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Abstract: *In this paper a didactical model is presented. The goal of the model is to work as a didactical tool, or conceptual frame, for developing, carrying through and evaluating interdisciplinary activities involving the subject of mathematics and philosophy in the high schools. Through the terms of Horizontal Intertwining, Vertical Structuring and Horizontal Propagation the model consists of three phases, each considering different aspects of the nature of interdisciplinary activities. The theoretical modelling is inspired by work which focuses on the students abilities to concept formation in expanded domains (Michelsen, 2001, 2005a, 2005b). Furthermore the theoretical description rest on a series of qualitative interviews with teachers from the Danish high school (grades 9-11) conducted recently. The special case of concrete interdisciplinary activities between mathematics and philosophy is also considered.*

1. Introduction

There is worldwide consensus that the society we live in today gets increasingly more and more complex. Earlier the problem was often to gather information, whereas the knowledge society of today is characterized by the fact that much information is easy accessible. The problem nowadays is therefore to survey and filter the great amount of accessible information rather than to gain access to it. Thus, the schools have to aim at producing students who are prepared to deal with such a great complexity of knowledge, that is, scientifically literate students (Gräber et al., 2001). In the educational system knowledge is still in a very large scale separated into distinct blocks by different subjects. This separation of knowledge has shown itself to be very efficient in producing and teaching new knowledge, but does not necessarily provide the students with the skills necessary to navigate through the constantly increasing amount of accessible information. Interdisciplinary activities between different subjects can help to develop a broader context of meaning or understanding for the student, and in this way contribute to the ongoing scholarly development and provide the student with the tools necessary to deal with complex problem solving waiting in the future.

In spite of the fact that many different subjects and areas often contain more and more mathematics-rich elements, mathematics, as a subject, mathematics remains quite isolated. The objective importance of mathematics from a social point of view exists side by side with its subjective irrelevance experienced by many people. Niss et al. (Niss, Jensen, Wedege, 1998) have characterized this as the *relevance-paradox* of mathematics. One reason for this could be found in the fact that mathematical knowledge is hard to transfer to new domains of knowledge by the student. Although the subject of mathematics in its very nature often is described as a tool, and therefore should be able to establish obvious connections to other contexts, such transfers of

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mathematical knowledge between different domains seldom occur (Hatano 1996, Michelsen 2001). Most of the time subjects have their own specific use of language and system of terminology and this can prevent the desirable transfer of mathematical knowledge to other contexts and domains.

The purpose of using interdisciplinary elements in the teaching of mathematics is, as concluded from the description above, 1) an attempt to broaden the students' curricular perspective and general view by removing the discrete lens that characterizes most schools' separation of knowledge into curriculums and present to the students a more real picture of the role and importance of mathematics in extra-mathematical contexts[1] 2) an attempt to help the students' abilities to transfer mathematical knowledge between different curricular domains.

To be able to work with interdisciplinary aspects in the teaching of mathematics one has to consider what connections the subject mathematics has to other subjects and areas of knowledge. In *the first International Symposium of mathematics and its Connections to the Arts and Sciences*[2] (Beckmann, Michelsen & Sriraman, 2005) such connections were discussed, and a sketch of a didactical model for interdisciplinary activities between mathematics and philosophy presented (Iversen, 2005).

Afterwards, the modeling of such activities involving mathematics has continued and the purpose of this paper is to present a didactical model, a conceptual frame for the planning, completion and evaluation of successfully interdisciplinary activities involving mathematics. The model will function as a tool to help develop activities that can facilitate a reasonable transfer of mathematical knowledge to other subjects and domains. The model is inspired by the work of Michelsen (2001, 2005a, 2005b) and is further developed through the special case of mathematics and philosophy and a section is therefore devoted to this specific topic. The section will also work as a demonstration of how the model should be understood and applied.

2. Theoretical Framework

Working with interdisciplinary activities implies a belief that there exist elements that is general and somewhat identical between the knowledge presented in different subjects. We assume that such an intersection of knowledge contains more elements the more related[3] the subjects are to one another, and at least *not-empty* (Dahland, 1998). There are different ways of trying to describe such assumed curricular intersections. In the development of the didactical model presented here a notion of *competencies* is used to identify and characterize the possible intersection of knowledge between mathematics and other subjects.

