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MODELING THE EFFICIENCY OF EXTERNAL GEAR PUMPS BASED ON SIMILARITY CONSIDERATIONS

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ABSTRACT

Gear pumps are used in numerous different applications and industrial sectors. However, when selecting a suitable gear pump for a specified application, manufacturers are often confronted with a lack of comparable measurement data for the desired combination of operating conditions and pumping fluid. Consequently, an estimation of the volume flow rate and the power consumption of a pump under the operating conditions of the application is necessary.

In this context, this paper discusses the application of similarity on external gear pumps and presents its validation by means of measured pump characteristics. Seven gear pumps of different displacement volume are measured at different operating conditions varying pressure, rotational speed and the viscosity of the pumping fluid. The validation results prove that similarity is useful to represent a pump's characteristic over a wide operating range. The prediction of the volume flow rate and the power consumption at a changed viscosity show good accuracy. However, the scaling of the pump characteristic based on the displacement volume show contradictory results.

INTRODUCTION

Gear pumps are characterized by their wide operating range including diverse operating pressures, volume flows and pumping fluids. This motivates their utilization in numerous different applications and industrial sectors. The selection of an appropriate pump for a given application is always based on a compromise between application-related requirements and costs [1]. At the same time, it is mandatory to fulfill the function

of the pump, i.e. providing a required volume flow under a specified pressure.

Although robustness, price and available space are typically the major selection criteria, increasing attention is paid to efficiency considerations due to the reduction of power consumption and thus, savings in operation costs. However, when selecting a suitable gear pump for a specified application, manufacturers are often confronted with a lack of comparable measurement data for the desired combination of operating conditions and pumping fluid. Especially varying viscosities of the pumping fluid immensely affect the efficiency behavior of pumps. Consequently, an estimation of the volume flow and the power consumption of a pump under the operating conditions of the application is necessary. Highly precise calculations based on numerical simulations or experimental tests usually are too costly and time consuming and are therefore neglected. A useful estimation needs to be easy to apply, physically based and sufficiently accurate. To reduce the complexity it is advantageous to focus on the major influence parameters.

Against this background, similarity in gear pumps promises to be a simple and effective approach to estimate the volume flow and power consumption considering the above conditions. On this basis, the Pelz et. al. introduced a dimensionless and type-independent efficiency model of positive displacement pumps [2]. The model considers the operating conditions, i.e. pressure difference Δp and rotational speed n , the fluid properties, i.e. kinematic viscosity ν , density ρ and compressibility κ , and the machine parameters, i.e. displacement volume V and the average gap height \bar{s} .

As a result, four dimensionless variables are identified that describe the efficiency behavior: (i) specific pressure Δp^+ , (ii) Reynolds number Re , (iii) specific compressibility κ^+ , and (iv) relative gap ψ :

$$\begin{aligned}\Delta p^+ &:= \frac{\Delta p}{v^2 \varrho V^{-2/3}}, & Re &:= \frac{nV^{2/3}}{v}, \\ \kappa^+ &:= \kappa \Delta p, & \psi &:= \frac{s}{V^{1/3}}.\end{aligned}\quad (1)$$

From this previous work and the mentioned context of gear pumps, we derive the following three research questions of this paper:

- (i) How accurate can a gear pump characteristic be represented based on similarity?
- (ii) How accurate is a prediction of the volume flow (function) and the power consumption (cost) based on similarity?
- (iii) Is the pump characteristic scalable by means of the displacement volume?

To answer these research questions, an experimental study on seven external gear pumps is carried out. These gear pumps belong to three model series and are typically used for low pressure applications, e.g. lubrication and fuel. The efficiency measurements are performed compliant to ISO 4409 [3] for three different viscosities and six rotational speeds, respectively. The parameter range of the operating variables is summarized in table 1.

OPERATING VARIABLES	RANGE
pressure Δp	1 ... 25 bar
rotational speed n	600 ... 3600 rpm
kin. viscosity ν	22, 46, 100 cSt
density ϱ	835 ... 850 kg/m ³

Table 1: Parameter range of the operating variables at the efficiency measurements.

This paper begins with a brief literature overview on efficiency models. Afterwards similarity with regards to external gear pumps is analyzed and the experimental setup for this study is described. Subsequently, the three research questions are discussed based on the experimental results. The paper closes with the conclusion and outlook.

LITERATURE

The literature gives numerous studies on the efficiency of positive displacement pumps. The most comprehensive overview give Ivantsyn and Ivantysynova [4] and Kohmäscher et. al [5]. They differentiate the efficiency models into physical, empirical and data-driven models.

The physical models describe the volumetric and mechanical-hydraulic losses in positive displacement pumps. On this basis, Wilson [6] developed an efficiency model in the 1940s that assumes the leakage to be a laminar flow and describes the

mechanical-hydraulic losses as dependent on viscous friction only. Schlösser and Hilbrands [7,8,9,10] extend the leakage model with a turbulent flow component. The mechanical-hydraulic losses are complemented with a pressure-related and an inertia-related loss. Thoma [11] und Bravendik [12] further develop these models focusing on pumps with adjustable displacement volumes.

