Modeling Path Duration Distributions in MANETs and Their Impact on Reactive Routing Protocols

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***Abstract*—We develop a detailed approach to study how mobility impacts the performance of reactive mobile ad hoc network routing protocols. In particular, we examine how the statistics of path durations including probability density functions vary with the parameters such as the mobility model, relative speed, number of hops, and radio range. We find that at low speeds, certain mobility models may induce multimodal distributions that reflect the characteristics of the spatial map, mobility constraints and the communicating traffic pattern. However, this paper suggests that at moderate and high velocities the exponential distribution with appropriate parameterizations is a good approximation of the path duration distribution for a range of mobility models.**

Analytically, we show that the reciprocal of the average path duration has a strong linear relationship with the throughput and overhead of dynamic source routing (DSR), which is also confirmed by simulation results. In addition, we show how the mathematical expression obtained for the path duration distribution can also be used to prove that the nonpropagating cache hit ratio in DSR is independent of velocity for the freeway mobility model. These two case studies illustrate how various aspects of protocol performance can be analyzed with respect to a number of significant parameters including the statistics of link and path durations.

***Index Terms*—Communication system, modeling, simulation.**

I. Introduction

Availability of small, inexpensive wireless communicating devices has played an important role in moving ad hoc networks closer to reality. Consequently, mobile ad hoc networks (MANETs) are attracting a lot of attention from the research community. MANETs are advantageous because of their readily deployable nature as they do not need any centralized infrastructure. Various MANET routing protocols are proposed in the recent years. Some of them are categorized as *proactive* routing protocol, because the routing table of node is periodically exchanged and updated. Some others are classified as *reactive* routing protocol, since, for each node, the route to destination is on demand discovered only when it is needed.

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Since the field of MANET is still in its developing stage, not many MANETs have been deployed yet. Thus, most of the research in this area is simulation-based. These simulations have several parameters such as the mobility model, traffic pattern, propagation model, etc., to name a few. We acknowledge that these and other factors like channel characteristics, medium access control (MAC) effects, etc., do impact the protocol performance. However, we realize that the study of the interplay of these factors may be very complex. Hence, in this paper, we only focus on developing a detailed approach to study the effect of mobility on the performance of reactive MANET routing protocols like dynamic source routing (DSR) [3] and ad hoc on-demand distance vector (AODV) [6].

Unlike the existing internet, the MANET environment is resource-constrained, i.e., the mobile nodes have the limited-bandwidth and constrained power. Thus, it seems that providing guaranteed quality of service in MANETs is not a trivial task. Moreover, mobility is also expected to affect the service quality significantly. For example, the frequent topology changes caused by node movement in high mobility scenarios may result in the disruption of established routes, leading to packet losses and substantial degradation of service quality. Thus, in order to design paradigms of service quality in MANETs, it is essential to understand the impact of mobility on performance. We believe that the mobility factor needs to be taken into consideration at the design phase, and not just considered as an after thought, e.g., during evaluations. In this paper, as the first step, we aim to gain a deeper insight into the characteristics of mobility itself.

This paper proposes a novel approach to understand the effect of mobility on protocol performance. It uses statistical analysis (of simulation data) to obtain detailed statistics of link and path duration including their probability density functions (pdfs). Through simple analytical models, using the case study of DSR, it shows a strong correlation between the reciprocal of **average path duration** and the throughput and overhead of reactive protocols. Further, the approach uses a case study of DSR to demonstrate how the **path duration pdf** can be used to analytically determine the nonpropagating cache hit ratio in the freeway mobility model.

Recently, there has been a greater focus on a systematic study of the effect of mobility on the performance of routing protocols. Reference [21] proposed the IMPORTANT framework to systematically analyze the effect of mobility on routing protocols. In this framework, the authors proposed to evaluate the MANET routing protocols using a "test-suite" of mobility models that span several mobility characteristics like spatial dependence, geographic restrictions, etc. These models included the random waypoint (RW), reference point group mobility (RPGM), freeway (FW), and Manhattan (MH). They found that mobility significantly impacts the performance of the protocols, which is in agreement with several other studies. Moreover, they also proposed a reason for *why* mobility impacts performance: Mobility impacts the connectivity graph (average link duration in particular), which in turn impacts the protocol performance.

To explain *how* mobility impacts the performance, [22] introduced the BRICS methodology. It proposed that a protocol could be considered to be made up of **parameterized** "building blocks" or basic mechanisms. The effect of mobility on the entire protocol can be explained in terms of its effect on these "building blocks." Some of the "building blocks" proposed by BRICS for reactive protocols were flooding, caching, error detection, error notification, and error recovery. Both DSR and AODV use these "building blocks" in their operation. However, they still behave differently for a given mobility model. BRICS suggested that a possible reason for this difference might be the different parameter settings for the "building blocks" in AODV and DSR. This leads to different impacts of mobility on these mechanisms. A brief overview of the work done in [21] and [22] is given in the Section III.

In this paper, we develop an approach that combines statistical analysis of simulation data and analytical modeling to get a deeper understanding of the protocol performance in the presence of mobility. Reference [21] concluded that **average** link duration is a useful metric for relating mobility with protocol performance. At the same time, intuitively, the protocol performance depends on the duration of a path between the source and the destination. Path duration is significantly related to link duration. It is actually the minimum link duration along a path. In general, longer the path duration, better the performance in terms of throughput and overhead. However, the relationship between the path duration and protocol performance (throughput and overhead) has not been categorized yet. In this paper, we examine the detailed statistics of link and path duration including pdfs across the "test-suite" of mobility models proposed in [21]. We then attempt to categorize the relationship between the average path duration and protocol performance as either strongly (or weakly) linearly (or nonlinearly) related. We also relate the path duration pdfs to the impact of mobility on the "building blocks" of reactive protocols. The contributions of this study are the following.

1) Characterizing the statistics of link and path durations including pdfs for the different mobility models used in our study using simple statistical analysis. This also leads to a characterization of link and path durations based on the communicating traffic pattern.
2) Investigating possible distributions to approximate the path duration pdf across the mobility models used. At moderate to high mobility, we suggest that an exponential distribution with an appropriate parameterization is a reasonable approximation for the path duration pdf across most of the models used in our study.
3) Establishing a linear relationship, through simple first-order analytical models (that are validated by simulation results), between the reciprocal of the path duration and protocol performance, that helps explain several performance trends under various mobility models.
4) Illustrating the use of the path duration pdf to analytically model protocol performance (using the case study of the nonpropagating cache hit ratio in DSR for the FW model).

The rest of the paper is organized as follows. Section II gives an overview of the related work. Section III sets our work in context with the recent work in this area. Link and path duration are formally defined in Section IV. Section V discusses our simulation setup, while the results of these simulations and analytical models for path duration are discussed in Section VI. Section VII gives first-order analytical models relating the path duration statistics (pdfs and averages) and the protocol performance of reactive protocols using the case study of DSR. Our conclusions and future work are listed in Section VIII.

II. Related Work

In this paper, we study the detailed statistics of link and path duration including their pdfs across a rich set of mobility models. As mentioned in Section I, we believe such a study might help in formulating analytical models for protocol performance across these mobility models. However, such a thought was inspired by other pioneering work done in MANET research.

A. Mobility Models

Mobility models for simulations have been one of the early topics of research in this field. One of the early contributions was made by Broch *et al.*, where they evaluated DSR, AODV, destination sequenced distance vector routing (DSDV) [4] and temporally ordered routing algorithm (TORA) [20] using the RW model [2]. They concluded that mobility does impact the performance of routing protocols. To evaluate these protocols over a wider range of scenarios, Johansson *et al.* proposed the scenario-based performance analysis [14]. In this paper, they proposed mobility models for disaster relief, event coverage and conferences. Haas [9] introduce a mobility model in which the current velocity of a node may depend on its previous velocity. Hong *et al.* proposed the RPGM model in [11]. One of the main applications of this model is in battlefield communications. The authors give several other applications of RPGM in [11]. While defining their framework, [21] proposed to evaluate the protocols under a richer set of mobility models. Apart from using the RW and RPGM, they used two other mobility models, i.e., the FW and the MH models. In this paper, we use these four models for our simulations.

B. Protocol Independent Metrics

Apart from analyzing the effect of mobility on protocol performance, it is useful to characterize mobility independent of the protocols. Hence, there have been several attempts to propose mobility metrics. Johansson *et al.* proposed the relative motion between mobile nodes to distinguish the different mobility models used for their scenario-based study in [14]. Reference [21] used the metrics of relative motion and average degree of spatial dependence to characterize the different mobility models used in their study. They also proposed the connectivity graph metrics as a "bridge" relating the mobility metrics to the protocol performance. They found that average link duration at the graph level could explain this relationship. Hong *et al.* proposed the rate of link change as a metric to differentiate the various kinds of RPGM and RW models in [11]. We agree with [11] and [21] that the connectivity graph characteristics might help in relating mobility with protocol performance. As mentioned in Section I, we believe that the path duration can also be added to this set of connectivity graph metrics. Moreover, unlike other studies, we not only examine the averages, but also focus on the detailed statistics including the pdfs of link and path duration across several mobility models.

C. Reactive Protocols

In this paper, we focus on evaluating the reactive MANET routing protocols like DSR and AODV. There have been several studies to compare both proactive and reactive routing protocols. References [3], [5], and [15]–[17] give a very good exposition of this subject. Here, we discuss the work that focus completely on reactive protocols. Johnson *et al.* proposed DSR in [3], while AODV was proposed by Perkins in [6]. Maltz *et al.* gave a very comprehensive analysis of DSR in terms of its basic mechanisms of route discovery and caching [5]. They proposed several optimizations for reducing the route discovery overhead. Most of these optimizations are now part of the DSR implementation in the network simulator $(ns-2)$ [19]. Das *et al.* compared the performance of AODV and DSR in [16]. They observed that DSR outperformed AODV in less demanding situations, while AODV outperformed DSR at heavy traffic load and high mobility. To explain these differences, the BRICS methodology was proposed to decompose protocols into basic "mechanisms" [22]. It illustrated an approach for this decomposition by suggesting a common architecture that encompassed both AODV and DSR. Though both AODV and DSR consist of similar mechanisms or "building blocks" (that are parameterized), they behave differently in the presence of mobility. Some of these mechanisms are caching, flooding, etc. BRICS claimed that this difference arises due to the differing impact of mobility on the mechanisms of the protocols. The difference in impact on the mechanisms seems to arise from the different parameters chosen by these protocols for these mechanisms. In this study, using the case study of DSR in the FW model, we propose an analytical model relating the path duration pdf to the performance of one of the "building blocks," i.e., the nonpropagating cache hit ratio. Both [5] and [22] consider this mechanism to play an important role in determining the routing overhead of DSR (and reactive protocols in general). Moreover, through first-order analytical models, using the case study of DSR, we also show the relationship between the average path duration and the reactive protocol performance.