In the educational system of Denmark a huge step forward is taken with the completion of the *KOM-report* for mathematics (Niss et al., 2002). In this Niss lists eight mathematical competencies, valid for all steps of education, which is a meant to work as an overall frame for

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[1] This follows Sriraman (2004) who argued that students are used to viewing knowledge through the discrete lens of disjoint school subjects.

[2] The symposium took place 18-21 May 2005 in Schwäbish Gmünd, Germany. See Beckmann, A., Michelsen, C., & Sriraman, B (Eds.)., (2005). *Proceedings of the 1st International Symposium of Mathematics and its Connections to the Arts and Sciences.* The University of Education, Schwäbisch Gmünd, Germany, Franzbecker Verlag.

[3] Related should here be understood in a common way. The subject of mathematics is e.g. is supposed to be more *related* to physics than to English.

description of the education in mathematics in Denmark. The concept of a mathematical competence is here understood as some sort of *mathematical expertise*, and is more formally defined as *an insightful readiness to act appropriate in situations which contains a certain kind of mathematical challenges*.[4] The report has been a starting signal to similar competence descriptions of other subjects in the Danish educational system.

A description of mathematics by the means of competencies focuses more on the *purpose* of learning mathematics than to the specific curriculum. This description expresses a broader minded view on the teaching of mathematics than a normal curricular-dependent view. But Niss describes (Niss et al., 2002, p. 66) the eight mathematical competencies as strictly belonging to the sphere of mathematics thereby partly closing down the newly constructed bridge to other subject domains. Michelsen et al. (2005a) instead argues that some of the competences put forward by Niss et al (2002) are actually *interdisciplinary competences*, and mentions the modeling and representational competence as examples.

In this paper we will try to make use of the interdisciplinary potential inherent in a competence approach to mathematics on a theoretical didactical level suggested by Michelsen et al (2005a). A less bounded description of mathematical competences can then be substratum that enables an entanglement of mathematics with other subjects both on an educational theoretical level and on a practical level in the classrooms. It is here suggested that the notion of a mathematical competence should contain both a *narrow* and a *broad* dimension, by means of which such characterization of mathematical expertise in the student can both work as a description internally in mathematics and as a link to the rest of the world. As an example Niss (2002) mentions *the ability to reason mathematically* i.e. to be able to follow and judge mathematical argumentation, as one of the eight described competences. But the ability to be able to follow and judge a reasoning is far from restricted to the sphere of mathematics. It is the kind of expertise that is important to master in all the school's different subjects, and it could therefore be argued that some sort of *reasoning competence* is just as essential in physics or philosophy as it is in mathematics. Obviously arguments and reasoning often appear in different use of language and forms in different subjects, and therefore a reasoning competence is here suggested to be characterized by *the ability to follow and judge a reasoning in different curricular domains*, AND *being able to distinguish and characterize different types of arguments* thereby having the ability to go deeply into a certain subject and follow and judge a reasoning characteristic for this one subject.

Within mathematics valid arguments often have character of a proof, while arguments in other subjects, as e.g. philosophy or history, often are marked by less cogency and more contingent elements. In this context mastering the reasoning competence will be understood as the ability to distinguish different kinds of arguments but at the same time know why the different arguments work in different contexts, and to be able to dive into a specific argument, as e.g. a mathematical proof, and follow its string of reasoning.

This broad minded approach to the notion of competences should be understood as an attempt to, over time, change the educational practice which makes it possible that

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[4] My own translation from Danish (ibid.).

> *"Although critical thinking, problem solving and communication are real world skills that cut across the aforementioned disciplines students are led to believe that these skills are context dependent."(Sriraman, 2004, p.14).*

3. Interviewing high school teachers

During May 2005 a series of qualitative interviews were conducted. Six high school teachers were interviewed individually. The main purpose was to find out: *Which didactical (and practical) possibilities and obstacles exist for interdisciplinary activities between mathematics and other subjects (especially philosophy) in the Danish high school (grade 9-11)?*[5] The interviewees were teachers from different high schools in Denmark and varied both in age and seniority. They were chosen so that each one taught either mathematics or philosophy (or both) on a daily basis and moreover most of them had been engaged in relevant interdisciplinary activities. The hope was to be able to incorporate some of this real-life information into the development of the didactical model. In the following I will reproduce some of the, for this paper, relevant conclusions one can draw from the conducted interviews.[6]