All these physical model rely on dimensionless loss coefficients, comparable with pressure loss coefficients. These loss coefficients need to be determined empirically and are assumed to be constant. However, Zarotti and Nervegna [13], Rydberg [14] as well as McCandlish and Dorey [15] conclude from their studies that this assumption can be violated, e.g. by changing gap height due to varying operating conditions. Consequently, this may lead to inaccuracy and model uncertainty. That is why they complete the physical description with empirical formulas based on experimental findings and increase the complexity of the models according to Ivantsyn und Ivantysynova [4].

Following Kohmäscher et. al. [5], another model based approach are the numerical or data-driven models of Ivantsyn and Ivantysynova [4], Huhtala [16] and Baum [17]. These models require a large amount of measurement data that is approximated by different numerical methods, e.g. non-linear polynomial approximation or neural networks.

In summary, a lot of research on the efficiency of positive displacement pumps was carried out in the past decades, leading to more complex and detailed models.

However, regarding the requirements for an estimation, i.e. being easy to apply, physically based and sufficiently accurate, physical models are still of high value. In this context, Corneli et al. [18] and Pelz et. al. [2] prove that the leakage behavior of a spindle screw pump can be modeled based on dimensional analysis [19] only, despite the complexity due to a variety of different gap geometries. Hence, a similarity consideration for gear pumps and its experimental validation is of high interest to meet the above requirements for a useful efficiency estimation.

SIMILARITY OF GEAR PUMPS

The isentropic efficiency η represents a measure of the energetic quality of a pump. Based on the first law of thermodynamics, for a time averaged stationary and thermally isolated machine the isentropic efficiency is defined as the hydraulic power divided by the shaft power P_s . The hydraulic power is obtained as the product of the volume flow Q_1 at the inlet of the pump, the discharge pressure Δp and a correction factor that depends on the compressibility κ . Considering the shaft power is the product of the shaft torque M_s and the rotational speed n , one obtains the well-known definition of the total efficiency η as (cf. [2])

$$\eta := \frac{Q_1 \Delta p}{2\pi M_s n} \left(1 - \frac{\kappa \Delta p}{2}\right). \quad (2)$$

In this study, the influence of the compressibility can be neglected due to low pressure, i.e. $\kappa \Delta p \ll 1$, and, thus, is

excluded in the further consideration. Consequently, the volume flow rate $Q_1 = Q$ is assumed to be constant for $\kappa\Delta p \ll 1$.

Extending equation (2) with the displacement volume V , the efficiency can be written as the product of the volumetric efficiency η_{vol} and the mechanical-hydraulic efficiency η_{mh}

$$\eta = \eta_{vol}\eta_{mh}, \quad \eta_{vol} := \frac{Q}{nV}, \quad \eta_{mh} := \frac{\Delta p V}{2\pi M_s}. \quad (3)$$

Both partial efficiencies can be represented as a function of the respectively responsible loss: for the volumetric efficiency, this is the leakage Q_L . Taking the theoretical volume flow rate $Q_{th} = nV = Q + Q_L$ into account, the volumetric efficiency η_{vol} can be written as

$$\eta_{vol} := \frac{Q}{nV} = 1 - \frac{Q_L}{nV}. \quad (4)$$

The friction torque M_{mh} represents the mechanical-hydraulic losses. Considering the shaft torque is the sum of the hydraulic torque $M_{hyd} = \Delta p V / 2\pi$ and the friction torque M_{mh} , gives the mechanical-hydraulic efficiency η_{mh}

$$\eta_{mh} := \frac{\Delta p V}{2\pi M_s} = \frac{1}{1 + 2\pi \frac{M_{mh}}{\Delta p V}}. \quad (5)$$

Following the above approach, a description of the losses, i.e. the leakage Q_L and the friction torque M_{mh} , results in a description of the volumetric and mechanical-hydraulic as well as total efficiency.

On this basis, we continue performing a dimensional analysis, the basis of our modeling and of similarity. The procedure is as follows:

firstly, all major influencing variables on the losses are determined. The following six influencing variables are considered: the operational parameters discharge pressure Δp and rotational speed n , the properties of the pumping medium density ρ and kinematic viscosity ν , and the geometric parameters displacement volume V and average gap height \bar{s} of the pump. The average gap height \bar{s} is a newly introduced size representing an average height all gap heights of the several different gaps of a gear pump. It is motivated by the analogy of a hydrodynamic journal bearing with the average height of the lubrication gap \bar{h} , i.e. \bar{s} is interpreted as \bar{h} . The characteristic length of the pump is defined as $V^{1/3}$.