D. Analytical Models

Apart from simulation-based studies, the MANET research literature also contains analytical work on mobility and protocol performance modeling. One of the earliest analysis of mobility was done by Mc Donald and Znati in [7]. They used a RW like mobility model and derived expressions for the probability of path availability and link availability for different initial conditions. Stochastic properties of the RW model were studied recently in [24]–[26]. Su *et al.* exploited the nonrandom movement of mobile nodes during intervals to predict its location in [12]. They proposed a model for link duration and evaluated it using the RW model. In this paper, we examine the detailed statistics of link and path duration including pdfs across several mobility models used in our study. Gruber and Li presented a very detailed analysis of link duration times for a two-hop MANET in [27]. In this paper, the distribution of the link duration appeared to be exponential. Their analysis assumed that the source and destination are fixed, while the intermediate hop is moving using the RW model. The exponential distribution of link duration also comes up in the analysis of single path and multipath DSR by Nasipuri *et al.* in [23]. They assumed that the link durations are exponentially distributed independent random variables (i.i.d.) and analytically derived the distributions for path duration, which turns out to be exponential as well. The underlying mobility model was not very clearly specified. Moreover, the exponential distribution assumption was not validated by simulation or real data. Inspired by these studies, in this paper, we examine the detailed statistics of link and path duration including pdfs across the RW, RPGM, FW, and MH models. We observe that under certain conditions, the path duration pdfs can be approximated by exponential distributions for the models used in our study. We demonstrate the effect of the number of hops, the transmission range and the relative speed of the mobility model on the path duration pdf. Using the case study of DSR, we propose simple analytical models that relate the average path duration to the performance of reactive protocols. These models can also be extended for the performance of AODV. For DSR, we also show how the path duration pdf can be analytically related to the performance of the nonpropagating cache hit ratio "building block" in the FW model.

III. Background

Our approach of evaluating the protocols across mobility models was inspired by the IMPORTANT framework proposed in [21]. This framework made an attempt toward the systematic evaluation of the impact of mobility on MANET routing protocols. It defined protocol independent metrics like the average degree of spatial dependence $(\bar{D}_{\text{spatial}})$ and the average relative speed $(\bar{\text{RS}})$ to capture certain mobility characteristics. One of these characteristics was the extent to which the motion of a node is influenced by nodes in its neighborhood (which is captured by $\bar{D}_{\text{spatial}}$). Another characteristic was the presence of geographic restrictions on mobility. Once these metrics were defined, mobility models that spanned these mobility characteristics were chosen. These models were the following.

1) Random Waypoint (RW): As simulation starts, a node randomly chooses a speed and destination, and moves toward it. Each node moves independently of other nodes. Upon reaching the destination, the node stops for T_{pause}

time before moving to the next destination. This procedure repeats until simulation ends.[1]

2) Reference Point Group Mobility (RPGM): Nodes move in either single or multiple groups. The movement of a node in a group is strongly influenced by the leader of the group.
3) Freeway (FW): Each node moves in its lane on the FW. Its movement is constrained by nodes moving ahead of it in the same lane.
4) Manhattan (MH): Nodes move on a grid. As in the FW model, each node is constrained by nodes moving ahead of it. However, at the cross points of the grid, a node is free to change its direction unlike the FW model.

Different mobility patterns following the above mobility models were generated by varying the maximum speed of the mobile nodes. The mobility metrics of these mobility patterns were evaluated. Using these patterns, simulations were run in the network simulator $(ns-2)$ environment with the CMU wireless ad hoc networking extension to evaluate the performance of DSR, AODV, and DSDV in terms of throughput and routing overhead. To explain the relationship between the mobility metrics and the protocol performance, certain connectivity graph metrics were defined. Some of these metrics were the number of link changes, the path availability and the average link duration. For their study, the most useful of these graph metrics was the average link duration $(\overline{\mathrm{LD}})$, which could help in relating the mobility metrics to the protocol performance metrics.

The study observed that, given a communication traffic pattern, the underlying mobility pattern does have a significant impact on the performance of routing protocols. Moreover, it concluded that there is no clear performance-based ranking of the protocols across these mobility models.

To explain *why* mobility affects the protocol performance, [22] proposed the BRICS methodology to systematically decompose routing protocols into basic mechanisms or "building blocks." This methodology claimed that the difference in the protocol performance comes from the fact that the basic mechanisms (or "building blocks") of these protocols are different. For example, DSR and AODV are reactive, while DSDV is proactive. However, although DSR and AODV belong to the class of reactive protocols, they behave differently for a given mobility model. To understand this difference better, BRICS proposed the following possible decomposition of the reactive routing protocols:

Reactive protocols consist of two major phases

1) *Route Setup Phase*: In this phase, a route between the source and destination is setup on demand. The basic mechanisms (and their parameters) used in this phase are the following.
   a) *Flooding*: It is responsible for distributing the source's route request in the network. Its parameter is the range of flooding, which is specified by the time-to-live (TTL) field in the Internet protocol (IP) header.
   b) *Caching*: Caching is an optimization to reduce the overhead of flooding. If a node has a cached route to the destination, it will reply to the source's route request. Its parameter is whether aggressive caching should be used, i.e., should the nodes use all the overheard route replies and should they cache multiple routes to the destination.
2) *Route Maintenance Phase*: This phase is responsible for maintaining the path between the source and the destination. The basic mechanisms used in this phase are *error detection, error notification* and *error recovery*.

Both DSR and AODV make different choices for the parameters of the "building blocks" mentioned above. For example, in the caching "building block," DSR performs aggressive caching, while AODV does not. In the flooding "building block," before flooding a route request in the network, DSR issues a route request with a TTL of 1 (nonpropagating route request). On the other hand, AODV performs an expanding ring search (with $\mathrm{TTL} = 1$, 3, 5, and 7) before initiating the flooding.[2] As in [22], we define the **nonpropagating cache hit ratio** as the ratio of the route requests which are answered by the one-hop neighbors to the total number of route requests. Reference [22] observed that the "building blocks" are impacted differently by a given mobility model, depending on their choice for the parameters. Moreover the performance of the entire protocol is determined by the performance of these building blocks. For example, the overhead of the protocol is affected by the nonpropagating cache hit ratio. Higher the ratio, lower will be the frequency of route request flooding. Since both AODV and DSR use different caching strategies, this nonpropagating cache hit ratio for the two protocols might be different, which leads to different routing overheads for these protocols for a given mobility model.

Against this background, in the next section, we formally define the link and path duration metrics.

IV. Connectivity Graph Metrics

One of the main challenges for routing in MANETs is to deal with the topology (connectivity graph) changes resulting from mobility. The performance of a protocol is greatly determined by its ability to adapt to these changes. Realizing this, researchers have proposed metrics to characterize the effect of mobility on the connectivity graph with an aim to explain the effects of mobility on protocol performance. We define the link duration and path duration metrics in this section.

First, we mention some commonly used symbols in this section. Let

- N be the total number of nodes;
- $D_{ij}(t)$ be the Euclidean distance between nodes i and j at time t;
- R be the transmission range of the mobile nodes.

The connectivity graph is the graph $G=(V,E)$, such that $|V|=N$. At time t, a link $(i,j) \in E$ iff $D_{ij}(t) \le R$.

[1]In a recent publication [13], Yoon, Liu, and Noble pointed out that the original RW model is unable to reach a steady state in terms of the level of mobility. Correspondingly, they proposed a modified RW model. However, in our simulation, we still use the original RW model distributed within network simulator $ns-2$.

[2]Although, the initial design does not specify the expanding ring search, the $ns-2$ implementation of AODV uses the expanding ring search.

Let $X(i, j, t)$ be an indicator random variable, which has a value $\mathbf{1}$ iff there is a link between nodes i and j at time t. Otherwise, $X(i, j, t) = 0$.

1) *Link Duration*: For two nodes i and j, at time t_1, duration of the link (i, j) is the length of the longest time interval $[t_1, t_2]$ during which the two nodes are within the transmission range of each other. Moreover, these two nodes are not within the transmission range at time $t_1 - \epsilon$ and time $t_2 + \epsilon$ for $\epsilon > 0$. Formally

$$\mathrm{LD}(i, j, t_1) = t_2 - t_1$$

iff $\forall t\ t_1 \le t \le t_2, \epsilon > 0$: $X(i, j, t) = 1$ and $X(i, j, t_1 - \epsilon) = 0$ and $X(i, j, t_2 + \epsilon) = 0$. Otherwise, $\mathrm{LD}(i, j, t_1) = 0$.

2) *Path Duration*: For a path $P = \{n_1, n_2, \ldots, n_k\}$, consisting of k nodes, at time t_1, path duration is the length of the longest time interval $[t_1, t_2]$, during which each of the $k-1$ links between the nodes exist. Moreover, at time $t_1 - \epsilon$ and time $t_2 + \epsilon$, $\epsilon > 0$, at least one of the $k - 1$ links does not exist. Thus, path duration is limited by the duration of the links along its path. Specifically, at time t_1, path duration is the minimum of the durations of the $k - 1$ links $(n_1, n_2), (n_2, n_3) \ldots (n_{k-1}, n_k)$ at time t_1. Formally

$$\mathrm{PD}(P, t_1) = \min_{1 \le z \le k-1} \mathrm{LD}(n_z, n_{z+1}, t_1)$$

Thus, both link and path durations are a function of time. Link duration has been studied before across the "test-suite" of mobility models in [21]. However, that study was based on average values. Here, we also examine the pdfs of the link and path duration across these mobility models. We believe that this approach might give a deeper understanding of the impact of mobility on the protocol performance. PDFs are estimated using simple statistical analysis of the simulation data. The simulation settings for estimating the pdfs are discussed in the next section.

