

Modelling the errors of multi-axis machines : a general methodology

Citation for published version (APA):

Schellekens, P. H. J. (1992). Modelling the errors of multi-axis machines : a general methodology. *Precision Engineering*, 14(1), 5-19. [https://doi.org/10.1016/0141-6359\(92\)90137-L](https://doi.org/10.1016/0141-6359(92)90137-L)

DOI:

[10.1016/0141-6359\(92\)90137-L](https://doi.org/10.1016/0141-6359(92)90137-L)

Document status and date:

Published: 01/01/1992

Document Version:

Publisher's PDF, also known as Version of Record (includes final page, issue and volume numbers)

Please check the document version of this publication:

- A submitted manuscript is the version of the article upon submission and before peer-review. There can be important differences between the submitted version and the official published version of record. People interested in the research are advised to contact the author for the final version of the publication, or visit the DOI to the publisher's website.
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Modeling the errors of multi-axis machines: a general methodology

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A methodology is presented to obtain a generalized error model for multi-axis machines of arbitrary configuration. The model accounts for errors due to inaccuracies in the geometry, finite stiffness, and thermal deformation of the machine's components. Special statistical techniques are applied to the calibration data to obtain an empirical model for each of the errors. In a feedback loop, the identified significant parameters in these models are used for the computer-aided optimization of artefact-based test procedures. Results are shown for a five-axis milling machine and a three-axis coordinate measuring machine.

Keywords: multi-axis machines; volumetric accuracy; nonrigid body effects; least squares estimation; calibration; performance test

Introduction

Characterization and improvement of a machine tool's performance in terms of accuracy have become increasingly important in modern manufacturing. This is mainly due to the increased dimensional accuracy requirements of parts to be produced and a tendency toward small batch sizes of relatively expensive products, with associated high costs for trial runs to establish correct manufacturing parameters. Also the increased automation in all stages of manufacturing has contributed to this effect. The latter trend is accompanied by a tremendous growth in the application of numerically controlled multi-axis machines like metal cutting machine tools, industrial robots, and coordinate measuring machines (CMMs).

The accuracy of multi-axis machines is to a large extent determined by systematic errors in the relative location (i.e., position and orientation) of the tool (e.g., probe, cutter, robot end effector) for any two positions of the machine's carriages. Many studies have assessed the problem of describing this relationship.¹⁻⁹ The methods used range from correlation models to trigonometric analysis to "error matrix" representations. In recent reports, the use of rigid body kinematics is prominent. It results for a three-axis machine in a linear dependency of 21 measurable errors, which describe the difference between the actual and nominal relative location of two adjacent

bodies as a function of the intermediate carriage position.

Besides including revolute joints (e.g., turntables), machine tools with more than three axes and nonrigid body effects, this paper describes a general methodology to generate a model that relates the various errors to the status of the machine and its environment. By reducing these models to significant terms, a powerful tool is obtained for

- identification and analysis of major machine tool errors,
- software error compensation,
- evaluation and optimization of a calibration method's efficiency.

Design of the model

The proposed design procedure constitutes an integral part of the machine tool's quality control system with respect to its (quasi) static errors (*Figure 1*). The core of this system, and subject of this paper, is the so-called individual model. This model describes the error structure of an individual machine tool at a certain time and place. With the model, the machine's accuracy can be unambiguously assessed and, if desired, improved by software error correction. In the system, this model is preceded by two intermediate models: the general and type-dependent models. The general model relates errors in the relative location between tool and workpiece to errors in the relative location of coordinate frames attached to succeeding components of the machine. In order to reduce the calibration effort

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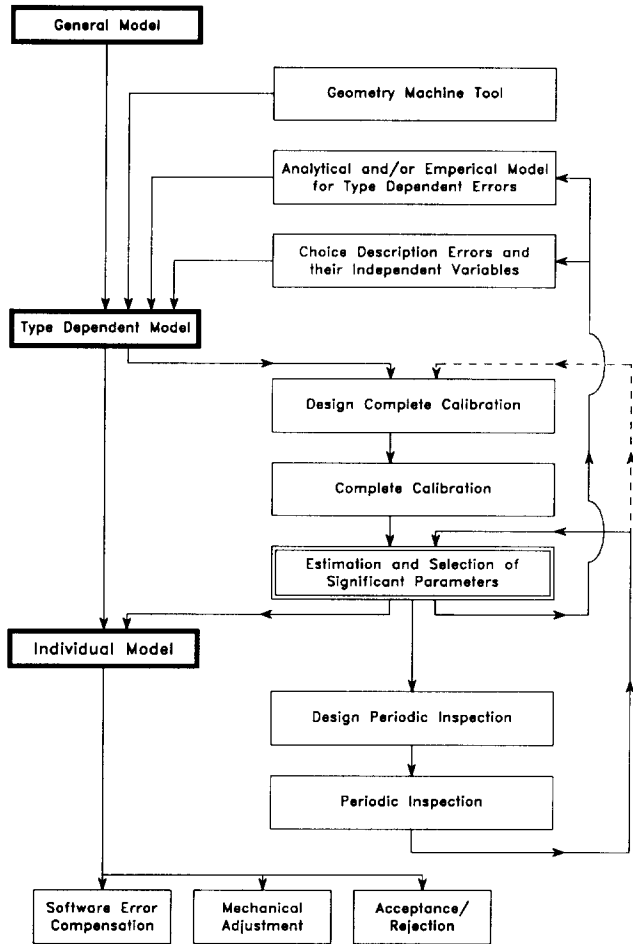


