Modeling of the TOA-based Distance Measurement Error Using UWB Indoor Radio Measurements

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***Abstract*— Time of Arrival (TOA) estimation used with ultra wideband (UWB) transmission is currently the most popular technique for accurate indoor geolocation. Due to severe indoor multipath conditions these techniques often suffer from significant inaccuracy in location estimation. In this paper we introduce a model for the error in estimated distance as measured from the estimated TOA of the direct path (DP) in a typical multipath indoor environment. The TOA estimation error has two components, (1) the errors caused by the multipath dispersion affecting any signal path and (2) the errors caused by undetected direct path (UDP) conditions. The statistical behavior of this error is also a function of the system bandwidth. The empirical data from UWB indoor measurements in an office building are used to design a model for the distance measurement error. This model relates the behavior of the two components of the TOA estimation error to the bandwidth of the system.**

***Index Terms*— Radio propagation, positioning, ultra wideband.**

I. INTRODUCTION

RECENTLY , location-based applications for *ad-hoc* wireless personal area networks (WPANs) and wireless local area networks (WLANs) have attracted considerable attention. In addition to traditional indoor geolocation user applications, a number of important location-based algorithms are becoming popular in network operation. Emerging *ad-hoc* WPAN technologies employ location-based methods to facilitate hand-off and to improve the effectiveness of their routing algorithms [1]. Next generation corporate WLAN networks also utilize location-based techniques to improve security [2] and privacy [3].

The most frequently used distance measurement method for accurate indoor positioning is time-of-arrival (TOA) estimation of the direct path (DP) using ultra-wideband (UWB) technology [4] [5] [6]. Due to severe multipath conditions in indoor areas, estimation of TOA of DP results in small random and sometimes large errors. The small random errors are caused by paths arriving close to the detected first path. The large errors occur when the DP goes below the detection threshold so the detected first path in the received multipath profile is erroneously considered to be the DP. We refer to these situations as undetected direct path (UDP) conditions [7] [8]. Figure 1-(a) shows the occurrence of a small distance measurement error of 30cm for a channel profile with a bandwidth of 1GHz when the DP is detected. Figure 1-(b) shows the occurrence of the UDP scenario for another channel profile with the same bandwidth. Since the power of the DP is less than the detection threshold, we have a clear UDP situation, in which the first path is detected and declared as the DP, resulting in a 5.23m distance measurement error.

This letter introduces a framework and a statistical model for both small and large distance measurement errors in a typical indoor multipath area and relates the statistical behavior of these errors to the bandwidth of the system. We examine the validity of the model using 405 UWB measurements taken in Atwater Kent Laboratory, Worcester Polytechnic Institute, with the measurement system described in [9].

II. MODELING OF THE TOA ESTIMATION ERROR

A. Distance Measurement Error

Assuming the actual distance between the transmitter (Tx) and the receiver (Rx) is *d*; the estimated distance in a TOA positioning system is given by $\hat{d}_W = c\hat{\tau}_W$, where *c* is the speed of light and τ_W is the estimate of the TOA of the DP. The actual distance *d* is measured through physical measurement between Tx and Rx, and $\hat{d}$ is obtained by detecting TOA of the first peak above the detection threshold. Based on the above assumptions we define the distance measurement (estimation) error $\epsilon_W(d)$ as

$$\epsilon_W(d) = \hat{d}_W - d \tag{1}$$

the subscript W refers to the system bandwidth.

In modeling the distance measurement error we differentiate the small errors caused by multipath from the large errors produced by the occurrence of UDP conditions. We refer to the small distance errors caused by multipath as the multipath distance measurement error, $\epsilon_{M,W}(d)$, and to the large errors caused by the UDP condition as UDP distance measurement error, $\epsilon_{U,W}(d)$. The multipath error caused by neighboring paths always exists and the UDP error exists only when the UDP condition occurs. We model the occurrence of the UDP condition with the random variable $\xi_W(d)$ that takes value of ”1” when a UDP condition occurs and ”0” otherwise. Therefore, our model for $\epsilon_W(d)$ is given by,

$$\epsilon_W(d) = \epsilon_{M,W}(d) + \xi_W(d)\epsilon_{U,W}(d) \tag{2}$$

There is also another type of error involved in this process caused by physical measurement of *d*, similar to $\epsilon_{M,W}(d)$ this type of error always exists but compared to $\epsilon_{M,W}(d)$ it has negligible value. Therefore we don't consider measurement error in here. In the rest of this section we use the empirical

