

# Modeling Sensor Performance for Model-Based Vision

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The model-based vision requires object appearances in the computer. How an object appears in the image is a result of interaction between the object properties and the sensor characteristics. Thus, in model-based vision, we ought to model the sensor as well as modeling the object. In the past, however, the sensor model was not used in the model-based vision or, at least, was contained in the object model implicitly.

This paper presents a framework between an object model and the object appearances. We consider two aspects of sensor characteristics: sensor detectability and sensor reliability. Sensor detectability specifies what kind of features can be detected and in what area the features are detected; sensor reliability specifies how reliable detected features are. Commonly available sensors are briefly examined in terms of their sensor characteristics. We define the configuration space to represent sensor characteristics. We propose a representation method of the sensor detectability in the configuration space. Sensor reliability distribution is also discussed in the configuration space. Under this framework, we characterize the photometric stereo and the light-stripe range finder as examples.

## 1. INTRODUCTION

The model-based vision requires object models in the computer. Various researchers propose many kinds of object models, ranging from generic models such as generalized cylinders [5, 28, 9, 37], extended Gaussian images [38, 19, 18], and super quadric models [35] to specific models such as aspect model [25, 11, 20], region-relation model [4, 34, 6], and smooth local symmetry [7, 8].

The object appearances, however, are determined by a product of an object model with a sensor model. Thus, in the model based vision, it is insufficient to consider only an object model; it is essential to exploit a sensor model as well. On the other hand, modeling sensors for model-based vision has attracted little attention; quite often, researchers who are familiar with the sensors they use tended to construct object appearances by implicitly incorporating their sensor behavior. This paper, in contrast, explores a general framework for explicitly incorporating sensor models which govern the relationship between object models and object appearances.

A sensor model must be able to specify two important characteristics: sensor detectability and sensor reliability. The sensor detectability specifies what kind of features can be detected and in what condition the features are detected. The sensor reliability specifies how reliable the detected features are. This paper, thus, presents a method for modeling sensors with sensor detectability and sensor reliability. Commonly available sensors are briefly examined in terms of their sensor characteristics. Then, representation techniques for sensor characteristics are explored. We define the configuration space to represent sensor characteristics. Finally, we consider two aspects of sensor characteristics: sensor detectability and sensor reliability. We propose a representation

space on which a sensor's detectability is expressed in the uniform way. Sensor reliability analysis consists of reliability distribution and error propagation from observed data to geometric features. Under this framework, we characterize the photometric stereo and the light-stripe range finder as examples.

## 2. SENSORS IN THE MODEL BASED VISION

This section gives a brief survey of commonly available sensors in the model-based vision. We include both passive and active sensors. The following sensors are often used: edge detector [36, 27, 10], shape-from-shading [17, 22], binocular stereo [29, 14, 3, 33], time-of-flight range finder [24, 15], light-stripe range finder [1, 34], trinocular stereo [32], photometric stereo [40, 21], polarimetric light detector [26], and SAR (Synthetic Aperture Radar) [12, 39, 31].

Each sensor is a mapping function from object features to sensor features. Object features such as faces, edges, and vertices exist in the three-dimensional object space. These object features in the object space are mapped to sensor features such as regions, lines, and points in the sensor space. Note that the sensor features have no meaningful interpretation in the sensor space. For example, a sensor feature such as a region should be considered as a collection of points which are not grouped as one meaningful unit by a sensor. The conversion process from sensor features to geometric features will group a collection of points into a geometric feature such as a region.

Table 1 gives the summary of detectable features in the object space by commonly available sensors. For example, an edge detector can detect some edges in the object space as lines in the sensor space. Since it does not need special light sources, it is classified as a passive sensor.

Table 1 Detectable Object Features				
Sensor	Vertex	Edge	Face	active/passive
Edge Detector	no	yes	no	passive
Shape-from-shading	no	no	yes	passive
SAR	yes	yes	yes	active
Time-of-FlightRange Finder	no	no	yes	active
Light-stripe Range Finder	no	no	yes	active
Binocular Stereo	no	yes	no	passive
Trinocular Stereo	no	yes	no	passive
Photometric Stereo	no	no	yes	active
Polarimetric light detector	no	no	yes	active

Since the brightness change does not occur over any face, passive sensors have usually difficulty in detecting faces. An active sensor projects lights over the scene; stronger reflection can be obtained from faces; weaker or no reflection is given from edges and vertices. Thus, most active sensors are good in detecting faces, while they can detect neither edges nor vertices.