Figure 1 A machine tool's quality control system with respect to its (quasi) static errors

necessary for the estimation of the latter errors, a type-dependent model is developed. Optimal efficiency is achieved when the type-dependent model contains the common properties of the error structures belonging to machine tools of the same factory type (e.g., deformation due to the finite static stiffness of the machine's components and their thermal expansion).

The selection of significant parameters that describe the machine's errors constitutes an essential component of the proposed modeling procedure. In addition to an improvement of the individual model's quality in describing the machine tool's error structure, identification of these parameters greatly enhances its diagnostic potential. This information can be used to update the type-dependent model (e.g., to reduce the number of temperature sensors used for thermal compensation).

Due to the inherent sensitivity of multi-axis machine tools, the validity of the individual model is restricted to a limited time domain that has to be expanded by periodic performance evaluations. In these tests, a compromise has to be engineered between the efficiency in verifying characteristic errors and the

possible detection of new errors. A method is presented to use components of the various models to evaluate and optimize the performance evaluation or calibration.

The general model

The general model can be applied to multi-axis machines composed of revolute and prismatic joints in an arbitrary serial configuration. It relates errors between the actual and nominal location of the tool with respect to the workpiece to errors in the relative location of coordinate frames attached to succeeding components of the machine. Such errors describe the difference between the nominal and actual geometry of machine parts enclosed by two frames. In association with the complexity and efficiency of the final model and the effort needed to estimate it, the number and position of the coordinate frames are determined by the way in which errors introduced by the machine's components vary differently with respect to the status of the machine and its environment. Accuracy variations due to machine usage and between machine tools of the same factory type are generally caused by errors introduced in the machine's carriages. Since the position of these carriages also represents an important component of the machine's status, the number and position of the coordinate systems are chosen such that there is one kinematic element between every two frames.

Starting with the global coordinate system 0 attached to the machine tool's foundation, these orthogonal frames are successively numbered. As depicted in *Figure 2*, a prefix is added to this number in order to identify the corresponding frame as being part of kinematic chain *a* from foundation to tool or

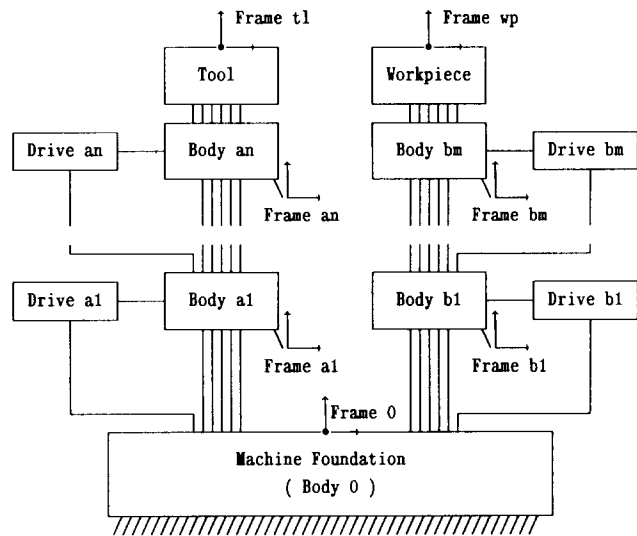


Figure 2 Nomenclature of the coordinate frames attached to a multi-axis machine with $n + m$ kinematic elements

chain b from foundation to workpiece. Two additional frames, wp and tl , are introduced, which are attached to workpiece and tool, respectively. The nominal relation between the homogeneous coordinates ${}_k\mathbf{p}$ and ${}_l\mathbf{p}$ of a point p in frames k and l , respectively, can be described by a 4×4 transformation matrix ${}_kT_l$:¹⁰

$${}_k\mathbf{p} = {}_kT_l {}_l\mathbf{p} \quad (1)$$

where

$${}_kT_l = \begin{bmatrix} {}_kR_l & {}_k\mathbf{t}_l \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (2)$$

$${}_k\mathbf{p} = [{}_k p_x \quad {}_k p_y \quad {}_k p_z \quad 1]^T \quad (3)$$

In this transformation, the 3×3 matrix ${}_kR_l$ describes the orientation of frame l with respect to frame k . The 3×1 vector ${}_k\mathbf{t}_l$ contains the coordinates of the origin of frame l in frame k .