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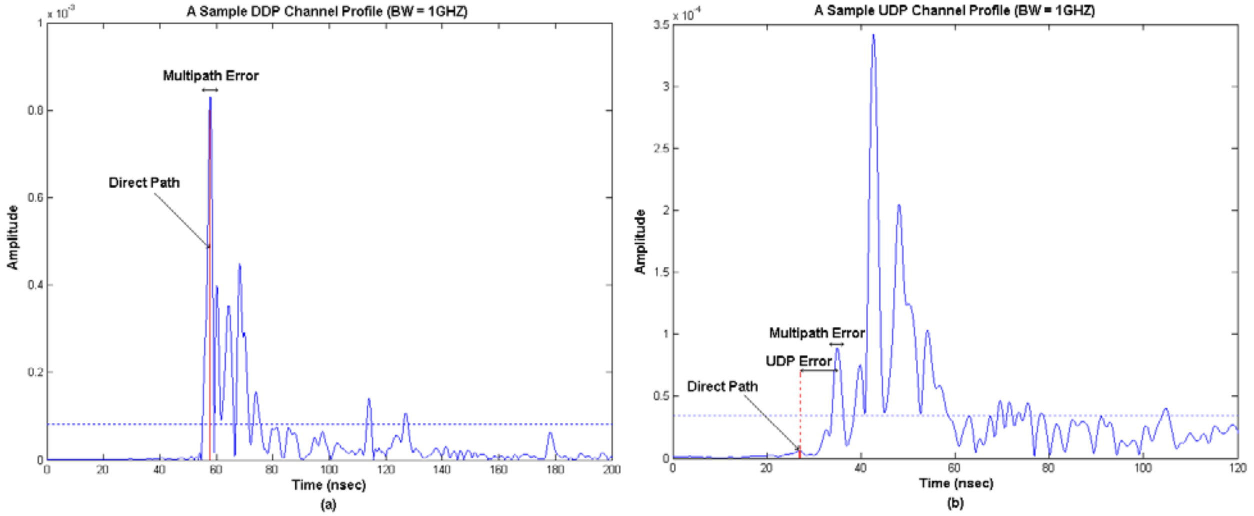

Fig. 1. Distance measurement error in two different types of channel profiles (BW=1GHz), (a) DDP, (b) UDP

data to develop a statistical model for all components of (2) as a function of system bandwidths and the distance between Tx and Rx.

B. Modeling of Multipath Error

To determine the model parameters from the empirical data we have partitioned our data base of the distance measurement errors for a given bandwidth into DDP (detected DP) and UDP. The DDP distance measurement error database represents cases which are only disturbed by multipath distance measurement errors and in this section we use only those to model multipath error.

Naturally one expects an increase in the distance measurement error with the increase of distance between Tx and Rx. Our observation of the empirical measurement results showed that this increase has a nonlinear relation. Therefore, to proceed with our modeling of $\epsilon_{M,W}(d)$, in a manner similar to [10], we introduce parameter γ_W, the normalized distance error, given by,

$$\gamma_W = \frac{\epsilon_{M,W}}{\log(1+d)} \tag{3}$$

The logarithm in the denominator is used to accommodate the increase pace of error below linear. Our measurement results show that this normalization causes the resulting normalized error to form a Gaussian distribution. Therefore, the multipath error has been modeled as

$$\begin{aligned} \epsilon_{M,W}(d) &= \gamma_W \log(1+d) \\ &= G(m_{M,W}, \sigma_{M,W}) \log(1+d) \end{aligned} \tag{4-a}$$

$$f_{\gamma_W}(x) = \frac{1}{\sigma_{M,W}\sqrt{2}} e^{-\frac{(x-m_{M,W})^2}{2\sigma_{M,W}^2}} \tag{4-b}$$

where, $G(m_{M,W}, \sigma_{M,W})$ is a Gaussian random variable with mean $m_{M,W}$ and variance $\sigma_{M,W}^2$. Table 1 displays typical values of these parameters. It should be noted that due to its nature, $\epsilon_{M,W}(d)$ can be positive or negative [10] [11]. Therefore the Gaussian model with a mean close to zero can model this error.