Secondly, performing a dimensional analysis reduces the number of model variables and, thus, simplifies the model while maintaining the physical significance [19]. This yields five dimensionless variables that characterize the operating state of a pump: the specific pressure Δp^+ , Reynolds number Re , and relative gap size ψ are the independent dimensionless variables and are defined as

$$\Delta p^+ := \frac{\Delta p}{\nu^2 \rho V^{-2/3}}, \quad Re := \frac{nV^{2/3}}{\nu}, \quad \psi := \frac{s}{V^{1/3}}. \quad (6)$$

Furthermore, both the leakage Q_L and the friction torque M_{mh} are also represented by dimensionless variables, the specific leakage Q_L^+ and the specific friction torque M_{mh}^+ . These are the dependent dimensionless variables and are defined as

$$Q_L^+ := \frac{Q_L}{\nu V^{1/3}}, \quad M_{mh}^+ := \frac{M_{mh}}{\Delta p V}. \quad (7)$$

Finally, the specific leakage $Q_L^+ = Q_L^+(\Delta p^+, Re, \psi)$ and the specific friction torque $M_{mh}^+ = M_{mh}^+(\Delta p^+, Re, \psi)$ are functions of the specific pressure, Reynolds number and relative gap, which need to be determined. This leads directly to the descriptions of the volumetric, of the hydraulic-mechanical and of the total efficiency

$$\begin{aligned} \eta_{vol} &= 1 - \frac{1}{Re} Q_L^+(\Delta p^+, \psi) \\ \eta_{mh} &= \frac{1}{1 + \frac{2\pi}{1 - \kappa\Delta p/2} M_{mh}^+(\Delta p^+, Re, \psi)} \\ \eta &= \frac{1 - \frac{1}{Re} Q_L^+(\Delta p^+, \psi)}{1 + \frac{2\pi}{1 - \kappa\Delta p/2} M_{mh}^+(\Delta p^+, Re, \psi)}. \end{aligned} \quad (8)$$

On the basis of measurement data provided by pump manufacturers, Pelz et. al. [2] illustrate that the specific leakage Q_L^+ can be approximated by a semi empirical model in terms of a power law.

$$Q_L^+ = L * (\Delta p^+ \psi^3)^m. \quad (9)$$

The dimensionless leakage coefficient L and the exponent m are the only model parameters. The leakage coefficient includes the ratios of leakage specific geometric parameters of the pump to the characteristic length of the pump $V^{1/3}$. At the same time, it was shown, that the influence of the Reynolds number is negligible.

On the other hand, Schlösser and Hilbrands [9] introduced a physically based approach for the estimation of the friction torque M_{mh} that represents a linear combination of a pressure-related loss, a viscous friction-related loss and inertia-related loss:

$$M_{mh} = C\Delta p V + R_\mu \frac{\mu n V}{s} + R_\rho \rho n^2 V^{5/3}. \quad (10)$$

Applying the dimensionless quantities given by the equations (6) and (7) to this approach yields a description of the specific friction torque

$$M_{mh}^+(\Delta p^+, Re, \psi) = C + R_\mu \frac{Re}{\Delta p^+ \psi} + R_\rho \frac{Re^2}{\Delta p^+}. \quad (11)$$

C , R_μ and R_ρ are the dimensionless loss coefficients of the different loss terms. Furthermore, these loss coefficient also include the ratios of loss specific geometric parameters to the characteristic length of the pump $V^{1/3}$ (see [7,8,9,10]).

Altogether, there are five model parameters, namely L , m , C , R_μ and R_ρ , that need to be identified using measurement data. The model parameter identification is based on robust linear regression. The semi-analytical equations (9) and (11) together with equation (8) give the mathematical formulas which are applied on gear pumps in this paper.

Before that, we take a closer look on the prerequisite of a useful application of the concept of similarity and dimensional analysis. This is the presence of geometric similarity being a part of full similarity. For this study, this is crucial for two reasons:

firstly, the average gap height \bar{s} of a series of identical pumps will always vary from the nominal value due to manufacturing (including assembly) uncertainty. Hence, full geometric similarity in gear pumps is never present. Furthermore, from the practical point of view, it is advantageous to set a detailed geometrical modelling of the several different gaps of a gear pump aside in favor of determining the average gap height by means of the measured pump characteristic. For this purpose, Pelz et al. [2] introduce the relative gap $\psi := \bar{s}/V^{1/3}$ and the class of gap ψ/ψ_{ref} to provide an approach for relative considerations. On the one hand, the approach allows a characterization of the manufacturing uncertainty and, on the other hand, a comparison of pumps based on one single characteristic quantity is possible. In this paper we follow this approach.

Secondly, the seven gear pumps considered in this paper need to be investigated in regard to their geometric similarity. It turns out that a model series of gear pumps usually vary the gear width whereas the other main geometry parameters, e.g. number of teeth and diameters, and the pump housing remain unchanged. This also applies on the gear pumps of this study. These pumps are provided by two manufacturers, one model series with three pumps and two model series (see table 2: a and b) with two pumps, respectively. Table 2 shows the displacement volume and width ratio of these gear pumps.

	PUMP	DISPLACEMENT VOLUME in cm ³	WIDTH RATIO OF PUMPS b/b_{PUMP1}
MANUFACTURER	A	1	16.1
		2	20.1
		3	25.1
	B	1a	17.1
		2a	21.5
		1b	25.4
		2b	32.0

Table 2: Displacement volume and width ratio of gear pumps.