Sensor features in the sensor space are summarized in Table 2. In Table 2, a line means a line-shaped collection of detected points and a region means a region-shaped collection of detected points. For all sensors except SAR the correspondence between the object feature and the sensor feature is one-to-one. For example, an edge detector generates one line-shaped sensor feature corresponding to either one edge or one reflectance discontinuity line of a object feature in the object space. Photometric stereo generates a surface orientation distribution as sensor features which corresponds to one physical face in the object space. On the other hand, SAR generates either line-shaped sensor feature or point sensor feature from one edge in the object space depending on the sensor configuration. A precise discussion of SAR will be found elsewhere [12, 39, 31].

Table 2 Detected Sensor Features			
Sensor	Vertex	Edge	Face
Edge Detector	-	line	-
Shape-from-shading	-	-	region
	point	point/line	line
Time-of-FlightRange Finder	-	-	region
Light-stripe Range Finder	-	-	region
Binocular Stereo	-	line	-
Trinocular Stereo	-	line	-
Photometric Stereo	-	-	region
Polarimetriclight Detector	-	-	point

While this summary tells in general what object features are detectable in what forms by various sensors, it is also important to characterize in what viewing conditions those features are actually detectable and how reliable the detected features are. For that, we need to develop a representation tool for relation between object coordinates and sensor coordinates.

### 3. REPRESENTING SENSOR CONFIGURATION

This section defines the sensor configuration space on which sensor detectability and sensor reliability are specified. Sensor detectability and reliability depend on various factors: distance of an object, configuration of an object, reflectivity of an object, transparency of air, and background noise such as the sun's brightness. In the model-based vision, since the target object and its rough distance is a prior known, mainly angular freedom of object affects on detectability and reliability. Thus, we will define a space to specify the relationship between the sensor coordinate and the object coordinate.

The relationship between the sensor coordinate and the object coordinate can be specified by three degrees of freedoms; two degrees of freedom in the sensor direction and one degree of freedom in the sensor rotation. Since the relationship between two coordinates is relative, for the sake of convenience, we fix the sensor coordinate and discuss how to specify the object coordinate with respect to the sensor coordinate.

We will depict an object coordinate as a point in the following sphere. The point direction from the center denotes the z axis of the object coordinate,  $(\theta, \phi)$ , where  $\theta$  and  $\phi$  specify the zenith angle and the azimuth angle of the z axis. The distance from the spherical surface to the point denotes the axis rotation around the z axis,  $\psi$ . On the spherical surface, we set points such that corresponding coordinates satisfy  $\phi + \psi = 0$ . The north pole of the sphere corresponds to the sensor coordinate.

Due to the bias of z axis rotation, this sphere has no discontinuity around the north pole. Actually, this sphere is a three dimensional projection of four dimensional quaternion space. Precise discussion will be found [23]. We will refer to the sphere as the configuration space.

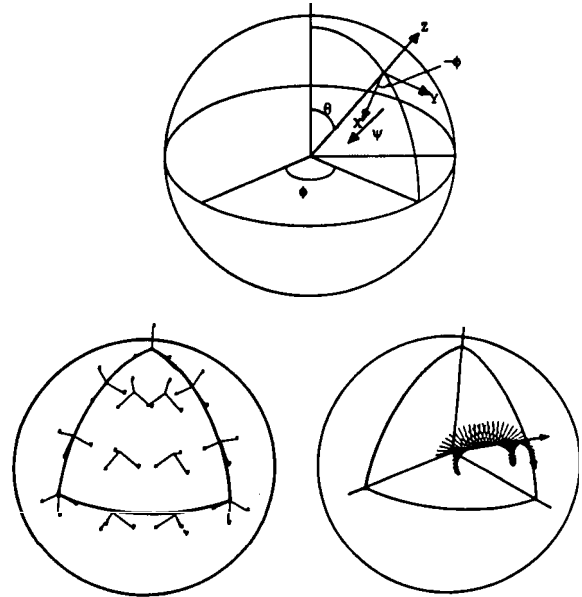


Figure 1: The configuration space

## 4. DETECTABILITY OF SENSORS

In the previous section, we have defined the way to represent the relationship between the sensor coordinate and the object coordinate. In this section, we will develop a constraint to determine whether an object feature can be detected at each point of the configuration space.

### 4.1. Constraints in Configuration Space for Feature Detection

Each sensor has two components: sources and detectors. For example, both a time-of-flight range finder and a light-stripe range finder have one source and one detector. Binocular stereo has one source and two detectors; photometric stereo has three sources and one detector. Table 3 summarizes the number of sources and detectors of each sensor.

Table 3 Source and Detector		
Sensor	Number of sources	Number of detectors
Edge Detector	1	1
Shape-from-shading	1	1
SAR	1	1
Time-of-Flight Range Finder	1	1
Light-stripe Range Finder	1	1
Binocular Stereo	1	2
Trinocular Stereo	1	3
Photometric Stereo	3	1
Polarimetric light detector	n	1

One source only illuminates one part of an object; one detector only observes one part of the object. Each sensor which consists of sources and detectors, only detect one part of the object. Thus, in order to specify the detectable area of each sensors, we need to define each source's illuminated area and each detector's visible area. We also need to define a operation method on illuminated areas and visible areas.

