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Modelling for the assessment of the long-term behaviour of prestressed concrete box girder bridges

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Abstract: Large-span PC box girder bridges suffer excessive vertical deflections and cracking. Recent serviceability failures in china show that, the current modelling approach of the Chinese standard (JTG D62) fails to accurately predict long term deformations of large box girder bridges. This hinders the efforts of inspectors to conduct satisfactory structural assessments and make decisions on potential repair and strengthening.

This study presents a model updating approach aiming to assess the models used in JTG D62 and improve the accuracy of numerical modelling of the long-term behaviour of box girder bridges, calibrated against data obtained from a bridge in service. A three-dimensional FE model representing the long-term behaviour of box girder sections is initially established. Parametric studies are then conducted to determine the relevant influencing parameters and to quantify the relationships between those and the behaviour of box girder bridges. Genetic algorithm optimization, based on a Response-Surface Method, is employed to determine realistic creep and shrinkage levels and prestress losses. The modelling results correspond well with the measured historic deflections and the observed cracks. The approach can lead to more accurate bridge assessments and result in safer strengthening and more economic maintenance plans.

Author Keywords: Prestressed Concrete Girder Bridge; Creep; Shrinkage; Effective Prestress Forces; Response-Surface Method; Parameter Identification.

Introduction

Prestressed concrete (PC) box girder bridges are widely used in spans 100-300m, due to their structural efficiency and economy. In recent years, many concrete box girder bridges have been reported to suffer from excessive mid-span deflections, which affects their safety and serviceability (Bažant et al. 2012 a; Bažant et al. 2012 b; Elbadry et al. 2014; Křístek et al. 2006), for example the Koror–Babeldaob Bridge in Palau. Measured displacements often exceed predicted values calculated according to conventional design methods, especially for box girder bridges spanning more than 200 m. Possible reasons behind this problem include: (1) inaccuracy of existing creep and shrinkage models; (2) existing design approaches underestimate the long-term prestress loss and degradation of prestressing tendons; (3) conventional design approaches analysing isolated beam elements neglect the effects of shear lag and additional curvature due to differential shrinkage and creep between different parts of a box girder section; (4) unsuitable numerical solution strategies for multi-decade prediction of PC box girder bridges with large span.

To reduce the difference between calculated and measured long-term deflections, previous studies propose two approaches. One is based on uncertainty analysis which utilizes certain confidence intervals to consider variations in material properties such as concrete creep, shrinkage and prestress loss, so that the confidence intervals can potentially envelope the measured data (Pan et al. 2013; Tong Guo et al.2011; Yang et al. 2005). This approach has produced a closer agreement with the field monitoring the Jinghang Canal Bridge in China (Guo and Chen 2016). The other approach is to reduce the difference between the analytial results and measured values by adjusting the inputs (i.e., material properties) using scaling parameters. This method is widely used by researchers and practitioners and has shown its effectiveness in different bridges, in particular predicting the long-term behaviour of the North Halawa viaduct, Hawaii (Robertson 2005). This method was further improved by using creep and shrinkage values obtained through in-situ testing during the construction and was used for the monitoring and analysis of the V2N viaduct in Portugal (Sousa et al. 2013). However, since beam (line) elements were adopted in the above mentioned studies, the effects of shear lag and non-uniform distribution of shrinkage and creep throughout box-girder sections have been ignored. More precise FE modelling, using ether shell and solid elements have also been used(Malm and Sundquist 2010; Norachan et al. 2014). In current bridge design and assessment practice, the fact that shrinkage and creep are very much dependent on section thickness is often ignored, since thickness varies within each box girder cross-section as well as along the span of a bridge. However, to accurately simulate the behaviour of an entire bridge, a large number of different geometries and shrinkage and creep models are needed, which makes the analysis computationally demanding, especially when solid and shell elements are used. This approach has led to a closer agreement between the prediction and long-term measurements for the Koror–Babeldaob Bridge in Palau(Bažant et al. 2012 a,b).

A new shrinkage and creep prediction model, B4, recommended by RILEM Committee (RILEM Technical Committee TC-242-MDC (Bažant 2015), was developed based on the model B3(2000). A new prediction model for creep and shrinkage is also adopted in fib Model Code 2010(2013), which differentiates between the drying and basic creep. However, existing codes such as the JTG62 (2004) still rely on simpler and thus less accurate models, which result in costly underestimation of deflections. In this study, the current prediction model of shrinkage and creep in JTG62 (2004), which uses similar formulations to fib90, is assessed to identify if suitable modifications can be made to enhance its performance.

This paper utilises data from the Jiang Jing Bridge in China which showed a significant deflection at mid-span after ten years of service well above what was expected, as well as many inclined cracks. To assess the structural integrity of the bridge, an in-depth structural inspection was performed, and historical displacements are reviewed and analysed. An analysis of the real internal force condition and stress distribution within the structure based on the available measurements is necessary to enable proper decision-making with regards to structural strengthening or retrofitting.

In this study, a model updating approach numerical method is developed to improve the accuracy of the numerical modelling of the long-term behaviour of box girder bridges calibrated against data obtained from the Jiang Jing Bridge. A comprehensive three-dimensional FE model representing the long-term behaviour of box girder sections is used. Parametric studies are then conducted to determine the relevant influencing parameters and to quantify the relationships between these parameters and the behaviour of box girder bridges. Genetic algorithm optimization, based on a Response-Surface Method, is employed to determine realistic creep and shrinkage levels and prestress losses of the model.

Description of the bridge

Bridge design and performance

The Jiang Jin Bridge, a continuous prestressed concrete box girder bridge, was segmentally constructed over the Jialing River in Chongqing, China in 1997. The main span and the side span of the bridge are 240 m and 140 m, respectively. The cross-section of the bridge consists of a single cell box girder with cantilevered slabs, of a total transverse width of 22 m, as shown in Fig. 1. The girder depth varies from 3.85 m at mid-span to 13.42 m at the main piers. The bottom slab thickness varies from 1.2 m (at main piers) to 0.32 m (at mid-span), and the web thickness varies from 0.8 m (at main piers) to 0.5 m (at mid-span). The symmetrical cantilevered cast-in-situ construction method was adopted for the segmental construction of the bridge. A total of 64 cantilevered segments with various lengths (2.5 m, 3.5 m and 4.4 m) were cast-in-situ. 25 15.2 mm diameter tendons (for top slabs) and 19 15.2 mm diameter tendons (for bottom slabs) were used, designed for initial tensioned forces of 4888 kN and 3715 kN, respectively.