Some of the interviewed teachers have conducted interdisciplinary activities between mathematics (or physics) and philosophy earlier on in their daily teaching. It has not been possible to find any writings about conducted activities between mathematics and philosophy in the Danish high school, but some of the interviewed teachers have been involved in documented activities involving physics and philosophy. Generally the experiences from these courses were positive

> *" It's easy for me to register that the students have been going through these activities (involving physics and philosophy) and other teachers can easily do so to. … They [the students] own more academically concepts than students usually have. They are really good at thinking different subjects together, and they also get very good at working together in little groups … I think they simply have a greater cultural and historical horizon." - Teacher 1*

The purpose of these activities involving physics and philosophy was primarily to strengthen the subject of physics. To embody the abstractness of physics as one of the interviewed teachers told me. This goal was in some sense achieved according to the teacher quoted above and the reports of evaluation carried out by the involved students and teachers afterwards. Besides the registered positive cognitive effects the students realized that physics can not be reduced to a mere collection of dead facts. Physics is a human activity that evolves and therefore argumentation actually do count. This shift in the students' perception of the subject physics from being a dusty collection of facts, to being relevant, is an experience that another of the involved teachers believe can be re-produced in the case of mathematics.

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[5] The fact that some of the asked questions particularly involved a reference to the Danish high schools(as opposed to any high schools) was because I wanted to find out which effect a forthcoming reform of the Danish high schools would have on the daily teaching practice. Most of questions asked involved only general educational components, and did not hold any particular reference to any Danish conditions.

[6] All the interviews were conducted in Danish, and the quotes given in the text is therefore my own translation. The text in the brackets is my insertions. They are there to give the right coherence in the teachers statements. The interviewed teachers are here given only a number, but all the quotes given in this paper are approved by the particularly teacher concerned.

> *"We can re-create the part about discovering in the case of mathematics … For example the students often only see the end-product when they see a proof for some mathematical relation. For them it's often a strange thing; How have "they" found out you are supposed to do like that? They ask themselves. The process from a proof starts to crystallize and right to the final version of the proof which needs polishing before it appears in a textbook, nice and rounded. That whole process one should try in teaching mathematics, I believe it would be very beneficial for the students." -Teacher 2*

Besides using philosophy as a tool to illustrate the world and methods of physics the teachers involved report how at the same time the activities created the perfect interdisciplinary context for developing central concepts from the philosophy of science. Ideas such as: *induction*, *empirically investigations* and *verification* were easy for the students to acquire and work with in this expanded domain. In this way the activities held the possibility that both involved subjects could engage in the work of developing the students' scientific literacy, but at the same time use the cross-curricular context to discover and develop relevant aspects specific to the different curriculums.

Others of the interviewed teachers had themselves planned and conducted interdisciplinary activities involving mathematics and philosophy. In both cases the activities had been carried out in relation to the daily teaching of mathematics, and both set of activities centered about *argumentation and proof in mathematics*. The purpose of the different activities varied slightly but fundamentally they both tried to illustrate characteristics of mathematical argumentation and how this often is worked out.

> *"When we speak about method, we did something about; When do you examine something and when do you actually construct a proof? And also, what is needed to construct a proof and what is the nature of a mathematical proof? These issues are very philosophical I think, and the activities were a great success for the students."- Teacher 3*

> *"We worked with paradoxes and reasoning and things like that … The overall theme was argumentation. It was a very good course, and the students were very fond of it." - Teacher 4*

The work with these topics in mathematics was carried through based on a wish to equip the students with some general tools, or concepts, which could function as some sort of cognitive scheme for their ongoing daily struggle for learning mathematics.

> *"A part of the teaching is about giving them [the students] a set of concepts which they can use to relate to what the are doing concretely. When they engage in a specific task in mathematics, they now have some concepts, some work habits,*

> *some patterns, some ways of thinking which they can use to throw light on what they are actually doing." - Teacher 3*

Mathematics propagates through a large and branching taxonomy of concepts and ideas. Several of the interviewed teachers pointed out that, cross-curricular activities between mathematics and a subject as philosophy should deal with concepts placed fairly high in the mathematical taxonomy used in the high school. To illustrate this point we can consider the relative position of two mathematical concepts in the taxonomy. Look for example at say the concept of *function* and a specific function as $f(x)=sin(x)$. Both entities can be considered as a concept that a student in the high school should become acquainted with at some point. The concept of function however will be placed highest of the two in a taxonomy of mathematical concepts, and we will therefore regard this as a *meta-concept* in comparison with $f(x)=sin(x)$. This way there also exists meta-concepts in comparison with the concept of function. The concept of *functional* is an example of a such, and the use of the name meta-concept will therefore always be relative.