V. SIMULATION SETTINGS

Having defined the metrics, as mentioned in the introduction, we focus our attention on obtaining the detailed statistics of the link and path duration across the different mobility models used in our study. We simulate the node movement according to the "test-suite" of mobility models proposed in [21]. For each mobility model, we collect the detailed statistics of the link and path duration at both the connectivity graph level and the routing protocol level. The details of the mobility models used are mentioned in Section V-A. The collection of statistical data on link and path duration at the connectivity graph level is mentioned in Section V-B, while the collection of the corresponding data at the routing protocol level is discussed in Section V-C. Finally, the method to estimate the pdf is introduced in Section V-D.

A. Mobility Patterns

The mobility patterns are obtained from the mobility scenario generator mentioned in [21]. This scenario generator produces the different mobility patterns following the RPGM, FW, and MH models according to the format required by $ns - 2$. In all these patterns, 40 mobile nodes move in an area of 1000 m × 1000 m for a period of 900 s. The values for the transmission range will be mentioned in Sections V-B and V-C when the link and path durations at the connectivity graph level and the protocol level are measured. RW mobility pattern is generated using the *setdest* tool, which is a part of the $ns - 2$ distribution. For RPGM, we use two different mobility scenarios: single group of 40 nodes and four groups of ten nodes each moving independent of each other and in an overlapping fashion. Both speed deviation ratio and angle deviation ratio are set to 0.1.[3] For the FW and MH models, the nodes are placed on the Freeway lanes or local streets randomly in both directions initially. Their movement is controlled as per the specifications of the respective models. The maximum speed $V_{\max}$ is set to 1, 5, 10, 20, 30, 40, 50, and 60 m/s to generate different movement patterns for the same mobility model.

Once, the mobility patterns are obtained, we measure the link and path duration across them. For path duration, it can be either calculated as the duration of shortest path between source-destination pairs based on their location information, or measured by monitoring the status of traffic flow in $ns-2$ simulation. We define the path duration (of the shortest path) calculated from the mobility trace file, as the *path duration at connectivity graph level*, while the path duration collected under $ns - 2$ simulation environment is the *path duration at routing protocol level*. Our procedure for measuring these two path durations is described in the next two sections.

B. Measuring Link Durations and Path Durations at Connectivity Graph Level

For the purpose of measuring the link and path duration distributions at connectivity graph level, we use the following values for the transmission range R of the mobile nodes: 50, 100, 150, 200, and 250 m. Across all these values, the link and path durations at the connectivity graph level are measured using our mobility trace analyzer program. Given a mobility trace file, this program calculates the link and path durations between mobile nodes based on their location information. This calculation might get complicated due to node mobility. A common way to simplify the procedure is to take a series of "snapshots" of the network connectivity graph during the simulations. For each snapshot, the connectivity graph can be considered static and analyzed. In our paper, we take a snapshot once every second. Once the snapshot of the network connectivity graph is taken, the link and path durations can be readily measured as follows.

1) *Link Durations at Connectivity Graph Level*: The status of a link between every pair of nodes within the transmission range of each other is monitored during the simulation. The link duration is calculated as the interval between the time when the link is created and time when it breaks. This is done for every link that comes into existence during the simulation. The different link durations are then sorted into bins of 1, 2,. . .,900 s (simulation time).

[3]Speed deviation ratio and angle deviation ratio are defined in [21]. They control the extent to which the group members can deviate from the leader in speed and direction.

2) *Path Durations at Connectivity Graph Level*: The status of a path between every source-destination pair in the network is monitored. The path duration is counted as the interval between the time when the path is set up and the time when the path is broken. However, there can be potentially exponential paths between any specific source-destination pair. Analyzing the duration of all these paths might not be feasible. As an approximation, we define the path duration at connectivity graph level as the duration of the shortest path.[4] The shortest path between the source and the destination is computed by the breadth first search (BFS) algorithm [28]. The path duration is measured for all source-destination pairs in the network. The different path durations thus obtained are then sorted into bins of 1, 2,...,900 s (simulation time).

C. Measuring Path Durations at Routing Protocol Level

Most existing MANET protocols, in general, select shortest (min-hop count) paths. Hence, one may argue that the path duration of the shortest paths at the connectivity level is a good approximation of path duration collected under $ns-2$ simulation environment. However, the behavior of paths in the $ns-2$ simulations may differ from the behavior of shortest paths because routing protocols may not choose the shortest path always. In addition, the path duration at the protocol level may not be the same for the various routing protocols since their detailed mechanisms differ (e.g., in reacting to network dynamics and error recovery). Thus, we develop our own packet tracer program under standard $ns-2$ environment to analyze the path duration at the routing protocol level. In the simulation, the packet tracer program follows each data packet on its way from source to destination and records the necessary information including the whole sequence of visited nodes along the path in the header of data packet. For a specific traffic flow, the path duration at routing protocol level can be easily estimated by measuring the interval between consecutive packet arrivals at the destination and comparing the paths taken by these packets. Then, the path duration is measured for all ongoing traffic flows in the network. Finally, the different path durations thus obtained are sorted into bins of 1, 2,...,900 s.

Our simulations to collect path duration at routing protocol level were run in the $ns-2$ environment. In these simulations, the transmission range R is set to 250 m. The traffic consisted of 20 constant bit rate (CBR) sources and 30 connections. The source destination pairs were chosen at random. The data rate used was 4 packets/s and the packet size was 64 bytes. The mobility patterns generated in Section V-A were used for these simulations.

D. PDF Estimation

After having sorted the samples of link and path durations into bins as mentioned above, we plot a histogram of these durations for the mobility scenarios mentioned in Section V-A. For link durations, we plot the histograms for the different mobility models and different maximum velocities $V_{\max}$ for each model. For path durations at the connectivity graph level, we plot the histograms *vis-a-vis* the number of hops h in the path for the various mobility models, various maximum velocities $V_{\max}$ for each model and different values of R.

For path duration at the routing protocol level, most of the above procedure is repeated for a fixed value of R, i.e., 250 m (as mentioned at the end of Section V-C).

Having collected a large set of samples for link and path durations, we use the relative frequency approach (from standard probability theory) to estimate the pdfs of the link and path duration across the different mobility models used in our study [29].

Once the pdfs are determined, we compute the average path duration for all the sample sets.

Our observations from these simulations and measurements are discussed in the following section.

[4]Thus, in general, the one-hop path duration is not the same as the link duration. If a path of more than one-hop already exists between the source and the destination before they come within range of each other, we still monitor the original shortest path until it breaks.

VI. Observations and Analytical Model

The purpose of examining the detailed statistics of link and path duration across the "test-suite" of mobility models was to gain a deeper understanding of the impact of mobility on the protocol performance. At the connectivity graph level, we observe that for low $V_{\max}$, some models like FW and RPGM (four groups) have multimodal distributions for both link duration and path duration. However, for moderate and high values of $V_{\max}$, path durations at the connectivity graph level can be approximated as exponential distributions for most of the models used in our study. Moreover, we also learn some lessons about the effect of traffic pattern on these distributions. We first discuss the link duration pdfs and follow it up with a discussion of the path duration pdfs at connectivity graph level. Then, we show that the conclusions drawn from the observations of path duration at connectivity graph level are also valid for their counterpart at routing protocol level. In this section, because of space restriction, we only present a number of pdfs of link duration and path duration for the representative cases, among many others.

A. Link Duration PDFs at the Connectivity Graph Level

When $V_{\max}$ is small, i.e., 1 or 5 m/s, the link duration pdf has a multimodal distribution for the FW and the RPGM model (with four groups). For the rest of the section, we will refer to a "peak" as a cluster in the pdf. For example, as shown in Fig. 1, there is a big peak in the link duration pdf for the FW model (at around 100 s). Through simulation, we identify that this peak accounts for the links between mobile nodes moving in the opposite directions. There are several small peaks centered at larger values of link duration (for example, at around 250 s). These peaks account for the links between mobile nodes moving in the same direction. The peak on the left side dominates the pdf, i.e., the area under the peak on the left side is much larger than the area under the peak on the right side. This is because the links between nodes traveling in opposite directions are frequently broken and the number of such instances is larger, compared to the links between nodes traveling in the same direction.

A similar phenomenon is also observed for RPGM (with four groups) at small velocities as shown in Fig. 2. However, in this

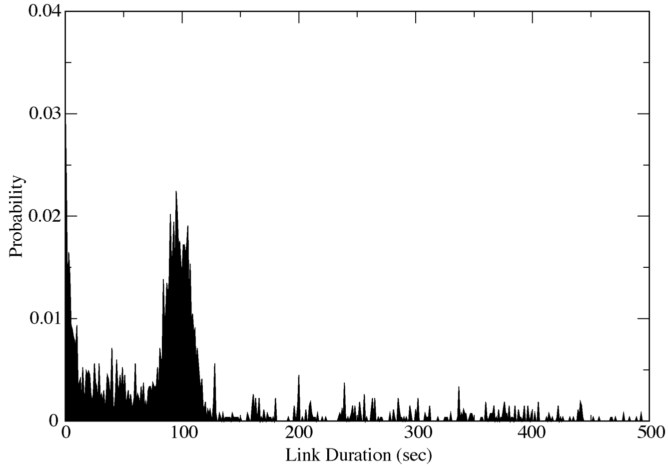

Fig. 1. PDF of the link duration for the FW model. Here, $V\max = 5$ m/s and $R = 250$ m.

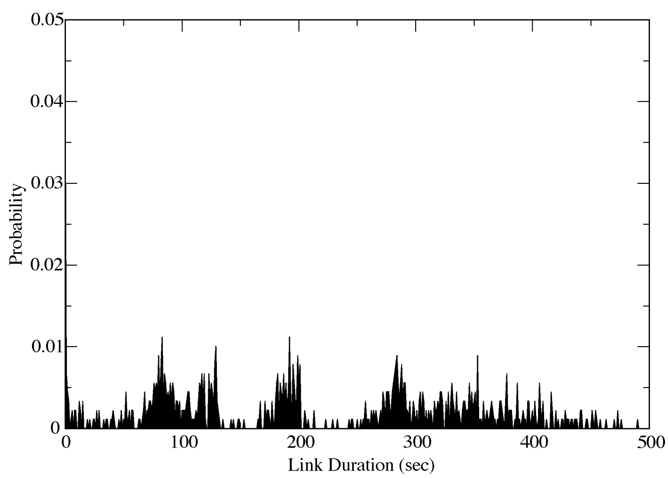

Fig. 2. PDF of the link duration for the RPGM model with four groups. Here, $V\max = 5$ m/s and $R = 250$ m.

case, we observe multiple peaks of almost similar size (for example, at around 100, 200, 280, and 350 s). The peak to the left of around 300 s are slightly larger and are due to the links between the nodes from different groups, and the peaks to the right of 300 s are due to the links between nodes within the same group. The area under the left peaks is more due to a larger number of intergroup links as compared to the intragroup links in our scenarios.