Long-term deflections were measured by a relative elevation survey at specific points placed on the pavement after the Jiang Jin Bridge opened to traffic. Based on the initial design calculations, monitoring of vertical deflections at midspan was expected to stop within three to five years after the opening. However, this was not the case as deflections continued to increase and reached 33 cm 10 years after opening (4 times more than expected), causing significant downward deflection of the top slab and pavement. (see Fig. 1a). Structural inspections also revealed a large number of cracks on both the webs and slabs. Inclined cracks were observed on the surface of both sides of the webs with a maximum crack width of 0.8 mm, 40 m away from the centre of the main span, with an inclination angle varying from 30° to 60° (See Fig. 16). Bending cracks were observed at the bottom of the closure segment concentrated within 3 m from the centre of the main span, with a maximum crack width of 0.3 mm (Also see Fig. 16).

An in-service inspection of the grouting and the prestressing anchorages of ten prestressing tendons was conducted. This revealed that one prestressing wedge was missing from one tendon, as shown in Fig. 2. The elastic wave velocity method was employed to evaluate the condition of the grouting. Voids were detected in the grout, and these increase the risk of corrosion of the prestressing tendons and a potential reduction in bond strength.

Preliminary analysis

Preliminary analysis is carried out to investigate the structure and to find the reasons for the excessive vertical deflection. Possible reasons for this problem could include inaccuracies in material modelling of creep and shrinkage. To check the influence of different creep and shrinkage models, several prediction models are examined (including JTG D62 (2004), CEB FIP90 (1990)，ACI 209(2008) and B3(1995)). The material properties of the Jiang Jin Bridge are shown in Table 1. An FE model of the bridge with 1D elements is analysed by using the FEA package Midas Civil®(2011) to assess these prediction models with default parameters. The size of the 1D elements varies from 2.5 to 4.4m depending on the length of each segment. For simulation of the actual procedure of construction, the construction stages were modelled by activation and deactivation of the elements, structural boundary and load groups at each construction stage.

Results of vertical deflection at the middle of the main span (Fig.3 a) show that these models cannot predict accurately the deflections for this bridge. This is still the case even if a scaling coefficient is added to amplify the influence of creep as shown for example in Fig. 3b for the JTG model.

Another reason for excessive deflection may be due to inaccuracies in the prestressing forces. Through a parametric analysis, the initial prestressing forces were reduced parametrically from 100% to 50% (using the original JTG D62 values of creep and shrinkage). Even if the initial prestressing forces are decreased to 50% of the design value, the deflection results are still 30% smaller than the measurements at day 3700 (Fig.3 c).

To achieve the measured response, modification coefficients can be found by scaling the creep and effective prestressing forces separately through parametric analysis. Several feasible combinations of the modification coefficients are shown in Fig.3 d. However, none of these combinations can capture the development of deflection over time. The calculated results indicate a decreasing trend in the growth of the deflection with time, while the measured deflections show a continually increasing trend over time.

Other possible factors that can affect vertical deflections include shear lag in the box girder. A 3D element model was established to consider the above effect. The results from this model show 22% higher deflections than those of the 1D element model (Fig.3 e). The influence of the existing cracks on the box girder section is also considered in a new 1-D model by decreasing the thickness of webs according to the location and depth of these cracks. This modification only increases deflection by 1%. Differential shrinkage was also considered in the 3-D model for the slabs, according to the JTG D62 code and that increased deflections by up to 20%, but not enough to reach the actual deflections measured.

According to this preliminary analysis, the initial conclusions are: (1) 3D element modelling is necessary for analysis of large span box girder bridges as it can produce more accurate deflection results; (2) To predict the deflection history and improve the design of new bridges, a more sophisticated model is needed for creep development with time; (3) Besides mid-span deflection, more measurements at other locations of high deformation are needed to understand the behaviour of these structures; This paper aims to analytically examine these bridges and address some of the issues identified.

FEA modelling - geometry and material models

Geometry of the FE models

The FE package ADINA® (2001) is employed for the numerical analysis of this study. 3D solid elements are employed to account for the shear lag effect. The model geometry is as shown in **Fig. 4**. A quarter **(**half width and half length) of the bridge is modelled using symmetric boundary conditions.

The 1D rebar element, a type of truss element in ADINA, is used to model the prestressing tendons. The prestressing tendons in the top and bottom slabs of the model are illustrated in **Fig. 5.** The prestressing force is applied to the rebar elements as an initial strain.

To determine the long-term behaviour of the bridge, four main construction steps are considered in the model, including:

(1) Casting of the ends of the cantilevers and tensioning of the top prestressing tendons (t = 300 days);

(2) Casting of the closure segment of the side span and tensioning of the bottom prestressing tendons (t = 310 days);

(3) Casting of the closure segment of the main span of cantilever and tensioning of the bottom prestressing tendons (t = 320 days).

(4) Casting of the pavement and parapet (t=350 days).

The model is divided into different parts, which are activated sequentially according to the construction order described above. Both self-weight and prestressing forces are applied to the model. It worth mentioning that the simplification of the construction process for the first 300 days of the construction process prior to the casting of the closure segment was necessary to reduce computational effort. However, this approach can provide acceptable prediction of the long term behaviour of the complete bridge.

The non-uniform distribution of drying shrinkage within the box girder section, due to the variation in thickness among different parts of the section, is considered one of the causes of excessive vertical deflections (Křístek et al. 2006). To consider this effect, the webs and the top and bottom slabs of the box girder section are assigned different shrinkage properties according to the actual nominal thickness. The thickness of the top slab also varies along its width, and this is reflected in the geometry of the model (Fig.5). In addition, the nominal thickness used in the shrinkage model is calculated for each element according to the actual thickness of the part of the box girder section modelled. It should be noted that the nominal thickness given by JTG D62, is used in the shrinkage model.