For high school students the concept of *proof* will be regarded as a meta-concept most of the time and a direct investigation of this in the classroom by the students will often involve several problems. According to Dreyfuss (1999) most of the students on this educational level has a very restricted knowledge about what constitutes a mathematical proof. Also Hazzan and Zazkis (2005) point to the importance of trying to help the students acquire relevant mathematical meta-concepts as e.g. the proof.

According to Niss (1999) a major finding of research in mathematics education is students' alienation from proof and proving. Students' conceptions of the mathematical proof and those held by the mathematical community is separated by a huge gap. Niss concludes that

> *"Typically, at any level of mathematics education in which proof or proving are on the agenda, students experience great problems in understanding what a proof is (and is not) supposed to be, and what its purposes and functions are, as they have substantial problems in proving statements themselves, except in highly standardized situations." (Niss , 1999, p. 18).*

Instead the students' consider proofs and proving as strange rituals performed by professional mathematicians that are not really meant to be understood by ordinary human beings. The activities referred to above by the interviewed teachers are exactly concerned with these problems and shows how other subjects such as philosophy can be used in the struggles.

The interviewed teachers generally believed that interdisciplinary activities involving mathematics were very relevant for the students. Focusing on the special case of mathematics and philosophy some of teachers suggested that relevant activities could take as a starting point the purpose of illuminating the structure of mathematics, its fields of study and its characteristic form of argumentation. It comes as no surprise that the examples mentioned here are of a very general character. Engaging in interdisciplinary activities should hold the possibility of gaining something for all the involved subjects, and this would indeed be a very difficult premise to fulfill for *both* mathematics and philosophy if the activities centered about the quadratic equation and Socrates' famous Defence. Both are examples of a far to narrow approach to interdisciplinary activities determined too much by curricular considerations.

In spite of a general optimism shared by the interviewed teachers towards integrating the teaching of mathematics with other subjects, several of them also point to a number of difficulties with the subject of mathematics that must be overcome if the interdisciplinary activities should be rewarding.

The subject of mathematics is regarded as a subject that holds great technical difficulties for the students. According to the interviewed teachers exciting problems and topics in mathematics often demands a severe amount of preparation from the students before they can engage with the activities thereby losing the immediate interest that is so important for the learning process (Mitchell, 1993). Other subjects, e.g. philosophy, is for most students easier to engage in and this often leads to a shift in the students attention away from the mathematical content of the chosen topic. For that reason the development of successful interdisciplinary activities involving mathematics needs the development of a working culture among teachers and students where it is respected that a subject as mathematics can be hard accessible and show this problem extra attention in the classroom.

Most of the interviewed teachers highlighted the fact, that in many cases interdisciplinary activities end up bringing in the mathematics teacher to simply help the students read of some values on a prefabricated curve or similar. Here the actual mathematical content is far from challenging or relevant for the students (or the teacher). To avoid this situation one of the interviewed teachers point out that

> *"There's an interaction between the other subject [than mathematics],, the way it asks its questions and the areas of mathematics you can point out and work with. Sometimes mathematics and the other subject actually pose the same kinds of questions but they each give different kinds of answers. … The problems that the activities are meant to center on must have double-relevance, and that means that they should have relevance both in the reality to which they belong and also in mathematics. As a thought I think that is very correct because often they [the other teachers] say; Yes, this topic is really interesting could the mathematics teacher please come in here and help reading of the curve! I answer: No, no that's not really interdisciplinary activities." - Teacher 4*

The subject domains involved in the activities must in some sense meet and use each other properly. Subjects are not actually co-operating when the co-operation is reduced to a parasitic process where one of the subjects *de facto* is not gaining anything as described in the above quote.

