Online Supplement for "Modeling for the Equitable and Effective Distribution of Donated Food under Capacity Constraints" by Sengul Orgut, I., Ivy, J.S., Uzsoy, R., & Wilson, J.R.

APPENDIX A

Proposition 1. *An optimal solution to a given instance of the Food Distribution Model is as follows:*

Case 1: *If $\sum_{l=1}^{n} C_l \geq S$ and $J_0 = \emptyset$, i.e., the instance is **partially equity constrained**, then the optimal objective function value is:*

$$P^* = 0, \tag{1}$$

and the individual distributions to the counties $\left(X_j^*\right)$ have multiple optimal solutions.

Case 2: *If $J_0 \neq \emptyset$ and $\sum_{l=1}^{n} C_l \geq S \geq R\sum_{l=1}^{n} D_l$, i.e., the instance is **capacity and equity constrained**, then the optimal objective function value is:*

$$P^* = S - R\sum_{l=1}^{n} D_l. \tag{2}$$

In an optimal solution, the bottleneck counties receive an amount of food equal to their capacity,

$$X_j^* = C_j \; for \; all \; j \epsilon B. \tag{3}$$

Food shipments to all remaining counties will have multiple optimal solutions.

Case 3: *If $J_0 \neq \emptyset$, $\sum_{l=1}^{n} C_l \geq S$, and $R\sum_{l=1}^{n} D_l > S$, i.e., the instance is **supply and equity constrained**, then the optimal objective function value is:*

$$P^* = 0. \tag{4}$$

The individual distributions to the counties $\left(X_j^*\right)$ will have multiple optimal solutions.

Proof of Proposition 1. The proof will consider each of the three cases separately. Let $\Delta = \sum_{l=1}^{n} D_l$.

Case 1: In this case we have $\frac{D_j}{\Delta} \leq K$ for all counties $j \in J$. This implies that $-K + \frac{D_j}{\Delta} \leq 0$ for all j, i.e., $J_0 = \emptyset$. Constraint (2) can be written as:

$$-K + \frac{D_j}{\Delta} \leq \frac{X_j}{\sum_{l=1}^{n} X_l} \leq K + \frac{D_j}{\Delta} \qquad j \in J. \tag{A1}$$

which, due to the nonnegativity of X_j, (A1) can be rewritten as:

$$0 \leq \frac{X_j}{\sum_{l=1}^{n} X_l} \leq K + \frac{D_j}{\Delta} \qquad j \in J. \tag{A2}$$

Combining (A2) with the capacity constraint (4) yields

$$0 \leq X_j \leq \min\left(C_j, \left(K + \frac{D_j}{\Delta}\right) \sum_{l=1}^{n} X_l\right) \qquad j \in J, \tag{A3}$$

where constraints (3) and (5) ensure that

$$\sum_{l=1}^{n} X_l \leq S. \tag{A4}$$

We will prove that there always exists a feasible solution set $\{X_j^*, j \in J\}$ such that $\sum_{l=1}^{n} X_l^* = S$.

For the given value of S, let

$$X_j = \min\left(C_j, S\frac{D_j}{\Delta}\right) \qquad j \in J. \tag{A5}$$

Let

$$X_C \equiv \{j \in J: X_j = C_j\}, \tag{A6}$$

$$X_S \equiv \left\{j \in J: X_j = S\frac{D_j}{\Delta}\right\}. \tag{A7}$$

If, $X_C = \emptyset$ and $X_S \neq \emptyset$, i.e., for all j, $X_j = S\frac{D_j}{\Delta}$; then $\sum_{l=1}^{n} X_l = S$ and an optimal solution has been obtained.

Next, we show that the case when $X_S = \emptyset$ and $X_C \neq \emptyset$ is not possible. This case would imply that for all j, $C_j < S\frac{D_j}{\Delta}$. Summing over all j, we get $\sum_{l=1}^{n} C_l < S$, which is a contradiction since we assume that $S \leq \sum_{l=1}^{n} C_l$.

The last case we consider is $X_S \neq \emptyset$ and $X_C \neq \emptyset$. For counties $l \in X_C$, define

$$E_l = S\frac{D_l}{\Delta} - C_l \qquad l \in X_C. \tag{A8}$$

The value E_l represents the extra amount of supply that must be allocated to some county $j \in X_S$ with idle capacity in order to achieve $\sum_{l=1}^{n} X_l = S$. The total idle capacity available must be greater than or equal to the total extra pounds of food to be shipped in order for this solution to hold, implying that

$$\sum_{l \in X_C} E_l \leq \sum_{l \in X_S} \left(C_l - S\frac{D_l}{\Delta}\right), \tag{A9}$$

where $C_l - S\frac{D_l}{\Delta}$ represents the idle capacity at county $l \in X_S$.