The inverse transformation ${}_lT_k$ can be expressed as

$${}_lT_k = {}_lT_k {}_k\mathbf{p} \quad (4)$$

where

$${}_lT_k = \begin{bmatrix} {}_kR_l^T & -{}_kR_l^T {}_k\mathbf{t}_l \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (5)$$

For a multi-axis machine composed of n kinematic elements in chain a and m elements in chain b , successive application of these transformations yields the following expression for the nominal location ${}_{wp}T_{tl}$ of the tool coordinate system tl in the workpiece coordinate system wp .

$${}_{wp}T_{tl} = {}_{wp}T_{00} T_{tl} \quad (6)$$

$$= {}_{wp}T_{bm} \prod_{k=m}^1 ({}_{bk}T_{bk-1}) \prod_{k=1}^n ({}_{ak-1}T_{ak}) \quad (7)$$

In the actual machine tool, errors can be identified in the relative location of subsequent frames as well as the location of the tool with respect to the last frame an of the kinematic chain. Because none of the contemporary multi-axis machines shows an absence of Abbe offsets, the relevant errors in the relative location of two subsequent frames are not limited to those in the moving direction of the enclosed kinematic element. Consequently, the following errors are to be considered in the location of frame k with respect to frame $k-1$:

- translational errors ${}_{k-1}e_{kx}$, ${}_{k-1}e_{ky}$, and ${}_{k-1}e_{kz}$ along the X, Y, and Z axes of frame k , respectively.
- angular errors ${}_{k-1}\varepsilon_{kx}$, ${}_{k-1}\varepsilon_{ky}$, and ${}_{k-1}\varepsilon_{kz}$ about the X, Y, and Z axes of frame k , respectively.

In the analysis of the effect of angular errors on the machine tool's accuracy, a first-order approximation is used. Application of this approximation yields additive and commutative properties for the various errors. This results in the following relationship between the actual

transformation ${}_{k-1}Ta_k$ and its nominal ${}_{k-1}T_k$:

$$\begin{aligned} {}_{k-1}Ta_k &= {}_{k-1}T_k \text{Trans}[x, {}_{k-1}e_{kx}] \cdot \\ &\quad \text{Trans}[y, {}_{k-1}e_{ky}] \cdot \\ &\quad \text{Trans}[z, {}_{k-1}e_{kz}] \cdot \\ &\quad \text{Rot}[x, {}_{k-1}\varepsilon_{kx}] \cdot \\ &\quad \text{Rot}[y, {}_{k-1}\varepsilon_{ky}] \cdot \\ &\quad \text{Rot}[z, {}_{k-1}\varepsilon_{kz}] \end{aligned} \quad (8)$$

$$= {}_{k-1}T_k (I + {}_{k-1}\delta T_k) \quad (9)$$

where

$${}_{k-1}\delta T_k = \begin{bmatrix} 0 & -{}_{k-1}\varepsilon_{kz} & {}_{k-1}\varepsilon_{ky} & {}_{k-1}e_{kx} \\ {}_{k-1}\varepsilon_{kz} & 0 & -{}_{k-1}\varepsilon_{kx} & {}_{k-1}e_{ky} \\ -{}_{k-1}\varepsilon_{ky} & {}_{k-1}\varepsilon_{kx} & 0 & {}_{k-1}e_{kz} \\ 0 & 0 & 0 & 0 \end{bmatrix} \quad (10)$$

and I : 4×4 identity matrix

The transformations $\text{Trans}[x, {}_{k-1}e_{kx}]$ and $\text{Rot}[x, {}_{k-1}\varepsilon_{kx}]$ represent, respectively, a translation along the local X-axis by a distance ${}_{k-1}e_{kx}$ and a rotation about the same axis by an angle ${}_{k-1}\varepsilon_{kx}$. Similarly, the actual location ${}_{wp}Ta_{tl}$ of the tool coordinate system with respect to the workpiece coordinate system can be expressed as

$${}_{wp}Ta_{tl} = {}_{wp}T_{tl} (I + {}_{wp}\delta T_{tl}) \quad (11)$$

where

$${}_{wp}\delta T_{tl} = \begin{bmatrix} 0 & -{}_{wp}\varepsilon_{tlz} & {}_{wp}\varepsilon_{tly} & {}_{wp}e_{tlx} \\ {}_{wp}\varepsilon_{tlz} & 0 & -{}_{wp}\varepsilon_{tlx} & {}_{wp}e_{tly} \\ -{}_{wp}\varepsilon_{tly} & {}_{wp}\varepsilon_{tlx} & 0 & {}_{wp}e_{tlz} \\ 0 & 0 & 0 & 0 \end{bmatrix} \quad (12)$$

