MODELING OF MULTILAYERED STRUCTURES INCLUDING POROUS MATERIALS USING HIERARCHICAL ELEMENTS AND NON COINCIDENT MESHES

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1. INTRODUCTION

The finite element modeling of 3D poroelastic materials often leads to an important number of unknowns. The use of a mixed displacement-pressure {u,P} formulation [1] allows for an accurate description of porous media using only 4 degrees of freedom per node. Yet, 3D linear poroelastic elements based on this formulation have a slow convergence rate and are subject to numerical locking. An important refinement of the mesh is hence needed to get satisfactory results. As a result, the modeling of multilayered structures including porous materials and using coincident meshes implies the design of a refined mesh for the whole structure. Recently, the implementation of the {u,P} formulation using hierarchical elements has been presented [2]. The so called hierarchical poroelastic elements have proved to solve the problems of numerical locking related to the classical finite element implementation of the considered formulation, to increase its convergence rate in a significant way, and leads to a reduced number of degrees of freedom required for an accurate modeling of the porous medium. In this paper, the coupling of hierarchical elements with an elastic domain is considered. The enforcement of continuity equations between the two subdomains using continuous Lagrange multiplyiers is presented. Incompatible meshes are assumed. The present approach is then compared to the classical finite element representation of the multilayered structure using coincident meshes. Validation results are showed for the configuration of a porous material backed by an elastic plate and subjected to an acoustical excitation. The performance of the proposed approach is then underlined regarding the number of degrees of freedom required for the correct modeling the multilayered structure.

2. Theory

In the following, a 3D system composed of an elastic plate and a porous material is considered. The coupling between the two subdomains consist in insuring the continuity of the solid displacement vector and the continuity of the normal stresses at the interface. Also, the normal component of the relative flow along the interface has to be set to zero. These two conditions can actually be directly taken into account in the weak formulation of the multilayered system. Yet, the continuity of displacement has to be enforced. In the present approach, the theory of continuous Lagrange multipliers is used to construct a two field hybrid formulation [3] for the multilayer. This formulation is given in equation (1) where Ω_{el} (resp. Ω_p) and $\partial\Omega_{el}$ (resp. $\partial\Omega_p$) represent the elastic (resp. porous) subdomain and its boundary. σ^{el}, ε^{el} and $\underline{u}^{el}$ are the stress and strain tensor related to the elastic domain and its displacement vector, ρ^{el} is its mass density. and are the strain and stress tensor related to the solid phase of the porous material *in vacuo*. is the total strain tensor in the porous material. $\underline{u}^s$ and p stand for the displacement vector of the solid phase and the pressure in the pores, and are the complex dynamic mass density of the solid and fluid phase respectively. h denotes porosity, γ, and are complex poroelastic coefficients [1]. Vector $\underline{\lambda}$ in equation (1) stands for the continuous Lagrange multiplier and can be physically interpreted as the force vector applying at the interface between the two subdomains and which allows for the continuity of displacement. In the present approach, each component of $\underline{\lambda}$ is then interpolated on an orthogonal basis of polynomials, namely Legendre polynomials, defined globally on the interface. The structure considered here has a planar geometry. Any point on the interface between the elastic plate and the porous material is located by its x and y coordinate. Component i of $\underline{\lambda}$ is interpolated in directions x and y using the following formula:

$$\begin{aligned}
&\int_{\Omega_{el}} \underline{\underline{\sigma}}^{el}(\underline{u}^{el}):\underline{\underline{\varepsilon}}^{el}(\delta\underline{u}^{el})\,d\Omega - \rho_{el}.\omega^2 \int_{\Omega_{el}} \underline{u}^{el}\cdot\delta\underline{u}^{el}\,d\Omega + \int_{\Omega_p} \underline{\underline{\tilde{\sigma}}}^{S}(\underline{u}^{S}):\underline{\underline{\tilde{\varepsilon}}}^{S}(\delta\underline{u}^{S})\,d\Omega - \tilde{\rho}.\omega^2 \int_{\Omega_p} \underline{u}^{S}\cdot\delta\underline{u}^{S}\,d\Omega \\
&\int_{\Omega_p}\left[\frac{h}{\omega^2\tilde{\rho}_{22}}\underline{\nabla}p\cdot\underline{\nabla}\delta p - \frac{h^2}{\tilde{R}}p\cdot\delta p\right]d\Omega + \left[\gamma + h\left(1+\frac{\tilde{Q}}{\tilde{R}}\right)\right]\int_{\Omega_p}\left(\underline{\nabla}p\cdot\delta\underline{u}^{S} + \underline{\nabla}\delta p\cdot\underline{u}^{S}\right)d\Omega \qquad (1)\\
&-h\left(1+\frac{\tilde{Q}}{\tilde{R}}\right)\int_{\Omega_p}\left(p\cdot\mathrm{div}(\delta\underline{u}^{S}) + \delta p\cdot\mathrm{div}(\underline{u}^{S})\right)d\Omega - \int_{\partial\Omega_{el}\backslash\partial\Omega_p}\left[\underline{\underline{\sigma}}^{el}\cdot\underline{n}^1\right]dS - \int_{\partial\Omega_p\backslash\partial\Omega_{el}}\left[\underline{\underline{\tilde{\sigma}}}^{t}\cdot\underline{n}^2\right]dS \\
&-\int_{\partial\Omega_p\backslash\partial\Omega_{el}} h(u_n - U_n)\,dS + \underbrace{\int_{\partial\Omega_p\cap\partial\Omega_{el}}\delta\lambda\cdot\underline{u}^{el}\,dS}_{I'_1} - \underbrace{\int_{\partial\Omega_p\cap\partial\Omega_{el}}\delta\lambda\cdot\underline{u}^{S}\,dS}_{I'_2} + \underbrace{\int_{\partial\Omega_p\cap\partial\Omega_{el}}\lambda\cdot\delta\underline{u}^{el}\,dS}_{I_1} - \underbrace{\int_{\partial\Omega_p\cap\partial\Omega_{el}}\lambda\cdot\delta\underline{u}^{S}\,dS}_{I_2} = 0
\end{aligned}$$

$$\lambda_i(x,y) = \sum_{m,n}\lambda^i{}_{mn}\cdot P_m(x)\cdot P_n(y) \qquad (2)$$

where P_m is the Lengendre polynomial of order m. Integrals I_1 and I_2 (as well as I'_1 and I'_2) are discretized on the mesh of the elastic plate and the porous domain respectively. The global definition of $\underline{\lambda}$ allow for an easy computation of the discretized form these integrals, because all the shape functions have a global definition on one element of the considered mesh. In the result section, the present approach is applied to the study of a baffled porous coated plate and subjected to an imposed pressure condition on the porous material. Two indicators are considered.

First, the mean quadratic velocity of the plate is computed using the approximated formula:

$$\langle v_z \rangle^2 = \frac{\omega^2}{2N}\sum_{i=1}^{N}|u_z^i|^2 \quad (3)$$

where N is the number of nodes for the plate and u^i_n the displacement component along thickness for node i. Also the transmission loss (TL) is chosen as a power indicator. One defines:

$$TL = 10 * \log_{10}\left(\frac{P_{tr}}{P_{inc}}\right) \quad (4)$$

where P_{inc} is the incident power and P_{tr} is the transmitted power. Given the amplitude P_0 of the imposed pressure, the incident power is given by:

$$P_{inc} = \frac{P_0^2}{2\rho_0 c} \quad (5)$$

where ρ_0 and c are the density and celerity of air. Since the multilayered structure is baffled, the transmitted power is given by :

$$P_{tr} = \frac{1}{2}\Re\left(\int_S p \cdot u^* dS\right) \quad (6)$$

where * means complex conjugate. This integral is discretized on the F.E. mesh of the plate and computed using the values of the normal displacement and the pressure at the nodes of the mesh. The pressure is obtained from the normal displacements by the use of Rayleigh formula.

