for weighing a driver's different alternatives. The model is implemented and tested to assess its properties and those of the resulting traffic stream behavior.

Acceleration models are at the core of operational driving behaviors and include car-following models that capture interactions between a lead and following vehicles. The main assumption in these models is that the behavior of the following vehicle (e.g., change in acceleration) is directly related to a stimulus observed or perceived by the driver, defined relative to the lead vehicle (e.g., difference in speeds or headways). This idea was adopted in the car-following models of Chandler et al. (*1*), Gazis et al. (*2*), and Herman et al. (*3*), known as the General Motor (GM) models. These first models are not complete in the sense that they are applicable to all traffic situations. Later investigations proposed improved models by introducing a safe time headway and a desired velocity. The Gipps model (*4*) and the intelligent-driver model (IDM) (*5*) contain intuitive parameters that can be related to the driving style, such as desired accelerations, comfortable decelerations, and a desired safe time gap. Furthermore, they include braking strategies that prevent accidents under a given heuristic. Subsequent studies have extended these models by introducing additional parameters intended to capture dimensions such as anticipation, learning, and response to several vehicles ahead. The Wiedemann model captures the indifference of drivers to small changes in stimuli. It also allows different execution modes, including emergency braking (*6*). As these models continue to evolve and gain in behavioral realism, a greater range of cognitive phenomena could be incorporated. An important aspect missing from previous formulations pertains to the modeling of the cognitive processes used by drivers, such as perception, judgment, and execution while driving. Previous studies that recognized these dimensions remained in a qualitative framework, with limited mathematical specificity and hence no calibration effort.

A wide spectrum of car-following and lane-changing models has been presented in the literature and in some cases incorporated in traffic simulation tools. Existing models aim to capture driver behaviors under a variety of traffic conditions that range from free-flow conditions to extreme situations. However, few models can claim to fully capture driver behavior in these different driving environments, especially in phase transitions, traffic breakdowns, and incident occurrences. These conditions call for a richer representation of the cognitive processes underlying driver behavior. In particular, explicit representation of drivers' risk attitudes is expected to provide greater insight into the role of risk-taking behaviors in accident-prone and other extreme situations.

This paper explores and evaluates a car-following model that reflects the psychological and cognitive aspects of the phenomenon and captures risk-taking behavior under uncertainty. In this model, Kahneman and Tversky's prospect theory provides a theoretical and operational basis with which to weigh a driver's alternatives (*7*).

BACKGROUND REVIEW

Major developments in human decision-making research offer a solid base for many models in the domains of psychology, marketing, and economics. However, the influence of these theories on modeling traffic and driver behavior has been limited. This may be because of the initial normative intent of early decision theories, that is, to help decision makers reach better decisions rather than seeking to describe the often suboptimal ways in which people actually make decisions in everyday situations. Following the pioneering theoretical contributions of Bernoulli to classical utility theory (*8*), Von Neumann and Morgenstern introduced a rigorous axiomatiation (vNM's axioms) that provided the formal basis for expected utility theory (*9*). The latter lies at the core of modern decision theory, the primary technical approach for operational decision aiding under risk. Inconsistencies between choices actually made by humans and those predicted by the theories led to recognition of the limitations of strict utility theory for describing many practical decision situations (*10, 7*). Refinements of utility theories, including subjective variants and prospect theory, advanced by Kahneman and Tversky as a descriptive model of how humans make decisions under risk, have been proposed. Attempts to identify and formalize these cognitive and decision processes have

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resulted in the identification of a large range of heuristics and biases that appear to be prevalent in human decision making.

Prospect Theory

Prospect theory postulates two phases while making decisions in complex situations: a framing and editing phase followed by an evaluation phase (*7*). The first phase is a preliminary analysis of the decision problem to subjectively frame the effective alternatives. The decision maker may mentally edit the alternatives, resulting in assigning subjective utilities to the outcomes, which reflect asymmetries between attitudes toward losses versus gains. Figure 1*a* shows a typical subjective utility function used in prospect theory.

The evaluation phase produces a prospect index calculated in a similar manner to an expected utility, albeit with the major difference that the probabilities of the different possible outcomes are replaced by subjective decision weights assumed by decision makers (Figure 1*b*). The weighting function is characterized by overweighing probability differences near certainty and impossibility, relative to comparable differences in the middle of the scale (overestimation during extreme situations) (*11*). At the end, the alternative with the highest prospect (not expectation) is selected. In general, the probability weighing function corresponds to an inverse S-shaped function with steep gradients near the beginning and near the end of the curve. However, in Figure 1*b,* these steep gradients are replaced by discontinuities or probability jumps near 0 and 1. This kind of curve is favorable for lotteries and insurance companies. These companies are considered as utility distribution transformers, either by accumulating a given amount of utility to an extremely rare event (lottery) or by redistributing an extremely rare big disutility to a small but certain disutility (the insurance payment).

Heuristics and Information Processing

In decision theory, heuristics are simplified models of the world or shortcuts that can produce decisions efficiently. Because of humans' limited information processing abilities, heuristics were considered as a strategy to adapt to a complex environment. In seminal works, Tversky and Kahneman pointed to the prevalence of heuristics in everyday decisions and grouped heuristic rules into three main categories, identifying common biases associated with each category, as follows (*12*):

- Representativeness. In estimating the probability that Object A is part of B, the degree to which A is representative of B or the degree to which A resembles B affects the assessment. The biases resulting from this heuristic include
  - Insensitivity to prior probability of outcomes or base-rate frequency of the outcomes,
  - Insensitivity to sample size,
  - Misconception of chance,
  - Insensitivity to predictability,
  - Illusion of validity, and
  - Misconception of regression (to the mean).
- Availability. This judgment heuristic is represented by the tendency of people to assess the probability of occurrence of an event by the "ease with which instances or occurrences can be brought to mind" (*13*). The biases associated with this heuristic are caused by the following:
  - Retrievability of instances (familiarity),
  - Effectiveness of a search set where different tasks will give different search sets,
  - Imaginability, and
  - Illusion of correlation.
- Adjustment and anchoring. In estimating probabilities (values in general), different starting points will lead to different estimates, biased toward the initial estimates. This is called anchoring. The associated biases are
  - Insufficient adjustment,
  - In evaluation of conjunctive and disjunctive events, and
  - Anchoring in the assessment of subjective probability distributions.