Hence, geometric similarity for these gear pump model series is not given, which negatively effects the accuracy when scaling the pump characteristic dependent on the displacement volume. Therefore, the incomplete geometric similarity must be considered in the context of the third research question of this paper.

As mentioned above, the leakage coefficient L and the loss coefficients C , R_μ and R_Q include ratios of loss specific geometric parameters to the characteristic length $V^{1/3}$. Is geometric similarity fulfilled, these ratios and, thus, the loss coefficients remain constant and independent of the displacement volume. However, since the displacement volume of the pump model series of this study scales solely to the gear width b (see table 2), all ratios of geometric parameters, and

therefore all of the above loss coefficients will change when varying the displacement volume. This is the reason for taking the following path for the discussion of the third research question instead:

all model parameters L , m , C , R_μ and R_Q for all seven gear pumps are identified based on the measured pump characteristics (see section ‘Experimental Setup’). However, for this purpose, in the definition of the dimensionless variables (cf. eqn. 6 and 7) the characteristic length $V^{1/3}$ is replaced by the addendum circle diameter d of each pump. This diameter is constant within each model series. From this point of view, all ratios of loss specific geometric parameters to the characteristic length d , that are included in the model parameters L , m , C , R_μ and R_Q , remain constant. Only the increase of the gear width within a pump model series influences the model parameters. Consequently, the influence of the increasing gear width is evaluated by means of the identified model parameters L , m , C , R_μ and R_Q of the gear pumps in this study. In this context, the following hypotheses are raised:

(i) the loss coefficients C and R_μ increase within a pump model series. Due to the increased gear width, the pressure acts on a larger area of the gear and generates higher forces in the rolling bearings. On the other hand, the sliding area between top land of gear and housing is increased and leads to higher values of friction torque. At this point, a change of the gap height, e.g. due to manufacturing uncertainty, would also influence the loss coefficient R_μ .

(ii) The loss coefficient R_Q decreases within a pump model series. The pressure losses due to cross-section change, e.g. Carnot shock loss, reduces with an increasing gap width.

(iii) The leakage coefficient L increases within a pump model series. The cross-section of the gap between top land of the gear and housing increases with a larger gap width. Like above, a change of the gap height, e.g. due to manufacturing uncertainty, will also influence the leakage coefficient L .

(iv) The exponent m remains unchanged.

As mentioned above, the change of the gap height due to manufacturing uncertainty can have a major influence on the leakage and on the viscous friction-related losses. This complicates the interpretation of the change of the model parameters within a model series. Nonetheless, all four hypotheses are evaluated on the measured pump characteristics in the ‘result’ section.

EXPERIMENTAL SETUP

The experimental setup as well as the methods of testing are in accordance with ISO 4409 [3]. Pressure and temperature are measured at the inlet and outlet of the pump. A piezoresistive sensor and a Pt-100 resistance thermometer are used, respectively. A torque meter with built-in speed sensor operating on the strain gage principle measures the shaft torque and rotational speed of the pump. Furthermore, a screw type flow meter is used to measure the volume flow rate at the outlet of the pump. The pressure variation is induced by means of an electric

ball valve in combination with a needle valve. Figure 1 and figure 2 show the hydraulic circuit diagram and the test bench for the efficiency measurements.

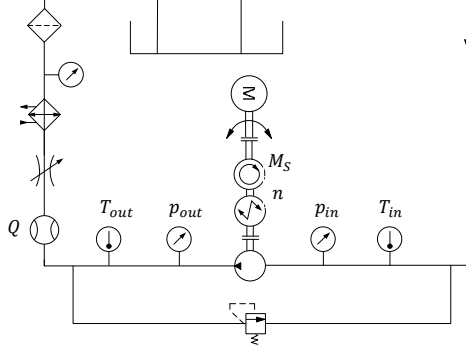


Figure 1: Hydraulic circuit diagram of the test bench.

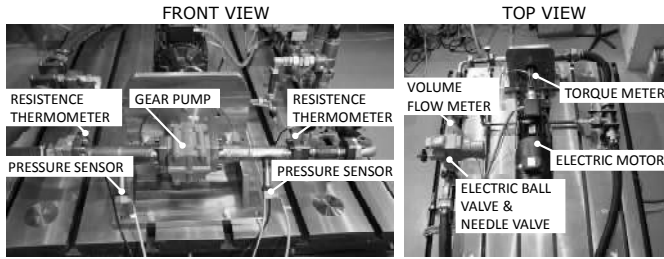


Figure 2: Test bench for the efficiency measurements.

The measurement equipment used in this study predominantly meets the accuracy requirements of class A (see ISO 4409 [3]). Solely for measurements with a low shaft torque, i.e. at low discharge pressures, the measurement accuracy of the shaft torque meter decreases and drops to class B or C respectively. Table 3 summarizes the measurement range and accuracy of the used measurement equipment.