In the following discussion, we will consider both sources and detectors as generalized sources (G-sources). Each G-source has two properties: the illumination direction and the illuminated area. In the source case, the illumination direction and the illuminated area are the same as the nominal meanings. In the case of detectors, the illumination direction corresponds to the line of sight of the detector, and the illuminated area corresponds to the visible area from the detector.

In order to define the sensor detectability, we will use the configuration space previously defined. The illumination direction of a G-source is specified as a line in the configuration space; its illuminated area is specified as a volume in the configuration space. We will define two kinds of G-sources in terms of the distribution of illuminated areas: the uniform G-source and the directional G-source. A uniform G-source distributes its light evenly in all directions. An example of a uniform G-source is a usual light source whose illuminated area is located as a hemispherical cone of the sensor space. The center direction of the cone corresponds to the source direction.

We specify a uniform G-source as

(NS type direction angle)

The first argument, *type*, specifies what kind of feature the G-source illuminates, and takes one of the values: *face*, *edge*, and *vertex*. The second argument, *direction*, denotes the G-source illumination direction as a vector. The third argument, *angle* defines the illuminated area by specifying the spherical angle between the illumination direction and the surface normals associated with object features. If *type* is *face*, this angle defines the maximum allowable angle between the face surface normal and the illumination direction. If *type* is *edge*, this angle defines the maximum allowable angle of the smaller one of the two angles between the illumination direction and the two normals of incident surfaces to the edge. That is, if either or both faces are well illuminated, then the edge is considered to be illuminated. If *type* is *vertex*, we have to consider at least three faces incident to the vertex. This angle defines the maximum allowable angle of the smallest angle of those angles between the illumination direction and the normals of incident surfaces. That is, if any of the incident faces of the vertex is illuminated, the vertex is considered to be illuminated.

Another kind of G-source is a directional G-source which projects light depending on the rotation around the light source direction. We specify a directional G-source as

(DS type direction angle spec-direction spec-angle)

The first argument, *type*, specifies one of the object features: *vertex*, *edge*, and *face*. The second argument, *direction*, denotes the G-source illumination direction as a vector. The third argument, *angle*, defines the spherical angle of the illuminated area, as for the uniform G-source. The fourth argument, *spec-direction* defines the constraint direction to be used in the following argument. The fifth argument, *spec-angle* defines the maximum allowable angle between the constraint direction and the principal direction such as the surface normal of a face, the edge direction of an edge, and the average surface orientation around a vertex.

An example of a directional source is a directional edge detector. As mentioned before, a detector is also considered as a source, and its illuminated area corresponds to the detectable area. Since the directional edge detector only detects edges with certain orientations, it is regarded as a directional source. The illuminated area of a directional source becomes a thin slice of the configuration space.

We can specify the sensor characteristic with AND and OR operations of these formal definitions of all component G-sources of the sensors. Figure 2 shows feature detection constraints represented by this method for all sensors listed in table 3.

### 4.2. Use of Feature Detection Constraints

The feature detection constraints are used together with a geometric modeler to predict how the object appears relative to the sensor. A geometric modeler generates possible attitudes of an object corresponding to each point in the configuration space. Then, detectability of each component face, edge, or vertex of the object

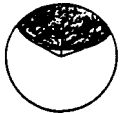
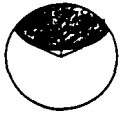
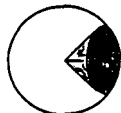






Sensor	Constraints in the formal definition	Constraints in the sensor space
Edge Detector	(AND (NS edge V d) (NS edge V d)) = (NS edge V d)	
Shape-from-shading	(AND (NS face V d) (NS face V d)) = (NS face V d)	
SAR	(OR (NS face V d) (NS edge V d) (NS vertex V d)) (needs postprocess)	
Time-of-Flight Range Finder	(AND (NS face V d) (NS face V d)) = (NS face V d)	
Light-strip Range Finder	(AND (NS face V1 d) (NS face V2 d))	
Binocular Stereo	(AND (NS edge V1 d1) (DS edge V2 d2 VE de) (DS edge V3 d3 VE de))	
Trinocular Stereo	(AND (NS edge V1 d1) (DS edge V2 d2 VE de) (DS edge V3 d2 VE de) (DS edge V4 d2 VE de))	
Photometric Stereo	(AND (NS face V d1) (NS face V1 d2) (NS face V2 d2) (NS lam V3 d2))	
Polarimetric Light Detector	(OR (AND (NS face V d) (NS bo V1 d1)) (AND (NS face V d) (NS bo V2 d1)) ...) where $V \cdot V = \cos 2d$	

Figure 2: Feature detection constraints

under this attitude is determined using the constraint. The illumination direction constrains the surface shape to be detected, and the illuminated area constrains the surface orientation to be detected.

