Modeling of Age Specific Fertility Rates of Jakarta in Indonesia: A Polynomial Model Approach

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Abstract—The purpose of this study is to build mathematical models to age specific fertility rates (ASFRs) and forward cumulative ASFRs for Jakarta, Indonesia. For this, the secondary data of ASFRs have been taken from Muhidin (2005). It is observed that ASFRs and forward cumulative ASFRs follow polynomial models. To examine whether they are valid or not, model validation technique, cross-validity prediction power (CVPP) and F-test are applied to those mathematical models.

Keywords— Age Specific Fertility Rates (ASFRs), Polynomial Model , Cross- Validity Prediction Power (CVPP), t-test, F-test.

1 INTRODUCTION

In Asian region, mathematical modeling in Population Science especially in Demography has been used in a very limited scale. Mathematical model, in modern era, is a very sophisticated mechanism to express data in mathematical formulation. To investigate the relationships among various demographic phenomena, mathematical models are very helpful not only for demographers but also for all social science researchers in understanding the process for distinguishing among various important and unimportant variables. Finally, model is very essential tools for not only population projections but also for population estimations. Indeed, model is essentially an effort to find out structural relationships and their dynamic behaviors among the various components or elements in demographic processes. Traditionally, one can figure out some graphs of the demographic parameters. But, in context of Demography, very few of us know what types of mathematical function are more apt for the parameters.

It was showed that age specific fertility rates (ASFRs) follow slightly modified biquadratic polynomial model where as forward and backward cumulative ASFRs follow quadratic and cubic polynomial model, respectively in the rural area of Bangladesh [1]. It was also reported that ASFRs follow 3rd degree polynomial model where as forward cumulative ASFRs follow quadratic polynomial model, respectively for Bangladesh [2].

Therefore, an effort has been made here to find what types of models are more appropriate for the case of Jakarta, Indonesia. Thus, the main objectives of this study are as follows:

i) to build up mathematical models to ASFRs and forward cumulative ASFRs of Jakarta in Indonesia and

ii) to employ cross-validity prediction power (CVPP) and F-test to these models to check how much these models are valid.

2 SOURCES OF DATA

In the present study, to fulfill the above objectives, the secondary data of ASFRs for Jakarta, Indonesia taken from [3] have been used as raw materials that is presented in Table 1.

3 METHODOLOGY

In this portion, polynomial model is briefly discussed to understand for the convenience of the readers in the following:

3.1 Polynomial

A mathematical expression of the form

$$y = f(x) = a_0 + a_1 x + a_2 x^2 + a_3 x^3 + \ldots + a_n x^n$$

$(a_n \neq 0)$ [4]

where a_i is the coefficient of x^i (i =1, 2, 3, ..., n) but $a_1, a_2, \ldots, a_n$ are also constants, a_0 is the constant term and n is the positive integer,

is called a polynomial of degree n and the symbol x, in this case, is called an indeterminate. If n=0 then it becomes constant function. If n=1 then it is called polynomial of degree 1 i.e. simple linear function. If n=2 then it is called polynomial of degree 2 i.e. quadratic polynomial, etc. [5].

3.2 Model Fitting

Using the scattered plot of ASFRs for Jakarta, Indonesia by ages in years (Fig. 1 and Fig. 2), it appears that ASFRs can be fitted by polynomial model with respect to ages. Therefore, an nth degree polynomial model is considered and the form of the model is

$$y = a_0 + \sum_{i=1}^{n} a_i \, x^i + u \quad [6]$$

where, x is age group in years; y is ASFRs; a_0 is the constant; a_i is the coefficient of x^i (i =1, 2, 3, ..., n) and u is the stochastic error term of the model. Here, a suitable n is chosen so that the error sum of square is minimum.

Using the dotted plot of forward cumulative ASFRs for Jakarta, Indonesia by ages (Fig. 3 and Fig. 4), it seems that forward cumulative ASFRs follows an nth degree polynomial model and the form of the model is

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$$y = a_0 + \sum_{i=1}^{n} a_i \; x^i + u$$

where, x is age group in years; y is forward cumulative ASFRs; a_0 is the constant; a_i is the coefficient of x^i (i =1, 2, 3, ..., n) and u is the error term of the model. In this case, a suitable n is selected so that the error sum of square is minimum.