Accordingly, these heuristics have helped explain the so-called fallacies contrasting human judgment with probability theory. However, they do not individually or collectively define a comprehensive theory

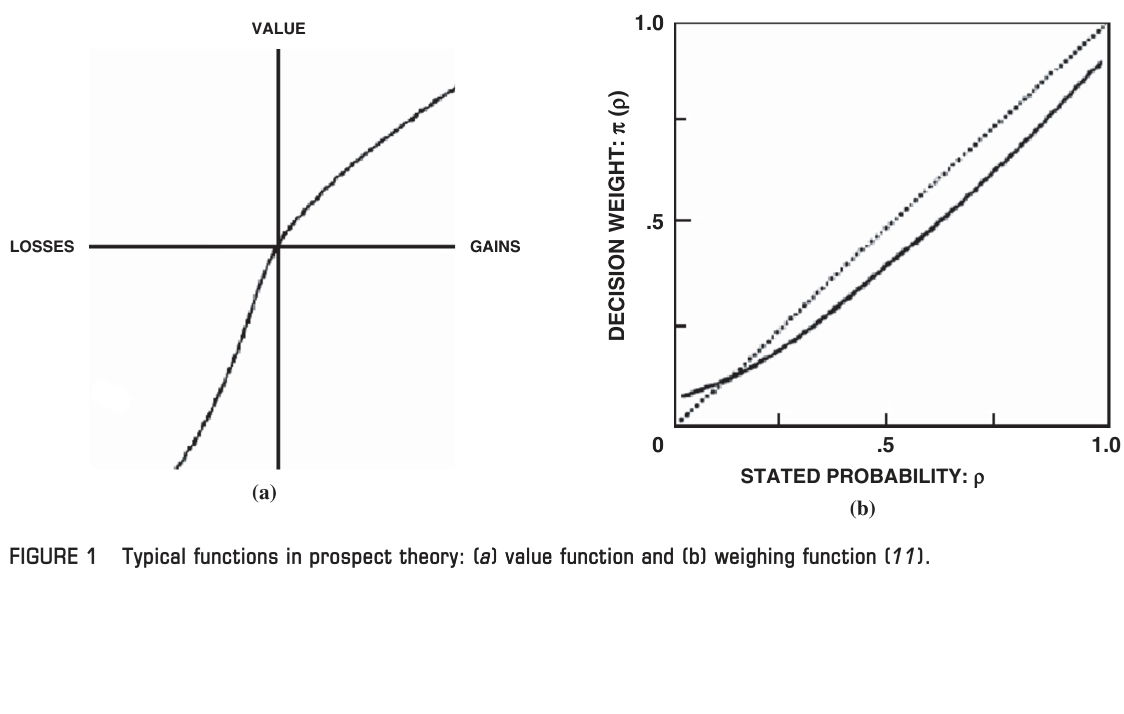


FIGURE 1 Typical functions in prospect theory: (*a*) value function and (b) weighing function (*11*).

of decision making, nor do they provide a sufficient basis for formulating decision mechanisms for a variety of general decision situations (*14*).

In the model of driver behavior formulated in the next section, the prevalence of such heuristics is reflected implicitly, with the main focus on explicitly incorporating prospect theoretic concepts in the model formulation. This allows a comprehensive inclusion of major characteristics found in the human decision-making research to the field of traffic modeling. Such characteristics contain (*a*) stochastic behavior under uncertain conditions and (*b*) disrespect of safe rational rules in the form of risk taking.

CAR-FOLLOWING MODEL FORMULATION

In the car-following process, three behaviors are possible:

1. Drivers accelerate,
2. Drivers decelerate, and
3. Drivers keep the same speed.

It is assumed that time is divided into different acceleration instances $i = 1, 2, \ldots$. car-following model formulation is currently defined by the reaction time where a driver considers accelerating, decelerating, or keeping the same speed. The main variable of interest is the subjective probability of being involved in a rear-end collision with the car in front.

The main assumptions are as follows:

1. Decision makers believe that—at all instances—they will follow the same stochastic process: given an assumed distribution of the future velocity of the leader, a driver will have a probability density function of the acceleration he or she will adopt.
2. The subjective probabilities are updated optimally to increase the velocity (up to a given desired value)—thus decreasing the travel time—while taking into consideration the risk of being involved in a collision.

Following the logic in an earlier work (*15*), the n's decision maker's representation of the probability of being involved in a rear-end collision in the acceleration instance i is denoted by $p_{n,i}$. This representation is not equal to the objective collision probability $O_{n,i}$ that is not known at this stage. In other words, $p_{n,i}$ represents the decision maker's representation of the task and depends on his or her prior distribution of $p_{n,1}$ and how he or she updates that prior distribution with experience (driving history). On the other hand, $O_{n,i}$ represents how the environment behaves and is structured for the car-following task. Linking $p_{n,i}$ to the surrounding driving conditions is a current object of investigation.

Four possible submodels can be considered in this framework:

1. Nonstationary submodel with decreasing probability. In this model, $p_{n,i}$ decreases as i increases, that is, the longer one follows a car, the less likely one is to be involved in a rear-end collision. This model is not adopted here because it does not consider possible driver fatigue.
2. Stationary submodel: $p_{n,i}$ remains constant when i increases. This model lacks consideration of the driving process and the inconsistency of drivers and therefore is not considered further.
3. Nonstationary submodel with increasing probability: there are infinitely many possibilities in which $p_{n,i}$ might increase with i. However, the driver's perception of being involved in a rear-end collision is unlikely to be monotonically increasing since it depends on the acceleration state of the leading drivers. This volatility is the main reason the next submodel is introduced.
4. Nonstationary submodel with mixed-behavior probability: there is a large number of mechanisms in which $p_{n,i}$ might increase or decrease with i.

Submodel 4 is the one considered in this paper. A prior probability of collision is assumed to be held by the driver ($p_{n,1}$). This probability is updated while conditioned on the behavior (acceleration, deceleration, etc.) of the leader and the follower. This probability is also conditioned on the fact that a driver is not involved in a crash in the previous acceleration instance. Note that the behavior of the leading driver is not known at the beginning (type of the driver assumed at beginning, like n), and it is assumed that it is related to the driver population in which a given study is being conducted. The term $p_{n,i}$ is formulated in the following.