A user subroutine has been developed in ADINA to provide access to the node coordinates of every element, which can be utilized to calculate the notational size for all concrete elements. The nominal thickness h, given by the JTG D62 design code, is defined as two times the ratio of the cross-sectional area to the perimeter of a structural member that is in contact with the atmosphere, and it can also be calculated by using the equivalent ratio of volume-to-surface area. In the FE model, the nominal thickness, h, of each hexahedron element is calculated as follows:

(1) Identify the location and the surface in contact with the atmosphere for each element;

(2) Calculate the exposed surface area A and volume V of each element by using its nodal coordinates;

(3) Calculate the nominal thickness h according to V/A.

To identify the location of an element, a shape function is defined to reflect the geometry of the model. The value of h is assumed to be uniform throughout the thickness of the slab or web. The cross-section at mid-span is analysed to validate this method. The nominal thickness h calculated for the entire section by the conventional method (JTG D62), without considering the effect of thickness variation, is 51 cm; the values of h calculated using the proposed method are shown in Fig. 6.

Material models

To reduce the difference between calculated and measured long-term deflections, it is essential to adjust the input parameters (i.e., material properties) used in conventional models. This approach has also been used for the prediction of the long-term behaviour of the Leziria Bridge(Sousa et al. 2013), by using the modification coefficients in the models of the EC2 for shrinkage and creep. Robertson (2005) introduced scaling constants to modify the shrinkage, creep and prestress loss, which significantly influenced the long-term deflections of the North Halawa viaduct, Hawaii.

This research assesses the shrinkage and creep model adopted by JTG D62 (concrete code of China), which uses similar formulations to fib90. To represent the long-term development of the vertical deflection of Jiang Jin Bridge over its entire span, additional parameters are introduced into the JTG models, to enable it to capture the response of the studied bridge.

The creep coefficient $\phi^*(t, t_0)$ is modified using three additional coefficients k_{c1}, k_{c2} as,

$$\phi^*(t, t_0) = k_{c1}\phi_{RH}\beta(f_{cm}) \left(\frac{1}{0.1 + (t_0)^{0.2}}\right) \left[k_{c2}\left(\frac{t - t_0}{\beta_H + (t - t_0)}\right)^{0.3} + (1 - k_{c2}) \frac{(t - t_0)^{0.5}}{t_e}\right] \quad (1)$$

Where ϕ_{RH} is the notional creep coefficient; β_H is the coefficient that describes the influence of the relative humidity and the notational size of member; $\beta(f_{cm})$ is the coefficient that is dependent on the strength of concrete f_{cm}; k_{c1} is a modification parameter for the amount of creep and k_{c2} is a modification parameter to reflect the evolutionary history of creep. Shrinkage strain is calculated by,

$$\varepsilon^*_{sh}(t, t_0) = k_s \varepsilon_{cso} \beta_s(t - t_s) \quad (2)$$

where k_s is the shrinkage modification parameter, ε_{cso} is the notional shrinkage coefficient, β_s is the coefficient that describes the development of shrinkage with time, and t_s is the age of concrete (days) at the beginning of shrinkage or swelling.

The time-dependent strains of concrete consist of both creep and shrinkage strains. The evolution of shrinkage strains is not dependent on the applied load, and can be directly calculated by the predictive model. To avoid the need to record the entire history of the creep stress evolution, the exponential series and continuous retardation spectrum has been used to represent creep compliance. In this study, the explicit method based on the exponential series is adopted to obtain the incremental strain and stress by a time step-by-step procedure. This approach has been modified and widely applied (Zhu 2014; Lou et al. 2014; Norachan et al. 2014). The long-term creep strain ε consists of the creep strain ε_c and the elasticity strain ε_e,

$$\varepsilon = \varepsilon_e + \varepsilon_c \tag{3}$$

During the explicit iteration process, the stress remains unchanged in each time step (from τ_i to $\tau_i + \Delta\tau_i$ with $\Delta\tau_i$ as the size of each time step), and is subsequently updated at the beginning of the next time step (at τ_{i+1}.). Consequently, the elasticity strain at the end of the nth step (at τ_n), considering the effect of concrete ageing, can be expressed as,

$$\varepsilon_e^{\ n} = \sum_{i=0}^{n-1} \frac{\Delta\sigma_i}{E_i} \tag{4}$$

where $\Delta\sigma_i$ is the stress increment from τ_i to τ_{i+1}, and E_i is the modulus of elasticity at τ_i, which contributes to the aging effects of concrete, is expressed as,

$$E_i = E_{28} exp\left\{s[1 - (\frac{28}{\tau_i})^{0.5}]\right\} \tag{5}$$

where E_{28} is elasticity of modulus at age of 28 days, s is an adjusting coefficient which depends on the strength class of cement. The creep strain at the end of the nth time step considering the effect of concrete ageing can be expressed as,

$$\varepsilon_c^{\ n} = \sum_{i=0}^{n-1} \frac{\Delta\sigma_i}{E_i} \sum_{j=1}^{m} A_j(\tau)[1 - e^{-p_j \Delta\tau i}] \tag{6}$$

where τ is the loading age of concrete and $A_j(\tau)$ is the jth age coefficient, and p_j is a coefficient considering the development of creep with time. From Eq. (6), the creep strain increments from τ_i to τ_{i+1} are given by

$$\Delta\varepsilon_c^{\ \Delta\tau_n} = \sum_{j=1}^{m} B^j{}_n (1 - e^{-p_j \Delta\tau_n}) \tag{7}$$

where

$$B^j{}_n = \sum_{i=0}^{n-2} \Delta\sigma_i A_j(\tau_i) e^{-p_j(t-\tau_i-\Delta\tau_n)} + \Delta\sigma_{n-1} A_j(\tau_{n-1}) \tag{8}$$

From Equations (6) to (8), the incremental relationship can be established as,

$$B^j{}_{n+1} = B^j{}_n e^{-p_j \Delta\tau_n} + \Delta\sigma_n A_j(\tau_n) \tag{9}$$

To accomplish the above mentioned creep incremental analysis, the creep coefficient expression needs to be converted the exponential series according to the format of Eq. (6), so that parameters $A_j(\tau)$ and p_j can be determined. Eq. (1), for calculating the creep coefficient, includes two time-dependant parameters, time t and age of initial loading of concrete t_0. Thus, the creep coefficient can be simply modified to $\phi^*(\tau, \Delta\tau)$ and approximated as,

$$\phi^*(\tau, \Delta\tau) = k_{c1}\phi_{RH}\beta(f_{cm}) \left(\frac{1}{0.1 + (\tau)^{0.2}}\right)\sum_{j=1}^{m} q_j(1 - e^{-p_j\Delta\tau}) \qquad (10)$$

