Modeling Permanent Deformation of Unbound Granular Materials under Repeated Loads

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Abstract: Finite-element analysis on a pavement structure under traffic loads has been a viable option for researchers and designers in highway pavement design and analysis. Most of the constitutive drivers used were nonlinear elastic models defined by empirical resilient modulus equations. Few isotropic/kinematic hardening elastoplastic models were used but applying thousands of repeated load cycles became computationally expensive. In this paper, a cyclic plasticity model based on fuzzy plasticity theory is presented to model the long-term behavior of unbound granular materials under repeated loads. The discussion focuses on the model parameters that control long-term behavior such as elastic shakedown. The performance of the constitutive model is presented by comparing modeled and measured permanent strain at various numbers of load cycles. Calculated resilient modulus from the complete stress-strain curve is also discussed.

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Introduction

Permanent deformation is one of the important factors used to evaluate the performance of a pavement structure under a service load. Depending on the stress history in a pavement due to a moving wheel load, the accumulated permanent deformation could keep increasing with load cycles or reach a stabilized value which is called elastic shakedown (Werkmeister et al. 2004; García-Rojo and Herrmann 2005). For moderate stress levels, elastic shakedown is expected where the material response is typically characterized by the resilient elastic modulus after numerous cycles. It is preferred so that the life of the pavement lasts longer. Through laboratory and field measurements and observations, empirical equations have been widely used in predicting permanent deformation under repeated load (Lekarp et al. 2000a). It is straightforward to use for practical design; however, they normally lack a physical framework to be formulated and a well-defined stress-strain relationship. Because of that, finite-element analysis has been popular in analyzing and modeling a pavement structure under traffic load. In order to carry out an accurate analysis, a constitutive model capable of describing material behaviors under repeated loading is desired. Most of the constitutive models used in finite-element analysis for pavement structures are nonlinear elastic models defined by empirical resilient modulus equations (Lekarp et al. 2000b; Kim and Tutumluer 2008). Few isotropic/kinematic hardening elastoplastic models were used (e.g., Chazallon et al. 2006; Johnson and Sukumaran 2009) because applying thousands of repeated load cycles became computationally expensive.

A majority of the models for granular materials is based on classical plasticity theory, where kinematic hardening or mixed hardening (isotropic and kinematic hardening) is normally used to mimic hysteretic phenomena such as reverse plastic flow and memory of particular loading events. However, they are sophisticated and often difficult to implement in constitutive drivers within reliable finite-element codes.

Multisurface plasticity, bounding surface plasticity, and generalized plasticity theories have been successfully used to model cyclic behavior of granular materials. In multisurface plasticity (e.g., Prévost 1982), multiple yield surfaces take the shape of nested subspaces in stress space, where the stress-strain behavior within the innermost surface is assumed to be elastic. The instantaneous configuration of the field of yield surfaces was established by computing the parameters and equations that govern the translation, expansion, or contraction of individual surfaces during proportional as well as nonproportional loading and unloading. In bounding surface plasticity (e.g., Manzari and Dafalias 1997), the plastic strain occurs for stress states within the bounding surface. It is also possible to have a very flexible and smooth variation of the plastic modulus during straining, unlike the multisurface plasticity model which assumes piecewise constant plastic moduli. As for generalized plasticity (e.g., Pastor and Zienkiewicz 1986), both plastic flow direction and plastic modulus for loading and unloading are defined explicitly where dilatancy was approximated by a linear function of stress invariant ratio, as proposed by Nova and Wood (1979).

A cyclic plasticity model based on fuzzy set plasticity theory is presented in this paper to model the accumulated permanent axial strain and shakedown behavior of unbound granular materials under repeated loads. The concept of the fuzzy set plasticity was

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first introduced by Klisinski et al. (1988) and its theory and formulation have been described by several researchers (e.g., Klinsinski 1988; Klisinski et al. 1991; Arduino and Macari 2001; Ge and Sture 2003a,b). The model presented in this paper is capable of simulating realistic stress-strain behaviors under repeated load cycles including nonlinear dilatancy, material memory, accurate reverse loading feature, nonproportional loading, and long-term cyclic effects. In the following sections, the framework of the fuzzy set plasticity theory is first introduced, followed by its model formulation, calibration, and performance.