Estimation of Collision Probability

Assume that at time t the subjective representation of driver n for the future speeds of the leader $n - 1$ (Figure 2) during the anticipated time span τ_n follows a normal distribution with a given standard deviation $\sigma(v_{n-1})$, and the mean is given by the actual velocity of the leader. This means that the estimated velocity $v_{n-1}^{\text{est}}(t)$ of the leader has a probability density $f(v|t)$ given by

$$f(v|t) = \frac{1}{\sqrt{2\pi}\sigma(v_{n-1})}\exp\left[\frac{-(v - v_{n-1}(t))^2}{2\sigma(v_{n-1})^2}\right] \quad (1)$$

For the leading vehicle, a constant velocity heuristic is adopted, that is, its velocity distribution given by Equation 1 is assumed to be valid during the entire anticipated time interval between t and $t + \tau_n$.

The driver under consideration (i.e., the follower) estimates the probability of a rear-end collision at the end of the anticipation time horizon $t + \tau_n$ for candidate accelerations a_n in a range $a_{\max}$ to $a_{\min}$. See Table 1, where the parameters above the line are actual model parameters, and those below the line are secondary parameters needed only for numerical implementation. Since the driver is in control of his or her acceleration, a constant acceleration heuristic is assumed for determining the crash probability: once chosen, the acceleration a_n will not be changed during the anticipation time horizon. The crash probability $p_n(t + \tau_n)$ is given by the probability that the gap ($s_n(t) = x_{n-1}(t) - x_n(t) - L_{n-1}$) is negative at time $t + \tau_n$, that is, $p_n(t + \tau_n) = P(s_n(t + \tau_n) < 0)$.

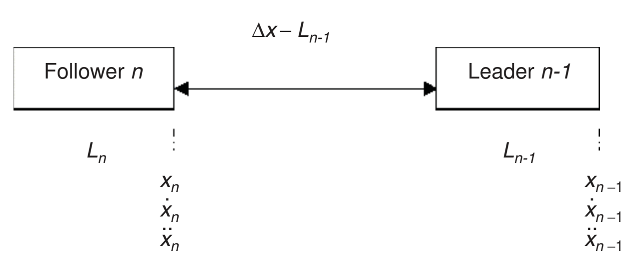


FIGURE 2 Representation of vehicles in standard car-following model.

TABLE 1 Parameters and Typical Values of Model

Parameter	Typical Value
Maximum anticipation time horizon	$\tau_{max} = 5$ s
Velocity uncertainty variation coefficient	$\alpha = 0.1$
Logit uncertainty parameter (higher for smaller uncertainty)	$\beta = 3$
Accident weighing factor	$w_c = 40$
Exponents of the PT utility	$\gamma = 0.5$
Weighing factor for the negative PT utility	$w^- = 2$
Minimum acceleration	$a_{min} = -8$ m/s^2
Maximum acceleration	$a_{max} = 4$ m/s^2
Acceleration normalizing factor	$a_0 = 1$ m/s^2

On the basis of the constant-velocity heuristic for the leader, it is known that $x_{n-1}(t+\tau_n) = x_{n-1}(t) + \tau_n\, v^{\text{est}}_{n-1}(t)$. The constant-acceleration heuristic for the follower gives

$$x_n(t+\tau_n) = x_n(t) + v_n(t)\tau_n + \frac{1}{2}a_n\tau_n^2$$

The crash probability can be written as

$$p_n(t+\tau_n) = P\left(v^{\text{est}}_{n-1}(t) < \frac{v_n(t)\tau_n + \frac{1}{2}a_n\tau_n^2 - s_n(t)}{\tau_n}\right) \tag{2}$$

In the last step, the stochastic variable $v^{\text{est}}_{n-1}(t)$ is written in terms of the standardized normal distribution by setting $v^{\text{est}}_{n-1}(t) = v_{n-1}(t) + \sigma(v_{n-1})Z$, where Z is the standardized Gaussian stochastic variable (mean 0, variance 1):

$$p_n(t+\tau_n) = P\left(Z < \frac{\Delta v_n(t)\tau_n + \frac{1}{2}a_n\tau_n^2 - s_n(t)}{\sigma(v_{n-1})\tau_n}\right)$$

$$= \Phi\left(\frac{\Delta v_n(t)\tau_n + \frac{1}{2}a_n\tau_n^2 - s_n(t)}{\sigma(v_{n-1})\tau_n}\right) \tag{3}$$

where Δ = duration and $\Delta v_n(t) = v_n(t) - v_{n-1}(t)$ denotes the approaching rate and $\Phi(z)$ is the tabulated cumulative distribution function for the standardized Gaussian.

Evaluation Process

Once the $p_{n,i}$ are known, a driver enters the evaluation process. Prospect theory is adapted for this purpose. The gain and losses are expressed here in term of gains and losses in speed from the previous acceleration instance $i - 1$. The gain is, however, limited by the maximum desired velocity of the driver, and the losses are limited by the nonnegative velocity constraint. If the gain and losses are expressed in terms of an abscissa $\Delta\dot{x} = \Delta v = a_n \times \tau_n$, the value function $U_{\text{PT}}(a_n)$ is defined as follows:

$$U_{\text{PT}}(a_n) = \frac{\left(w^- + (1-w^-) * \left(\tan\text{h}\left(\frac{a_n}{a_0}\right) + 1\right)\right)}{2} * \left(\frac{\left(\frac{a_n}{a_0}\right)}{1 + \left(\frac{a_n}{a_0}\right)^2}\right)^\gamma \tag{4}$$

where $0 < \gamma$ and w^- are parameters to be estimated and a_0 is used for normalizing purposes only. Without loss of generality, $a_0 = 1$ m/s^2 (see Table 1); other values for a_0 would only rescale the crash weighting factor w_c. The value function used in the initial model (based on the values of parameters shown in Table 1) is illustrated in Figure 3. The value function captures three characteristics: (*a*) loss aversion seen in the steeper slope with losses than with gains, (*b*) diminishing sensitivity to increasing gains and losses, and (*c*) evaluation of outcomes relative to a reference point, taken as the speed in the previous acceleration instance.