Since we have $S \leq \sum_{l=1}^{n} C_l$ by assumption,

$$S\left(\sum_{l \in X_C} \frac{D_l}{\Delta} + \sum_{l \in X_S} \frac{D_l}{\Delta}\right) \leq \sum_{l \in X_C} C_l + \sum_{l \in X_S} C_l, \tag{A10}$$

$$S\sum_{l \in X_C} \frac{D_l}{\Delta} - \sum_{l \in X_C} C_l \leq \sum_{l \in X_S} C_l - S\sum_{l \in X_S} \frac{D_l}{\Delta}, \tag{A11}$$

$$\sum_{l \in X_C} E_l \leq \sum_{l \in X_S} C_l - S\sum_{l \in X_S} \frac{D_l}{\Delta}. \tag{A12}$$

so inequality (A9) always holds.

Therefore, by assigning this extra $\sum_{l \in X_C} E_l$ pounds of food among the counties in set X_S in an arbitrary manner, we can obtain an optimal solution with $\sum_{l=1}^{n} X_l^* = S$. Furthermore, since there can be different assignments to the counties with idle capacity, there can be multiple optimal solutions to the Food Distribution Model for Case 1. An algorithm for generating these alternative allocations is given in Figure 3 of the paper.

Case 2: For Case 2 we have at least one county $j \in J$ such that $\frac{D_j}{\Delta} > K$, and $S \geq R\Delta$. Hence, constraint (2), in combination with the capacity constraint (4), can be written as:

$$\max\left(0, \left(-K + \frac{D_j}{\Delta}\right)\sum_{l=1}^{n} X_l\right) \leq X_j \leq \min\left(C_j, \left(K + \frac{D_j}{\Delta}\right)\sum_{l=1}^{n} X_l\right) \qquad j \in J_0. \tag{A13}$$

where constraints (3) and (5) ensure that

$$\sum_{l=1}^{n} X_l \leq S. \tag{A14}$$

For feasibility, we must have

$$\left(-K + \frac{D_j}{\Delta}\right)\sum_{l=1}^{n} X_l \leq C_j \qquad j \in J_0, \tag{A15}$$

$$\sum_{l=1}^{n} X_l \leq \frac{C_j \Delta}{D_j - K\Delta} \qquad j \in J_0, \tag{A16}$$

$$\sum_{l=1}^{n} X_l \leq \min_{j \in J_0}\left\{\frac{C_j \Delta}{D_j - K\Delta}\right\}, \tag{A17}$$

$$\sum_{l=1}^{n} X_l \leq R\Delta. \tag{A18}$$

Since this case satisfies the condition that $S \geq R\Delta$, we only need $\sum_{l=1}^{n} X_l \leq R\Delta$.

We will prove that there always exists a feasible solution set $\{X_j^*, j \in J\}$ that satisfies $\sum_{l=1}^{n} X_l^* = R\Delta$.

For the given value of $R\Delta$, let

$$X_j = \min(C_j, RD_j) \qquad j \in J. \tag{A19}$$

Let

$$X_C \equiv \{j \in J: X_j = C_j\}, \tag{A20}$$

$$X_S \equiv \{j \in J: X_j = RD_j\}. \tag{A21}$$

If, $X_C = \emptyset$ and $X_S \neq \emptyset$, i.e., for all j, $X_j = RD_j$, then $\sum_{l=1}^{n} X_l = R\Delta$ is obviously an optimal and feasible solution.

Next, we show that the case when $X_S = \emptyset$ and $X_C \neq \emptyset$ is not possible. This case would imply that for all j, $C_j < RD_j$. Summing over all j, we get $\sum_{l=1}^{n} C_l < R\Delta \leq S$ due to the assumption of Case 2. This is a contradiction since we assume that $S \leq \sum_{l=1}^{n} C_l$.