Rewriting Eq. (10) in the format of Eq. (6), $A_j(\tau)$ is expressed as,

$$A_j(\tau) = k_{c1}\phi_{RH}\beta(f_{cm}) \left(\frac{1}{0.1 + (\tau)^{0.2}}\right)q_j \qquad (11)$$

In this study, a calibration approach is adopted to determine the parameters p_j and q_j, indicating that the exponential expression, with the number of fitting items m=4, can accurately reproduce the creep model given by the JTG D62, as well as deal with the interaction between creep stress and strain. These modified shrinkage and creep models have been implemented into subroutine CUSER3 for the 3D solid elements of ADINA. It is worth mentioning that since the creep coefficient in JTG D62 is expresses as the product of functions according to the loading age and age of concrete, the fitting method can be directly applied to provide acceptable approximations. The continuous retardation spectrum, as was proposed by Bažant and Xi (1995), can also be used to accurately approximate various creep models (ACI, CEB, B3 and JSCE) (Jirásek and Havlásek 2014).

The effective prestress forces in the prestressing tendons directly affect the elastic and time-dependent deformations, as well as the distribution of internal forces. However, there is no reliable non-destructive measurement method for monitoring the prestressing force in tendons embedded in concrete during the service life of PC bridges. Hence, predictive models are normally used to calculate prestress losses in practice. Long-term prestress loss is mainly caused by intrinsic tendon relaxation as well as concrete shrinkage and creep. For this purpose, various calculation methods are given by design codes and guides, such as ACI and Eurocode. The prestress loss due to creep, shrinkage and relaxation can be accounted for by time-dependent analysis or a simplified approach using age-adjusted elastic modulus (Elbadry et al. 2014). The overall relaxation of the prestressing tendons can be determined through detailed FE modelling using viscoelastic material models (Malm and Sundquist 2010). However, the actual prestress level is also affected by the ambient environment and construction quality of the prestressing process. For instance, the measured prestress loss of the KB Bridge in Palau reached approximately 50% of the design prestress level after 19 years, which is much lower than can be predicted using available calculation methods. A predictive model for the prestress loss due to steel relaxation has been proposed(Bažant and Yu 2013) on the basis of viscoplastic constitutive relation, for arbitrarily variable strain and temperature. Corrosion of the tendons can also cause prestressing force loss, as it reduces the cross-sectional area of the tendons. Robertson (2005) and Barthélémy (2015) introduced scaling constants to modify the calculated prestress level to account for these effects (e.g. thermal, corrosion). However, these effects are time dependent. In this research, two new parameters k_{p1} and k_{p2} have been added to the ACI relaxation model to explicitly consider the effects of construction quality and the time-dependent characteristics of prestress loss. k_{p1} is the initial prestress force modification coefficient that accounts for the effect of construction quality on the initial prestress force, and k_{p2} considers the time dependence of the prestress loss by modifying the amount of the prestress loss caused by relaxation. The effective prestress at time t is, therefore, expressed as,

$$\sigma(t) = k_{p1} f_{si} - 0.1 k_{p2} f_{si} \log_{10}^{t} \left\langle \frac{f_{si}}{f_y} - 0.55 \right\rangle \quad (12)$$

where f_{si} is the initial tendon stress, and f_y is the specified yield strength of the prestressing tendon. Eq. (12) has been converted into the format of Eq. (13) and input into the FE models as a viscoelastic material function.

$$\sigma(t) = \varepsilon_0 E_\infty + \varepsilon_0 \sum_{i=1}^{n} E_i e^{-t/\tau_i} \quad (13)$$

where ε_0 is the initial strain of the tendon caused by tension, E_∞ is the long-term modulus, E_i is the ith modulus for the Prony series, and τ_i is the ith relative time. The Prony series can be calculated according to Eq. (12) using the least-square method. To accurately simulate the real distribution of prestress losses along the length, a refined contact model is needed with consideration of the tension stage before grouting of the tendons, which makes the analysis computationally demanding and practically unfeasible for this study. To simplify the FE model, the average prestress loss caused by friction is assumed to be uniform along the length of each prestress tendon, which can provide acceptable approximations on long term behaviour of Jiang Jin Bridge. Taking T64, the longest prestress tendon in the top slabs and of the largest friction losses, as an example, the instantaneous deflection at the end of the cantilever, due to the tensioning of T64, given by the simplified model is 2.5% larger than when considering the actual distribution of friction forces. The initial strain ε_0 is calculated based on the tension control stress of each tendon (design value for the bridge analysed is 1395 MPa), subtracting by the immediate prestress loss which is calculated based on the design code.

Results of parametric studies

The effect of the targeted parameters (k_c, k_s, k_p) on the following structural responses is examined: (1) overall deflection shape; (2) curvature due to time-dependent deflection; and (3) crack distribution. The ranges of these parameters are selected to represent the expected physical limits and rate of occurrence. A series of FE models with different combinations of the targeted parameters was established and analysed.

Creep

The parameters k_{c1} and k_{c2} in the concrete creep model are varied within the ranges, 1 to 2 and 0.6 to 1, respectively. To isolate the effect of these two parameters, only one parameter changes at a time. Concrete shrinkage and prestressing force variations are also neglected to isolate the effect of creep, and so k_s and k_{pi} were set to '0'. The long-term structural responses of the Jiang Jin Bridge up to 30 years after its completion are simulated. The permanent loading considered in the model is from the self-weight of the bridge. Two typical locations, 100 m from the main pier at the side span (Location 1) and the middle of the main span (Location 2) are examined, and the ratio of the deflections at these two locations is used to indicate the overall deflected shape of the entire bridge.

The results indicate a linear relationship between parameter k_{c1} and the vertical deflections of the bridge. The deflections at both Locations 1 and 2 at Year 30 double as k_{c1} increases from 1 to 2, as shown in Fig. 7a. However, k_{c1} does not influence the trend of the deflection-time relationship, as shown in Fig. 7b. This figure also shows that the deflection develops very rapidly during the first 2000 days after completion and then stabilises. The ratio of the deflections at Location 1 and Location 2 is approximately 3.25, which remains roughly unchanged over time.