How a driver determines his or her behavior is by sequentially evaluating the outcome of a candidate acceleration before each acceleration or deceleration opportunity. If driver n decides to use a positive a_n at instance i, he may either be able to increase his velocity by $(a_n \times \tau_n)$ (gain) or will be involved in a rear-end collision. In a collision, the loss is assumed to be related to a seriousness term $k(v, \Delta v)$ weighted by w_c: when the seriousness of the driver increases, $k(v, \Delta v)$ increases. As for w_c, it represents the sensitivity to the loss caused by an accident. A higher w_c corresponds to conservative individuals, whereas a lower w_c corresponds to drivers willing to take a higher risk with little concern for crashing their vehicles. Conversely, if the driver decelerates, he will lose a corresponding amount of speed $(a_n \times \tau_n)$ (<0). That is,

$$U(a_n) = (1 - p_{n,i})U_{\text{PT}}(a_n) - p_{n,i}w_c k(v, \Delta v) \qquad a_n \geq 0 \tag{5}$$

where $U_{\text{PT}}(a_n)$ is the value function derived from Equation 4 and $p_{n,i}$ is the probability of colliding with the rear-end bumper of the lead vehicle given that no collision took place in the $i - 1$th acceleration instance.

To reflect the stochastic response adopted by the drivers, the acceleration of vehicle n $a_{n,\text{car-following}}(t + \Delta t_i)$ is retrieved from the following probability density function:

$$f(a_n) = \begin{cases} \dfrac{\exp[\beta \times U(a_n)]}{\int_{a_{min}}^{a_{max}} \exp[\beta \times U(a')]da'} & a_{min} \leq a_n \leq a_{max} \\ 0 & \text{otherwise} \end{cases} \tag{6}$$

where β is a free parameter ($\beta > 0$) that reflects the sensitivity of choice to the utility $U(a_n)$. It can also reflect different preferences

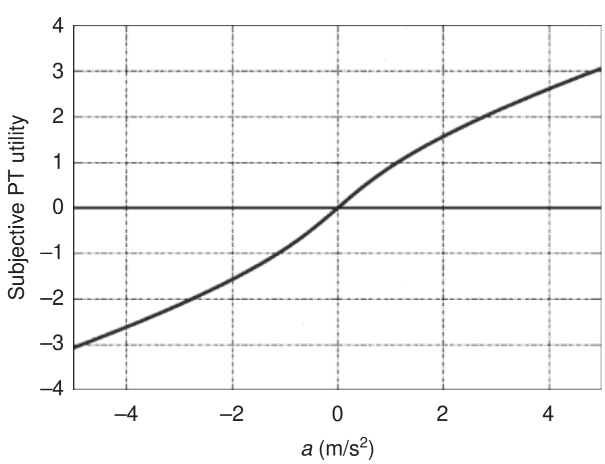


FIGURE 3 Value function (Equation 4) for evaluating different acceleration alternatives.

of the drivers or estimating uncertainties other than that of the velocity of the leader. It should be noted that β can change with the experience *i,* reflecting a given learning process. For example, β can be higher for more experienced drivers, thereby reflecting a more stable driving style.

Free-Flow Model

For driver behavior during free-flow conditions, the logic adopted in the Gipps model is adopted here (*4*). Each driver has a desired velocity $V_{n,\text{desired}}$. In each time step of duration Δt, the acceleration applied by the driver n to reach this velocity is given by

$$a_{n,\text{free-flow}}\left(t+\Delta t\right)=\frac{V_{n,\text{desired}}-v_n\left(t\right)}{\Delta t} \tag{7}$$

Finally, the chosen acceleration is

$$a_n\left(t+\Delta t\right)=\min\left[a_{n,\text{free-flow}}\left(t+\Delta t\right),a_{n,\text{car-following}}\left(t+\Delta t\right)\right] \tag{8}$$

MODEL IMPLEMENTATION

Before being calibrated against empirical data, the unconventional structural form of the model requires a thorough study of its physical characteristics. This study includes implementing the model and testing its asymptotic properties so it can be assessed for feasibility. The findings of the testing process in terms of sensitivity analysis allow one to conclude if the model is valid.

At this stage, all $N+1$ drivers ($n = 0, 1, \ldots, N$) are assumed to have identical parameters where s is the corresponding gap and Δv is the relative speed ($\Delta v > 0$ when approaching). The basic model developed in the previous section is implemented by making the following specifications:

1. The estimation uncertainty $\sigma(v_l) = \alpha v_l$ of the velocity of the leader is proportional to the velocity itself, that is, the relative error (variation coefficient) α is constant (see Table 1).
2. The anticipation time horizon τ is assumed to be the minimum between the time to collision $\tau_{TTC} = (s/\Delta v)$ and some maximum value τ_{max}:

$$\tau=\tau\left(s,\Delta v\right)=\begin{cases}\dfrac{s}{\Delta v} & \Delta v\geq\dfrac{s}{\tau_{max}}\\ \tau_{max} & \text{otherwise}\end{cases} \tag{9}$$

Initial Plots

The model was tested by using the parameter values presented in Table 1. The resulting plots are shown in Figures 4 and 5. Remarkably, in stochastic equilibrium, approximate time headways of 1.5 s are kept constant in the car-following regime. These headway values are mainly influenced by the term $\alpha\tau_{max}$, where higher values of this quotient lead to higher time headways (Figures 4*a* and 4*b*). The sensitivity to relative speeds is influenced by α alone (the higher α is, the higher the sensitivity, the string stability, and the acceleration variations) (Figure 4*c*). Accordingly, the time headway and the sensitivity to the velocity differences can be influenced separately.

Conversely, if lower acceleration uncertainties are desired, the value of $\beta \propto 1/\sigma_a$ should be decreased (Figure 5). Moreover, to increase the skewness (third normalized moment), the crash weight w_c has to be increased. This can be also accomplished by slightly decreasing β for a constant variance. However, values of the order $w_c < 20$ lead to bimodal and unrealistic distributions: a new peak appears for very high accelerations leading to crashes, and the prospect theory utility can outweigh the crash penalty even for a crash probability $p_c = 1$.