The last case we consider is $X_S \neq \emptyset$ and $X_C \neq \emptyset$. For counties $l \in X_C$, define

$$E_l = RD_l - C_l \qquad l \in X_C. \tag{A22}$$

The value E_l represents the extra amount of supply to be allocated to any county $j \in X_S$ with idle capacity in order to achieve $\sum_{l=1}^{n} X_l = R\Delta$. This E_l pounds of food can be allocated. The total idle capacity available should be greater than or equal to the total extra pounds of food to be shipped in order for this solution to hold. So, we must have:

$$\sum_{l \in X_C} E_l \leq \sum_{l \in X_S} (C_l - RD_l), \tag{A23}$$

where $C_l - RD_l$ represents the idle capacity at county $l \in X_S$.

By using the main assumption of $S \leq \sum_{l=1}^{n} C_l$ and the condition of this case that $S \geq R\Delta$, we get $\sum_{l=1}^{n} C_l \geq R\Delta$. It follows that,

$$R\left(\sum_{l \in X_C} D_l + \sum_{l \in X_S} D_l\right) \leq \sum_{l \in X_C} C_l + \sum_{l \in X_S} C_l, \tag{A24}$$

$$R\sum_{l \in X_C} D_l - \sum_{l \in X_C} C_l \leq \sum_{l \in X_S} C_l - R\sum_{l \in X_S} D_l, \tag{A25}$$

$$\sum_{l \in X_C} E_l \leq \sum_{l \in X_S} C_l - R\sum_{l \in X_S} D_l. \tag{A26}$$

So, inequality (A23) always holds.

Therefore, by assigning this extra $\sum_{l \in X_C} E_l$ pounds of food to the counties in set X_S in an arbitrary manner, we can obtain the optimal solution of $\sum_{l=1}^{n} X_l^* = R\Delta$. Furthermore, since there can be different assignments to the counties with idle capacity, this shows that there can be multiple optimal solutions to the Food Distribution Model for Case 2.

Proof for distribution to bottleneck counties: According to the definition of a bottleneck county given in Proposition 1, bottleneck counties are those with the minimum $\left\{\frac{C_j}{D_j - K\Delta}\right\}$ ratio among the counties $j \in J_0$. Then, for $j \in B$, since we have shown that $\sum_{l=1}^{n} X_l^* = R\Delta$, from Constraint (2), we have

$$\left(-K + \frac{D_j}{\Delta}\right) R\Delta \le X_j^* \le \min\left(C_j, \left(K + \frac{D_j}{\Delta}\right) R\Delta\right) \qquad j \in B, \tag{A27}$$

$$\left(-K + \frac{D_j}{\Delta}\right)\left(\frac{C_j\Delta}{D_j - K\Delta}\right) \le X_j^* \le \min\left(C_j, \left(K + \frac{D_j}{\Delta}\right)\left(\frac{C_j\Delta}{D_j - K\Delta}\right)\right) \qquad j \in B, \tag{A28}$$

$$C_j \le X_j^* \le C_j \qquad j \in B. \tag{A29}$$

It follows that

$$X_j^* = C_j \;\; for\; j \in B. \tag{A30}$$

<u>Case 3:</u> Case 3 considers the situation where there exists at least one county $j \in J$ such that $\frac{D_j}{\Delta} > K$, and $S < R\Delta$. The proof for this case follows from a combination of the proofs for Cases 1 and 2. We can apply the equations (A13)-(A18) exactly to this case. However, due to the condition of $S < R\Delta$ that this case satisfies, we only need $\sum_{l=1}^{n} X_l \le S$. The remaining argument follows along the lines of Case 1 as given in (A5)-(A12).

APPENDIX B

Proof of Proposition 2. Let $\Delta = \sum_{l=1}^{n} D_l$. If we associate dual variables π_l with the constraints (18) and π_0 with constraint (19), then the dual of the Capacity Allocation Model can be written as

$$\min \sum_{l \in J_0} C_l \pi_l + \pi_0 \tag{B1}$$

$$\sum_{l \in J_0} (D_l - K\Delta)\pi_l \geq 1 \tag{B2}$$

$$-Y\pi_j + \pi_0 \geq 0 \qquad j \in J_0 \tag{B3}$$

$$\pi_j \geq 0 \qquad j \in J_0 \cup \{0\} \tag{B4}$$

From (B3), we see that

$$\pi_j \leq \pi_0 / Y \qquad j \in J_0 \tag{B5}$$

and thus from (B2), we have

$$\sum_{l \in J_0} \frac{(D_l - K\Delta)\pi_0}{Y} \geq \sum_{l \in J_0} (D_l - K\Delta)\pi_j \geq 1. \tag{B6}$$