It is to be mentioned here that these models are fitted using the software STATISTICA.

3.3 Model Validation

To verify how much these models are stable, the cross validity prediction power (CVPP), ρ^2_{cv}, is applied. Here

$\rho^2_{cv} = 1 - \dfrac{(n-1)(n-2)(n+1)}{n(n-k-1)(n-k-2)}(1-R^2)$; where, n is the number of classes, k is the number of regressors in the model and the cross-validated R is the correlation between observed and predicted values of the dependent variable [7]. The shrinkage of the model is $\left| \rho^2_{cv} - R^2 \right|$; where ρ^2_{cv} is cross validity prediction power & R^2 is the coefficient of determination of the model. Moreover, the stability of R^2 of the model is equal to 1-shrinkage. The estimated CVPP, ρ^2_{cv}, corresponding to their R^2 and information of model fitting are shown in Table 2.

3.4 F-test

The F-test is used to the model to verify the overall measure of the significance of the model as well as the significance of R^2. The formula for F-test is given by

$$F = \frac{R^2 / (m-1)}{(1-R^2)/(n-m)}$$ with (m-1, n-m) degrees of freedom (d.f.);

where m is the number of parameters of the fitted model, n is the number of cases and R^2 is the coefficient of determination in the model [8].

3.5 Velocity curve

To draw the velocity and elasticity curves we fit the polynomial regression model. The velocity curve is just the first derivative of the fitted polynomial regression with respect to age [9] , [10] , [11].

Now, the velocity curve is the first derivative of the fitted polynomial and so we obtain

$$\frac{dy}{dx} = f'(x) = a_1 + 2a_2 x + 3a_3 x^2 + \cdots + n a_n x^{n-1}.$$

Actually, velocity is the rate of change of y with respect to x.

3.6 Elasticity curve

The elasticity is estimated using the formula mentioned by [12] [13] and [14] as

$$\varepsilon = \frac{d \log y}{d \log x} = \frac{x\, dy}{y\, dx} = \frac{x}{y} f'(x)$$

3.7 Area of ASFRs

If f(x) is a bounded (i.e., there exists a positive number k such that |f(x)|≤k for all values of x) single-valued continuous function defined in the interval (a, b), a and b being both finite quantities and b>a; then the area is defined as the definite integral of f(x) with respect to x within the limits a and b, is expressed by the following way:

$$Area = \int_a^b f(x)\, dx$$ [15]

Where a is lower limit and b is upper limit. In this case f(x) is the fitted model of ASFRs or forward cumulative ASFRs and a=15 years, b=49 years, that is, range of integration is 15 to 49 years. In this section, *f(x)* is termed as integrand. It is to be noted that area has been estimated using the software Maple 9.5.

4 NUMERICAL RESULTS AND DISCUSSION

The polynomial model is assumed for ASFRs of Jakarta, Indonesia in 1994 and the fitted model is:

$y = (-1.00032)+(0.101537)x+(-0.00296)x^2+(0.000027)x^3$ (i)

t-stat (-7.08343) (7.028244) (-6.36297) (5.628602)

with coefficient of determination R^2 = 0.97793 and ρ^2_{cv} =0.873886. In this case, it is known cubic polynomial as n is 3.

The another polynomial model is assumed for ASFRs for Jakarta, Indonesia in 1990 and the fitted model is:

$y = (-1.13506)+(0.115071)x+(-0.00335)x^2+(0.00003)x^3$ (ii)

t-stat (-7.24857) (7.19076) (-6.51223) (5.76155)

with coefficient of determination R^2 is 0.97884 and ρ^2_{cv} =0.879086. In this case, it is known as cubic polynomial since n is 3.