However, the link duration pdfs for the RW, MH, FW and the RPGM (with four groups) do not exhibit the multimodal behavior for $V_{\max} > 10$ m/s. The link duration pdf for the RW model, RPGM (four groups) and the FW model at $V_{\max} = 30$ m/s are shown in Figs. 3–5.

Moreover, for the RPGM (single group) model, it is observed that most of the links have a duration of around 900 s (simulation time), i.e., most of the links last for the entire duration of the simulation. Since it does not convey any new information, we do not show the link duration pdf for the single group case.

Having examined the link duration pdfs across the models used in this paper, we discuss the path duration pdfs at connectivity graph level in the next section.

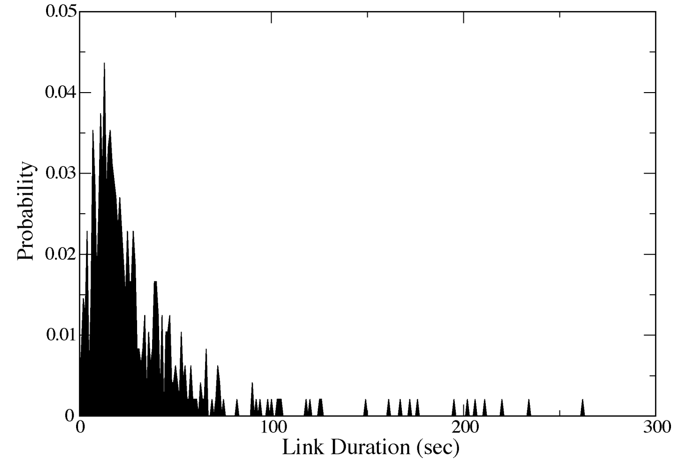

Fig. 3. PDF of the link duration for RW model. Here, $V\max = 30$ m/s and $R = 250$ m.

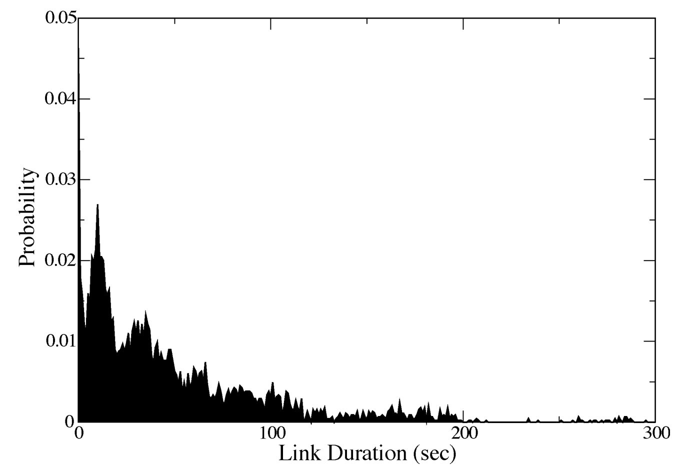

Fig. 4. PDF of the link duration for RPGM (four groups) model. Here, $V\max = 30$ m/s and $R = 250$ m.

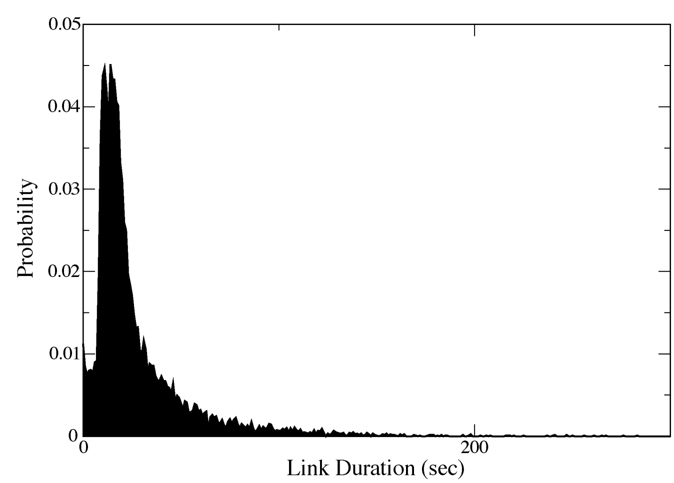

Fig. 5. PDF of the link duration for FW model. Here, $V\max = 30$ m/s and $R = 250$ m.

B. Path Duration PDFs at Connectivity Graph Level

We observe the multimodal behavior for the FW model and the RPGM model (with four groups) when $V_{\max}$ is small (i.e., around 1 or 5 m/s) and path length is short (i.e., only one or two

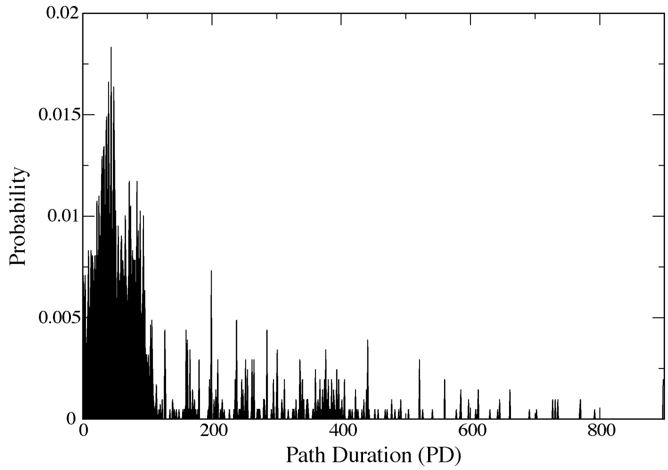

Fig. 6. PDF of path duration (at connectivity graph level) for the FW model. Here, $V\max = 5$ m/s and $h = 1$ hop.

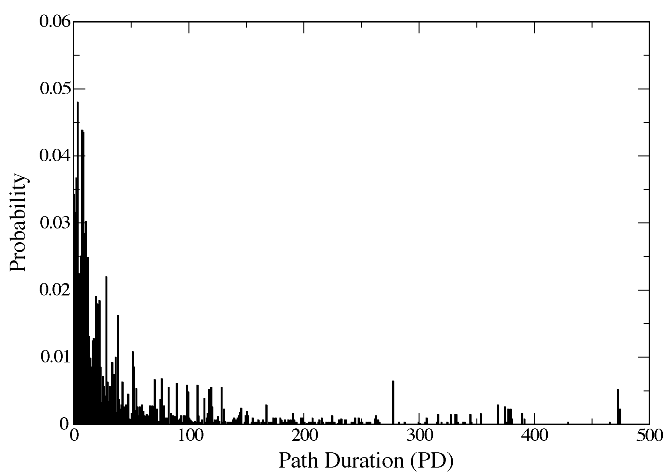

Fig. 7. PDF of path duration (at connectivity graph level) for the RPGM model with four groups. Here, $V\max = 5$ m/s and $h = 2$ hops.

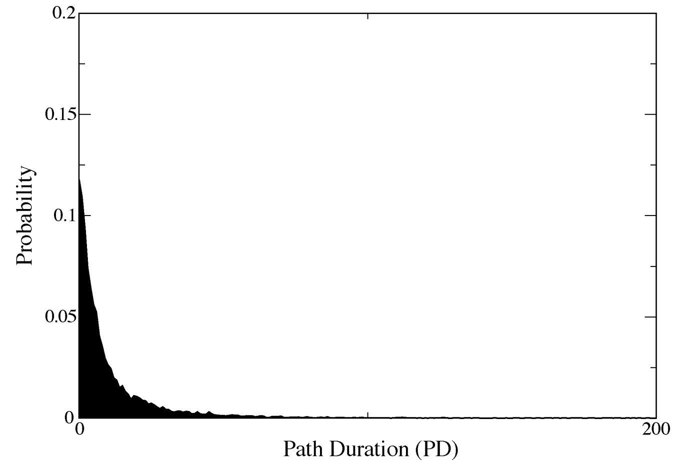

Fig. 8. PDF of path duration (at connectivity graph level) for RW model. Here, $h = 2$ hops, $V\max = 30$ m/s, and $R = 250$ m.

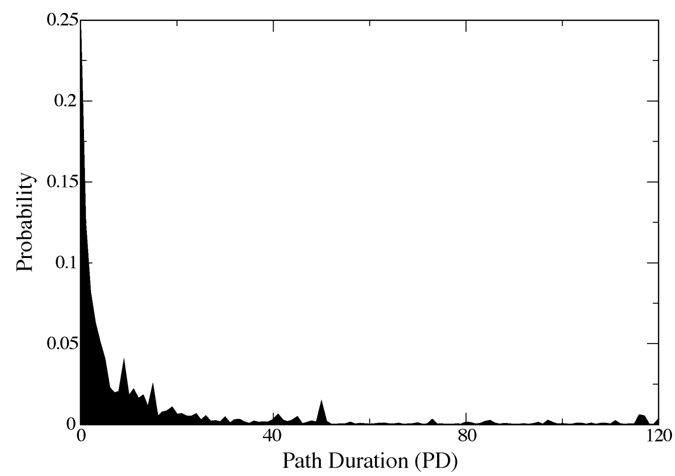

Fig. 9. PDF of path duration (at connectivity graph level) for the RPGM model with four groups. Here, $h = 4$ hops, $V\max = 30$ m/s, and $R = 250$ m.

hops). For example, as shown in Fig. 6, two strong concentrations of values exist in the path duration pdf for the FW model. The concentration of values on the left (at around 75 s) with large area seems to consist of paths containing nodes going in the opposite direction. The concentration of values on the right (at around 400 s) with a smaller area seems to consist of paths containing the nodes going in the same direction. We also notice a similar multimodal behavior for RPGM (with four groups) as shown in Fig. 7. The concentration of value with larger area on the left (at around 30 s) consists of paths having intergroup links, while the concentration of value with smaller area on the right (at around 110 s) is composed of paths containing intragroup links. Similar to the link duration pdf, the concentration of value on the left dominates the pdf for both FW and RPGM (with four groups) models.