From (B6), we have

$$\pi_0 \geq \frac{Y}{\sum_{l \in J_0} (D_l - K\Delta)} > 0 \tag{B7}$$

and therefore by the complementary slackness theorem (Bertsimas & Tsitsiklis, 1997) constraint (19) must be satisfied at strict equality. ■

APPENDIX C

Proof of Proposition 3. Let $\Delta = \sum_{l=1}^{n} D_l$. Direct inspection of the Capacity Allocation Algorithm suggests that the solution obtained from the Capacity Allocation Algorithm is optimal. Here, we will use the solution obtained from the algorithm and prove that it is optimal. By the operation of the Capacity Allocation Algorithm, if we terminate the algorithm with η bottleneck counties, the algorithm must terminate at iteration η. All bottleneck counties have MCD ratios of at least $\frac{C_\eta}{D_\eta - K\Delta}$. Since the algorithm terminated at iteration η, there was not sufficient capacity to perform the next iteration, so the optimal number of bottleneck counties is

$$\eta = \underset{1 \le \xi \le |J_0|}{\text{argmax}} \left\{ \sum_{l=1}^{\xi-1} \left(\frac{C_\xi (D_l - K\Delta)}{D_\xi - K\Delta} - C_l \right) \le Y \right\} \tag{25}$$

In order to show how equation (25) is obtained, let W_j denote the total additional capacity to be allocated to county j as a result of the Capacity Allocation Algorithm where $j \le \eta$. Then, to reach iteration $\eta > 1$, we need

$$\frac{C_j + W_j}{D_j - K\Delta} = \frac{C_\eta}{D_\eta - K\Delta} \qquad j < \eta \tag{C1}$$

$$W_j = \frac{C_\eta (D_j - K\Delta)}{D_\eta - K\Delta} - C_j \qquad j < \eta \tag{C2}$$

This implies that the total additional capacity needed to reach iteration η is:

$$\sum_{l=1}^{\eta-1} W_l = \sum_{l=1}^{\eta-1} \left(\frac{C_\eta (D_l - K\Delta)}{D_\eta - K\Delta} - C_l \right) \tag{C3}$$

and should satisfy

$$\sum_{l=1}^{\eta-1} W_l = \sum_{l=1}^{\eta-1} \left(\frac{C_\eta (D_l - K\Delta)}{D_\eta - K\Delta} - C_l \right) \le Y. \tag{C4}$$

The equation (25) follows directly. The summation in (25) is taken to be zero when $\xi = 1$, from which it follows that we stop at iteration $\eta = 1$.

The solution $\left[Q^*, \left(\rho_j^*;\ j \in J_0\right)\right]$, as given in Proposition 3 is feasible for the Capacity Allocation Model. Let $\left[\left(\pi_j, j \in J_0\right),\ \pi_0\right]$ represent the corresponding dual solution. The vectors $\left[Q^*, \left(\rho_j^*;\ j \in J_0\right)\right]$ and $\left[\left(\pi_j, j \in J_0\right),\ \pi_0\right]$ are optimal solutions for the two respective problems if and only if, by the Complementary Slackness Theorem (Bertsimas & Tsitsiklis, 1997), $\left[\left(\pi_j, j \in J_0\right),\ \pi_0\right]$ is a feasible dual solution and they satisfy the following:

$$\pi_j\left(Q^* D_j - KQ\Delta - \rho_j^* Y - C_j\right) = 0 \quad j \in J_0 \tag{C5}$$

$$\pi_0\left(\sum_{l \in J_0} \rho_l^* - 1\right) = 0 \tag{C6}$$

$$\left(\sum_{l \in J_0} (D_l - K\Delta)\pi_l - 1\right) Q = 0 \tag{C7}$$

$$\left(-Y\pi_j + \pi_0\right)\rho_j^* = 0 \quad j \in J_0 \tag{C8}$$

Since by Proposition 2, Constraint (19) is satisfied at equality in an optimal solution, we obtain no additional information from (C6).