4. Modeling interdisciplinary activities involving mathematics and philosophy

The purpose of developing a didactical model for interdisciplinary activities involving mathematics and philosophy is, as mentioned earlier on, multiple. The model should function as a link between educational theory and the daily teaching practice in mathematics, both in the development of new activities, the carrying through of already planned ones and the evaluation of completed activities. The model gets inspiration from the work of Michelsen (2001, 2005a, 2005b), and a former version was presented at MACAS 1 and described in Iversen (2005). The didactical model consists of three phases - *the horizontal intertwining*, *the vertical structuring* and *the horizontal propagation*. Freudenthal (1991) introduced the idea of two different types of mathematization in an educational context – horizontal and vertical mathematization. In the horizontal mathematization students develop mathematical tools that help them organize and work with mathematical problems situated in real-life situations. The process of reorganizing the mathematical system itself Freudenthal designates vertical mathematization. Also Harel & Kaput (1991) sees a distinction between horizontal and vertical growth of mathematical knowledge. They associate the term horizontal growth with the translation of mathematical ideas between extra-mathematical situations (and models of these) and across other representation systems. By vertical growth is understood the construction of new mathematical conceptual systems.

5. The Horizontal Intertwining

As mentioned by some of the interviewed teachers interdisciplinary activities involving mathematics very often end up as fictitious constructs without much relevant mathematical content. In the first phase of a cross-curricular collaboration the attention should be centered on the importance of obtaining a real intertwining of the involved subjects. Such a curricular intertwining involves considerations about which fields of study, problems and methods in mathematics and the other subjects involved that have potentiality as interdisciplinary elements. Such elements must not originate from oversimplified lingual similarities among the subjects, but instead from considerations about how these elements can be used later in the continued learning of e.g. mathematics. This kind of intertwining of the subjects' core subject matter the students will often experience as "the meeting of different subjects", and the term of *horizontal* refers therefore to the students pre-understanding of the chosen curricular element as belonging to both mathematics and another involved subject, but not necessarily as an subject-exceeding element. Often the students do not consider ideas to be related because of their logically connection, but because they are being used together in the same kind of problem solving situations (Lesh & Doerr, 2003; Lesh & Sriraman, 2005). Michelsen et al. (2005a) suggest the term horizontal linking to describe the process of identifying contexts across mathematics and other subjects of science that are suitable for integrated modeling courses. I will here suggest the notion of *horizontal intertwining* to describe a related process of identifying and characterizing interdisciplinary problems and context suitable for integrating the subjects of mathematics and philosophy, thereby emphasizing the broader scope the integration of mathematics with a subject not from the natural sciences demands.

The interdisciplinary activities should be chosen so they set up non-routine problems, which in order to be solved properly, need the involvement of all the involved subjects. A competence approach to the subject of mathematics contains a possibility to identify such relevant subject-exceeding elements, because this approach focuses on what the students master after going through the courses, and not on concrete curricula. As argued in the theoretical section of this paper such an approach demands a broadminded view on the notion of competencies to be able to work as an educational tool.

A horizontal intertwining of the subjects designates a weaving together of the involved subjects' core subject matter by identifying non-routine problems and contexts suitable for integrating mathematics and philosophy. In order to be able to do this one needs to clarify what constitutes such core subject matters. Furthermore, such a weaving together of subjects demands a clarification of the overall purpose of the activities. The purpose must have relevance for both mathematics and the other subjects involved in order to be justified. In practice it can span a wide field of areas; from helping the cognitive growth of the individual student (e.g. in relation to concept formation), trying to strengthen the motivation for the involved subjects or even trying to create a unified view of knowledge and science in the students.

6. The Vertical Structuring

A reasonable intertwining of the involved subjects facilitates the possibility that the student can identify with the cross-curricular aspects of the chosen problems, and thereby engage meaningfully in the activities. A clarification of the overall purpose with the activities will from the beginning help the teacher to follow the students' cognitive development along the activities. Such observations will often involve that the mathematics teacher abandons the usual authoritarian role and take on a more *guide-like* function instead.[7] From a combination of the involved subjects' core subject matter the student should under suitable guidance and activity go through a cognitive development – a so called vertical structuring - that will root the cross-curricular phenomenon concerned conceptually. It is crucial for a successful interdisciplinary engagement that the involved phenomena are central for the further learning of mathematics. If the purpose of the activities is the formation of new mathematical concepts the vertical structuring could be described as the construction of a new mathematical *concept image* (in the sense of Tall and Vinner, 1981). More theories describe how the formation of a new concept image in the student involves a qualitative change in the students perception of the specific concept. The change of perception is registered as a cognitive shift between perceiving the mathematical concept as an activity (or a process) and viewing the concept as an entity in itself i.e. a kind of structure or object (Dubinsky 1991, Sfard 1991, Tall 1997, 2001).