More precisely, we can imagine putting the configuration space on each point of the object in order checked whether it can be detected by the sensor. If the illumination direction from that point intersects with any of the surfaces of the object, the point cannot be detected. If the illumination direction does not intersect with any of the

surface, the point is detectable. If the surface orientation on the point is contained in the illuminated area, the point is detectable. If the surface orientation is outside of the illuminated area, the point cannot be detected. Figure 3 illustrate the outline of this operation for the illumination direction and illuminated area using this constraint.

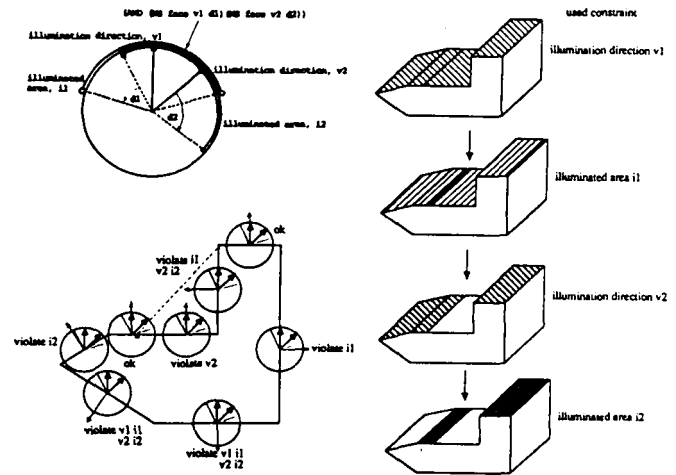


Figure 3: How to use the constraints

### 4.3. Detectability Distribution

The feature detection constraint gives the upper bound of the detectable areas in the configuration space. In some cases, however even though a object feature exists within the detectable area, the feature may be undetected due to noise. We define the detectability distribution such that a feature in the detectable area is actually detected. The probability is usually high in the central part and low in the peripheral part of the detectable area.

The detectability distribution can be described by multiplication of detectability distributions of the component G-sources. Namely, each G-source has a detectability distribution defined over the illuminated area. In the previous subsection, the constraint was either illuminated or non illuminated or either detectable or non-detectable. We will expand this idea to the continuous case. Namely, each G-source has its own continuous detectable distribution over its illuminated area defined in the configuration space.

Since all sensors detect features based on a brightness distribution, the detectability distribution also depends on a brightness distribution which is detected and converted to sensor features. However, there are two types of sensors in terms of the conversion method; direct sensors and indirect sensors. The direct sensor measures the brightness value and converts it to sensor features, such as surface orientation, directly from the brightness value. The indirect sensor measures the brightness value and positional information of the bright spot if the brightness value is greater than some threshold. The indirect sensor then converts the positional information to sensor features such as depth. Table 4 shows a classification of sensors based on this difference.

Since a TV camera is a most typical input device, we will examine its performance before exploring the detectability distribution. Let  $P(x|d)$ ,  $P(d|x)$ , and  $P(x)$  be the conditional probability of a real value  $x$

Table 4 Measurement method	
sensor	direct/indirect
Edge Detector	direct
Shape-from-shading	direct
SAR	indirect
Time-of-Flight Range Finder	indirect
Light-stripe Range Finder	indirect
Binocular Stereo	indirect
Trinocular Stereo	indirect
Photometric stereo	direct
Polarimetric light detector	indirect

under the observed value  $d$ , the conditional probability of the observed data  $d$  under a real value  $x$ , and the probability of  $x$ , respectively. Then, TV camera performance can be described using Bayes' theorem as

$$P(x|d) = \frac{P(d|x)P(x)}{\int P(d|x)P(x)dx}$$

If we assume that  $P(x)$  is constant, namely the brightness distribution occurs randomly.

$$P(x|d) = P(d|x).$$

The conditional probability of observed data  $d$  under the real value  $x$  is assumed as the Gaussian distribution whose mean value is  $x$  and standard deviation is  $\sigma$ ,

$$P(x|d) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{(x-d)^2}{2\sigma^2}}.$$

We can obtain  $\sigma$  from experiments. Our SONY CCD camera has  $\sigma=3$ , which roughly coincides with a result elsewhere [2].

Since the detectability distribution depends on sensing methods, we will develop the distributions for the photometric stereo as a representative case of the direct sensor, and for the light-stripe range finder as a representative case of the indirect sensor.