And, the polynomial model is assumed for forward cumulative ASFRs for Jakarta, Indonesia in 1994 and the fitted model is

$y=(-0.59651)+(0.041783)x+(-0.00044)\, x^2$ (iii)

t-stat (-7.95034) (8.504611) (-5.86797)

giving proportion of variance explained (R^2) = 0.99119 and ρ^2_{cv} is 0.984897. Here n is 2, so, it is known as quadratic polynomial.

Another polynomial model is considered for forward cumulative ASFRs for Jakarta, Indonesia in 1990 and the fitted model is

$y=(-0.67207)+(0.047071)x+(-0.00049)\, x^2$ (iv)

t-stat (-7.96027) (8.514592) (-5.87298)

giving proportion of variance explained (R^2) = 0.99122 and ρ^2_{cv} is 0.974914. Here n is 2, so, it is known as quadratic polynomial.

It should be noted here that usual models i. e. Gompertz, Makeham, log-linear, semi-loglinear and logistic were also applied but seemed to be worse fitted in terms of their

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shrinkages. Therefore, the findings of those models were not shown here.

The estimated CVPP, ρ^2_{cv}, corresponding to their R^2 is shown in Table 2. From this table it is seen that all the fitted models (i) - (iv) are highly cross- validated and their shrinkages are 0.104044, 0.099754, 0.006293 and 0.016306, respectively. These fitted models (i)-(iv) will be stable more than 87%, 87.9%, 98.48% and 97%, respectively. Moreover, from this table, it is shown that the parameters of the fitted models (i) - (iv) are highly statistically significant with more than 97%, 97%, 99% and 99% of variance explained respectively. The stability of R^2 of these models are more than 89%, 90%, 99% and 99%, respectively.

The calculated values of F-test for the models (i) - (ii) are 44.31 with (3, 3) d.f. and 46.26 with (3, 3) d.f., respectively where as the corresponding tabulated values for both cases is 29.5 at 1% level of significance. The calculated values of F-test for the models (iii) - (iv) are 225.015 with (2, 4) d.f. and 225.79 with (2, 4) d.f., respectively where as the corresponding tabulated values for both cases is only 18.00 at 1% level of significance. Therefore, from these statistics it is seen that these models and their corresponding R^2 are highly statistically significant. Hence, these models are fit well. Predicted and residual values of these fitted models are also presented in Table 1.

The velocity and elasticity curve only for ASFRs in 1994 have been estimated and shown in Fig. 5 and Fig. 6 respectively.

Thereafter, proper definite integral is also employed to these fitted models to find out the area bounded by these fitted curves. The areas of these fitted curves are 1.86, 2.28, 8.42 and 9.70 respectively.

TABLE 1
OBSERVED, PREDICTED AND RESIDUAL VALUES OF ASFRS AND CUMULATIVE ASFRS FOR JAKARTA, INDONESIA

Age Group a – a+5	Age Specific Fertility Rates (ASFRs)					
	Observed 1994	Predicted 1994	Residual 1994	Observed 1990	Predicted 1990	Residual 1990
15-19	0.0152	0.0128	0.0024	0.0171	0.0141	0.0030
20-24	0.0808	0.0896	-0.0090	0.0911	0.1015	-0.0100
25-29	0.1196	0.1089	0.0107	0.1348	0.1237	0.0111
30-34	0.0910	0.0936	-0.0030	0.1025	0.1033	-8E-04
35-39	0.0510	0.0551	-0.0040	0.0575	0.0630	-0.0060
40-44	0.0252	0.0224	0.0028	0.0285	0.0257	0.0028
45-49	0.0121	0.0126	-5E-04	0.0137	0.0140	-3E-04
Age Group a – a+5	**Cumulative Age Specific Fertility Rates (ASFRs)**					
	Observed 1994	**Predicted 1994**	**Residual 1994**	**Observed 1990**	**Predicted 1990**	**Residual 1990**
15-19	0.0152	0.0002	0.0150	0.0171	0.000209	0.016891
20-24	0.0960	0.1213	-0.0250	0.1082	0.136646	-0.02845
25-29	0.2156	0.2204	-0.0050	0.2430	0.248353	-0.00535
30-34	0.3066	0.2908	0.0158	0.3455	0.327745	0.017755