Assume that the number of bottleneck counties obtained from (25) is η. From (C5), using the proposed optimal solution in Proposition 3, for $j \leq \eta$,

$$Q^* D_j - KQ^*\Delta - \rho_j^* Y - C_j = Q^* D_j - KQ^*\Delta - \frac{Q^*\left(D_j - K\Delta\right) - C_j}{Y} Y - C_j = 0. \tag{C9}$$

From Equations (B4) and (C5),

$$\pi_j \geq 0\ for\ 1 \leq j \leq \eta. \tag{C10}$$

For $\eta + 1 \leq j \leq n$, from Equation (C5) and using the proposed optimal solution in Proposition 3,

$$\begin{aligned} Q^* D_j - KQ^*\Delta - \rho_j^* Y - C_j &= \frac{D_j\left(Y + \sum_{g=1}^{\eta} C_g\right)}{\sum_{l=1}^{\eta}(D_l - K\Delta)} - \frac{K\Delta\left(Y + \sum_{g=1}^{\eta} C_g\right)}{\sum_{l=1}^{\eta}(D_l - K\Delta)} - C_j \\ &= \frac{D_j\left(Y + \sum_{g=1}^{\eta} C_g\right) - K\Delta\left(Y + \sum_{g=1}^{\eta} C_g\right) - C_j \sum_{l=1}^{\eta}(D_l - K\Delta)}{\sum_{l=1}^{\eta}(D_l - K\Delta)} \end{aligned} \tag{C11}$$

Based on the termination condition of the Capacity Allocation Algorithm, if $\eta < |J_0|$, then we have

$$\frac{C_{|J_0|}}{D_{|J_0|} - K\Delta} > \cdots > \frac{C_{\eta+1}}{D_{\eta+1} - K\Delta} > \frac{Y + \sum_{l=1}^{\eta} C_l}{\sum_{l=1}^{\eta}(D_l - K\Delta)} = R_\eta; \tag{C12}$$

and therefore we have,

$$\frac{C_j}{D_j - K\Delta} > \frac{Y + \sum_{l=1}^{\eta} C_l}{\sum_{l=1}^{\eta}(D_l - K\Delta)} \; for \; \eta + 1 \le j \le |J_0|. \tag{C13}$$

It follows that

$$D_j\left(Y + \sum_{l=1}^{\eta} C_l\right) - K\Delta\left(Y + \sum_{l=1}^{\eta} C_l\right) - C_j \sum_{l=1}^{\eta}(D_l - K\Delta) < 0 \; for \; \eta + 1 \le j \le |J_0|. \tag{C14}$$

Since this is the numerator of equation (C11), from (C5), it follows that

$$\pi_j = 0 \quad for \;\; \eta + 1 \le j \le |J_0|. \tag{C15}$$

If $\eta = |J_0|$, equations (C11) – (C15) are not needed.

We can assume that $Q > 0$ because it represents the minimum MCF ratio after capacity allocation. Then, from (C7), we have

$$\sum_{l \in J_0} (D_l - K\Delta)\pi_l = 1. \tag{C16}$$

Using (C15), we can rewrite (C16) as

$$\sum_{l=1}^{\eta} (D_l - K\Delta)\pi_l = 1. \tag{C17}$$

By using the proposed solution, without loss of generality, we can assume that $\rho_j \ge \varepsilon$ for $j \le \eta$, where ε is a small positive number. Then, from (C6), it follows that

$$-Y\pi_j + \pi_0 = 0 \quad for \; j \le \eta \tag{C18}$$

$$\pi_j = \frac{\pi_0}{Y} \quad for \; j \le \eta \tag{C19}$$

By inserting (C19) into (C17),

$$\sum_{l=1}^{\eta}(D_l - K\Delta)\frac{\pi_0}{Y} = \frac{\pi_0}{Y}\sum_{l=1}^{\eta}(D_l - K\Delta) = 1 \tag{C20}$$

$$\pi_0 = \frac{Y}{\sum_{l=1}^{\eta}(D_l - K\Delta)}. \tag{C21}$$

By inserting (C21) into (C19), we get

$$\pi_j = \frac{1}{\sum_{l=1}^{\eta}(D_l - K\Delta)} \quad for\ j \leq \eta. \tag{C22}$$

This result is also intuitively meaningful; π_j is the marginal benefit of increasing C_j by one unit. If we examine the structure of Q^* in Equation (26), we can see that if C_j for $j \leq \eta$ is increased by one unit, Q^*, which is the optimal objective function value, increases by $\left(\frac{1}{\sum_{l=1}^{\eta}(D_l - K\Delta)}\right)$. This solution, $[(\pi_j, j \in J_0)$, $\pi 0$, as given by equations (C14), (C20) and (C21) is also feasible for the dual problem since it satisfies constraints (B2)-(B4).