Model Formulation

The original fuzzy set plasticity model consists of deviatoric and locking fuzzy surfaces to account for material responses under purely deviatoric shearing and isotropic compression/extension conditions. In this study, the model has been simplified, and only deviatoric component of the original fuzzy set plasticity model was used for the simulation. Further and detailed information about fuzzy set plasticity can be found at Ge (2003).

The cone fuzzy surface in compression is expressed as

$$F_c = r - a_0 - a_1 p = 0 \tag{1}$$

which represents a three-stress-invariant yield surface. p is the mean stress and is one-third of first stress invariant I_1. r is the multiplication of q and g, where q is defined in a way similar to second deviatoric stress invariant J_2 and g=Willam-Warnke function (Willam and Warnke 1974). For proportion loading, g is 1 so that $r=q$.

The coefficient a_1 in the cone fuzzy surface for triaxial compression is a density dependent parameter

$$a_1 = M_c + \kappa\langle -\psi\rangle \tag{2}$$

where M_c=stress ratio q/p at critical state condition; κ =constant; ψ=state parameter which is defined as the difference between the current void ratio e and the void ratio at its critical state at a given mean stress; and $\langle\,\rangle$=Macaulay brackets. For loose granular materials, the current void ratio is always greater than its value at critical state, i.e., $\psi \geq 0$, so $a_1 \equiv M_c$ and the fuzzy surface remains fixed in the stress space. For dense materials, the fuzzy surface moves along with the volumetric strain which is controlled by the shear dilatancy rule described later.

Membership Function

The membership functions γ from fuzzy set theory were introduced to construct reversal plastic loading without resorting to a kinematic hardening rule. A value of the membership function ranging from 0 to 1 is assigned and associated with a given stress state. The stress state with the value of the Membership Function 1 is at a fully elastic state and 0 at the fully plastic state. The basic rules of kinematic mechanism of the membership functions are as follows:

1. Plastic loading: $\dot{\gamma} < 0$;
2. Plastic unloading: $\dot{\gamma} < 0$;
3. Elastic loading: $\dot{\gamma} \geq 0$; and
4. Elastic unloading: $\dot{\gamma} \geq 0$.

Although the value of the membership function is 1 at a fully elastic state and 0 at the fully plastic state, the assignment of the value in elastoplastic state is deterministic and can be arbitrarily defined as needed. A linear variation with respect to stress state was adopted in this study. For illustration purposes, the kinematic mechanism of the cone membership function γ is shown in Fig. 1, which represents plastic loading from Point a to Point b, followed by elastic unloading from Point b to Point c. After Point c is reached, unloading with associated decrease of the value of the membership function results in plastic deformation, which is the feature of the membership function. The degree of material memory α can be simulated by predetermining the location of Point c. Furthermore, by controlling the location of Point c, one can also model elastic shakedown behavior, where Point c moves toward a point as cyclic loading proceeds. Plastic unloading, or reversal loading, is shown from Point c to Point d, followed by elastic loading from Point d to Point e and plastic loading from Point e to Point b again, as shown in Fig. 1.

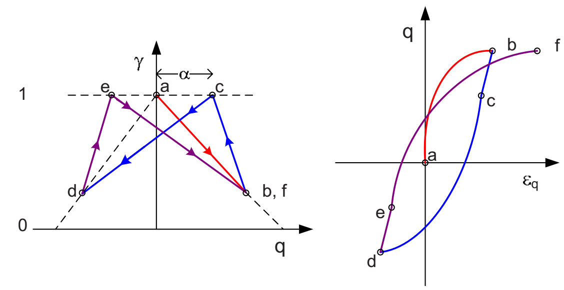

Fig. 1. Membership function and its corresponding stress-strain state

Material Memory

The material memory parameter α represents the material degree of memory and it shows the evolution of elastic and plastic deviatoric behaviors during the entire loading and unloading (reversal loading) process. For $\alpha=0$, it represents that the material has no memory and it shows fully elastic behavior during the entire unloading process. For $\alpha=1$, it represents that the material has maximum degree of memory and it shows fully plastic behavior during the entire unloading process. In order to describe the evolution of elastic and plastic behaviors of unbound granular materials under repeated loading, the material memory function is given as

$$\alpha = \alpha_0 (N_i)^{m_3} \tag{3}$$

where α_0=initial value of α; N_i=nth number of load application; and m_3=parameter controlling the evolution of material memory parameter with load cycles. Fig. 2 shows the evolution of memory function parameter under different m_3.