Here transformation ${}_{wp}\delta T_{tl}$ contains the errors in the location of the tool coordinate frame with respect to the workpiece. It consists of translational errors ${}_{wp}\mathbf{e}_{tl} = [{}_{wp}e_{tlx} \quad {}_{wp}e_{tly} \quad {}_{wp}e_{tlz}]^T$ defined along the X, Y, and Z axes of the nominal toolframe tl and angular errors ${}_{wp}\boldsymbol{\varepsilon}_{tl} = [{}_{wp}\varepsilon_{tlx} \quad {}_{wp}\varepsilon_{tly} \quad {}_{wp}\varepsilon_{tlz}]^T$ about these axes. Successive application of relation 9 yields the following expression for the actual location ${}_{wp}Ta_{tl}$ of the tool coordinate system with respect to the workpiece coordinate system:

$${}_{wp}Ta_{tl} = {}_{wp}T_{bm} \prod_{k=m}^1 ({}_{bk}Ta_{bk-1}) \prod_{k=1}^n ({}_{ak-1}Ta_{ak}) \quad (13)$$

$$= {}_{wp}T_{bm} \prod_{k=m}^1 [(I - {}_{bk-1}\delta T_{bk}) {}_{bk}T_{bk-1}]$$

$$\prod_{k=1}^n [{}_{ak-1}T_{ak} (I + {}_{ak-1}\delta T_{ak})] \cdot {}_{an}T_{tl} (I + {}_{an}\delta T_{tl}) \quad (14)$$

Transformation ${}_{an}\delta T_{tl}$ contains the errors in the location of the tool with respect to the last frame an of the kinematic chain. In case of CMMs, these errors (essentially the position of the measured point with respect to the ram) are generally related to the probe system and probe strategy used.^{11,12} For metal-cutting machine tools, errors such as spindle-induced errors, tool misalignment, tool wear, and thermal tool expansion can be included in this transformation.¹³ Note that the transformation from frame bm to the workpiece frame wp is included in relation 14 as a nominal transformation. This implies that only errors introduced by the machine are taken into account. Errors in the location of the workpiece with respect to the machine (i.e., ${}_{bm}\delta T_{wp}$) are not considered.

In the elaboration of relation 14, an approximation will be made by ignoring higher order effects consisting of the product of a matrix δT with one or more similar matrices. This approximation is valid since the difference between the actual and nominal machine structure usually does not significantly change the active arm of angular errors and the direction in which the various errors act. Combining relation 11 with relation 14 now yields the following expression for the error ${}_{wp}\delta T_{tl}$ in the relative location between tool and workpiece:

$$\begin{aligned} {}_{wp}\delta T_{tl} = & - {}_{tl}T_{bm} \sum_{k=m}^1 ({}_{bm}T_{bk} {}_{bk-1}\delta T_{bk} {}_{bk}T_{an}) {}_{an}T_{tl} \\ & + {}_{tl}T_0 \sum_{k=1}^n ({}_0T_{ak} {}_{ak-1}\delta T_{ak} {}_{ak}T_{an}) {}_{an}T_{tl} \\ & + {}_{an}\delta T_{tl} \end{aligned} \quad (15)$$

A more convenient description, which also provides more intuitive insight in the basic error relationships, can be obtained by decomposing the error transformations of relation 15 into their basic errors ε and \mathbf{e} . This procedure requires some vector algebra \mathbb{V} ,¹⁴ after which the angular and translational errors between tool and workpiece, contained in the 6×1 vector ${}_{wp}\mathbf{E}_{tl}$ can be expressed as similarly denoted errors in the relative location between succeeding frames:

$$\begin{aligned} {}_{wp}\mathbf{E}_{tl} = & - \sum_{k=1}^m ({}_{tl}F_{bk} {}_{bk-1}\mathbf{E}_{bk}) \\ & + \sum_{k=1}^n ({}_{tl}F_{ak} {}_{ak-1}\mathbf{E}_{ak}) + {}_{an}\mathbf{E}_{tl} \end{aligned} \quad (16)$$