Finally, calculating the dual objective function value,

$$Z_{dual} = \sum_{l \in J_0} C_l \pi_l + \pi_0 = \frac{\sum_{l=1}^{\eta} C_l}{\sum_{l=1}^{\eta}(D_l - K\Delta)} + \frac{Y}{\sum_{l=1}^{\eta}(D_l - K\Delta)} = Q^* = Z_{primal} \tag{C22}$$

Hence, by the Strong Duality Theorem (Bertsimas and Tsitsiklis, 1997), the proposed solution is optimal.

■

APPENDIX D

Analysis of different equity measures: We examine how the solutions proposed by our Food Distribution Model perform in terms of four alternative measures of inequity discussed by Marsh and Schilling (1994). First, we make the following definitions in accordance with Marsh and Schilling (1994):

$$E_j = \left| \frac{X_j}{\sum_{l=1}^{n} X_l} - \frac{D_j}{\sum_{l=1}^{n} D_l} \right| \tag{D1}$$

$$\bar{E} = \frac{\sum_{l=1}^{n} E_l}{n} \tag{D2}$$

As described in Section 3, through equations 2.a and 2.b, the inequity measure used in the Food Distribution Model is equivalent to constraining $\max_j E_j$, which is the first inequity measure discussed by Marsh and Schilling (1994), to be below a certain limit. By doing this, we enforce that this inequity measure remains below an equity deviation limit, K. We will compare our results from the Food Distribution Model to four alternative inequity measures. The measures we will use are: 1) Variance, $\frac{\sum_{j=1}^{n} (E_j - \bar{E})^2}{n}$; 2) Average absolute deviation from $\bar{E}$, $\frac{\sum_{j=1}^{n} |E_j - \bar{E}|}{n}$; 3) The range, $\max_j E_j - \min_j E_j$; and 4) Maximum absolute deviation from $\bar{E}$, $\max_j |E_j - \bar{E}|$. We have scaled some of the measures from Marsh and Schilling (1994) to normalize the inequity measures so that they all take values between zero and one. Since these are all measures of inequity, smaller values indicate a better equity level.

In terms of the experimental design, we use the same approach as explained in the previous section for uncertainty in capacities. We will again use the equity deviation limits, $K = 0, 0.002,$ and 0.004 since we would like to select K values corresponding to capacity and equity constrained instances. We then use the obtained optimal solutions for each instance and calculate the inequity levels for each of the four measures considered. The average inequity levels from the 1000 instances for Beta2 distribution are summarized in Table D1 where the values in parentheses show the corresponding standard deviations.

The remaining distributions are not shown here since this distribution has the highest level of variance and skewness among the considered distributions and hence exhibits the widest variability.

Table D1. Analysis of different inequity measures from Marsh and Schilling (1994) for Beta2 distribution.

Mean (S.D.)		*Equity Deviation Limit, K*		
		0	**0.002**	**0.004**
Inequity Measures	$\frac{\sum_{j=1}^{n}(E_j-\bar{E})^2}{n}$	0.00000 (0.00000)	0.00000 (0.00000)	0.00000 (0.00000)
	$\frac{\sum_{j=1}^{n}\lvert E_j-\bar{E}\rvert}{n}$	0.00000 (0.00000)	0.00008 (0.00009)	0.00031 (0.00019)
	$\max_j E_j - \min_j E_j$	0.00000 (0.00000)	0.00073 (0.00075)	0.00242 (0.00133)
	$\max_j \lvert E_j-\bar{E}\rvert$	0.00000 (0.00000)	0.00068 (0.00070)	0.00225 (0.00122)

The inequity measure used in the Food Distribution Model limits all E_j values to stay below a certain limit and hence, forces a certain equity level on each county. The perfect equity case, $K = 0$ requires that $E_j = 0$ for all j. Hence, all the other measures are also equal to zero indicating that our solutions are optimal for each inequity measure considered for $K = 0$. When $K > 0$, each E_j is required to be below K, and hence $\bar{E}$ is also required to be less than K. This causes all the measures containing the $(E_j - \bar{E})$ term to achieve low levels. The measure that behaves the worst is the range, $\max_j E_j - \min_j E_j$, but that measure is also constrained to be lower than the original measure since $\min_j E_j > 0$. The results show that the measure we use in our paper is a very strong equity measure and enforces a certain level of equity at each of the locations considered. This causes the resulting policies to behave well under different commonly used equity measures.

References

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