MEASURED VARIABLES	MEASUREMENT RANGE	MEASUREMENT ACCURACY
p_{in}	0 ... 2.5 bar	0.15 % FS
p_{out}	0 ... 25 bar	0.15 % FS
Q	120 l/min	0.5 % MW
M_s	50 Nm	0.1 % FS
n	12000 rpm	0.1 % MW
T_{in}, T_{out}	0 ... 100 °C	± 0.15 °C

Table 3: Measurement ranges and uncertainty of measured operation variables (MW= measured value, FS=full scale).

The pump fluids used in this study are the standard hydraulic oils Shell Tellus S2 MX 46 at a temperature of 40 and 56 °C, which means a viscosity of 44 and 22 cSt, and Shell Tellus S2 MX 100 at a temperature of 40 °C, which means a viscosity of 100 cSt. Thus, for all seven pumps, the characteristic at these three viscosities are measured. The viscosity-temperature curves and the density-temperature curves of both hydraulic oils were measured with a highly accurate glass capillary viscometer and a density meter.

RESULTS

As the introduction states, this paper raises three research questions: firstly, how accurate can a gear pump characteristic be represented based on similarity? Secondly, how accurate is the prediction of the volume flow rate and the power consumption for variations of pumping fluid viscosity (changed application)? Thirdly, can a pump characteristic be scaled by means of the displacement volume? The following section discusses the first two research questions in detail by means of one single gear pump: Pump 2 of Manufacturer A. However, the approach and the results are transferable to all seven gear pumps.

First of all, the model of the leakage flow rate is validated. Equation (9) gives a mathematical description by means of a power law. It is necessary to evaluate the influence of the Reynolds number and to decide whether it is negligible for the gear pumps used in this study. Figure 3 and 4 show the results for the two independent parameter variations that were conducted for this purpose: Separate variation of Reynolds number and specific pressure, respectively. As figure 3 illustrates, the specific leakage clearly is linearly depend on the Reynolds number. Furthermore the slope is nearly constant across the different curves, it varies between 0.0175 and 0.02. This motivates the extension of the leakage model, so far solely pressure-driven, in the following manner:

$$Q_L^+ = L_{\Delta p^+} * (\Delta p^+ \psi^3)^m + L_{Re} Re \psi. \quad (12)$$

A new dimensionless leakage coefficient L_{Re} is introduced which represents the slope in figure 3. The dimensionless leakage coefficient L (see eqn. 9) is renamed to $L_{\Delta p^+}$.

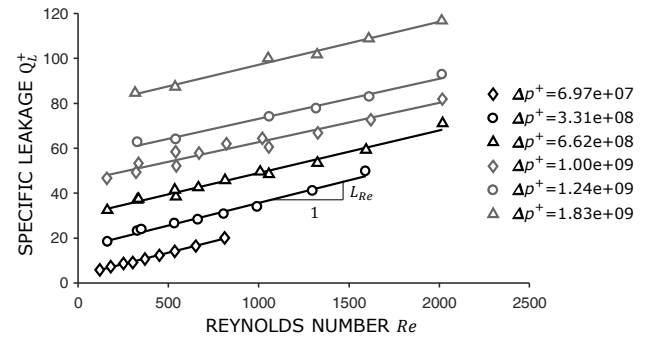


Figure 3: Specific leakage versus Reynolds number for six different specific pressures.

The linear influence of the Reynolds number is equivalent to the linear influence of the rotating speed, which represents a drag flow of the leakage. This drag flow can be found in the axial gap between the housing and the front and back side of each gear.

Figure 4 shows the specific leakage versus specific pressure for six different Reynolds numbers in a double logarithmic diagram. The power law in eqn. 12 describes the connection between the specific leakage and the specific pressure with good accuracy. However, the exponent m increases with the increasing Reynolds number.

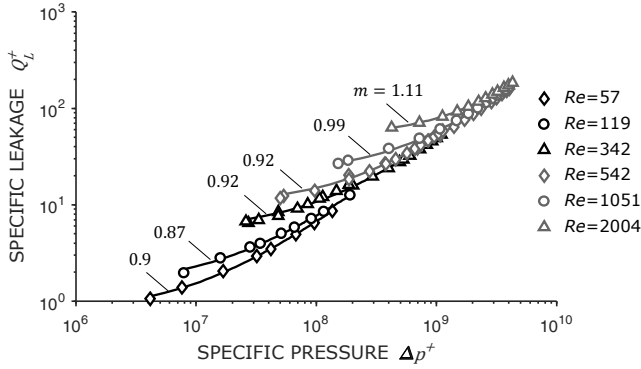


Figure 4: Specific Leakage versus specific pressure for six different Reynolds numbers.