Asymptotic Expansion

To illustrate better the behavior of the model, an asymptotic expansion of the acceleration probability distribution (Equation 6) of this model is useful. A series of straightforward steps leads to

$$\dot{v}=a\approx\sim N\left(a^*,\sigma_a^2\right) \tag{10}$$

that is, the distribution of accelerations is approximately given by a Gaussian distribution whose moments are

a^* arg (max [U(a)])

$$\sigma_a^2=\frac{-1}{\beta U''\left(a^*\right)} \tag{11}$$

$U'(a)$ (necessary for determining a^* by the condition $U(a^*) = 0$), and $U''(a)$ can be calculated analytically since this implies the derivative of $\Phi(z)$, which is just the density of a Gaussian. The value a^* itself needs to be calculated numerically. Because of the nonlinearities of the utility $U_{PT}(a)$, it is not guaranteed that a^* is unique. However, all investigations presented in this paper show that it is unique for the parameters of Table 1.

Efficient Implementation of Asymptotic Expansion

The major aim here is to calculate the acceleration a^* for which the utility is maximal conditioned to given values of s, v, and Δv.

Initial Estimate

At this early stage, it is useful to take the value of a^* for $\gamma^+ = \gamma^- = w^- = 1$, where it can be calculated analytically as follows. For $\gamma = w^- = 1$, the total utility (Equation 5) can be written as

$$U\left(a;s,v,\Delta v\right)=\frac{a}{a_0}-w_c\Phi\left[z\left(a\right)\right] \tag{12}$$

where

$$z\left(a\right)=\frac{\Delta v+\frac{1}{2}a\tau-\frac{s}{\tau}}{\alpha v} \tag{13}$$

and the prediction horizon is given by the minimum of the maximum prediction time τ_{max} and the time to collision:

$$\tau=\tau\left(s,\Delta v\right)=\begin{cases}\left(\dfrac{s}{\Delta v}\right) & \text{for }\Delta v>\left(\dfrac{s}{\tau_{max}}\right)\\ \tau_{max} & \text{otherwise}\end{cases} \tag{14}$$

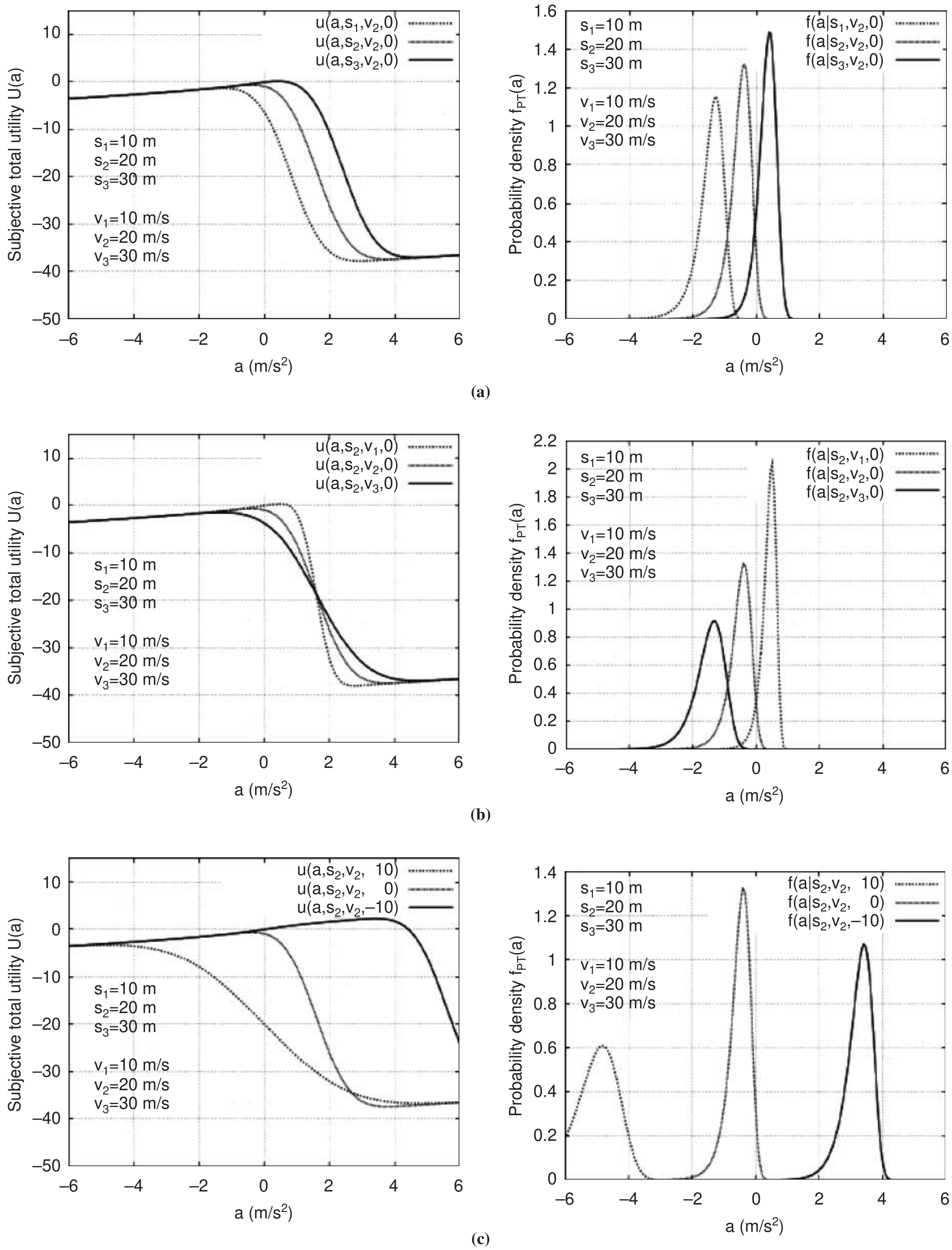


FIGURE 4 Utility and acceleration probability density for (*a*) $v = 20$ m/s, $\Delta v = 0$ m/s, and three values of gap s to leader; (*b*) $s = 20$ m, $\Delta v = 0$ m/s, and three values of velocity v to leader; and (*c*) $s = 20$ m, $v = 0$ m/s, and three values of approaching rate Δv.