#### 43.1. Detectability distribution of photometric stereo

An direct sensor such as photometric stereo can be modeled as

$$y=f(x)$$

where  $x$  is the input brightness.  $y$  is the output feature values. and  $f$  is the conversion function. Suppose  $X^*$  is the definition area of the function  $f$ , ie. the direct sensor outputs a feature value  $y$  from any  $x_i$  if  $x_i \in X^*$ . Then, the detectability distribution can be determined as the probability that the input brightness,  $x+\delta x$ , disturbed by  $\delta x$ , is still contained in the definition area,  $X^*$ . In order to be the problem more specific, we will examine the definition area of photometric stereo.

Photometric stereo determines the surface orientation from three images taken from the same position under different lighting directions.

$$\begin{aligned} I_1 &= S_1 \cdot N \\ I_2 &= S_2 \cdot N \\ I_3 &= S_3 \cdot N, \end{aligned}$$

where  $I_i, S_i, N$  are the brightness value under light source  $i$ , the  $i$  th light source direction vector, and the surface normal vector, respectively. Thus, expressing the brightness as a vector,  $I$ , and the light source as a matrix,  $S$ .

$$I=SN.$$

Applying  $S^{-1}$  to both sides, we obtain an explicit expression of  $N$ .

$$N=S^{-1}I.$$

This is the basic idea of photometric stereo [40].

Working photometric stereo has, however, two modification [21] to this theory.

1.  $S^{-1}$  is determined from calibration and stored in a lookup table rather than calculated from the ideal case.
2. Brightness values are normalized  $I/I$  so that we can cancel the albedo effect.

We will obtain the detectability distribution of the photometric stereo. At first, we consider light source 1's detectability distribution. Assume a brightness value moves from  $i$ , to  $i_1+\delta i_1$  due to sensor error. The normalization gives  $i'_1+\delta i'_1=(i_1+\delta i_1)/(i_1+i_2+i_3)$ . However, the normalized intensity  $(i'_1+\delta i'_1, i'_2, i'_3)$  exists on the same plane  $i'_1+i'_2+i'_3=1$ . Since a continuous area on the plane is the solution area for photometric stereo, we can obtain the solution from the new triple  $i'_1+\delta i'_1, i'_2, i'_3$ . Than is, we will always succeed to obtain the feature values, ie. we will have a unit detectability distribution for the light source 1. (Though of course the resultant value may be less reliable as will be discussed in the reliability section.) The same discussion is applicable to light source 2 and light source 3. Since the total detectability distribution is given as the multiple of all three detectability distributions of sources, the detectability distribution is a constant distribution over the detectable area in the configuration space. This analysis reveals that the normalization makes the detectability to be a unit value, and thus, helps to detect features in a stable manner.

#### 43.2. Detectability distribution of a light-stripe range finder

An indirect sensor projects light on the scene and determines the positional features from the observed image or signal. Thus, the detectability distribution depends on whether a sensor can detect the returned light or not. Usually, to avoid the confusion of the returned value with background noise, threshold operations are applied, such as

$$\begin{aligned} i \text{ is detected} & \quad \text{if } i \geq i_0 \\ i \text{ is not detected} & \quad \text{otherwise.} \end{aligned}$$

Let us consider the light-stripe range finder as an example. A light-stripe range finder projects light stripes on the scene and recovers the depth at a point from the distance between two adjacent light stripes. Thus, the detectability function depends on whether the TV camera observes the light stripes or not

Assuming that the surface is lambertian, the brightness of the stripe is determined by the angle between the surface normal,  $N$ , and the light source direction,  $S$ . Then, the brightness  $i$  is given by  $N \cdot S$ , while the disturbance factor  $\delta i$  is given by a Gaussian distribution,

$$p(\delta i) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{(\delta i)^2}{2\sigma^2}}.$$

In almost all illuminated areas.  $N \cdot S - i_0 > 3\sigma$  holds, and the viewer direction does not affect the observed brightness. Thus, the detectability distribution is constant over the most part of illuminated area of the light source. In the peripheral area, however,

$$P_{detectable}(i) = P(i + \delta i \geq i_0) = \int_{i_0-i}^{+\infty} \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{(\delta i)^2}{2\sigma^2}} d(\delta i)$$

See Figure 4.

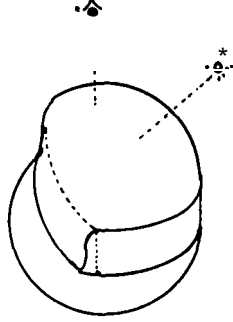


Figure 4: Detectability distribution of a light-stripe range finder

## 5. RELIABILITY OF SENSORS

Once a sensor feature is detected, then the next question is how reliable the sensor feature is. This section discusses two issues of sensor reliability. The first issue is the reliability of the detected sensor feature; data detected by a sensor always contains measurement error. To determine the bound of the error is important for model based vision. For example, suppose there is a sensor feature which the geometric model takes two nominal value 100 and 90 for two distinct situations or attitudes. If a sensor has an error range of plus/minus 1 for the sensor feature, we can use the feature from that sensor as one of reliable discriminators in the recognition stage. On the other hand, if a sensor has an error range of plus/minus 20, we cannot use the feature from that sensor.