After the consideration of these two separate influences on the specific leakage, in the next step the accuracy of the leakage model is determined. For this purpose, the relative deviation of the leakage model $\delta(Q_{L,Model})$ and the relative deviation of the volumetric efficiency model $\delta(\eta_{vol,Model})$ (see eqn. 8 and 12) are considered. Those relative deviations are defined as follows

$$\begin{aligned}\delta(Q_L) &:= \frac{|Q_{L,Model} - Q_{L,Measurement}|}{Q_{L,Measurement}}, \\ \delta(\eta_{vol}) &:= \frac{|\eta_{vol,Model} - \eta_{vol,Measurement}|}{\eta_{vol,Measurement}}.\end{aligned}\quad (13)$$

The model parameters $L_{\Delta p^+}$, L_{Re} and m are identified using all available measurement points. Hence, the relative deviation is a measure for the accuracy of the leakage model to represent the gear pump characteristic. Figure 5 shows both relative deviations between the measurement points and the model.

The relative deviation of the leakage model is below 5%, predominantly. Solely for low pressures, the deviation increases. Consequently, the relative deviation of the volumetric efficiency model is very low and mostly below 1%.

The following section shows the validation of the dimensionless friction torque model. The mathematical description is given in eqn. 11 and the accuracy of this model description can be determined in the same way as the accuracy of the leakage model.

Consequently, the relative deviation of the friction torque model $\delta(M_{mh,Model})$ and the relative deviation of the mechanical-hydraulic efficiency model $\delta(\eta_{mh,Model})$ (see eqn. 8 and 11) are examined. Following the approach of the leakage model, both deviations are defined as

$$\begin{aligned}\delta(M_{mh}) &:= \frac{|M_{mh,Model} - M_{mh,Measurement}|}{M_{mh,Measurement}}, \\ \delta(\eta_{mh}) &:= \frac{|\eta_{mh,Model} - \eta_{mh,Measurement}|}{\eta_{mh,Measurement}}.\end{aligned}\quad (14)$$

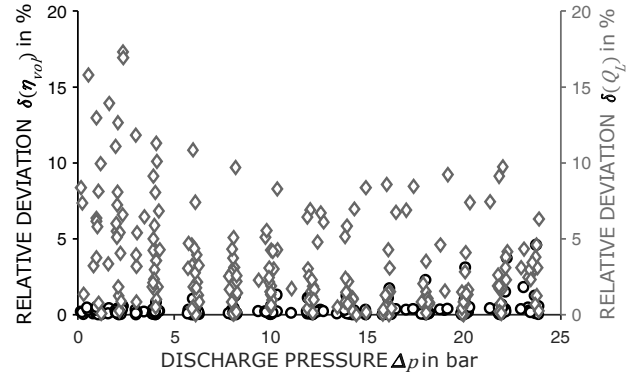


Figure 5: Relative deviation of the leakage model (grey diamond marker) and the volumetric efficiency model (black circle marker) versus the discharge pressure.

All model parameters of the friction torque model, namely C , R_μ and R_η are identified using all available measurement points with a discharge pressure above 4 bar. For discharge pressures below 4 bar, the hydraulic torque, and therefore also the shaft torque are low and the measurement uncertainty increases. At the same time, the dimensionless friction torque increases (see eqn. 7) due to the pressure independent losses. For this reason, these measurement points are neglected in the model parameter identification. Figure 6 shows both relative deviations between the measurement points and the model.

The relative deviation of the friction model is below 15%, predominantly. The relative deviation of the mechanical-hydraulic efficiency is also very low and mostly below 3%. In comparison with the leakage model, the friction torque model is less accurate. Nonetheless, the loss models and the corresponding partial efficiency models are able to represent the gear pump characteristic in the given operating range (cf. tab. 1).

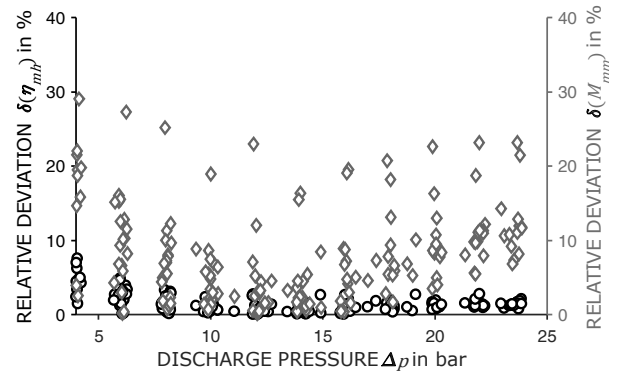


Figure 6: Relative deviation of the friction torque model (grey diamond marker) and the mechanical-hydraulic efficiency model (black circle marker) versus the discharge pressure.

The second research question is relevant for the practical use of the introduced similarity considerations. Measuring pump characteristics for more than one or two commonly used pumping fluids is costly for the manufacturers. Therefore, when selecting a suitable gear pump for a specified application, they are often confronted with a lack of comparable measurement

data for the desired combination of operating conditions and pumping fluid viscosity. For this reason, the practical benefit of the above leakage model and friction torque model is examined by means of the following case study: The pump characteristics measured at a viscosity of 22 cSt (Shell Tellus S2 MX 46 at 56°C) and 100 cSt (Shell Tellus S2 MX 100 at 40°C), are used to identify the six model parameters $L_{\Delta p+}$, L_{Re} , m , C , R_μ and R_Q . Subsequently, the calibrated models are used to estimate the pump characteristic at a viscosity of 44 cSt (Shell Tellus S2 MX 46 at 40°C).