Flow Rules

The plastic strain increments follow the flow rules in classical plasticity theory

$$\dot{\boldsymbol{\varepsilon}}^p = \dot{\lambda}\mathbf{m} \tag{4}$$

where $\dot{\lambda}$=magnitude and $\boldsymbol{m}$=direction of the plastic strain increments. In the fuzzy set plasticity, $\boldsymbol{m}$ is not determined through the gradient of plastic potential; instead, it is defined through a fourth tensor $\boldsymbol{T}$, i.e., $\mathbf{m}=\mathbf{T}:\mathbf{n}$.

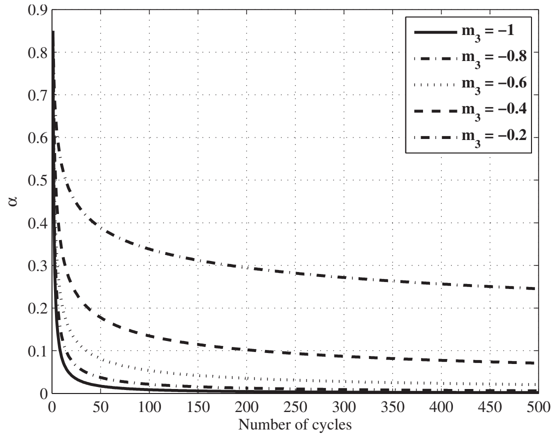

Fig. 2. Evolution of material memory parameter α under different m_3

In p-q space

$$\mathbf{T} = \begin{bmatrix} D & 0 \\ 0 & 1 \end{bmatrix} \quad \text{and} \quad \dot{\boldsymbol{\varepsilon}}^p = \begin{pmatrix} \dot{\varepsilon}_v^p \\ \dot{\varepsilon}_d^p \end{pmatrix} = \mathbf{T}:\mathbf{n} = \begin{bmatrix} D & 0 \\ 0 & 1 \end{bmatrix} \begin{pmatrix} \dfrac{\partial F_c}{\partial p} \\ \dfrac{\partial F_c}{\partial q} \end{pmatrix} \tag{5}$$

Shear dilatancy incorporating current stress state η and critical state condition M_c is defined as

$$D = A(-M_c\kappa_d + \eta) \tag{6}$$

where A and κ_d=model parameters and η=current stress ratio q/p.

Plastic Modulus

At each stress state, once the value of the membership function γ is defined, it is used to determine the plastic modulus on the loading surface, which is given as follows:

$$H = H^* + \frac{M\gamma^d}{1+\gamma^{d+1}} \tag{7}$$

where M and d=model parameters and H^*=plastic modulus at the image stress on the fuzzy surface. In this paper, the evolution of the deviatoric plasticity modulus function parameter d is introduced as the following equation to account for the long-term effects on the incremental plastic strain:

$$d = d_0\left[1 - m_1\left(\frac{N_i}{N_{\text{cyc}}-1}\right)^{m_2}\right] \tag{8}$$

where N_{cyc}=total number of load cycles; N_i=number of current load cycle; m_1 and m_2=exponential coefficients depending on the shear strain level; and d_0=initial values of the deviatoric plastic modulus function parameter. This can be used to simulate the elastic shakedown behavior when the material behaves elastically after certain amount of load cycles.

Model Calibration

With the advance of constitutive modeling, the parameters and constants required in elastoplastic models can be tremendous. Calibration of these constitutive models is not an easy task. In addition to different types of laboratory experiments demanded, a systematic approach for model calibration is desired. Conventional method, such as linear regression, is not sufficient to identify the model parameters because many of them cannot be described in linear fashion through laboratory testing data. Besides, some of these model parameters are with less physical meaning, which cause difficulties in model calibration. With that in mind, a numerical optimization technique of nonlinear least-squares regression is applied to the constitutive model calibration. Optimization problems are generally defined as minimizing the objective function $f(\boldsymbol{x})$ subject to decision variable vector $\boldsymbol{x}$. Numerical optimization algorithms can be categorized into three groups according to the type of information needed in searching for the minimum of the objective functions. The simplest way to minimize the objective function is to randomly choose a sufficiently large number of candidate vectors $\mathbf{x}$ and evaluate the objective function for each of them. In calibrating a fuzzy set plasticity model, the objective function is defined by the Euclidean distance between an experimental point and a theoretical point, and a random search method is used.