An approximately linear relationship between the parameter k_{c2} and vertical deflections is observed, as shown in Fig. 8a. Fig. 8b, indicates that k_{c2} does not influence the initial deflection up to 1000 days after completion, but it does affect the trend in the rate of deflection increase over time. Similar to k_{c1}, k_{c2} has little influence on the ratio of the deflections at Locations 1 and 2, which ranges from 3.22 to 3.28.

Prestress force

The effect of the prestress parameters k_{p1} and k_{p2} on the long-term behaviour of the box-girder bridges is discussed here. The original JTG D62-2004 creep model (Eq (1) when $k_{c1}= k_{c2}=1$) was adopted for the consideration of the interaction between prestress loss and concrete creep. The inspection of grouting and prestressing anchorages has revealed that the quality control during the construction of this bridge was poor and this has affected the initial prestress level and so k_{p1} is only possible to be less than 1. Therefore, the initial prestressing force modification coefficient k_{p1} varies from 1 to 0.6. The modification coefficient k_{p2} (which varies from 1 to 5) is used to consider the time dependency of all prestress losses (e.g. thermal, corrosion and relaxation) relating to the steel tendons. The results indicate that the time-dependent deflections of the Jiangjin bridge are sensitive to both k_{p1} and k_{p2}. As k_{p1} decreases (Fig. 9a), the deflection at Location 1 increases from 1.8 cm to 4.2 cm and the deflection at Location 2 increases from 14 cm to 24 cm at Year 30. As k_{p2}increases, the deflections at both Locations 1 and 2 increase (Fig. 9b). Figures 9c and 9d also indicate that the ratio of the deflections evolves linearly with log-time and remains almost constant at k_{p1}=0. The effects of the prestress parameters k_{p1} and k_{p2} on this ratio are significantly larger than those of the creep parameters k_{c1} and k_{c2}. It is, therefore, important to pay attention to the deflections at both main and side spans to distinguish the influence of prestess loss from that of creep on the long-term behaviour of box-girder bridges.

The total prestress force distribution of the top tendons at the main span is shown in Fig. 9e. The prestress tendon T10 location on the top slab of the box girder is selected to observe the evolution of the effective prestress force with time. As illustrated in Fig. 9f, the expected long-term loss of prestress caused by steel relaxation and concrete creep (k_{c1}=k_{c2}=1) is only 3% over the 30 years of observation period, the majority of which occurs before the 1st year. This value of prestress loss is only the incremental loss calculated from the first year, when the bridge opened to traffic, to the 30th year, and the effect of shrinkage is excluded. By using default parameters in JTG D62, the total loss (including the construction stage) due to creep, shrinkage and relaxation is 12.5%.By adjusting k_{p2}, the history of prestress loss development can be adjusted better.

Shrinkage

To isolate the effects of shrinkage, only shrinkage is considered and the effects of k_s varying from 1 to 2 are analysed. It is found that (See Fig. 10), both the axial shortening and the vertical deflection of the girder varies proportionally with k_s. As discussed above, the effect of thickness on shrinkage is considered using the self-developed subroutine CUSER3 in ADINA. Due to the variation of the slab thickness within the box girder section, the distribution of shrinkage within this section is also non-uniform. This causes an upward deflection in the middle of the main span (Location 2). This deflection increases over time until Day 2700, when it reaches its maximum value (Fig. 10a). After this peak point, this upward deflection, due to the non-uniform distribution of shrinkage, starts to decrease. However, the side span (Location 1) behaves differently; the upward deflection due to the differential shrinkage within the box girder section continuously increases within 30 years, as shown in Fig. 10a. The axial shortening of the girder pulls the main pier (see Fig.10 b), causes the pier to bend towards the centre of the span inducing a rotation of the girder on the top of the pier, as illustrated in Fig. 10c, which explains why the development of the vertical deflections at

Locations 1 and 2 follow different trends.

Parameter Identification

Process of Parameter Idenfitication

For the purpose of improving the existing creep, shrinkage and prestress models in JTG D62, additional parameters are required, as above described. The values of these parameters are calibrated using real-life measured data. The objective function used in the parameter optimization process needs to account for the time and location-dependency of the measurement data. The parameter identification model can be formulated using an optimization process. The relationships between the parameters and the structural response function $F(t)$ have been established, and the objective function can be specified as,

$$\text{Minimize: } f(\mathrm{X}) = \sum_{j=1}^{m} \omega_i (F_i(t_j) - M_i(t_j))^2$$

$$\text{Subject to: } \mathrm{X}_{Low} \leq \mathrm{X} \leq \mathrm{X}_{up} \tag{14}$$

where $f(\mathrm{X})$ is the total objective function, $M_i(t_j)$ and $F_i(t_j)$ are the values of the calculation and measurement at time t_i, and m represents the number of measurement times from t_0 to t_m, ω_i is the ith weighting coefficient, $\mathrm{X} = \{k_1, k_2 \ldots\ldots, k_5\}^T$ is the vector of the design variables, X_{Low} and X_{up} are the lower bound and upper bound, respectively, of the design variables.

Considerable computational effort is required to determine the relationships between the targeted parameters and structural response. During this process, different combinations of the targeted parameters are required. As ADINA does not provide access to interactive information, an efficient approximation approach is necessary to be used alongside the FE analysis. For this purpose, the response-surface method (RSM)(Chakraborty and Sen 2014; Shahidi and Pakzad 2014; Xu et al. 2016; Yao and Wen 1996) was adopted.