TABLE 2
INFORMATION ON MODEL FITTINGS AND ESTIMATED CVPP OF THE PREDICTED EQUATIONS OF ASFRS AND ITS FORWARD CUMULATIVE DISTRIBUTION FOR JAKARTA, INDONESIA

Models	n	k	R^2	ρ^2_{cv}	Parameters	Significant Probability (p)
Model 1	7	3	97.793	0.873886		
					a_0	0.00579
					a_1	0.005918
					a_2	0.00486
					a_3	0.011092
Model 2	7	3	97.884	0.879086		
					a_0	0.00542
					a_1	0.005543
					a_2	0.00736
					a_3	0.010391
Model 3	7	2	99.119	0.984897		
					a_0	0.00136
					a_1	0.001048
					a_2	0.00421
Model 4	7	2	99.122	0.974914		
					a_0	0.00135
					a_1	0.001044
					a_2	0.00420

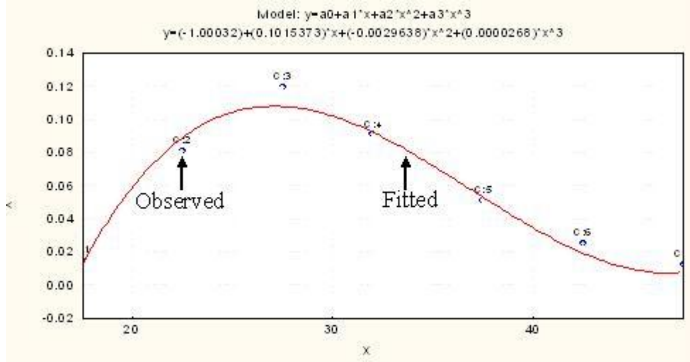

Fig. 1. Observed and Fitted ASFRs in 1994 for Jakarta, Indonesia . X: Age Group and Y: ASFRs.

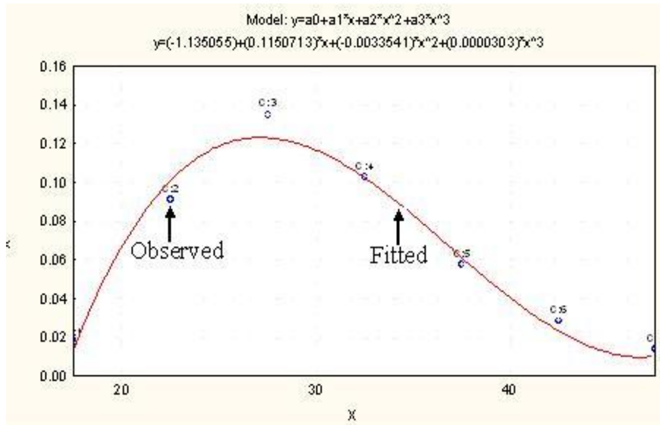

Fig. 2. Observed and Fitted ASFRs in 1990 for Jakarta, Indonesia . X: Age Group and Y: ASFRs.

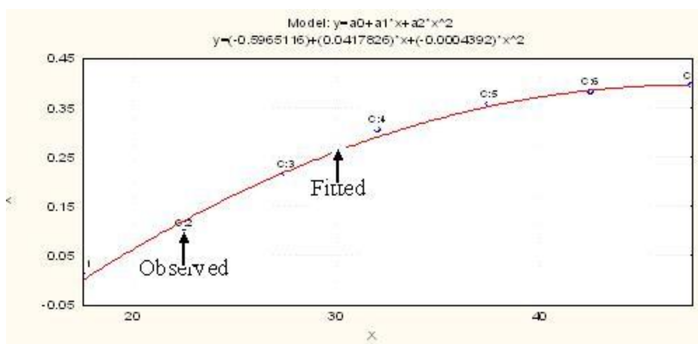

Fig. 3. Observed and Fitted Forward Cumulative ASFRs in 1994 for Jakarta, Indonesia . X: Age Group and Y: Forward Cumulative ASFRs.