Random search method is considered to be the most inefficient but most easily implemented among the zeroth-order methods. With that, random search method was adopted to calibrate the fuzzy set model in this paper. Moreover, the inefficiency can be overcome by the aid of modern high-speed computers. Assume that the variables x_i^* for $i=1-n$ lie between its lower bound x_i^l and upper bound x_i^u, there must exist a R_i such that $x_i^*=x_i^l+R_i(x_i^u-x_i^l)$ and minimize the objective function f.

Since constitutive models are being calibrated, it is intuitive to use stress and strain as variables in the objective function. It is then straightforward to formulate the objective function as the sum of distances from computed points to their adjacent experimental points in the stress-strain space. For each computed strain lies between $\varepsilon_j^{\text{exp}}$ and $\varepsilon_{j+1}^{\text{exp}}$, the distance between the computed and experimental strains can be calculated. The objective function is constructed as follows:

$$f = \sum_{i=1}^{n} \frac{1}{2}\left(\sqrt{\left(1-\frac{\sigma_j^{\text{exp}}}{\sigma_i^{\text{num}}}\right)^2 + \left(1-\frac{\varepsilon_j^{\text{exp}}}{\varepsilon_i^{\text{num}}}\right)^2} + \sqrt{\left(1-\frac{\sigma_{j+1}^{\text{exp}}}{\sigma_i^{\text{num}}}\right)^2 + \left(1-\frac{\varepsilon_{j+1}^{\text{exp}}}{\varepsilon_i^{\text{num}}}\right)^2}\right) \tag{9}$$

where n=number of computed strains.

Model Performance

Three sets of laboratory experimental data from Lekarp (1997) were chosen to calibrate the fuzzy set plasticity model. Due to the fact that the available laboratory data from Lekarp (1997) are not sufficient to calibrate the fuzzy set plasticity model and obtain a unique set of model parameters, appropriate assumptions were made to carry out the model calibration. The material types include Leighton Buzzard sands (S), the sand and gravel (S&G), and the slate waste (SW). The S&G and SW were tested in a triaxial cell apparatus, while the S was tested in a hollow cylinder apparatus. Lekarp's testing program was planned with the primary aim of characterizing the development of cumulative permanent strain with number of load applications, as listed in Table 1. The stress paths that were applied to the laboratory tests are also shown in Table 1, where the corresponding load applications in p-q stress space were converted when the stress-strain responses

Table 1. Selected Tests from Lekarp (1997) for Model Calibration

Material	Stress path code	N (number of cycles)	σ_3 (kPa) Min	σ_3 (kPa) Max	q (kPa) Min	q (kPa) Max
S	P1	10,000	70	70	0	80
	P3	10,000	70	70	0	105
S&G	P1	10,000	100	135	0	200
	P2	10,000	100	285	0	500
	P3	10,000	100	220	0	400
SW	P1	10,000	0	20	0	300
	P2	10,000	0	100	0	600
	P3	10,000	0	200	0	600

were computed through the fuzzy set plasticity model. As Lekarp's tests available for model calibration are limited, the fuzzy set plasticity model parameters given in Table 2 lead to one possible combination. Fig. 3 shows the entire stress-strain curves for S under the stress paths listed in Table 1. The sand subjected to Stress Path P3 where a higher maximum deviator stress (105 kPa) was applied shows more plastic deformation. Both curves show the tendency of being more elastic as more load cycles are applied. Essentially, the sand under both Stress Paths P1 and P3 was stabilized and elastic shakedown was reached. The entire stress-strain behaviors for S&G and SW are similar so the curves are not shown in the paper. The permanent strain versus the number of cycle curves for all three materials are presented in Figs. 4–6, respectively. It shows that the proposed cyclic plasticity model is capable of capturing the long-term behavior in permanent deformation under repeated loads. In particular, most permanent axial strains were taking place in the first 2,000 load cycles. When the materials were subjected to higher deviatoric load repetitions, more load cycles were needed to reach a stable state (shakedown). As for resilient modulus, it can be calculated at any given load cycle since the entire stress-strain curve is available from the fuzzy set plasticity model. Fig. 7 shows an example of the calculated resilient modulus for S if a range of confining pressure is applied. Each resilient modulus was calculated from each corresponding stress-strain curve at its 10,000th cycle. It shows the trend that for a given maximum deviator stress, the higher the confinement is, the higher the resilient modulus is. It also shows for a given confinement that the higher the maximum deviator stress is, the higher the resilient modulus is.