Considering the complexity of the parameter identification model, the genetic algorithm (GA) method was also adopted in this study. GA is an efficient method for solving complex problems of optimization by simulating the biological evolution of the survival of the fittest using three major processes: selection, crossover and mutation. The GA method has been extensively adopted in structural optimization design(Cheng 2010; El Ansary et al. 2010) and parameter identification(Caglar et al. 2015; Deng and Cai 2009). In this study, the parameter identification process was carried out using FEM, RSM and GA, as illustrated in Fig. 11:

1. Capture the influence of the targeted parameters on structural behaviour through sensitivity analyses in FEM;

2. Generate a database of modelling results from FEM with different combinations of parameters;

3. Using RSM, create a substitutive model based on the FEM results database;

4. Establish the objective functions and boundary conditions based on the substitutive model and measured data, then, use the GA method to seek the best parameter combinations;

5. Input the parameters found in Step (4) into FEM and compare against measured data.

Results of parameter identification

To accurately describe the relationship between the targeted parameters (k_{c1}, k_{c2}, k_{p1}, k_{p2} and k_s) and the structural response (deflections), the two-order RSM model is established, which contains 11 time-dependent regression coefficients. The accuracy of the RSM is dependent on a sufficient number of FE model runs with different combinations of adjusting parameters. A central composite design (CCD) is adopted in this study to decrease the number of parameter combinations and guarantee the precision for the substitute model. CCD, which is also known as the Box-Wilson design, is an efficient class RSM appropriate for calibrating full quadratic models (Yao and Wen 1996). Accordingly, 1/2 fractional factorial designs are defined with regards to the lower and upper bounds for each parameter. In this study, the CCD function ccdesign(fraction) in Matlab is adopted to generate a central composite design for the targeted parameters (k_c, k_c, k_p), for more details see MATLAB for Engineers (Moore 2014). To maintain all design points inside the regression domain and to enhance the accuracy of the parameter identification, the new data generated by GA are added to the original regression region.

To verify the total quality of the RSM, the R^2 statistics were employed and the calculation results throughout the entire time history for locations 1 and 2 are illustrated in Fig. 12. The results indicate that the R^2 statistics fluctuate with time and are close to 1, which indicates that the substitute model matches accurately the FE results. The relative error between the RSM and the FE models for each combination of the parameters is shown in Fig. 13, which includes 53 different combinations within the RSM region. With the exception of a few combinations at an early age, the majority of the error distributions are ±3%, which is acceptable for this study.

For the purpose of parameter identification, a GA optimization program is used to continuously evolve the parameters until the optimization targets are met, in order to seek the best combination of parameters. During the evolution of the parameters, the objective functions (Eq.14) are calculated by the RSM according to different attempted selections from the GA. As previously mentioned, the objective functions are calibrated using the measured data from the Jiangjin Bridge.

The measured data from Locations 1 and 2 within the entire observation period are implemented into the objective functions. Since the measured data are influenced by the environment temperature and moisture and measuring errors, trend lines are used to declutter the data. The weighting coefficients w_1 and w_2 are used to reflect the different contributions of the measured data at these two locations. Three sets of weighting coefficients are used, as summarised in Table 2. In Set 1, $w_1 = 1$ and $w_2 = 0$, meaning that only the measured data from Location 1 are considered in the objective functions and the time-dependent development of deflections at Location 1 is the single objective for the GA. Conversely, only the measured data from Location 2 are considered in Set 2. In Set 3, the measured data from both locations have the same weight in the objective functions, and so multi-objective GA optimizations are carried out to consider the measured data from both locations. For comparison purposes, the control model (Set 0) based on JTG D62-2004 without the implementation of the modification coefficients is also analysed. Table 2 summarises the lower and upper bounds of the modification coefficients and the optimisation results of the four sets.

All modification coefficients calculated by the GA are input into the FE models and the calculated deflection from different sets are shown in Fig. 14. As expected, without applying the modification coefficients (Set 0), the long-term deflection history at both Locations 1 and 2 is significantly underestimated. By applying the modification coefficients, a much better match with the measured data is obtained.

The values of the modification coefficients of Sets 1-3, which adopt different optimization objectives, are different, as shown in Table 2. If only one of the measured location is considered when establishing the objective function, a good comparison between the calculated and measured deflections can be obtained at this location only; however, the calculated deflections at other locations do not match the measured data at all. Set 3 considers both Locations 1 and 2 in the optimizing objective function and produces satisfactory results for both locations. The identified values of the modification coefficients (k_{p1} and k_{p2}) accounting for prestress loss of Set 3 are very different from those from Sets 1 and 2. This is because the prestress loss affects the ratio between the deflections at Locations 1 and 2. In addition, the calculated value of k_{p2}=3.76 indicates that the long term prestress losses of the Jiang Jin Bridge were significantly underestimated, and many other possible factors (e.g. thermal, corrosion, concrete creep and shrinakge) may have led to the additional prestress loss.

Discussion

The above method is used to identify the modification coefficients and calibrate the predictive models (JTG D62) for creep, shrinkage and effective prestress force, based on the measured vertical deflection data. Other measured data, i.e. crack distribution and crack time, can be used to validate the model. The updated model can be used to simulate the internal force condition and time-dependent stress distribution within the structure, which can help to perform structural assessments and to determine if strengthening or retrofitting is necessary.

The stress results, given by FE modelling of the updated model, indicate that two locations, i.e. the bottom of the web at mid span and top of the web at the supported end of a cantilever, are critical for the serviceability evaluation of the superstructure of the bridge. The calculated axial stresses are presented in Fig. 15. In JTG D62, the axial stress level is an important criterion for the long-term serviceability evaluation throughout the service life of a bridge. As the Jiang Jin Bridge was designed to be a fully prestressed structure, no tensile stress is allowed for the serviceability limit state of the bridge design. The characteristic concrete tensile strength of 2.65 MPa is used as the cracking limit in this study.

Flexural cracks were observed at the bottom flange at the centre of the main span with a maximum crack width of 0.3 mm 10 year after completion. As shown in Fig. 15a, the axial stresses along the bridge span of Set 0 (control model) are lower than the design limit, which is unconservative, as it does not predict well the vertical deflection. On the other hand, although Set 2 exhibits a satisfactory match with the measured data in terms of mid-span vertical deflection of the main span, it predicts the occurrence of cracking (axial stresses > cracking limit) significantly later (after 30 years) than in practice (after 3800 days). Set 3 offers a better simulation precision on both long-term deflection and axial stress development, indicating the importance of considering both the main span and side span in the analysis. The updated model also predicts long-term cracks at the top of the slab near the main column and this can lead to serviceability problems in 20 years' time (Fig. 15b).