C. Modeling of UDP Error

In order to model the UDP distance measurement errors we need to model two parameters, $\xi_W(d)$ and $\epsilon_{U,W}(d)$. The random variable $\xi_W(d)$ is a binary random variable with the probability density function shown in (5-a), where $P_{U,W}(d)$ is the probability of occurrence of UDP for a given bandwidth.

The $P_{U,W}(d)$, however, is also a function of distance. Based on measurements of UWB channel characteristics [12] and our observation of our empirical data we assume that the probability of UDP in locations close to Tx and locations far from that are substantially different and we need to partition the model. In an approach similar to [12] we choose 10m as the break point to differentiate the behavior of the error. Then for distances shorter than the break point $P_{U,W}(d)$ is modeled as $P_{closeU,W}$ and beyond the break point as $P_{farU,W}$.

$$f_{\xi_W}(x) = (1 - P_{U,W}(d))\delta(x) + P_{U,W}(d)\delta(x-1) \tag{5-a}$$

$$P_{U,W}(d) = \begin{cases} P_{closeU,W} & , d \leq 10m \\ P_{farU,W} & , d > 10m \end{cases} \tag{5-b}$$

Table 1 shows approximations to $P_{closeU,W}$ and $P_{farU,W}$ for different bandwidths. As Rx moves away from Tx the DP power decreases, resulting in an increase in the probability of occurrence of the UDP condition. To model the distribution of $\epsilon_{U,W}(d)$, similar to $\epsilon_{M,W}(d)$ we use a Gaussian distribution. But unlike $\epsilon_{M,W}(d)$ the UDP error can not be negative and we know that Gaussian has negative parts as well. In fact Gaussian with a positive mean has a negligible negative error and at the same time is a simpler and more practical solution. Therefore, the UDP error can be expressed as

$$\epsilon_{U,W}(d) = G(m_{U,W}, \sigma_{U,W}) \tag{6-a}$$

$$f_{\epsilon_{U,W}}(x) = \frac{1}{\sigma_{U,W}\sqrt{2}} e^{-\frac{(x-m_{U,W})^2}{2\sigma_{U,W}^2}} \tag{6-b}$$

TABLE I

TYPICAL VALUES OF MODEL PARAMETERS DERIVED FROM THE MEASUREMENTS

W(MHz)	m_W(m)	σ_W(cm)	$P_{closeU,W}$	$P_{farU,W}$	$m_{U,W}$(m)	$\sigma_{U,W}(cm)$
20	3.66	515	0	0.005	-12.83	0
50	1.57	205	0	0.009	24.48	21.1
100	0.87	115	0	0.091	5.96	358.5
200	0.47	59	0.006	0.164	3.94	289.0
500	0.21	26.9	0.064	0.332	1.62	80.9
1000	0.09	13.6	0.064	0.620	0.96	60.4
2000	0.02	5.2	0.070	0.740	0.76	71.5
3000	0.004	4.5	0.117	0.774	0.88	152.2

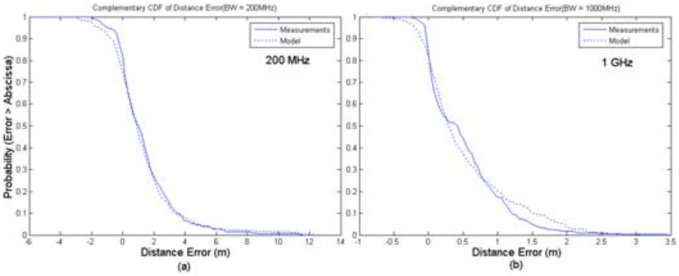

Fig. 2. CDF comparison between measurements and model, (a) BW=200 MHz, (b) BW = 1GHz

D. The General Model for Distance Measurement Error

If we combine our results of multipath and UDP modeling, the overall model for estimated distance measurement is

$$\begin{aligned}\hat{d} &= d + \epsilon_{M,W}(d) + \xi_W(d)\epsilon_{U,W} \\ &= d + \gamma_W \log(1+d) + \xi_W(d)\epsilon_{U,W} \qquad (7)\end{aligned}$$

where γ_W, $\epsilon_{U,W}$, and $f_{\xi_W}(x)$ are defined in (4-b), (6-a), and (5-a), respectively.