The second issue is propagation of error from sensor features to geometric features, hence the resulting reliability of those geometric features. In some cases, a detected sensor feature from a sensor is used directly as a feature; in most cases, however, geometric features are derived from sensor features and are used as features in model based vision. Thus, it is necessary to determine the error propagation mechanism.

### 5.1. Reliability Distribution of Sensor Feature

Table 5 shows the main sources that affect reliability of sensor features. In addition to these, various digitization such as phase digitization in a time-of-flight range finder and spatial digitization in binocular stereo must be considered but are omitted for the time being.

Table 5 Main factors of unreliability	
Sensor	Factor
Edge Detector	G-source brightness (TV camera)
Shape-from-shading	G-source brightness (TV camera)
SAR	G-source direction (camera direction)
Time-of-Flight Range Finder	G-source direction (mirror direction)
Light-Strip Range Finder	G-source direction (mirror direction)
Binocular Stereo	G-source direction (camera direction)
Trinocular Stereo	G-source direction (camera direction)
Photometric Stereo	G-source brightness (TV camera, light sources)
Polarimetric light detector	G-source direction (light source direction)

orientation  $N$ .

$$N = S^{-1} I'.$$

Let us denote the brightness disturbance distribution as  $N(0, \sigma^2)$ . Then the normalized brightness distribution is denoted as  $N(I, (\sigma f)^2)$ , where  $f$  is the first derivative of  $I$ . Figure 5a shows the distribution of  $f$  over the detectable area. Although it is possible to approximate the distribution with polynomial, we assume it is constant (0.004) over the detectable area for simplicity. Since  $\sigma = 3$ ,  $2\sigma f = 0.03$ . This value corresponds to a 1.5 mesh in the lookup table.

We determine  $S^{-1}$  from the real data, because  $S^{-1}$  is represented as a lookup table. Figure 5b shows the angular distance in terms of mesh number. Namely, the figure shows angular differences between two adjacent surface normals in the lookup table. By using this result and a 1.5 mesh error from the brightness distribution, the total error becomes 5 degrees over the detectable area. This agrees with the As shown in Table 5, the main error comes from G-source brightness in a direct sensor and from G-source direction in an indirect sensor. Thus, we will analyze the reliability of photometric stereo and the light-stripe range finder as representatives of the direct and indirect sensors, respectively.

#### 5.1.1. Reliability distribution of photometric stereo

For the direct sensor,  $y = f(x)$ , the disturbance of  $\delta x$  is propagated via  $f$ . Namely, the disturbance of the detected value,  $\delta y$  is

$$\delta y = f'(x) \delta x$$

Our photometric stereo can be described as two step processes. First a original brightness triple is converted to a normalized brightness triple.

$$I' = I / |I|.$$

Then, the normalized brightness triple is converted to a surface observation from the experiment, which has plus/minus 5 degrees error in determining surface orientations over the range of detectable surface orientations. See Figure 5c.

#### 5.1.2. Reliability distribution of a light-stripe range finder

In the case of indirect sensors, the main source of unreliability comes not from the G-source brightness but from the G-source direction. The indirect sensor can be modeled as

$$z = f(y(v_1, v_2, \dots, v_n)).$$

$v_i$  denotes the  $i$ th G-source direction, and  $y$  denotes the conversion function from G-source directions to the positional information, while  $f$  denotes the conversion function from the positional information such as a bright spot to the observed data, and  $z$  specifies

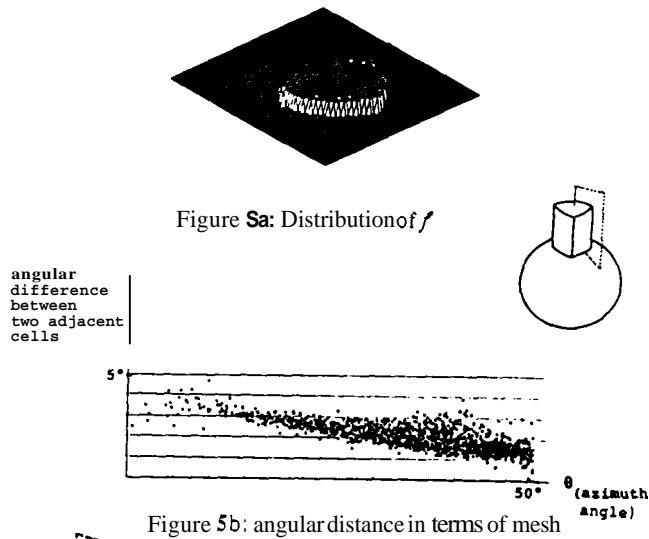


Figure 5a: Distribution of  $f$

Figure 5b: angular distance in terms of mesh

Figure 5c: angular error over the detectable area

Figure 5 Reliability of photometric stereo

the detected data such as depth or surface orientation.