Diagonal cracks are also observed on both webs of the box girder; the cracks are primarily located 40 m from the centre of the main span. The diagonal cracks are primarily due to shear forces and the loss of vertical prestressing force; this is commonly observed in large-span PC box bridges. As Set 3 predicts the cracking time better than the other sets, it is also used to check the crack distribution. An integer variable was defined in the material subroutine in ADINA; when the principal tensile stress reaches the cracking limit, the integer variable is set to be '1' to approximately display the crack locations. As shown in Fig. 16, the calculated crack location matches reasonable well with the observed one, confirming the reliability of Set 3.

Conclusions

This study presents a model updating approach aiming at improving the accuracy of numerical modelling of the long-term behaviour of box girder bridges using the Chinese standard models, calibrated against data obtained from the Jiang Jin Bridge in service. This work is important for assessing the predictive models of current standards so as to improve the long-term evaluation, monitoring and strengthening of such bridges. Based on the analytical results presented in this article, the following conclusions are drawn:

(1) For the case study bridge, the original prediction model in JTG D62 used for the design of the bridge is unable to predict the development of deflection over time. This shows that modifications on creep and shrinkage prediction model (i.e. parameters in Table 2) are needed to enhance the predicting accuracy of this and other design models. By adopting the proposed model updating approach, the predicting accuracy can be significantly improved.

(2) Creep and prestress losses influence significantly the calculated vertical deflections of both the main span and side span. However, prestress loss alters the ratio between the deflections of the main and side spans, hence, it is important to consider the performance of both the main and side spans, rather than only the main span.

(3) Based on FEM, RSM and GA, the updated models have been used in the modelling of the Jiang Jin Bridge, leading to much better agreement between the modelling results and measured data in terms of bridge deflection history and crack patterns. Although this method has been developed for and calibrated against a bridge, it is valid for other bridges of this kind whenever enough measured data are available.

(4) Future research should focus on monitoring and assessment methods to capture the behaviour for bridges of this kind throughout the service life, especially for the actual prestress loss and stress distribution on the structure.

Appendix.

Numerical Examples for Creep Analysis

To verify the accuracy of the method adopted in this paper, comparisons are made with the fib Model Code (CEB-fib90). For example, the notional thickness is 500mm, concrete class is C50, the relative humidity is 60%, and the loading age are 2, 10, 100, 1000, 5000 days. The derived parameters of the exponential series, according to Eq.10, are shown in Table3. Fig.17 and Fig.18 show that the present approach can accurately reproduce the results of the creep Model Code (CEB-fib90) predictions with acceptable relative error with maximum value 4.5%.

Response-Surface Model

In this study, five parameters have been defined: k_{c1} and k_{c2} are adopted to adjust the creep model, k_s is to adjust the shrinkage model and k_{p1} and k_{p2} are for adjusting the effective prestressing force. To simplify the RSM model, the targeted parameters are grouped according to their purposes. The grouping of the parameters are shown as,

$$\begin{cases} k_c = k_{c1}k_{c2} \\ k_p = k_{p1}k_{p2} \end{cases} \quad (15)$$

where k_c is for creep; k_p is for prestressing force . For the actual structure, concrete creep, shrinkage and prestress are interactive and make important contributions to the evolution of structural deflection and stress. The structural response $F(t)$ with cross terms can be defined as,

$$F(t) = \beta_0^t + \sum_{i=1}^{n} \beta_i^t\, k_i + \sum_{i=1}^{n} \beta_{2i}^t\, (k_i)^2 + \beta_{12}^t k_c k_s + \beta_{13}^t k_c k_p + \beta_{14}^t k_s k_p \quad (16)$$

Where k_i is the ith targeted parameters (k_{c1}, k_{c2}, k_{p1}, k_{p2} and k_s), β_i^t is the one-order regression coefficient at time t, β_{2i}^t is the two-order regression coefficient at time t, β_{12}^t is the regression coefficient for the interaction effect of shrinkage and creep, β_{13}^t is the regression coefficient for shrinkage and prestress, and β_{14}^t is the regression coefficient for prestress and creep. The structural responses $F(t)$ (e.g. deflections at time t) can be calculated through FE modelling.

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References

ACI Committee 209 (ACI). (2008). "Guide for modeling and calculating shrinkage and creep in hardened concrete." ACI Rep. 209.2R-08, ACI, Armington Hills, MI.

ADINA R & D, Inc. (2001). ADINA theory and modeling guide. Report 01–7, Watertown, MA.

Barthélémy, J. F., Sellin, J., and Torrenti, J. (2015). "The effects of long-term behavior of both concrete and prestressing tendons on the delayed deflection of a prestressed structure " Proc., 10th Int. Conf. Creep, Shrinkage and Durability Mechanics of

Concrete and Concrete Structures(CONCREEP-10), RILEM and the Engineering Mechanics Institute of ASCE, Vienna, Austria , 621-630.

Bažant, Z. P., and Baweja, S. (2000). "Creep and shrinkage prediction model for analysis and design of concrete structures-model B3." ACI Special Publications, 194, 1-84.

Bažant, Z. P., and Xi, Y. (1995). "Continuous retardation spectrum for solidification theory of concrete creep." J. Eng. Mech., 121(2), 281-288.

Bažant, Z. P., and Yu, Q. (2013). "Relaxation of prestressing steel at varying strain and temperature: viscoplastic constitutive relation." J. Eng. Mech., 10.1061/(ASCE)EM.1943-7889.0000533, 814-823.

Bažant, Z. P., Yu, Q., and Li, G. H. (2012 a). "Excessive long-time deflections of prestressed box girders. I: Record-Span bridge in palau and other paradigms." J. Struct. Eng., 10.1061/(ASCE)ST.1943-541X.0000487, 676-689.

Bažant, Z. P., Yu, Q., and Li, G. H. (2012 b). "Excessive long-time deflections of prestressed box girders. II: Numerical analysis and lessons learned." J. Struct. Eng., 10.1061/(ASCE)ST.1943-541X.0000375,687–696.

Caglar, N., Demir, A., Ozturk, H., and Akkaya, A. (2015). "A simple formulation for effective flexural stiffness of circular reinforced concrete columns." Eng. Appl. Artif. Intell., 38, 79-87.

Chakraborty, S., and Sen, A. (2014). "Adaptive response surface based efficient Finite Element Model Updating." Finite Elem. Anal. Des., 80, 33-40.