where

$${}_{wp}\mathbf{E}_{tl} = [{}_{wp}\varepsilon_{tlx}, {}_{wp}\varepsilon_{tly}, {}_{wp}\varepsilon_{tlz}, {}_{wp}\mathbf{e}_{tlx}, {}_{wp}\mathbf{e}_{tly}, {}_{wp}\mathbf{e}_{tlz}]^T \quad (17)$$

$$\begin{aligned} {}_{k-1}\mathbf{E}_k = & [{}_{k-1}\varepsilon_{kx}, {}_{k-1}\varepsilon_{ky}, {}_{k-1}\varepsilon_{kz}, {}_{k-1}\mathbf{e}_{kx}, {}_{k-1}\mathbf{e}_{ky}, {}_{k-1}\mathbf{e}_{kz}]^T \end{aligned} \quad (18)$$

$${}_{tl}F_k = \begin{bmatrix} {}_{tl}R_k & 0 \\ ({}_{tl}\mathbf{t}_k \times {}_{tl}R_k) & {}_{tl}R_k \end{bmatrix} \quad (6 \times 6) \text{ matrix} \quad (19)$$

In relation 16 the 6×6 matrices ${}_{tl}F_k$ describe how errors ${}_{k-1}\mathbf{E}_k$ in the relative location between two succeeding frames affect the errors ${}_{wp}\mathbf{E}_{tl}$ in the relative location between tool and workpiece. In these matrices, ${}_{tl}\mathbf{t}_k \times {}_{tl}R_k$ denotes a 3×3 matrix whose columns contain the vector product of vector ${}_{tl}\mathbf{t}_k$ with the respective columns of matrix ${}_{tl}R_k$. Thus, errors ${}_{wp}\mathbf{e}_{tl}$ in the relative position of the frames attached to tool and workpiece are expressed as a linear combination of the errors ${}_{k-1}\mathbf{e}_k$ in the relative position of two subsequent frames, and the effect of related angular errors ${}_{k-1}\varepsilon_k$ with active arm ${}_{k}\mathbf{t}_{tl}$. As with errors ${}_{wp}\varepsilon_{tl}$ in the relative orientation between tool and workpiece, each component k in this sum is transformed to the tool coordinate frame by a rotation ${}_{tl}R_k$. Finally error ${}_{an}\mathbf{E}_{tl}$ in the relative location of the tool with respect to the last frame an of the kinematic chain is added. Note that the errors ${}_{wp}\mathbf{E}_{tl}$ are defined in the nominal tool coordinate system. For some applications it is necessary to transform these errors to the workpiece coordinate system. This can be implemented in relation 16 by premultiplying each of the 3×3 submatrices of ${}_{tl}F_k$ with the appropriate orientation transformation ${}_{wp}R_{tl}$.

As already discussed, the chosen nominal location of the various frames seriously affects the efficiency of the final model. A generally useful model can be obtained by placing the frames in the centroid of the various kinematic elements, with one axis aligned with the respective axis of movement. Consistent application of this rule requires a distinction between kinematic elements whose corresponding frame moves with the carriage and those where this frame is fixed relative to the guide (Figure 3). This can be incorporated into the model by introducing so-called shape and joint transformations. The shape transformation ${}_{k-1}S_k$ describes the relative nominal location between frames $k-1$ and k , in case their respective kinematic elements are at home position. Joint transformation J_k describes the nominal angular or translational movement of kinematic element k . In accordance with these characteristics of the respective kinematic elements,

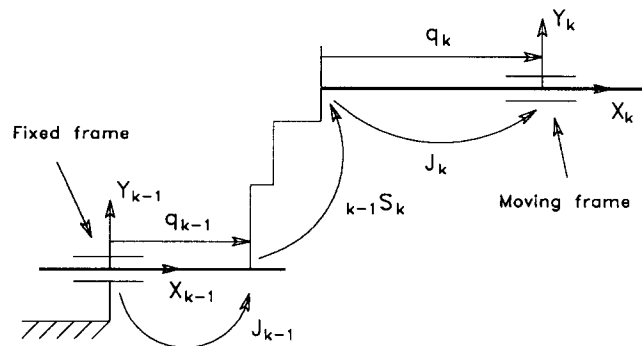


Figure 3 Connection between two kinematic elements with a fixed and moving coordinate frame