From the multimodal pdfs for link and path duration for the FW and RPGM (with four groups) models at low speeds and small path lengths, we can learn some useful lessons about the effect of the traffic pattern on the protocol performance. At small speeds, if most of the communication traffic is between nodes on the same lane (for the FW model) or between nodes in the same group (for the RPGM model), greater will be the path duration for this traffic, which will also result in higher average path duration. Thus, intuitively the throughput will be higher. On the other hand, if most of the communication traffic is between nodes in opposite lanes (for the FW model) or between nodes in different groups (for the RPGM model), the path duration for this traffic will be lower, leading to a lower average path duration. This would result in lower throughput and higher routing overhead. Although, these explanations seem intuitive, we now have strong evidence (based on the concentrations of values in the path duration pdf) to back these intuitions.

The path duration pdf for the RW, FW, MH, and RPGM models seems to be exponentially distributed when $V_{\max} \geq 10$ m/s and $h \geq 2$. Figs. 8–10 show the path duration pdfs for the RW, FW, and RPGM (with four groups).

Thus, from our analysis, we observe that if $V_{\max} \geq 10$ m/s and $h \geq 2$, then the path duration at connectivity graph level for the RW, MH, FW, and RPGM can be approximated by an exponential distribution. In the next section, we show that the above conclusion holds for the path duration at routing protocol level.

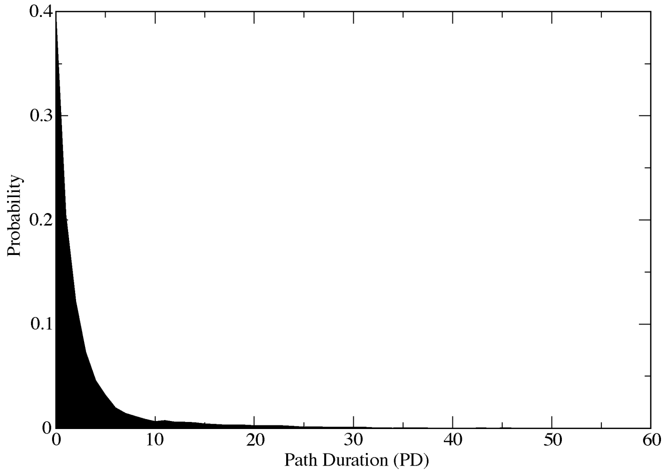

Fig. 10. PDF of path duration (at connectivity graph level) for the FW model. Here, $h = 3$ hops, $V\max = 30$ m/s, and $R = 250$ m.

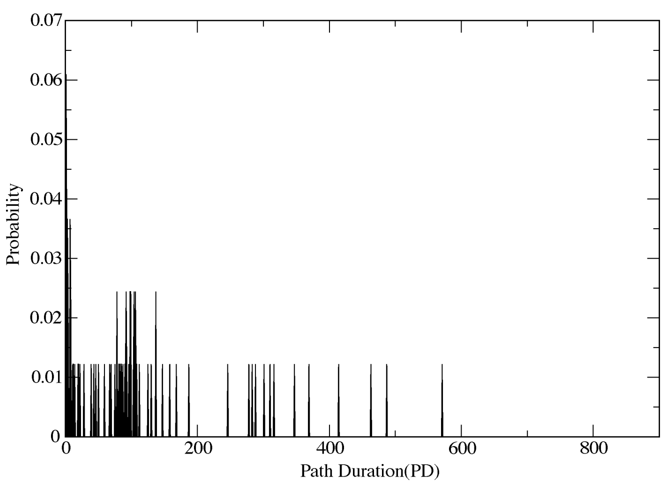

Fig. 11. Path duration pdf of DSR (at routing protocol level) for FW model. Here, $h = 1$ hop, $V\max = 5$ m/s, and $R = 250$ m.

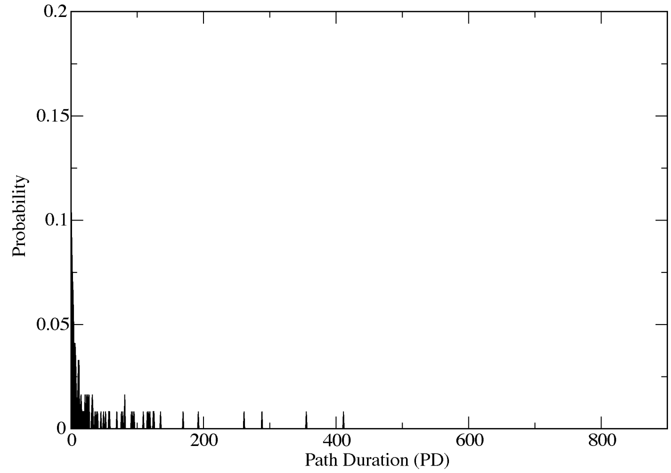

Fig. 12. Path duration pdf of AODV (at routing protocol level) for FW model. Here, $h = 1$ hop, $V\max = 5$ m/s, and $R = 250$ m.

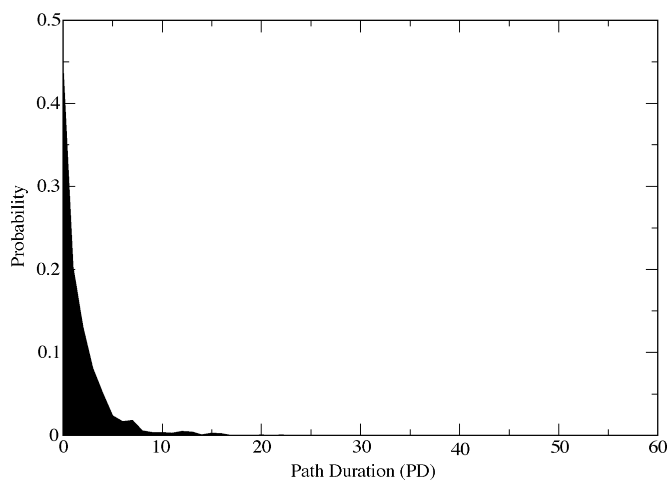

Fig. 13. Path duration pdf of DSR (at routing protocol level) for FW model. Here, $h = 3$ hops, $V\max = 30$ m/s, and $R = 250$ m.

C. Path Duration PDFs at the Routing Protocol Level

In addition to studying the path duration at connectivity graph through analyzing the mobility trace file, we also examine the duration of paths in the $ns-2$ simulations using the packet tracer program discussed in Section V-C. The results validate our conjecture that the path duration at connectivity graph level is a reasonable approximation of path duration at routing protocol level, even though they may differ from each other in some detailed aspects.

For both DSR and AODV in our $ns-2$ simulations, we again observe that the multimodal distribution for FW and RPGM model (with four groups) if path length is short and $V_{\max}$ is small. For example, as shown in Figs. 11 and 12, two concentrations of values can be observed in the pdfs of path duration at routing protocol level for FW model in DSR and AODV, respectively. Similarly, multimodal distribution is also found in RPGM model for DSR and AODV. Due to the limited space, we do not show the corresponding graphs. As the velocity and path length increase, i.e., $V_{\max} \geq 10$ m/s and $h \geq 2$, we observe that the pdfs of path duration at routing protocol level seems to be exponentially distributed as well. Figs. 13 and 14 show the path duration pdfs of DSR for FW and RW model, while Figs. 15 and 16 present the path duration of AODV for FW and RW model. Hence, the conclusions made for path duration at connectivity graph level in the previous section still hold for the path duration at routing protocol level.

When compared to the pdf of path duration at connectivity graph level calculated from mobility trace files, we find that pdfs of path duration at routing protocol level collected under $ns-2$ simulation are slightly "left-shifted," i.e., the center of the latter pdfs lie at the left side of the center of the former pdfs. It indicates that both DSR and AODV have a slightly smaller average path duration value than the average path duration calculated from the connectivity graph. Through tracing packets in the simulation, we identify the plausible reasons as follows.

1) Both DSR and AODV may choose the non-shortest paths in some scenarios, resulting in the lower path duration.
2) Both DSR and AODV may migrate the currently used path to another valid path with smaller metric or fresher sequence number even though the currently used path is not broken, which leads to a smaller path duration than that of connectivity graph where the currently used path does not migrate until it is broken.

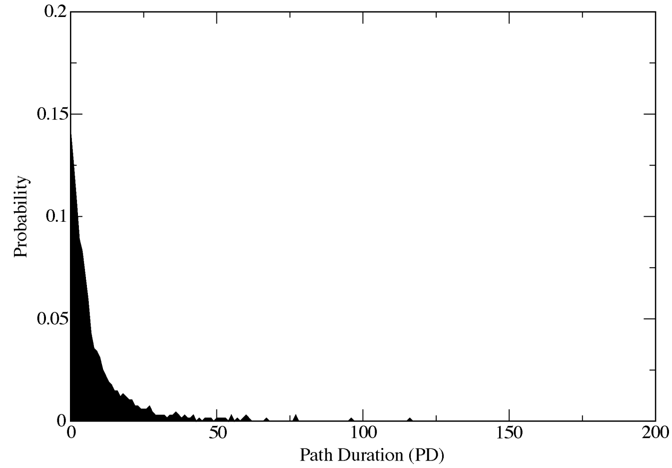

Fig. 14. Path duration pdf of DSR (at routing protocol level) for RW model. Here, $h = 2$ hops, $V\max = 30$ m/s, and $R = 250$ m.

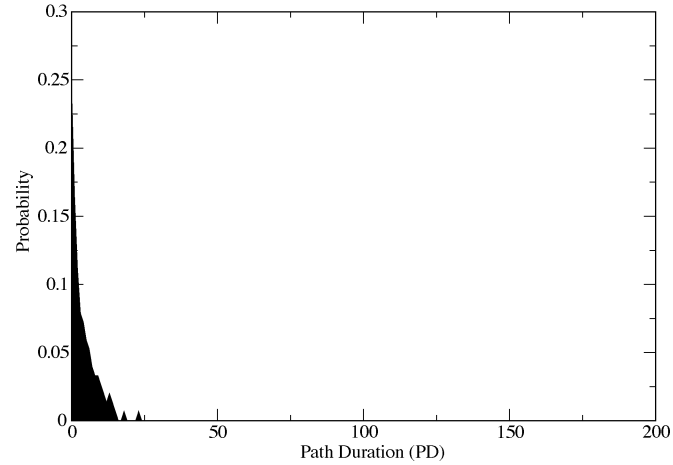

Fig. 16. Path duration pdf of AODV (at routing protocol level) for RW model. Here, $h = 2$ hops, $V\max = 30$ m/s, and $R = 250$ m.