$$\delta z = \sum f \frac{\partial y}{\partial v_i} \delta v_i.$$

Thus, we will examine the disturbance based on the G-source direction.

In the light-stripe range finder, we will calculate  $f \frac{\partial y}{\partial v_i}$  directly from the system analysis. The angular error in the mirror is propagated to the observed error with the physical conversion process. The propagation process can be divided into three parts: mirror error to positional difference in the stripe, positional difference in the stripe to image difference, and image difference to the converted distance difference. The propagation process can be obtained analytically as follows.

Let us denote the angular error as  $r\delta\theta$ , where  $r$  is the distance from the light source to the physical point. At the physical place A, the laser light is intercepted. Then, due to the angular error, the physical difference,  $\delta y$  occurs.

$$\delta y = \frac{r}{\cos \alpha} \delta \theta,$$

where  $\alpha$  is the angle between the light source,  $S$ , and the surface normal,  $N$ . See Figure 6a.

This physical difference is observed from the TV camera, and the image difference,  $\delta i$  occurs.

$$\delta i = (\cos \beta) \delta y,$$

where  $\beta$  is the angle between the surface normal,  $N$ , and the viewer

direction,  $V$ .

The third step is the propagation from the image plane to the distance. For the simplicity, we will assume that the camera model be orthographic projection. Then, the horizontal difference,  $\delta i$ , is propagated into the distance error  $\delta z$  as

$$\delta z = \frac{\delta i}{\tan \gamma},$$

where  $\gamma$  is the angle between the viewer direction,  $V$ , and the light source direction,  $S$ . See Figure 6a.

Finally, we obtain

$$\delta z = \frac{\cos \beta}{\cos \alpha \tan \gamma} r \delta \theta = \frac{(N \cdot V)(S \cdot V)}{(N \cdot S) \sqrt{1 - S \cdot V}} r \delta \theta.$$

Figure 6b shows the reliability distribution over the detectable area.

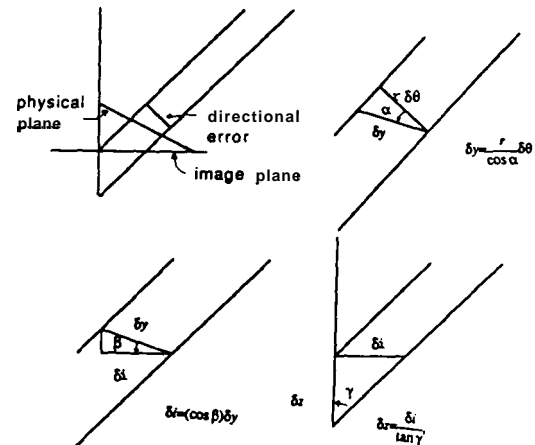


Figure 6a

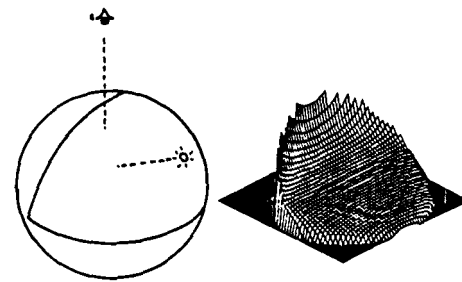


Figure 6b

Figure 6: Reliability distribution of light-stripe range finder

## 5.2. Propagation of Reliability to Geometric Features

Usually raw data detected by a sensor is converted into geometric features such as distance, area, and inertia. This process propagates errors into the geometric features due to two reasons: the detectability distribution and the reliability distribution. Since this conversion process depends on the detected data, we will concentrate on faces as detected data, because most of the active sensors detect faces as the primal features.

### 5.2.1. Error propagation from detectability distribution

Most active sensors detect physical patches as detected pixels. Usually, these detected pixels will be grouped and converted into isolated regions. If the sensor fails to find a detectable pixel, the measured area will be reduced from the nominal area size given by a model. This process can be modeled as follows:

Suppose the detectability probability is  $p$  over a region and the nominal area size of the region is  $n$ . Under this condition, the probability to observe  $x$  pixels out of  $n$  pixels over the region is

$$P(x) = \binom{n}{x} p^x q^{n-x}$$

Namely, this probability denotes that the system executes  $n$  trials and succeeds to detect  $x$  pixels under that condition that success probability is  $p$  and fail probability is  $q$ , where  $p+q=1$ . This process is a binomial distribution; the mean and variance of this distribution are

$$\begin{aligned} m &= np \\ \sigma^2 &= npq \end{aligned}$$

This gives the error propagation (area reduction ratio) for the detectability distribution.