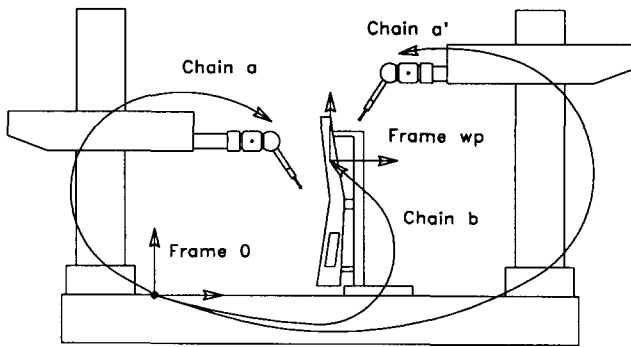


Figure 4 Dual CMM configuration with associated kinematic chains

application of these transformations yields the following expression for the relative nominal location ${}_{k-1}T_k$ between two succeeding coordinate frames:

- Moving → moving : ${}_{k-1}T_k = {}_{k-1}S_k J_k$ (20)

- Moving → fixed : ${}_{k-1}T_k = {}_{k-1}S_k$ (21)

- Fixed → moving : ${}_{k-1}T_k = J_{k-1} {}_{k-1}S_k J_k$ (22)

- Fixed → fixed : ${}_{k-1}T_k = J_{k-1} {}_{k-1}S_k$ (23)

As stated in the introduction of this section, the derived general model can be applied to multi-axis machines composed of revolute and prismatic joints in an arbitrary serial configuration. There exists however a class of CMMs where two machines are used simultaneously to inspect a part (Figure 4). Yet the position of a certain point on the workpiece's surface can be measured by only one machine at a time. Therefore, the modeling technique presented in this section can be applied to these CMMs by constructing an a-type kinematic chain for each of the two machines. In accordance with the measuring strategy used, the error structure of only one of these chains is active when measuring a certain point.

Example 1

As an example, the methodology described is applied to the five-axis milling machine depicted in Figure 5. This machine tool consists of one horizontal prismatic element in chain a from foundation to tool. Chain b from foundation to workpiece consists of two prismatic elements, one vertical and one horizontal, and two revolute joints with, respectively, a horizontal and a vertical axis. In the first stage of the modeling process, coordinate frames are located in the workpiece, the tool, and the centroid of each joint.

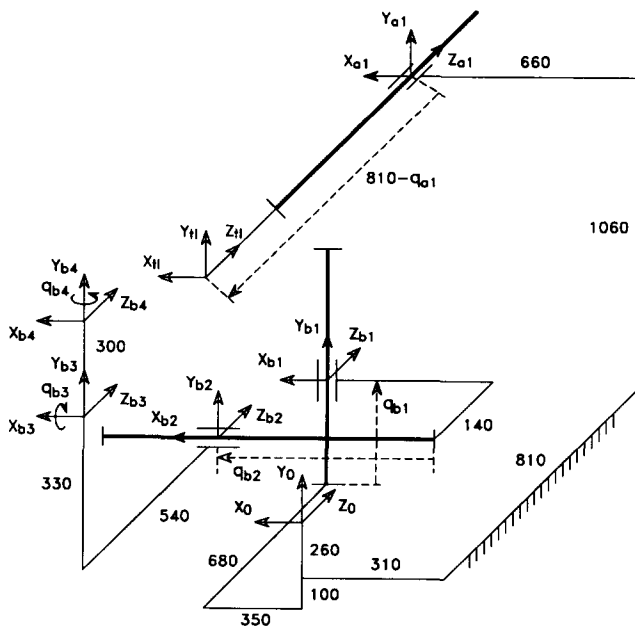
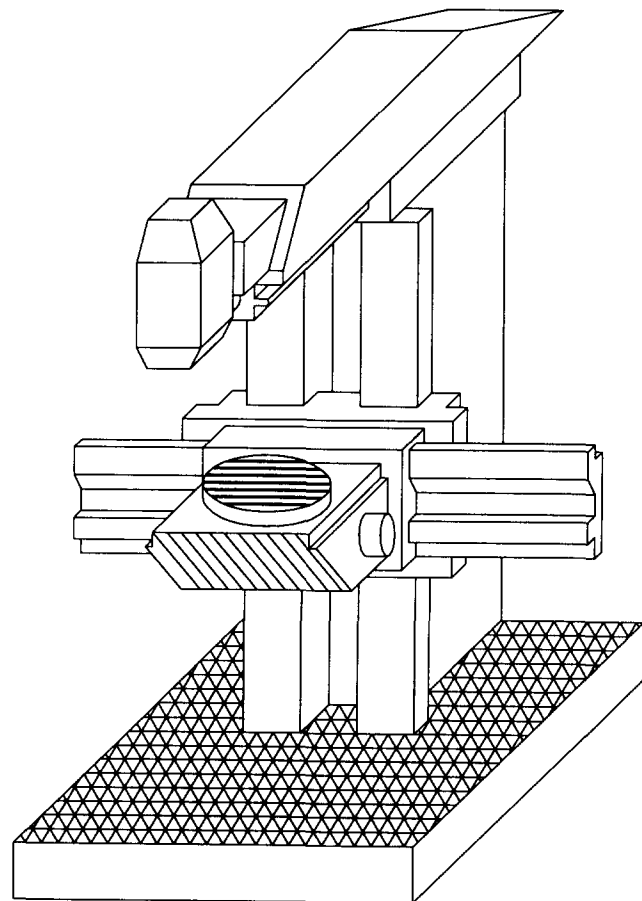