Figure 7 and 8 show the estimation of the leakage and the volume flow rate together with the measurements at 44 cSt at four rotating speeds. Both estimations match the measurements with high accuracy.

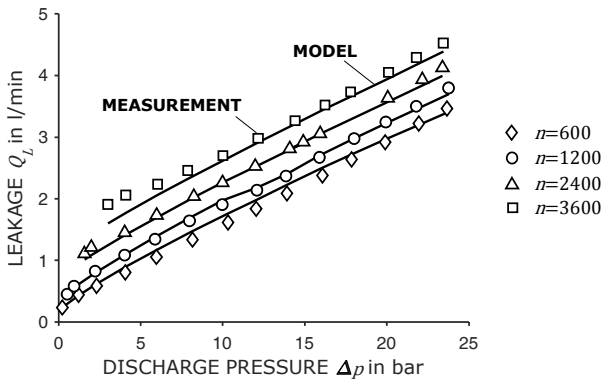


Figure 7: Leakage model and measurements at 44 cSt.

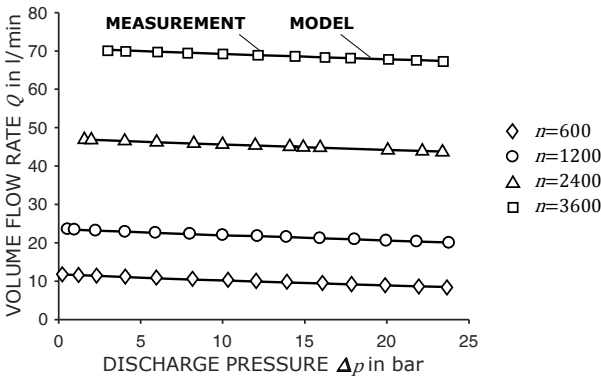


Figure 8: Volume flow rate model and measurements at 44 cSt.

Figure 9 and 10 show the estimations of the friction torque and the shaft power together with the measurements at 44 cSt at four rotating speeds. The estimation of the friction torque is of adequate accuracy for rotating speeds above 1200 rpm. The accuracy decreases with a lower rotating speed, where the slope of the model curve deviates noticeably from the measurements. As the rotating speed is constant, the slope deviation of the model curves must be caused by an inaccurate loss coefficient R_μ (cf. fig 9). The estimations of the shaft power approximate the measurements with high accuracy at all rotating speeds.

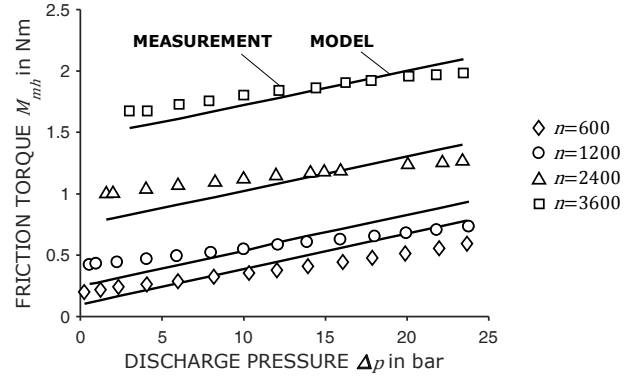


Figure 9: Friction torque model and measurements at 44 cSt.

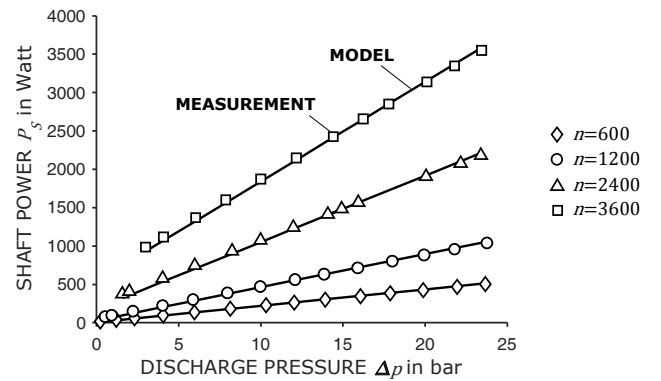


Figure 10: Shaft power model and measurements at 44 cSt.

In summary, the above results prove that the application of similarity and of the presented model is useful and beneficial to estimate the pump characteristic at a different viscosity. Following the approach of this paper, the prevalent lack of comparable measurement data for desired combinations of operating conditions and pumping fluid viscosity can be compensated.

The following section deals with the third research question which is the scaling of the pump characteristic based on similarity considerations. In the section ‘similarity of gear pumps’ the effect of the incomplete geometric similarity of the gear model series of this study are discussed and four hypotheses are raised. Figure 11 and 12 illustrate the results of the model parameter identification of the six model parameters $L_{\Delta p+}$, L_{Re} , m , C , R_μ and R_Q . The x-axis shows the pump of each model series, including three pumps of one model series of manufacturer A and two pumps per model series of manufacturer B (a and b; cf. table 2). In general, the gear width increases from pump 1 to pump 2 or 3 within each model series.