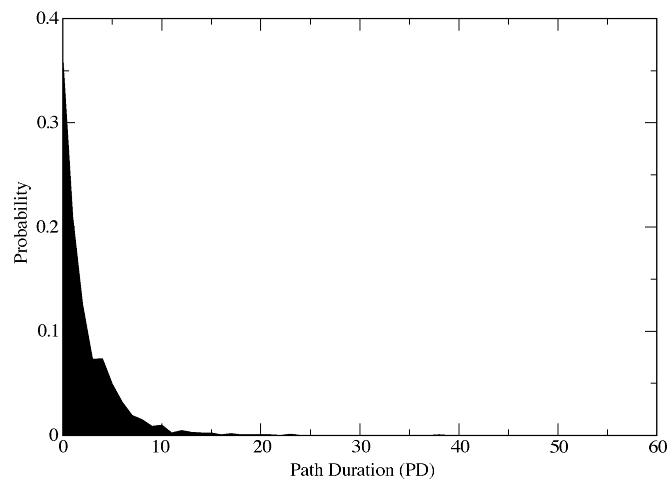

Fig. 15. Path duration pdf of AODV (at routing protocol level) for FW model. Here, $h = 3$ hops, $V\max = 30$ m/s, and $R = 250$ m.

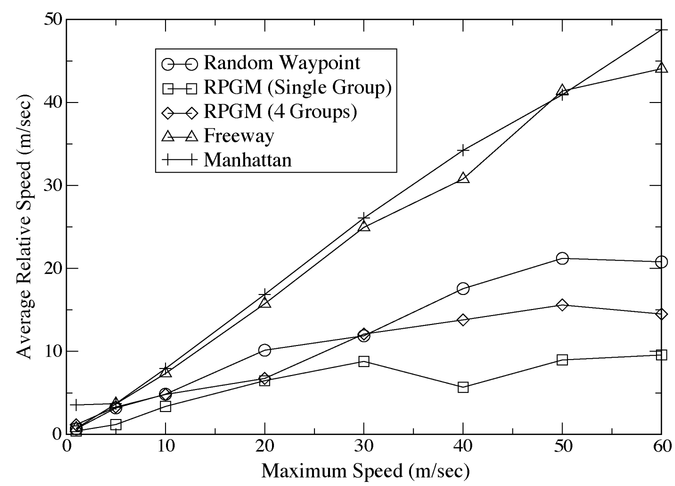

Fig. 17. Average relative speed.

Thus, since the path duration at routing protocol level is shown to be similar to the path duration at connectivity graph level, we only focus on the path duration at connectivity graph level in the remaining part of this paper. Without mentioning specifically, the term path duration means path duration at connectivity graph level.

In Section VI-D, we develop a simple analytical model to characterize the path duration pdf across these mobility models. We then relate the path duration statistics (PDF and average) to the performance of reactive protocols by using the case study of DSR in Section VII.

D. Analytical Model for the Path Duration PDF

For our study, we assume that the path duration for the mobility models is exponentially distributed. However, this assumption is valid only under the conditions mentioned at the end of Section VI-B. Now, we try to characterize this distribution for each mobility model, i.e., develop a model for the parameter λ_{path} of this distribution. Intuitively, λ_{path} has the following properties.

1) Greater the number of hops h in the path, the more likely a path is to break, thus, the average path duration decreases (i.e., λ_{path} increases). Hence, $\lambda_{\text{path}} \propto h$.
2) As the average relative speed V increases, link duration decreases and, hence, the average path duration decreases (i.e., λ_{path} increases). Hence, $\lambda_{\text{path}} \propto V$.
3) As the transmission range R increases, link duration increases, the average path duration increases (i.e., λ_{path} decreases). Hence, $\lambda_{\text{path}} \propto (1/R)$.

Thus

$$\lambda_{\text{path}} = \lambda_0 \frac{hV}{R} \tag{1}$$

where λ_0 is the constant of proportionality.

The above model for λ_{path} is verified by our simulations in Section V. Figs. 18–20 show that the average path duration estimated from the statistical analysis in Section V-B varies inversely as h, inversely as $V_{\max}$ and directly as R. In our analytical model, the average path duration is $1/\lambda_{\text{path}}$, since the path duration is assumed to be exponentially distributed with parameter λ_{path}. In Fig. 18, the curves for RPGM (four groups) and FW appear to be truncated. This is because in our scenarios, the longest path for these models has six and five hops, respectively,

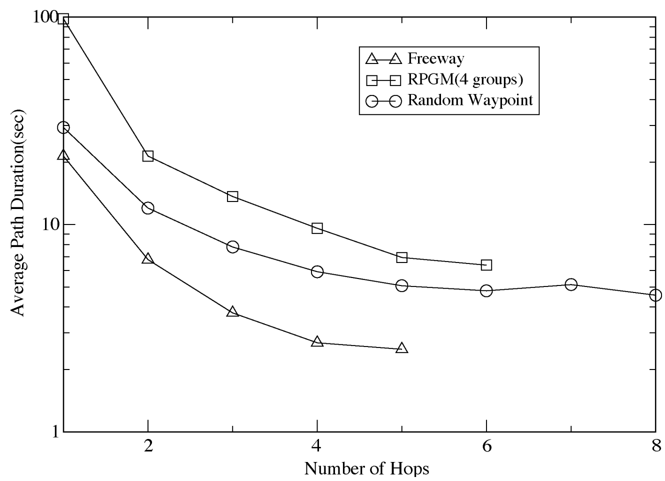

Fig. 18. Effect of h on the average path duration for $V\max = 30$ m/s and $R = 250$ m (inverse relationship).

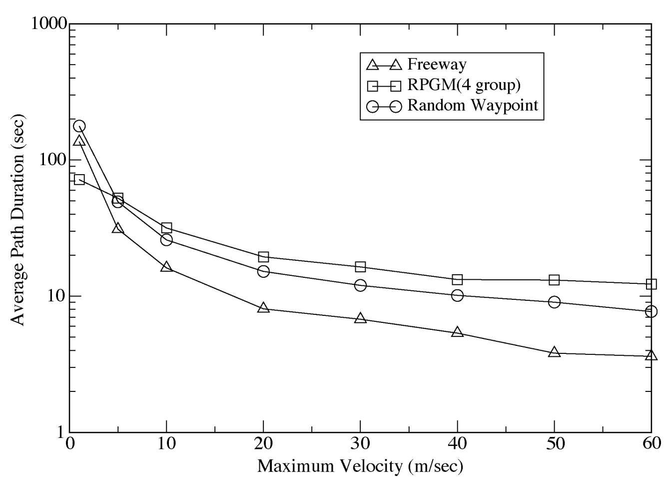

Fig. 19. Effect of Vmax on the average path duration for $h = 2$ and $R = 250$ m (inverse relationship).

while the RW model has a longest path of eight hops. Moreover, although we show the effect of $V_{\max}$ on the average path duration, the average relative speed and $V_{\max}$ are almost linearly related across all mobility models [21]. Hence, the relative speed will have a similar effect on the average path duration. The variation of average relative speed with $V_{\max}$ is shown in Fig. 17.

The constant λ_0 is independent of V, h, and R. The constant factor λ_0 is determined by the map layout, node density and other detailed parameters of mobility scenarios. Under the same mobility model, λ_0 remains same for various V, h, and R value. However, λ_0 is different for different mobility models; λ_0 is even different for the same mobility models with different map layout, node density or other parameters. In the Appendix, we show the values of λ_0 for some of our simulation scenarios. From these values, we observe that, in most cases, λ_0 remains almost constant for a given mobility model across different values of h, V, and R.

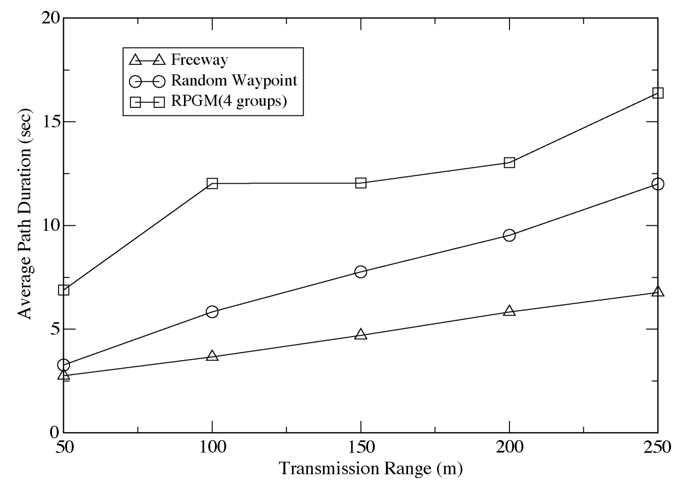

Fig. 20. Effect of R on the average path duration for $h = 2$ and $V\max = 30$ m/s (linear relationship).

Thus, the pdf of the path duration across most of the mobility models used in our study can be approximated as an exponential distribution

$$f(x) = \frac{\lambda_0 hV}{R} e^{\frac{-\lambda_0 hV}{R} x}. \tag{2}$$

The cumulative density function (CDF) of the path duration across the mobility models used in our study can be approximated as follows:

$$F(x) = 1 - e^{\frac{-\lambda_0 hV}{R} x}. \tag{3}$$

We conducted the Kolmogorov–Smirnov (K–S) test on these CDFs. The D-statistic for the pdfs shown in Figs. 8–10 is 0.13, 0.17, and 0.19, respectively, which shows that the exponential distribution is a reasonable approximation for the path duration pdf [29]. A detailed description of the K–S test and its results are shown in the Appendix.

In the next section, we show how this pdf can be related to trends in performance of reactive protocols.

VII. Utility of Path Duration Statistics

A. Relating the Average Path Duration to Performance of Reactive Protocols

As mentioned in Section I, one of the objectives of this study is to find whether the protocol performance is weakly (or strongly) linearly (or nonlinearly) related to the path duration. In this section, we give a simple first-order model that shows that the throughput and overhead are in a strong linear relationship with the reciprocal of the average path duration. We use the case study of DSR.

Before we derive the analytical model to study the relationship between path duration and protocol performance in terms of throughput and routing overhead, we first define the commonly used variables in this section. Let

N total number of nodes;

T total simulation time;

T_{flow} time during which actual data transfer takes place at maximum rate;
t_{repair} time spent to repair a broken path each time;
T_{repair} total time spent in repairing broken paths during the time T;
PD average path duration;
f frequency of path breaks $f = (1/(PD))$;
D total data transferred during simulation;
r data rate.