Since both photometric stereo and a light-stripe range finder give  $p=1$ , neither sensor causes an area reduction due to the detectability distribution.

### 5.2.2. Error propagation from reliability distribution

The Next factors to be considered is the reliability of detected data. We recover the geometric features such as distance, area, and inertia from skewed raw data by an affine transform, based on the observed surface orientation in either the photometric stereo or light-stripe range finder. Thus, if the raw data are erroneous, the obtained geometric features are also erroneous.

Let  $d$  be the real distance, and  $d+\delta d$  be the observed one. Then, the physical system generates an observed distance  $d \cos \theta$ , while due to the sensor error, we will measure this surface surface orientation as  $\cos(\theta+\delta\theta)$ , where  $\theta$  is the angle between the viewer and surface normal. Thus, for  $\delta\theta$  small, we get

$$d+\delta d = d \cos \theta / \cos(\theta+\delta\theta) = d(1+\delta\theta \tan \theta).$$

In the area case,

$$a+\delta a = a(1+2\delta\theta \tan \theta).$$

In the inertia case,

$$i+\delta i = i(1+3\delta\theta \tan \theta).$$

These formulas give error propagation from angular error to features at each pixel. We will obtain geometric features from a region which consists of  $n$  pixels. Thus, the system will execute  $n$  trials of measuring  $\delta\theta$ , which is approximated as a Gaussian distribution,  $N(0, \sigma^2)$ , and observe the total  $\Sigma \delta\theta$ . From the theorem of the Gaussian distribution,  $\delta\theta = \Sigma \delta\theta$  is a Gaussian distribution,  $N(0, n\sigma^2)$ .

By using these formulas, we calculate error ratio of areas and inertia for photometric stereo as shown in Table 6. Predicted results are obtained based on the reliability of photometric stereo developed previously and the formulas of section. We use  $n=70$ ,  $\sigma=0.045$ .

Observed results are obtained from the distribution of the real data sampled five times. Similar results are expected in the other sensors.

Table 6 Reliability of Geometric Features		
Feature	Observed	Predicted
Area	0.02	0.045
Inertia	0.05	0.067

## 6. CONCLUDING REMARKS

This paper discussed modeling sensors for model-based vision. Our sensor model consists of two characteristics: sensor detectability and sensor reliability. Sensor detectability specifies under what conditions a sensor can detect a feature, while sensor reliability denotes how reliable the obtained measurement is over the detectable area.

We have defined the configuration space which represents the relationship between sensor coordinates and object coordinates. The sensor detectability and the sensor reliability are expressed in this configuration space. Constraints in the configuration space involved in detecting features have been developed by using G-source illuminated area and G-source illumination direction. We have shown how to compute the sensor detectability distribution and the sensor reliability distribution for photometric stereo and a light-stripe range finder as examples.

In model based vision, expected values of various features can be computed from 3D geometric model. Those values are, however, nominal values that they should take in ideal cases or should be sensed by ideal sensors. On the other hand, actually observed sensor data contains noises and should be used accordingly. The sensor model bridges the discrepancy between these two values by modeling the distribution of the sensed value based on the characteristics of a given sensor. In model-based vision, it is possible to precompile a given 3D geometric model into a recognition strategy[20]. This precompilation cannot generate an optimal strategy without knowing each feature's reliability, because the strategy should use the most stable features at each recognition step. Thus, the sensor model is an essential component in model-based vision. We have to explore more reliable sensor models for this purpose.

We also have analyzed the error propagation mechanism from detected data to the geometric features. This is important, because quite often we are interested in geometric features derived from the detected sensor features. Once we establish the error propagation mechanism from detected sensor features to geometric features, we can also assess the reliability of the geometric features, hence we can construct a recognition system more systematically and reliably. Further study is required in this area.

To calculate detectable features of an object under the constraints of various sensors is a tedious job when we use a conventional geometric modeler. The better way is to interface a geometric modeler with the sensor model proposed. We call this a sensor



modeler. The traditional geometric modeler only allows users to generate a 3D object by combining primitive objects and to display its views. In this sense, the traditional modeling system has only one sensor model which is projection. The sensor modeler we propose can generate various 2D representations under given sensor specifications. Part of this facility is being implemented in our new geometric/sensor modeler VANTAGE[16].

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