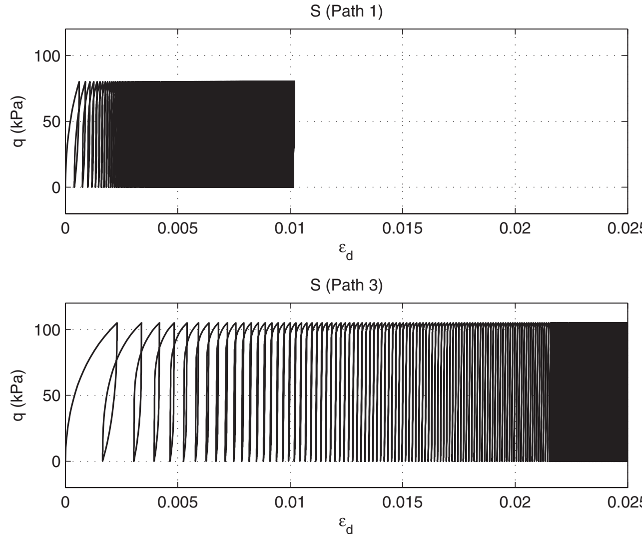

Fig. 3. Complete stress-strain response for S

Conclusions

A cyclic plasticity model based on fuzzy set plasticity theory is presented in this paper for modeling the permanent deformation behavior under repeated load cycles. The resilient modulus can also be calculated from the stress-strain response from the fuzzy set plasticity model. The model is capable of mimicking the behaviors of unbound granular materials under repeated loads which include reversal loading, nonlinear dilatancy, material memory, and long-term behavior such as elastic shakedown. The elastic shakedown is achieved by controlling the material memory and plastic modulus parameters. It is particularly attractive for finite-element analysis since a more realistic stress-strain response is available. However, it also has several drawbacks. Model calibration is challenging since it requires more laboratory test results. The triggering mechanisms for elastic shakedown, plastic shakedown, and progressive failure are not fully understood and not implemented into the model.

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Table 2. Fuzzy Set Plasticity Model Parameters

	K (MPa)	G (MPa)	M	d	a_0	a_1	A	κ_d	m_1	m_2	m_3	α
S-P1	580	700.9	105,000	1.8	30	1.3	1.7	0.35	1.5	0.15	−0.62	0.85
S-P2	580	700.9	56,000	1.8	30	1.3	1.7	0.35	1.5	0.21	−0.62	0.85
S&G-P1	580	700.9	405,000	1.12	50	1.55	1.95	0.35	1.5	0.02	−0.62	0.85
S&G-P2	580	700.9	95,000	1.8	50	1.55	1.95	0.35	1.5	0.46	−0.62	0.85
S&G-P3	580	700.9	50,000	1.85	50	1.55	1.95	0.35	1.5	0.23	−0.62	0.85
SW-P1	580	700.9	572,000	1.3	50	1.8	1.35	0.95	1.5	0.3	−0.62	0.85
SW-P2	580	700.9	572,000	1.97	50	1.8	1.35	0.95	1.5	0.6	−0.62	0.85
SW-P3	580	700.9	793,000	1.8	50	1.8	1.35	0.95	1.5	0.79	−0.62	0.85