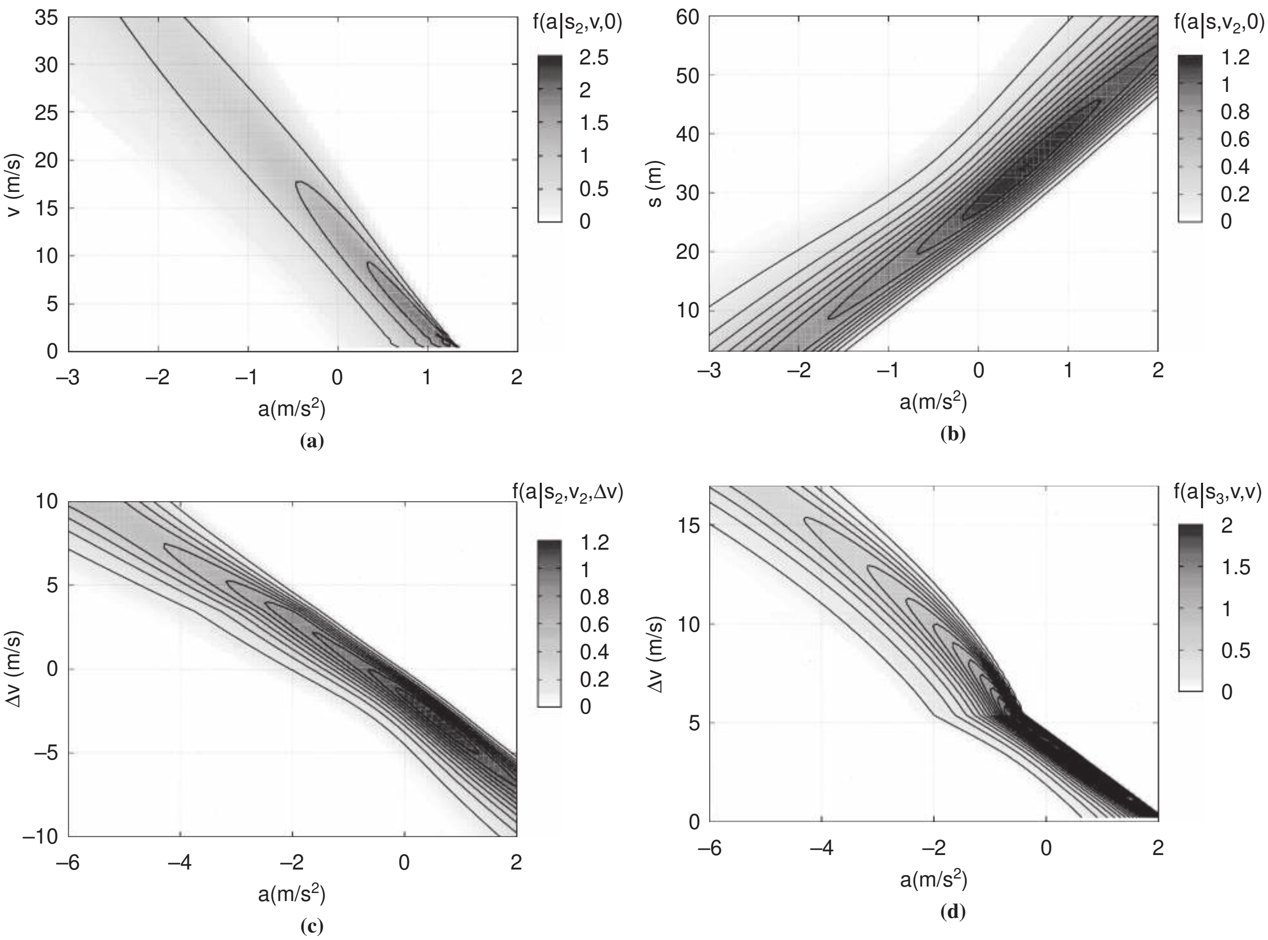


FIGURE 5 Contour plots of acceleration probability density (*a*) as function of *v* for *s* = 20 m, Δ*v* = 0 m/s; (*b*) as function of *s* for *v* = 20 m/s, Δ*v* = 0 m/s; (*c*) as function of Δ*v* for *v* = 20 m/s, *s* = 20 m; and (*d*) for situation with standing vehicle or red traffic light, *v* = Δ*v* for *s* = 30 m.

As a necessary condition for maximization and minimization problems, $U'(a)$ needs to be zero. Accordingly,

$$U'(a) = \frac{1}{a_0} - w_c f_N(z)(z'(a)) \tag{15}$$

with the density of the standardized normal distribution:

$$f_N(z) = \frac{1}{\sqrt{2\pi}} e^{-(z^2/2)} \tag{16}$$

and

$$z'(a) = \frac{\tau}{2\alpha v} \tag{17}$$

It is essential to the analytical solution that U' depends on a only by means of the argument $z(a)$ of the standardized Gaussian whereas $z'(a)$ does not depend on a at all. At maximum utility, one obtains for z value,

$$z^* = \arg\max(U(z)) = -\sqrt{2\ln\left(\frac{a_0 w_c z'}{\sqrt{2\pi}}\right)} \tag{18}$$

where z' is taken from Equation 17. Note that the negative square root is eliminated since this corresponds to the crashing probabilities smaller than 0.5, which is plausible in all cases. The positive root corresponds to a minimum of the utility. Inserting Equation 18 in Equation 15 finally gives the following expression for the initial estimate of the optimal acceleration $a^* = \arg\max(U(a))$:

$$a^* = \frac{2}{\tau_{\max}}\left(\frac{s}{\tau_{\max}} - \Delta v + \alpha v z^*\right) \tag{19}$$

This equation is exact for a linear prospect theory utility or, in other words, for a classical utility theory ($\gamma^+ = \gamma^- = w^- = 1$).

Numerical Approximation

Here, the maximum of the utility cannot be computed analytically. Since the analytical derivatives are easy to calculate and a good initial estimate is known for the optimal acceleration, one can use Newton's method to find the optimum; its $n + 1$th iteration is defined by

$$a_{n+1} = a_n - \frac{F(a_n)}{F'(a_n)} \qquad a_0 = a^* \tag{20}$$

where

$$F(a) = U'(a) = U''_{\mathrm{PT}}(a) - w_c f_N(z) z'(a) \tag{21}$$

and

$$F'(a) = U''(a) = U''_{PT}(a) - w_c f_N(z)\Big(z(a)\big(z'(a)\big)^2 + z''(a)\Big) \tag{22}$$

where

$f_N(z)$ = Gaussian density (Equation 16);
$z(a)$ = argument of the Gaussian, given by Equation 13;
$z'(a)$ = given by Equation 17, and $z''(a) = 0$; and
$U''_{\mathrm{PT}}(a)$ = second derivatives of the prospect theory utility $U_{\mathrm{PT}}(a)$.