In activities where the over-all purpose is to equip the students with a greater curricular perspective and overview we can describe the vertical structuring as the cognitive development of a new cross-curricular platform in the student, whereto new knowledge later can be attached to and grow from.

7. The Horizontal Propagation

A successful vertical structuring should be evaluated in a greater perspective. The development of new significant concepts and connections based on interdisciplinary elements should be further developed in the different curricular domains of mathematics and philosophy. According to Lesh & Doerr (2003) the real challenge of the teacher is not only to introduce new ideas and concepts but also to create situations where the students need to express their current ways of thinking so this can be further tested and revised in directions of stronger development. In the case of mathematics the student should be allowed to use the newly learned knowledge in different mathematical activities and thereby apply, test and approve the specific mathematical concepts in question for the purpose of developing a more firm and generalized mathematical structure in the end. This is only possible if the original purpose with the activities is aimed at such a propagation of the new knowledge in other contexts. In other words the vertical structuring should be followed up by a horizontal propagation of the newly acquired structures in

[7] For a more developed description of this shift in the teachers role in the classroom, see e.g. Gravemeijer (1997).

the students and thereby this knowledge can find its use in both mathematics and other involved subjects.

In this way the cross-curricular elements can work as a new basis, or context, for the student which can use it in the continued learning of mathematics furthermore in the development of new interdisciplinary connections between subjects thereby being able to overcome the crucial problems of *transfer* mentioned in the theoretical section of this paper. This is the true gain of such interdisciplinary activities.

After the carrying through of a longer cross-curricular course one of the interviewed teachers describes an example of what could be characterized as a horizontal propagation as follows

> *"I see the acquired competencies applied in many different places. They [the students] simply travel faster over the learning-ground. One can say that they fundamentally have a greater prerequisite for both conceptual entities and in working contexts." - Teacher 1*

8. Designing relevant activities involving mathematics and philosophy

After sketching the different components that make up the didactical model it should be illustrated how it can be used in the development of relevant interdisciplinary activities between mathematics and philosophy. Here we consider the special case of proof and proving in mathematics and philosophy.

First we need to identify relevant non-routine problems, topics or phenomena which can function as curricular-exceeding elements between the two subjects and thereby establish a reasonable horizontal intertwining. We can use a competence approach to the curriculums of mathematics and philosophy respectively, hereby focusing on what cognitive qualities the two subjects aim at developing in the students. Common to the two subjects is a (seemingly endless) search for logically healthy arguments and conclusions and the ability to follow and judge such kind of reasoning therefore belongs to the core subject matter in both mathematics and philosophy. In planning the activities we can therefore reasonably focus on developing some sort of *reasoning competence* as mentioned earlier. This involves an ability to compare and differentiate the different kinds of argumentation used by the two subjects, but also the ability to dive into specific arguments from each subject and be able to follow and judge such specific reasoning.

In all of the school's different subjects the students' ability to argue clearly and reason reasonably plays an important role, and a development of this capacity is a key area in both mathematics and philosophy. Philosophy is in fact often characterized as a subject that tries to generate and develop the students' ability to understand and use forms of argumentation and knowledge that cut across the school's different disciplines and dimensions.

Mathematical reasoning takes many forms but is in its clearest form crystallized as actual proofs. The power to give a definite proof for a certain conjecture is characteristic for the subject of mathematics and the students' knowledge about the meta-concept of *proof* is, as argued earlier in this paper, therefore central in the teaching activities in the high school. In philosophy the idea of proof also plays a key role. Earlier on, philosophers tried to transfer the mathematical (in some sense Euclidean) idea of proof to actual philosophical arguments. The most famous philosophical "proofs" are the proofs of the existence of God. These were put forward by e.g. Anselm of Canterbury and Thomas Aquinas, who both believed that giving a formal proof of the existence of God was actually possible. The high school teachers who took part in the interviews also highlighted argumentation and the concept of proof as phenomena that could transcend the gap between the subjects of mathematics and philosophy and thereby overcome the problem of transferring mathematical knowledge to other contexts and domains.

To sum up we have, starting from a wish to advance the students' ability to argue and reason within mathematics and philosophy identified *the concept of proof* as a concrete topic suitable for a curricular intertwining of the two involved subjects.