Now, we propose a simple first-order model relating the average path duration with throughput and routing overhead, respectively. We derive the following models based on DSR, but we believe these models can be applied to other reactive protocol like AODV with appropriate modifications.

Throughput: The throughput analysis is done as follows.

For each source-destination pair, the time T is composed of two parts: the time used to transfer data and the time used to repair the broken path. Thus

$$\begin{aligned} T &= T_{\text{flow}} + T_{\text{repair}} \\ &= T_{\text{flow}} + t_{\text{repair}} f T. \end{aligned} \tag{4}$$

Since $PD = (1/f)$, then

$$\begin{aligned} T_{\text{flow}} &= \left(1 - \frac{t_{\text{repair}}}{PD}\right) T \\ T &= \frac{T_{\text{flow}}}{1 - \frac{t_{\text{repair}}}{PD}}. \end{aligned} \tag{5}$$

Now

$$\begin{aligned} \text{Throughput} &= \frac{D}{T} \\ &= \frac{D}{\frac{T_{\text{flow}}}{1 - \frac{t_{\text{repair}}}{PD}}} \\ &= \left(1 - \frac{t_{\text{repair}}}{PD}\right) \frac{D}{T_{\text{flow}}} \\ &= \left(1 - \frac{t_{\text{repair}}}{PD}\right) r. \end{aligned} \tag{6}$$

Overhead: The overhead analysis is done as follows.

(T/PD) gives the number of route requests issued by DSR in time T. A fraction p (the nonpropagating cache hit ratio) of these requests is replied by the one-hop neighbors and, thus, needs only one route request transmission. For the remaining fraction $(1 - p)$, flooding of the route request will have to be done leading to N transmissions of the request. In general, the overhead of DSR (in terms of number of route request packets sent) can be given as follows:

$$\text{Overhead} = \frac{T}{PD}\left((p)1 + (1-p)N\right). \tag{7}$$

From (6) and (7), we make an interesting observation: *There exists a linear relationship between the reciprocal of the average path duration and the performance in terms of both throughput and routing overhead.* The correlation is positive between the reciprocal of the average path duration and overhead, while the correlation is negative between the reciprocal of the average path duration and throughput. Intuitively, higher path duration results in a higher throughput and lower overhead.

In order to validate the above models, we measure the Pearson coefficient of correlation between reciprocal of the average path duration and throughput we recorded in the experiments, we find that the coefficient between DSR throughput and the reciprocal of the average path duration for the same set of mobility patterns is −0.9165, −0.9597, and −0.9132 for RW, FW, and MH mobility models, respectively. Similarly, we also find that the coefficient between DSR overhead and the reciprocal of the average path duration for the same set of mobility patterns is 0.9753, 0.9812, and 0.9978 for RW, FW, and MH mobility models, respectively. The above facts indicate a strong correlation between the reciprocal of the average path duration and the performance of routing protocol DSR. Thus, the two simple analytical models (which do not capture MAC and physical layer effects) we propose are consistent with our experiment results (which do capture MAC and physical effects). For the RPGM model, such a strong correlation between the average path duration and protocol performance does not seem to exist. One plausible reason is that number of path changes is relatively small in RPGM model and, thus, the accuracy of estimation is affected.

Although the simple analytical models are derived based on DSR, we believe a similar approach can be extended to other reactive routing protocols such as AODV with some modifications. For example, when we analyze the overhead for AODV, the expanding ring search of AODV can also be modeled and used to predict the overhead of AODV for the FW model. In this case, in addition to nonpropagating cache hit ratio, the cache hit ratio at three, five, and seven hops will have to be taken into account.

The above first-order models do not include all factors affecting throughput and overhead, but they are useful in showing a strong linear relationship with $(1/PD)$, and can provide general trends that help in analyzing protocol behavior in many cases. These models, however, cannot be used to predict or estimate performance numbers. Nonetheless, we shall show an example of enriching the first-order model to provide prediction capability with low margins of error. Hence, in Section VII-B, we attempt to give a simple first-order model (by accounting for protocol convergence time) that may serve as a good predictor, particularly, for the throughput of DSR.

B. *Effect of Protocol Convergence Times*

Equations (6) and (7) assume that the frequency of path breakage is $f = (1/(PD))$. However, the time taken by a protocol to converge onto a path is also an important factor, while considering throughput (or overhead) of the protocol. Thus, although throughput (overhead) from (6) ((7)) has a strong correlation with the pat duration measured by simulations, the t_{repair} at the protocol level might impact the accuracy of prediction. In order to test this hypothesis, we propose a simple extension to the (6). In this case, we assume that the frequency of path breakage is $f = (1/(PD + t_{\text{repair}}))$. Thus,

TABLE I
1-NORMALIZED THROUGHPUT/SIMULATION THROUGHPUT * 100 FOR VARIOUS MOBILITY MODELS AND VELOCITIES

V_{max} (m/sec)	RW	RPGM (Single Group)	RPGM (Multiple Groups)	FW	MH
1	7.4647	0.00409	-0.043669	0.89971	3.88592
5	3.78451	-0.030021	0.09369	3.13604	5.57231
10	5.88434	0.00361	-0.169056	5.32633	7.41357
20	33.4858	0.00676	0.32781	7.02482	9.90664
30	8.69539	0.00149	0.25698	8.13975	11.5396
40	9.47452	0.0903	0.054981	9.6315	13.323
50	9.20675	0.23585	0.4586	11.7304	11.2949
60	9.90577	-0.003089	0.86554	11.6026	11.4947

replacing $(1/PD)$ by $(1/(PD+t_{\mathrm{repair}}))$ in (6), the normalized throughput for DSR can be given as follows:

$$\frac{\text{Throughput}}{r} = \left(1 - \frac{t_{\mathrm{repair}}}{PD + t_{\mathrm{repair}}}\right)$$
$$\text{Normalized Throughput} = \frac{PD}{PD + t_{\mathrm{repair}}} \qquad (8)$$

We measure the PD and the t_{repair} for DSR across the various mobility scenarios mentioned in Section V-A. We compare the normalized throughput computed by (8) with the throughput values obtained from $ns-2$ simulations. The difference between the normalized throughput and the throughput obtained from simulations for the various models used in our study is given in Table I.

Most of the error values are between 0%–12% (except for the RW model at 20 m/s, which is due to the occasional simulation crash) and, thus, (8) seems to be a good predictor of the DSR throughput for the scenarios used in our study. A similar model that takes t_{repair} into account can be developed for the routing overhead of DSR.

In these two sections, we have shown the utility of the average path duration in determining general trends in the performance of reactive MANET protocols. In the next section, we will discuss the utility of another statistic of path duration, i.e., pdf of the path duration.

C. Relating Path Duration PDF to Protocol "Building Blocks"

In this section, we demonstrate the utility of characterizing the pdfs of path durations for some mobility models. To be more specific, as a case study, we propose an analytical model for the nonpropagating cache hit ratio p (which is used in (7) in Section VII-A) of DSR in the FW model. This analysis assumes that whenever a route breaks, the source initiates a route discovery. Usually, in DSR, an intermediate node detecting a route break attempts to recover from the route error. This procedure is called salvaging. However, [22] observed that salvaging repairs a very small percentage of route breaks. Hence, in this study, we assume that a recovery from route error is done only by the source by reissuing a route request. Moreover, we also assume that a cache entry remains valid for a much longer period compared to the path duration.

Consider the following scenario.

Let the FW consists of two lanes, one lane going toward the right and another to the left.

Let

R	transmission radius of the mobile nodes;
S, D	source and destination, respectively; assume that S and D are on the same lane of the FW;
h	number of hops between S and D;
$d_{SD}(0) = hR$	assume the distance between S and D at time $t = 0$ when the path between S and D is created;
i	ith node around the path from S to D (on both the lanes), these are the nodes which can overhear D's route reply and cache the route to the destination D;
V_i	relative speed of node i with respect to S. $+V_i$ indicates that i is moving to the right, while $-V_i$ shows that i is moving to the left, i.e., in this frame of reference, S is the origin and is stationary;
$X_i(0)$	position of node i at time $t = 0$ when the path between S and D is created;
t_{pd}	path duration time;
$X_i(t_{pd}) = X_i(0) + V_i t_{pd}$	position of node i at time t_{pd};
l_s	set of nodes on the same lane as S and D that can overhear D's route reply;
l_o	set of nodes on the opposite lane that can overhear D's route reply;
p_s	probability that $\forall k_{k \in l_s} X_k(t_{pd}) > R$, i.e., none of the nodes having the cache in the same lane as S are within the transmission range of S at time $t = t_{pd}$;
p_o	probability that $\forall j_{j \in l_o} X_j(t_{pd}) < -R$, i.e., none of the nodes

having the cache in the opposite lane are in the range of S at time $t = t_{pd}$;

p probability that none of the nodes having the cache are within the transmission range of S at time $t = t_{pd}$.

$$p = p_o p_s. \tag{9}$$

We now evaluate p_o and p_s. Since the average relative speed of nodes on the same lane with respect to S is different from that in the opposite lane, both the lanes will have different pdf (and CDF) for the path duration as shown in (2) and (3). Hence, we consider them separately in this analysis.

Determining p_o: If the farthest node (having the cache) in the opposite lane moves out of range of S, all the nodes (having the cache) would have moved out of range of S. Thus, p_o is now the probability that the farthest node (having a cache) in the opposite lane moves out of the range of S. Let this farthest node be j'. Let $X_{j'}(0) = d_{SD}(0) + R$

$$\begin{aligned} p_o &= P\left(X_{j'}(t_{pd}) < -R\right) & (10)\\ &= P\left(d_{SD}(0) + R - V_o t_{pd} < -R\right) \\ &= P\left(-V_o t_{pd} < -2R - d_{SD}(0)\right) \\ &= P(V_o t_{pd} > 2R + hR) \\ &= P\left(t_{pd} > \frac{R(2+h)}{V_o}\right) \\ &= 1 - F_o\left(\frac{R(2+h)}{V_o}\right) & (11) \end{aligned}$$

where V_o is the average relative speed in the lane opposite to S and D. F_o is the CDF of the path duration on the opposite lane. Thus, from (3)

$$p_o = e^{-\lambda_0 h(2+h)} \tag{12}$$

i.e., p_0 is independent of the speed or relative speed of the nodes.

Next, we evaluate p_s.