Figure 5 Five-axis milling machine with its kinematic representation



The frames located in the various kinematic elements can be characterized as

- Frame b4 : fixed
- Frame b3 : fixed
- Frame b2 : moving
- Frame b1 : moving
- Frame a1 : fixed

The nominal coordinate transformations between succeeding frames can be expressed as

$${}_{a1}T_{tl} = J_{a1} S_{tl} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & q_{a1} \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (24)$$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & -810 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (24)$$

$${}_{0}T_{a1} = {}_{0}S_{a1} = \begin{bmatrix} 1 & 0 & 0 & 350 \\ 0 & 1 & 0 & 800 \\ 0 & 0 & 1 & 810 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (25)$$

$${}_{0}T_{b1} = {}_{0}S_{b1} J_{b1} = \begin{bmatrix} 1 & 0 & 0 & 350 \\ 0 & 1 & 0 & -360 \\ 0 & 0 & 1 & 680 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (26)$$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & q_{b1} \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (26)$$

$${}_{b1}T_{b2} = {}_{b1}S_{b2} J_{b2} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & -140 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (27)$$

$$\begin{bmatrix} 1 & 0 & 0 & q_{b2} \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (27)$$

$${}_{b2}T_{b3} = {}_{b2}S_{b3} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 330 \\ 0 & 0 & 1 & -540 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (28)$$

$${}_{b3}T_{b4} = J_{b3} S_{b4} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & cq_{b3} & -sq_{b3} & 0 \\ 0 & sq_{b3} & cq_{b3} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (29)$$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 300 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (29)$$

$${}_{b4}T_{wp} = J_{b4} S_{wp} = \begin{bmatrix} cq_{b4} & 0 & sq_{b4} & 0 \\ 0 & 1 & 0 & 0 \\ -sq_{b4} & 0 & cq_{b4} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} {}_{b4}S_{wp} \quad (30)$$

Where cq and sq, respectively, represent cos(q) and sin(q). Appropriate multiplication of the above transformations results in

$${}_{tl}R_{a1} = {}_{tl}R_{b1} = {}_{tl}R_{b2} = {}_{tl}R_{b3} = I (3 \times 3) \quad (31)$$

$${}_{tl}R_{b4} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & cq_{b3} & -sq_{b3} \\ 0 & sq_{b3} & cq_{b3} \end{bmatrix} \quad (32)$$

$${}_{tl}t_{a1} = [0, 0, -q_{a1} + 810]^T \quad (33)$$

$${}_{tl}t_{b1} = [0, q_{b1} - 1,160, -q_{a1} + 680]^T \quad (34)$$

$${}_{tl}t_{b2} = [q_{b2}, q_{b1} - 1,160, -q_{a1} + 540]^T \quad (35)$$

$${}_{tl}t_{b3} = [q_{b2}, q_{b1} - 830, -q_{a1}]^T \quad (36)$$

$${}_{tl}t_{b4} = [q_{b2}, q_{b1} + 300 \cdot cq_{b3} - 830, -q_{a1} + 300 \cdot sq_{b3}]^T \quad (37)$$

Application of relation 16 yields the following expression for the errors ${}_{wp}e_{tl}$ and ${}_{wp}e_{tl}$ in the orientation and position of the tool coordinate frame with respect to the workpiece coordinate frame:

$${}_{wp}e_{tl} = {}_{a1}e_{tl} + {}_{0}e_{a1} - {}_{0}e_{b1} - {}_{b1}e_{b2} - {}_{b2}e_{b3} - \begin{bmatrix} 1 & 0 & 0 \\ 0 & cq_{b3} & -sq_{b3} \\ 0 & sq_{b3} & cq_{b3} \end{bmatrix} {}_{b3}e_{b4} \quad (38)$$

$${}_{wp}e_{tl} = {}_{a1}e_{tl} + \begin{bmatrix} 0 & q_{a1} - 810 & 0 & 1 & 0 & 0 \\ -q_{a1} + 810 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} {}_{0}E_{a1} - \begin{bmatrix} 0 & q_{a1} - 680 & q_{b1} - 1,160 & 1 & 0 & 0 \\ -q_{a1} + 680 & 0 & 0 & 0 & 1 & 0 \\ -q_{b1} + 1,160 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} {}_{0}E_{b1} -$$