Cheng, J. (2010). "Optimum design of steel truss arch bridges using a hybrid genetic algorithm." J. Constr. Steel Res., 66(8-9), 1011-1017.

Comité Euro-International du Béton–Fédération International de la Précontrainte (CEB-FIP). (1990). CEB-FIP model code for concrete structures, Thomas Telford, London.

Deng, L., and Cai, C. S. (2009). "Identification of parameters of vehicles moving on bridges." Eng. Struct., 31(10), 2474-2485.

El Ansary, A. M., El Damatty, A. A., and Nassef, A. O. (2010). "A coupled finite element genetic algorithm technique for optimum design of steel conical tanks." Thin-Walled Struct., 48(3), 260-273.

Elbadry, M., Ghali, A., and Gayed, R. B. (2014). "Deflection control of prestressed box girder bridges." J. Bridge Eng., 10.1061/(ASCE)BE.1943-5592.0000564, 04013027.

Guo, T., and Chen, Z. (2016). "Deflection control of long-span psc box-girder bridge based on field monitoring and probabilistic FEA." J. Perform. Constr. Facil., 10.1061/(ASCE)CF.1943-5509.0000909, 04016053.

Guo,T., Sause R., Frangopol, D. M., and Li, A. (2011). "Time-Dependent Reliability of PSC Box-Girder Bridge Considering Creep, Shrinkage, and Corrosion." J. Bridge Eng., 10.1061/□ASCE□BE.1943-5592.0000135, 29-43.

International Federation for Structural Concrete (2013). fib Model Code for Concrete Structures 2010. Wilhelm Ernst & Sohn, Germany.

Jirásek, M., and Havlásek, P. (2014). "Accurate approximations of concrete creep compliance functions based on continuous retardation spectra." Comput. Struct., 135, 155-168.

Křístek, V., Bažant, Z. P., Zich, M., and Kohoutková, A. (2006). "Box girder bridge deflections." ACI Concr. Int., 28(1), 55-63.

Lou, T., Lopes, S. M. , and Lopes, A. V. (2014). "A finite element model to simulate long-term behavior of prestressed concrete girders." Finite Elem. Anal. Des., 81, 48-56.

Malm, R., and Sundquist, H. (2010). "Time-dependent analyses of segmentally constructed balanced cantilever bridges." Eng. Struct., 32(4), 1038-1045.

MIDAS Information Technology (2011). "on-line manual: civil structure design system."

<http://manual.midasuser.com/EN_TW/civil/791/index.htm> (Mar. 10, 2015).
Ministry of Transport of China (2004). Code for design of highway reinforced concrete and prestressed concrete bridges and culverts. JTG D62-2004, Beijing (in Chinese).
Moore, H. (2014). MATLAB for Engineers, Prentice Hall Press.
Norachan, P., Kim, K. D., and Oñate, E. (2014). "Analysis of segmentally constructed prestressed concrete bridges using hexahedral elements with realistic tendon profiles." J. Struct. Eng., 140(6), 10.1061/(ASCE)ST.1943-541X.0000923, 04014028.
Pan, Z., Li, B., and Lu, Z. (2013). "Re-evaluation of CEB-FIP 90 prediction models for creep and shrinkage with experimental database." Constr. Build. Mater., 38, 1022-1030.
RILEM Technical Committee TC-242-MDC (Zdeněk P. Bažant, chair (2015). "RILEM draft recommendation: TC-242-MDC multi-decade creep and shrinkage of concrete: material model and structural analysis." Mater. Struct., 48(4), 753-770.
Robertson, I. N. (2005). "Prediction of vertical deflections for a long-span prestressed concrete bridge structure." Eng. Struct., 27(12), 1820-1827.
Shahidi, S. G., and Pakzad, S. N. (2014). "Generalized response surface model updating using time domain data." J. Struct. Eng., 10.1061/(ASCE) ST.1943-541X.0000915, A4014001.
Sousa, H., Bento, J., and Figueiras, J. (2013). "Construction assessment and long-term prediction of prestressed concrete bridges based on monitoring data." Eng. Struct., 52, 26-37.
Xu, T., Xiang, T., Zhao, R., Yang, G., and Yang, C. (2016). "Stochastic analysis on flexural behavior of reinforced concrete beams based on piecewise response surface scheme." Eng. Fail Anal., 59, 211-222.
Yang, I. H. (2005). "Uncertainty and updating of long-term prediction of prestress forces in PSC box girder bridges." Comput. Struct., 83(25–26), 2137-2149.
Yao, H. J., and Wen, Y. K. (1996). "Response surface method for time-variant reliability analysis." J. Struct. Eng., 10.1061/(ASCE)0733-9445(1996)122:2(193), 193-201.
Zhu, B. F. (2014). Thermal stresses and temperature control of mass concrete, Tsinghua University Press, Printed in the United States of America.

TABLES

Table 1. Material properties of the Jiang Jin Bridge

Concrete		Prestressed tendons	
Girder $f_{cm,28d}$	48MPa	Tensile strength	1860MPa
Piers $f_{cm,28d}$	40MPa	Initial tendon stress	1395Mpa
Girder E_{28d}	34.5GPa	Elastic modulus	195GPa
Piers E_{28d}	32.5GPa	curvature friction	0.3/rad
Curing age	7 d	wobble coefficient	0.0066/m
Averaged RH	70%	Anchorage Slip	6mm
Averaged environmental temperature	20℃		

Table 2. Bounds and results of the updating parameters

	Updating parameters						
	k_{c1}	k_{c2}	k_s	k_{p1}	k_{p2}	w_1	w_2
lower bounds	0.6	0.01	0.8	0.6	1.0	/	/
upper bounds	2.0	1.0	2.0	1.0	4.0	/	/
set 0	1.0	1.0	1.0	1.0	1.0	/	/
set 1	1.224	0.105	1.823	0.961	1.519	1	0
set 2	1.06	0.102	1.610	0.999	1.797	0	1
set 3	0.848	0.100	1.500	0.900	3.759	1	1

Table 3. Parameters of the exponential series

i	p_i	q_i
1	1.1233	0.1535
2	0.0509	0.1863
3	0.0006	0.285
4	0.0047	0.3386