The activities originate from a study of which role the actual proving of statements and conjectures holds within the two subjects. What constitutes a proof? At what point can we say we actually have proven something? And what kind of knowledge does a proof give us? Is it true? Is it unchangeable? [8] In practice one could use simple proofs, easy for the students to master mathematically, such as small proofs from the classical *Elements* by Euclid himself (Euclid, 2002). E.g. using the proof that *the sum of the angles in a (Euclidean) triangle* is equal to the sum of two right angles or the proof of *the Pythagorean theorem*. Then comparing these to actual proofs of philosophical character e.g. a modern version of *Anselms Ontological proof* of the existence of God. It is important that the students subsequently are placed in different situations where they themselves are forced to work out small proofs thereby experiencing the process of trying to argue for a conjecture. This will enable the students to apply, test and further develop their understanding of the concept of proof. An understanding that (hopefully) in time will evolve further and be a useful tool for the students.

Through an experimenting approach, as described above, to the idea of proof a vertical structuring of the meta-concept *proof* should be developed. At the same time focus is on the students' ability to separate different kinds of argumentation. Most of the interviewed teachers agree that this would be of significant importance in the students' continued engagement with both mathematics and philosophy.

A vertical structuring of the concept of proof subsequently work as a structure which must be applied, re-valued and tested further in the daily teaching practice that follows within both subjects. Hereby obtaining a horizontal propagation of the newly acquired knowledge which results in a greater basis or context for the further learning and understanding of both mathematics and philosophy.

The Danish Ministry of Education has recently published an Education Manual for the high schools. The manual focuses on interdisciplinary activities and a large part is devoted to paradigmatic examples of concrete activities. In this manual I've contributed to more fully describe activities between mathematics and philosophy as the one sketched above.[9]

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[8] All questions Niss (1999) emphasized as extremely difficult for students to answer properly.
[9] The manual can be found at http:us.uvm.dk/gymnasie/vejl/?menuid=15 (unfortunately only in Danish).

9. Conclusion

In the paper a didactical model which should function as a concept frame for the development, completion and evaluation of interdisciplinary activities involving mathematics and philosophy, was presented. The model consists of three phases that these activities involve; *The horizontal intertwining*, *the vertical structuring* and *the horizontal propagation*. Although the model is presented as linear, the process of going through the different phases is in some sense to be understood as an iterative process that can be run through several times by each student.

In the description of the first phase it was argued that it is of great importance that the actual mathematical content in interdisciplinary activities is not reduced to simple instrumental activities. Instead one should seek to identify and characterize interdisciplinary phenomena and contexts which can facilitate a proper intertwining of the different subjects involved by setting up relevant non-routine problems which need the involvement of both mathematics and philosophy to be answered. This can be enabled by a competence-approach as to what constitute mathematical skills. Such an approach is broader than the usual curriculum-approach to mathematics which often works as a drag to the development of successful interdisciplinary activities.

The model's second phase describes how the students' engagement in the planned activities should facilitate a vertical structuring which leads to the development of new conceptual systems, objects or contexts in the student. This can appear as a formation of new mathematical concept images, by which the interdisciplinary phenomenon considered, conceptually is anchored. This can work as a further basis in the students' continued learning of both mathematics and philosophy.

Finally the third phase focuses on how ongoing activities involving the newly acquired constructions are the overall purpose with all interdisciplinary activities. Furthermore it is argued that the cross-curricular phenomenon should be applicable in the daily teaching practice through a horizontal propagation of the considered phenomenon in both mathematics and the other subjects involved.

An anchoring of the model in the daily teaching practice was sought through a series of qualitative interviews of Danish high school teachers. Furthermore the model was illustrated through a design of a concrete interdisciplinary activity between mathematics and philosophy, and it was thereby argued how the model can be used to develop concrete interdisciplinary activities between these two subjects. The sketched activities take the concept of proofs and proving as a starting point and centers themselves around argumentation and reasoning in both mathematics and philosophy.

As the modeling of such activities is still (and perhaps always) a work-in-progress the presented model is somewhat tentative in its nature. The model originates from a wish to develop a concept frame for interdisciplinary activities between mathematics and philosophy, and found inspiration in the work of Michelsen (2001, 2005a, 2005b) which centers about interdisciplinary activities between mathematics and physics. A further perspective is to continue the work of developing concrete teaching activities, as well as trying to adapt and evaluate the model's strengths and weaknesses as a didactical tool to integrating the subjects of mathematics and philosophy. The author, therefore, invites all interested readers to further test and revise the model as well as concrete realizations and afterwards sharing experiences which hopefully will lead to the improvement of the didactical model as a result.

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