Figure 6 illustrates Newton's method for fining an optimal acceleration.

Standard Deviation of Acceleration Function

With known analytical derivatives $U'(a)$ and $U''(a)$, the standard deviation of the acceleration is known if the acceleration at the optimal utility is known (see Equation 11):

$$\sigma_a^2(s, v, \Delta v) = \frac{-1}{\beta U''\big(a^*(s, v, \Delta v)\big)} \tag{23}$$

The results are plotted in Figure 7.

The approximation presented in this section provides a simplified and efficient method for implementing the formulated model.

MODEL ANALYSIS AND ASSESSMENT

At this stage, the model is structured so that the stimuli influencing driver behavior reflects the traffic conditions surrounding a given vehicle. These stimuli are (*a*) the predicted velocity distribution of the leading vehicle, (*b*) the relative speed between the leading vehicle and the vehicle in question, and (*c*) the gap between the end bumper of the leading vehicle and the front bumper of the vehicle in question. The sensitivity of the response (acceleration) to these stimuli is reflected by parameters that may be related to the driver's personality or the corresponding vehicle's characteristics:

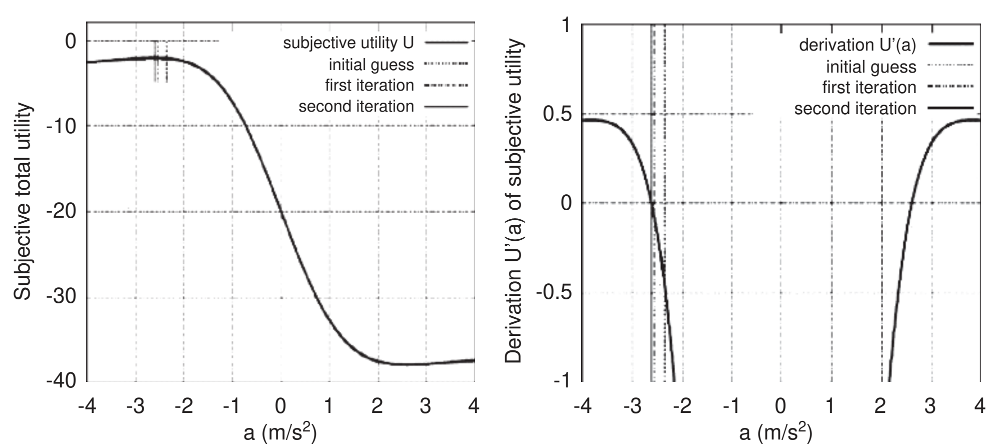
1. The parameters that may be related to the driver's personality are maximum anticipation time horizon, velocity uncertainty variation coefficient, logit uncertainty parameters (higher for higher uncertainty), accident weighing factor, prospect utility exponents, weighing factor for the negative prospect theory utility, and desired velocity (see Table 1).
2. The parameters that may be related to the vehicle's characteristics are maximum acceleration and maximum deceleration (see Table 1).

This structure reflects a trade-off between a simplicity facilitating the calibration task and a complexity imitating the stochastic and uncertain decision-making process adopted by drivers. If this structure is to be further complicated, additional stimuli can be added. For example, the model does not take into account the road geometry explicitly. However, this type of stimuli can be included implicitly by modifying some model parameters (maximum acceleration, maximum deceleration, desired velocity, etc.) on the basis of geometric factors (road curvature and smoothness, lane width, etc.).

Moreover, the stimuli emanating from the behavior of the leading drivers are only considered in this model. To consider the behavior of the following drivers, the collision probability of a given vehicle with its follower can be computed on the basis of Equation 2. The probability density function of the acceleration can then be calculated on the basis of the probability of colliding with the leading vehicle or with the back vehicle (Equation 6). Furthermore, clues for the deceleration of the leader given by braking lights or by the traffic situation several vehicles ahead are not considered.

The properties of the model for real-life driver behavior can be retrieved from Figures 4, 5, and 7. The model shows that during equilibrium, the time headway is kept constant at a value of 1.5 s, which is the value reported in different studies on headway distributions (*16, 17*). These headways increase when the drivers use higher anticipation time $\tau_{\max}$, thus taking more safety precautions. The intuitive relationships between different driving parameters expressed in Figure 4 can be summarized by Equation 19: higher acceleration values correspond to higher-distance headways and lower anticipation times (myopic view). Moreover, as a driver approaches a leading vehicle at a higher speed ($\Delta v > 0$), this driver tends to use higher deceleration rates.

Figure 5 shows the probabilistic side of the model, where different probability density functions of the acceleration term are plotted in



(a) (b)

FIGURE 6 Finding utility maximum by using Newton's method.