This model relates the distance measurement error to the distance and bandwidth of the system. Figure 2 compares the complementary CDF of the distance measurement error obtained from empirical UWB measurements and the overall model described by (7) and (8) for bandwidths of 200MHz and 1GHz. The bandwidth of the UWB measurements taken at 3-6GHz is adjusted to these two values to provide a fair comparison. The model shows close agreement with the empirical data.

III. CONCLUSION

In this paper, we have introduced a model for the estimated distance obtained from TOA of the first path in an indoor multipath environment typically used for WPAN applications. This model separates the distance measurement error into errors caused by multipath and errors occurring due to UDP conditions and relates these errors to the system bandwidth. Using empirical results of UWB channel measurements in a typical indoor area we have shown that the model fits closely with the results of measurements.

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REFERENCES

[1] E. M. Royer and C.-K. Toh, "A review of current routing protocols for adhoc mobile wireless networks," *IEEE Personal Commun. Mag.*, vol. 6, no. 2, pp. 46–55, Apr. 1999.

[2] [Online]. Available: http://www.tmcnet.com/enews/120602h.htm

[3] A. Smailagic and D. Kogan, "Location sensing and privacy in a context-aware computing environment," *IEEE Wireless Commun. Mag.*, vol. 9, no. 5, pp. 10–17, Oct. 2002.

[4] R. Fontana, "Advances in ultra wideband indoor geolocation systems," in *Proc. 3rd IEEE Wksp. WLAN*, Sept. 2001.

[5] X. Li and K. Pahlavan, "Super-resolution toa estimation with diversity for indoor geolocation," *IEEE Trans. Wireless Commun.*, vol. 3, no. 1, pp. 224–234, Jan. 2004.

[6] S. Gezici, Z. Tian, G. G. , H. K. amd A.F. Molisch, H. Poor, and Z. Sahinoglu, "Localization via ultra-wideband radios: a look at positioning aspects for future sensor networks," *IEEE Signal Processing Mag.*, vol. 22, no. 4, pp. 70–84, July 2005.

[7] K. Pahlavan, P. Krishnamurthy, and J. Beneat, "Wideband radio channel modeling for indoor geolocation applications," *IEEE Commun. Mag.*, vol. 36, no. 4, pp. 60–65, Apr. 1998.

[8] K. Pahlavan and A. H. Levesque, *Wireless Information Networks, Second Edition*. Wiley-Interscience, 2005.

[9] E. Zand and K. Pahlavan, "Measurement of toa using frequency domain characteristics for indoor geolocation," in *Proc. IEEE 14th Annual International Symposium on Personal Indoor and Mobile Radio Communications (PIMRC 2003)*, vol. 3, Sept. 7–10, 2003, pp. 2213–2217.

[10] B. Alavi and K. Pahlavan, "Bandwidth effect on distance error modeling for indoor geolocation," in *Proc. IEEE 14th Annual International Symposium on Personal Indoor and Mobile Radio Communications (PIMRC 2003)*, vol. 3, Sept. 7–10, 2003, pp. 2198–2202.

[11] ——, "Modeling of the distance error for indoor geolocation," in *Proc. IEEE Wireless Communications and Networking (WCNC 2003)*, vol. 1, Mar. 16–20, 2003, pp. 668–672.

[12] D. Cassioli, M. Z. Win, and A. F. Molisch, "The ultra-wide bandwidth indoor channel: from statistical model to simulations," *IEEE J. Select. Areas Commun.*, vol. 20, no. 6, pp. 1247–1257, Aug. 2002.