Hypothesis (i) is confirmed partially: Fig 11 shows that R_μ increases within all three model series, as it is expected. C increases within the model series of manufacturer A. However, the model series of manufacturer B show a contrary behavior.

Hypothesis (ii) does not apply on the model series of this study: R_Q increases within all three model series.

Hypothesis (iii) only applies to model series of manufacturer B where the leakage coefficient $L_{\Delta p+}$ increases (cf. figure 12). The leakage coefficient L_{Re} characterizes the axial gap between the front and back side of the gear and the housing. As the gear width does not directly affect the cross section of this gap (the manufacturing tolerance of the gear width effects the gap height of the axial gap, which is neglected here), no influence is expected. This is only confirmed by the two model series of manufacturer B.

Hypothesis (iv) is confirmed by all three model series. Figure 12 show that the deviations of the exponent m with the model series are low.

In summary, the hypotheses are only partially confirmed. The major reason for the deviation is expected to be a result of unknown changes of the gap height due to manufacturing uncertainty. At the same time, the applied model approach does not focus on pump specific details and may neglect influences that cause the partial deviations from the hypotheses.

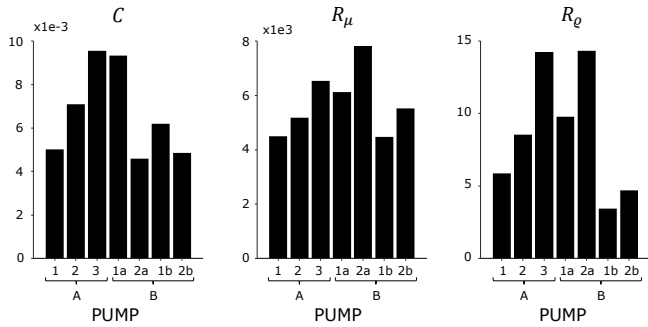


Figure 11: Model parameter identification of the specific friction torque models for three gear pump model series.

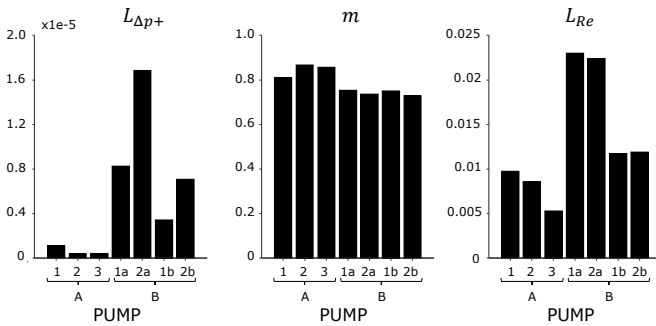


Figure 12: Model parameter identification of the specific leakage model for three gear pump model series.

CONCLUSION

The validation results prove that the presented model approach based on similarity is useful to represent a pump's characteristic over a wide operating range. The prediction of the volume flow rate and power consumption at a changed viscosity show good accuracy and are of high value for the manufacturer and operators. In fact, the authors have already received positive feedback to the presented leakage model from the industry.

However, the scaling of the pump characteristic based on the displacement volume is complex and show contradictory results. In particular the influence of the manufacturing uncertainty on the gap height affects the volumetric and mechanical-hydraulic losses that cannot be taken into account in detail.

Considering the requirements of a useful estimation of the pump characteristic at different operating conditions which are to be easy to apply, physically based and sufficiently accurate, the presented model approach meets all criteria.

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NOMENCLATURE

b	gear width
C	pressure-related loss coefficient
d	addendum circle
\bar{h}	average height of lubrication gap
L	dimensionless leakage coefficient
$L_{\Delta p+}$	dimensionless leakage coefficient
L_{Re}	dimensionless leakage coefficient
m	exponent for power law of specific leakage
M_{hyd}	hydraulic torque
M_{mh}	friction torque
M_{mh}^+	specific friction torque
M_S	shaft torque
n	rotational speed
Δp	discharge pressure
Δp^+	specific pressure
p_{in}	pressure at inlet of pump
p_{out}	pressure at outlet of pump
P_S	shaft power
Q	volume flow rate
Q_1	volume flow rate at inlet of pump
Q_L	leakage
Q_L^+	specific leakage
Q_{th}	theoretical volume flow rate
R_μ	viscous friction-related loss coefficient
R_q	pressure-related loss coefficient
Re	Reynolds number
\bar{s}	average gap height
V	displacement volume
$\delta(\cdot)$	relative deviation of a quantity
η	total efficiency
η_{mess}	measured efficiency
η_{mh}	mechanical-hydraulic efficiency
η_{vol}	volumetric efficiency
κ	compressibility
μ	dynamic viscosity

ν	kinematic viscosity
ρ	density
ψ	relative gap
ψ/ψ_{ref}	class of gap

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