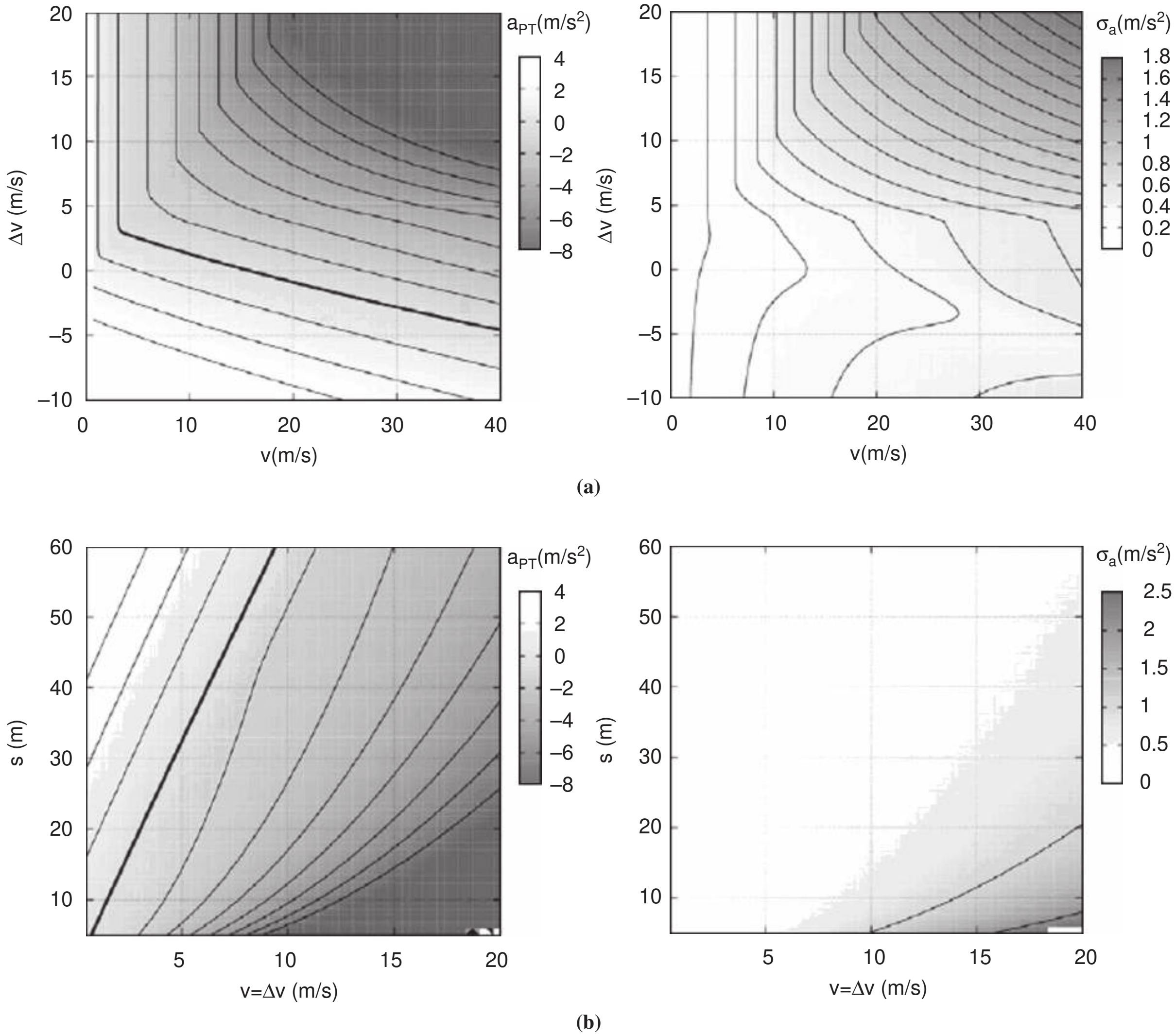


FIGURE 7 Acceleration at optimal utility and standard deviation (*a*) as function of velocity and approaching rate for gap *s* = 20 m and (*b*) when approaching standing obstacle as function of velocity and remaining distance *s*.

different driving scenarios. When the traffic is moving, in steady state ($\Delta v = 0$), for $s = 20$ m, the average velocity value is 10 m/s (36 km/h corresponding to $a = 0$). As this velocity value increases, the variance in the acceleration value increases (thicker probability density line), whereas the expectation value of the acceleration decreases. This decrease reflects that the driver wants to obtain her safety time gap again. As in other micromodels, this can lead to instability for higher velocities in dense traffic conditions ($s = 20$ m, $k = 50$ vehicles per kilometer) (*5*). The increase in the variance is a consequence of the value function of the prospect theory: near the anchoring point (zero acceleration) one acts more sensitively than far from the reference situation, that is, when accelerating or decelerating strongly. If the velocity value is fixed to $v = 20$ m/s, at steady state ($\Delta v = a = 0$), the headway values range between 20 and 60 m. When faced with the case of moving vehicle approaching a standing vehicle ($\Delta v = v$), the moving vehicle tries to decelerate stronger as Δv increases and s decreases. Furthermore, for $\Delta v = v > 10$ m/s, the deceleration increases nearly quadratically and its value is slightly higher than the kinematically necessary deceleration,

$$\frac{v^2}{2s}$$

When this same vehicle is at rest ($v = 0$), the applied acceleration values range only between 1 and 2 m/s², which is a realistic range for accelerations from a standstill on a road when only a small gap is provided ($0 < s < 20$) (*18*).

Finally, Figure 7 presents the deterministic side of the model, where the acceleration values computed at optimum utility (a^*) are provided with their corresponding variance. In the case of moving traffic (Figure 7*a*), all vehicles use the maximal deceleration values when 18 (m/s) $\leq v \leq$ 40 (m/s) and 7 (m/s) $\leq \Delta v \leq$ 20 (m/s). The lowest variances are observed when Δv increases below zero, allowing the vehicles to accelerate instead of decelerate. Figure 7*b* denotes the expected braking deceleration when approaching a standing vehicle or a red traffic light. For relatively small gaps or high velocities, the kinematically necessary braking decelerations are adopted, leading to a smooth and continuous braking maneuver to the standstill. For small velocities and comparatively large gaps, a continuous transition to the accelerating regime is observed. Again, this is the expected driver behavior. Notice that in contrast to most other models, such as the Gipps model, the driving properties of keeping a certain time headway and braking according to the necessary kinematical rules, is not introduced explicitly into the model equations. In fact, these are emergent properties resulting from the dynamics.

CONCLUSIONS AND FUTURE WORK

Existing car-following models are deterministic and do not sufficiently consider the cognitive aspects of the driving task. This paper introduced a car-following model that places greater confidence on the cognitive rationale of drivers. For that reason, prospect theory is adopted for the evaluation process of gains and losses while driving. This allows risk taking when a driver is uncertain of the leader's future behavior. Accidents will be possible and no artificial constraints will be needed to prevent them.

The model implemented showed promising results for stochastic equilibrium. The asymptotic extension of the car-following equations is possible analytically and allows more efficient implementations and faster execution. This makes such a cognitive-based stochastic model simple enough to compete with existing car-following models.

Decision-making theories such as prospect theory allow a more solid psychological background for the presented model, relating it to a rich literature not yet exploited in the traffic modeling domain: stochasticity, risk taking, and accidents are well incorporated in the modeled behavior of the drivers.

To test the validity of this model, a more complete implementation, including calibration and validation by comparison with real-life trajectory data, remains important. This will allow studying the resulting flow–density relationships as well as other macroscopic performance measures (average travel times, average delay, etc.). Moreover, the free-flow and the lane-changing behaviors are not fully developed in this stochastic framework.

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