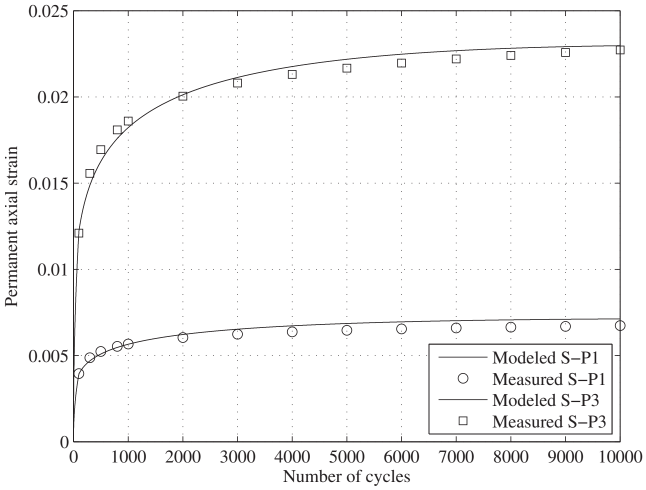

Fig. 4. Permanent axial strain versus number of cycles for S

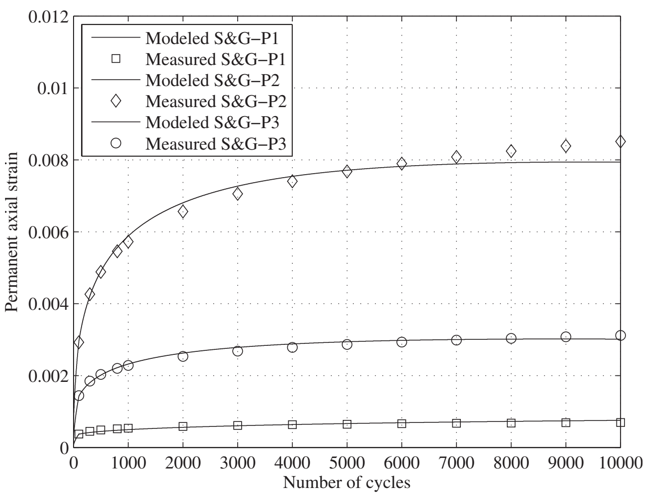

Fig. 5. Permanent axial strain versus number of cycles for S&G

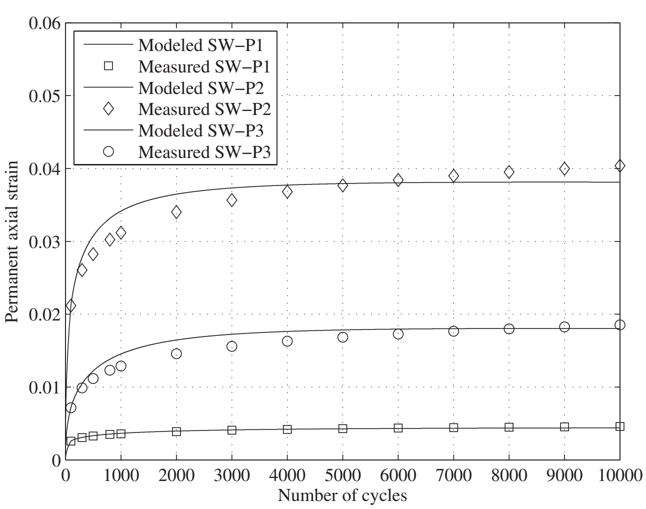

Fig. 6. Permanent axial strain versus number of cycles for SW

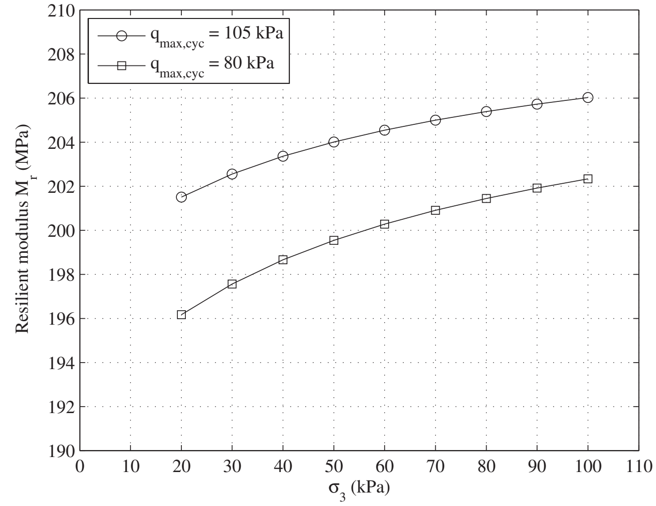

Fig. 7. Calculated resilient modulus at various confining pressures for S

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