Determining p_s: Now, in the case of the same lane, the node closest to S also has the cached route to D. If this node moves out of range, all nodes (having the cached entry) in the same lane have moved out of range. Thus, p_s is now the probability that the closest node (having the cached entry) in the same lane moves out of the range of S. Let this closest node be k', which may be arbitrarily close to S, i.e., $X_{k'}(0) \approx 0$

$$\begin{aligned} p_s &= P\left(X_{k'}(t_{pd}) > R\right) & (13)\\ &= P(V_s t_{pd} > R) \\ &= P\left(t_{pd} > \frac{R}{V_s}\right) \\ &= 1 - F_s\left(\frac{R}{V_s}\right) & (14) \end{aligned}$$

where V_s is the average relative speed in the same lane as S and D. F_s is the CDF of the path duration on the same lane. Thus, again from (3)

$$p_s = e^{-\lambda_0 h} \tag{15}$$

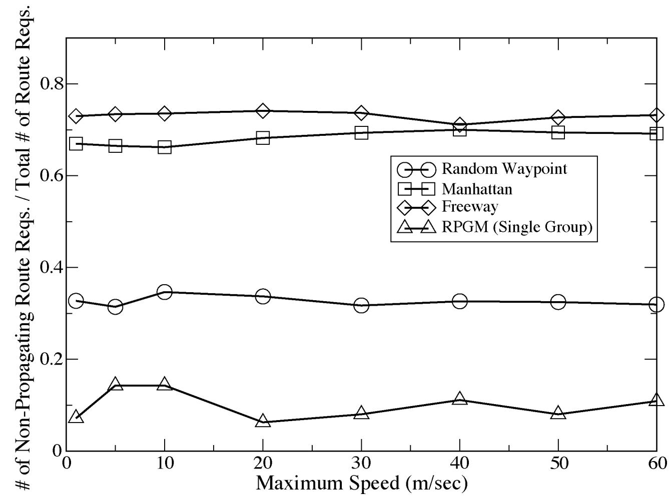

Fig. 21. Nonpropagating cache hit ratio in DSR, where $R = 250$ m and number of Nodes $= 40$.

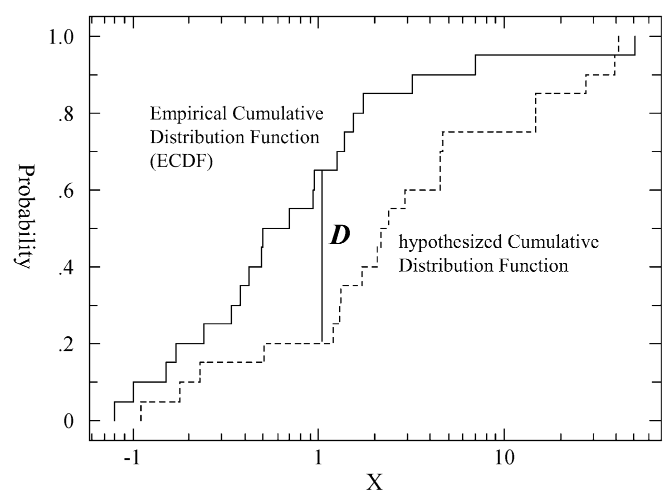

Fig. 22. Illustration of D-statistic in the K–S test.

i.e., p_s is independent of the speed or relative speed of the nodes.

Thus, from (9), (12), and (15)

$$p = e^{-\lambda_0 h(3+h)}. \tag{16}$$

Therefore, the cache hit ratio is given by

$$Hit = 1 - p = 1 - e^{-\lambda_0 h(3+h)}. \tag{17}$$

Thus, *the nonpropagating cache hit ratio in DSR is independent of the speed or the relative speed of the nodes for the FW mobility model*. This concurs with the simulation-based analysis in [22], where the nonpropagating hit ratio of DSR seems to be independent of the maximum speed (beyond 30 m/s). Moreover, this effect is not only observed for the FW but also for the RW, RPGM, and the MH models as shown in Fig. 21.

Thus, the exponential assumption of path duration pdfs turns out to be a good approximation for our study.

In this section, we gave simple first-order models relating the average path duration to the throughput and overhead of reactive protocols. However, from Section VI-D, for our simulations, we observed that the average path duration is directly proportional to the transmission range R and inversely proportional to the

TABLE II
VALUES OF THE D-STATISTIC, λ AND λ_0 ACROSS SEVERAL MOBILITY MODELS, VELOCITIES AND PATH LENGTHS. HERE $R = 250$ m

Mobility Model	Velocity (m/sec)	Path Length (hops)	D-Statistic	Best-fit λ	λ_0
RW	30	2	0.000284	0.122936	0.51221
RW	30	3	0.003362	0.16485	0.457908
RW	40	2	0.020663	0.184148	0.575433
RW	40	3	0.001166	0.277466	0.577984
RW	50	2	0.000070	0.183599	0.458961
RW	50	3	0.001182	0.300123	0.5002
RW	60	2	0.000202	0.243292	0.506818
RW	60	3	0.000449	0.338496	0.470096
FW	30	2	0.000537	0.306102	1.275246
FW	30	3	0.001190	0.500652	1.39061
FW	40	2	0.001134	0.360458	1.126364
FW	40	3	0.000699	0.654857	1.364122
FW	50	2	0.000893	0.569905	1.424757
FW	50	3	0.001010	0.960106	1.600176
FW	60	2	0.000601	0.601898	1.253936
FW	60	2	0.001208	0.975436	1.354715
MH	30	2	0.013150	0.282193	1.175776
MH	30	3	0.001277	0.460868	1.280102
MH	40	2	0.000846	0.386492	1.207742
MH	40	3	0.000586	0.602844	1.255865
MH	50	2	0.000779	0.419397	1.048416
MH	50	3	0.001424	0.73712	1.228444
MH	60	2	0.004663	0.540425	1.125792
MH	60	2	0.000141	0.872245	1.211386
RPGM (Multiple Groups)	30	2	0.000261	0.133234	0.555093
RPGM (Multiple Groups)	30	3	0.07315	0.04675	0.12988
RPGM (Multiple Groups)	40	2	0.012927	0.174228	0.544705
RPGM (Multiple Groups)	40	3	0.004441	0.280639	0.58463
RPGM (Multiple Groups)	50	2	0.000018	0.301081	0.752151
RPGM (Multiple Groups)	50	3	0.004441	0.280639	0.396321
RPGM (Multiple Groups)	60	2	0.000195	0.155777	0.324405
RPGM (Multiple Groups)	60	3	0.000217	0.478138	0.66374

number of hops h and average relative speed V. Thus, our first-order models also relate the protocol throughput and overhead to several factors including the transmission range, the average number of hops in the path, the relative speed of the mobility model used. Thus, this entire approach has given us a greater understanding of the impact of mobility and other factors on protocol performance.

VIII. CONCLUSION AND FUTURE WORK

We proposed an approach for a deeper understanding of the effect of mobility on MANET routing protocols. To begin with, this approach examined the detailed statistics (including pdfs) of link and path duration across a rich set of mobility models. For small velocities, these pdfs were observed to have a multimodal distribution across some of the models used. This observation showed the impact of the traffic pattern on the path duration pdf. For moderate and high velocities, across the mobility models used in our study, it was observed that the path duration pdfs for paths of two or more hops can be approximated by an exponential distribution, which is parameterized by the relative speed of the mobility model, the transmission range of the node and the number of hops in the path. We proposed simple analytical models that show that the reciprocal of the average path duration is strongly correlated with the throughput and overhead of reactive routing protocols. Simulations for DSR seemed to confirm this relationship. We also illustrated how the path duration pdf can be related to detailed protocol mechanisms by analytically deriving the nonpropagating cache hit ratio of DSR in the FW model. Our model concurred with the simulation results in a related study.

Thus, path duration seemed to be a good metric to predict the general trends in the performance of reactive routing protocols. At the same time, our analytical models showed the relationship between the path duration and other parameters like the average relative speed of the mobility model, the transmission range of the mobile nodes and the average number of hops in the path. These findings enabled us to relate the several parameters including mobility to the performance of reactive protocols using the detailed statistics of path duration.

As part of our future work, one of our immediate goals would be to develop an analytical model for the nonpropagating cache hit ratio for all the other models used in our study. It would also be interesting to see how this ratio is affected by the communicating traffic pattern. As a longer-term goal, we seek to use richer analytical models to predict the performance trends of the reactive MANET routing protocols like DSR and AODV. We believe that by analyzing the basic mechanisms of these protocols, we can develop a comprehensive model for the "whole" protocol. Moreover, this analysis can be readily applied to other protocols that use similar mechanisms.

APPENDIX I
KOLMOGOROV–SMIRNOV TEST AND D-STATISTIC

Usually, the Chi-Square test is used to verify the hypothesis that the given data is drawn from a particular probability distribution. However, the result of chi-square test is sensitive to the adequate choice of the number and size of the sample bins. Hence, in this paper, we used the Kolmogorov–Smirnov goodness-of-fit test (K–S test).

The K–S test is a rigorous test that does not depend on binning. The test is as follows.

Given $F_1(x)$, the expected hypothesized CDF derived in Section VI-D, K–S test compares $F_1(x)$ to $F_2(x)$, the empirical cumulative distribution function (ECDF) obtained by simulations. The result of K–S test is based on the value of the greatest discrepancy between the observed and expected cumulative distribution, which is called the D-statistic. The example of D-statistic is shown in Fig. 22.

The D-statistic is formally defined as follows:

$$D = \max_x \left\| F_1(x) - F_2(x) \right\|.$$

In order to test the hypothesis, the D-statistic value should be compared with a certain threshold value. However, in this study, since we aim to approximate the ECDF by the CDF (and not to prove that they are identical), we only examine the D-statistic value. If this value is small, it indicates that the ECDF can be approximated by the CDF.

In the next section, we show the D-statistic across several ECDFs, which indicate that they can indeed be approximated by the exponential CDF as shown in Section VI-D.

APPENDIX II
RESULTS OF K–S TEST

In this section, we compare the D-statistic of the ECDF and the best fit exponential distribution curve. The D-statistic values across several mobility models is observed to be quite small. For the best fit curve obtained via the maximum likelihood test, we compute the factor λ_0 (which is defined in Section VI-D) as follows:

$$\lambda_0 = \lambda \frac{R}{hV}. \tag{18}$$

We also notice that λ_0 is almost constant for a given mobility model across R, h. and V. Due to space constraints, we only show the data for the case when $R = 250$ m in Table II.

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