3. RESULTS

The porous coated plate has dimensions 0.35m*0.22m*0.005m is studied here. The plate is made up of aluminium and has simply supported edges constraints. The porous material is a foam with bonded lateral faces. The properties of the materials are given in tables 1 and 2. The excitation is an imposed pressure condition of amplitude 1Pa applied on the porous material front face. The computed indicators, either the mean square velocity or the transmission loss are compared to the results given by a FE code developed at the GAUS. This latter code, referred to as the classical approach in the following, is based on classical linear poroelastic elements for the porous medium and uses coincident meshes for the two subdomains. The mesh for each approach is chosen in order to insure the convergence of the solution. The classical approach uses a 16*11 nodes mesh in the lateral dimensions for the plate and the porous medium, and 2 elements along thickness for the porous subdomain. The present approach uses the same mesh for the plate but only a 3*2 brick element mesh in the lateral dimensions of the porous medium and a single element along thickness. The interpolation order for the solid and the fluid phase are 3 and 2 respectively. The results are represented on figure 1 and 2 for a frequency ranging from 10Hz to 500Hz. These figures show an excellent agreement between the two approaches for the mean quadratic velocity of the plate as well as for the transmission loss. Besides, the present approach allows for an important reduction of the number of degrees of freedom required for the accurate description of the multilayer. Actually, 1095 degrees of freedom are sufficient to insure convergence instead of 2190 for the classical approach.

E (kPa)	ν	η	ρ (kg/m³)
6.9*10⁷	0.33	0.007	2742

Table 1 : Properties of the plate

h	σ (kN.s/m⁴)	α∞	Λ (μm)	Λ' (μm)
0.98	13.5	1.7	80	160
N (kPa)	ν	η	ρs (kg/m³)	
200	0.35	0.1	1500	

Table 2 : Properties of the foam

4. CONCLUSION

This paper presented the coupling of the {u,P} formulation using hierarchical elements with a classical finite element plate modeling of an elastic domain assuming non coincident meshes. This approach has been used to predict the mean quadratic velocity and the transmission loss of a plate coated by a porous material. It gives accurate results with a reduced number of unknowns in comparison with a classical finite element modeling of the structure with coincident meshes.

5. REFERENCES

[1] Atalla N., Panneton R., Debergues P. "A mixed displacement-pressure formulation for poroelastic materials", J.A.S.A. **104**(3), 1444-1452 (1998)

[2] Rigobert S., Sgard F., Atalla N. "Investigation of the convergence of the mixed displacement-pressure formulation for 3D poroelastic materials using hierarchical elements", submitted for publication.

[3] Farhat C., Géradin M. "On a component mode synthesis method and its application to incompatible substructures", Computers and Structures **51**(5), 459-473 (1994).

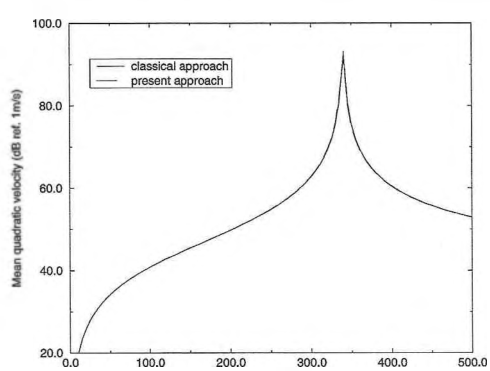

Figure 1: Mean quadratic velocity for the plate

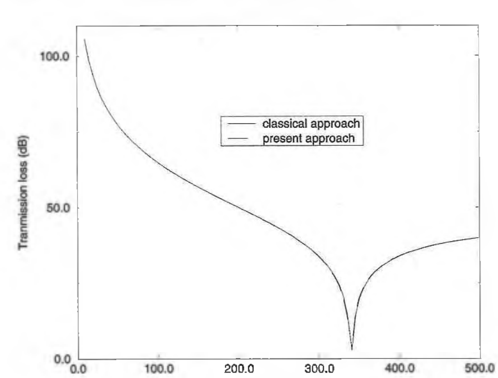

Figure 2: Transmission Loss for the multilayer