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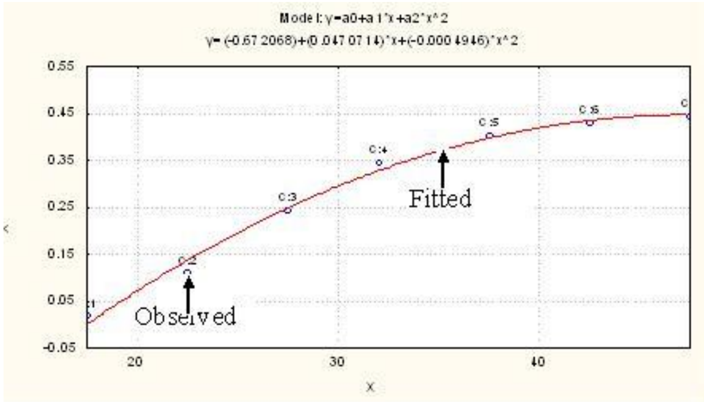

Fig. 4. Observed and Fitted Forward Cumulative ASFRs in 1990 for Jakarta, Indonesia . X: Age Group and Y: Forward Cumulative ASFRs.

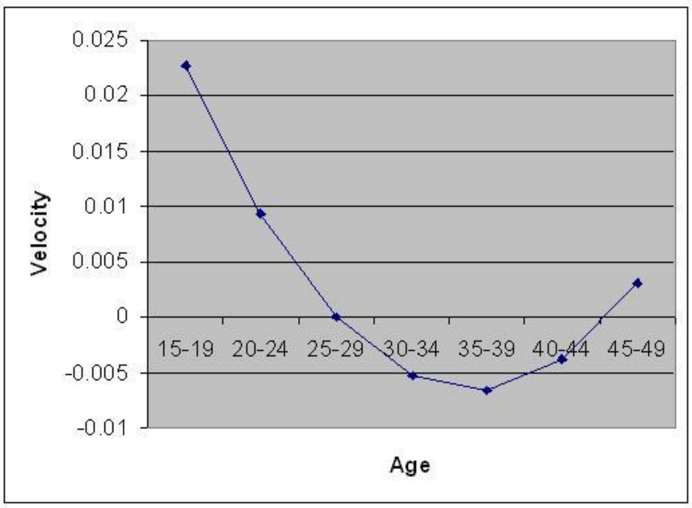

Fig. 5. Velocity Curve of ASFRs in 1994 for Jakarta, Indonesia . X: Age Group and Y: Velocity

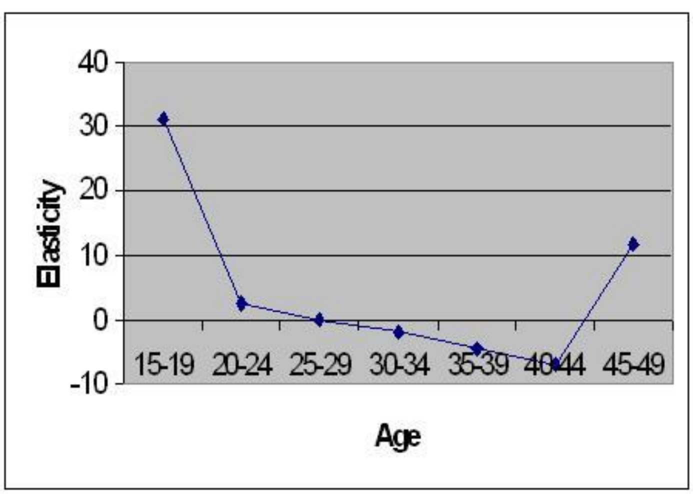

Fig. 6. Elasticity Curve of ASFRs in 1994 for Jakarta, Indonesia . X: Age Group and Y: Elasticity

5 CONCLUSION

Mathematical models of ASFRs for Jakarta, Indonesia and forward cumulative ASFRs are fitted. It is observed that ASFRs follows the 3rd degree polynomial model. On the other hand, forward cumulative ASFRs follows 2nd degree polynomial model. Hope that area might be an alternative approximate measure of fertility.

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