$$\begin{bmatrix} 0 & q_{a1} - 540 & q_{b1} - 1,160 & 1 & 0 & 0 \\ -q_{a1} + 540 & 0 & -q_{b2} & 0 & 1 & 0 \\ -q_{b1} + 1,160 & q_{b2} & 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 0 & q_{a1} & q_{b1} - 830 & 1 & 0 & 0 \\ -q_{a1} & 0 & -q_{b2} & 0 & 1 & 0 \\ -q_{b1} + 830 & q_{b2} & 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 0 & q_{a1} \cdot cq_{b3} + (q_{b1} - 830) \cdot sq_{b3} & 0 & 0 & 0 & 0 \\ -q_{a1} + 300 \cdot sq_{b3} & -q_{b2} \cdot sq_{b3} & 0 & 0 & 0 & 0 \\ -q_{b1} - 300 \cdot cq_{b3} + 830 & q_{b2} \cdot cq_{b3} & 0 & 0 & 0 & 0 \\ -q_{a1} \cdot sq_{b3} + (q_{b1} - 830) \cdot cq_{b3} + 300 & 1 & 0 & 0 & 0 & 0 \\ q_{b2} \cdot cq_{b3} & 0 & cq_{b3} & -sq_{b3} & 0 & 0 \\ -q_{b2} \cdot sq_{b3} & 0 & sq_{b3} & cq_{b3} & 0 & 0 \end{bmatrix}$$

(39)

The type-dependent model

In the former section, a so-called general model has been derived that relates errors in the actual relative location of frames attached to tool and workpiece to errors in the relative location of frames attached to succeeding components of the multi-axis machine. To construct the individual model, which describes the accuracy of a certain machine at a certain time and place, it is necessary to obtain a relation between the latter errors and the status of the machine and its environment. This usually requires an extensive calibration of the machine tool. In order to increase the efficiency of such a procedure, a type-dependent model is developed by implementing into the general model the common properties of error structures belonging to the same factory type of machine tools. Generally, these properties do not only comprise fully modeled errors but also qualitative information, such as an error's possible independent variables and an assembly of suitable function types to be used in its model. It should be noted that the ratio of type-dependent errors in the total error structure of the machine can be significantly improved by suitable design and assembly rules. The major type-dependent (quasi) static errors in the relative location of frames $k - 1$ and k can be affiliated with

- the finite stiffness of the machine tool's components (${}_{k-1}^s \mathbf{E}_k$),
- the expansion coefficient of the machine tool's components (${}_{k-1}^t \mathbf{E}_k$),
- properties of the machine tool's slideways, joints, and measuring systems (${}_{k-1}^m \mathbf{E}_k$).

In the analysis of these errors, we assume that deviations in the relative orientation of subsequent frames are small to such an extent that their effect on the direction of mass forces acting on the various

components does not affect the total accuracy of the machine. This allows for an independent treatment and analysis of the various errors. Thus, the errors ${}_{k-1} \mathbf{E}_k$ in the relative location of frames $k - 1$ and k can be formally expanded to

$${}_{k-1} \mathbf{E}_k = {}_{k-1}^s \mathbf{E}_k + {}_{k-1}^t \mathbf{E}_k + {}_{k-1}^m \mathbf{E}_k + {}_{k-1}^i \mathbf{E}_k \tag{40}$$

Where ${}_{k-1}^i \mathbf{E}_k$ describes the components that vary from machine to machine, which might include some thermal and finite stiffness-related errors. Note that this expansion is only required when subassemblies of the various components (e.g., temperature and finite stiffness effects) are analyzed or estimated independently of the other errors.

Due to the finite stiffness of the machine's components, errors in the relative location of succeeding frames are not only related to the machine tool's environment, internal heat sources, and the position of the enclosed kinematic element.^{5,12} Assuming small angular errors, as is the case with most contemporary machine tools, their relation with the position of other kinematic elements can, in the unloaded case, usually be summarized as:

- errors in the location of subsequent frames are not affected by the position of a revolute joint with vertical axis or a prismatic joint with an arbitrary axis, provided that they are lower in the kinematic chain that provides the connection with the machine tool's foundation.
- The position of a prismatic joint with vertical axis of movement only affects errors in the relative location of frames directly connected to it.

Example 2

As an example, we consider the gantry style coordinate measuring machine depicted in Figure 6. The machine's chain a from foundation to probe consists of three

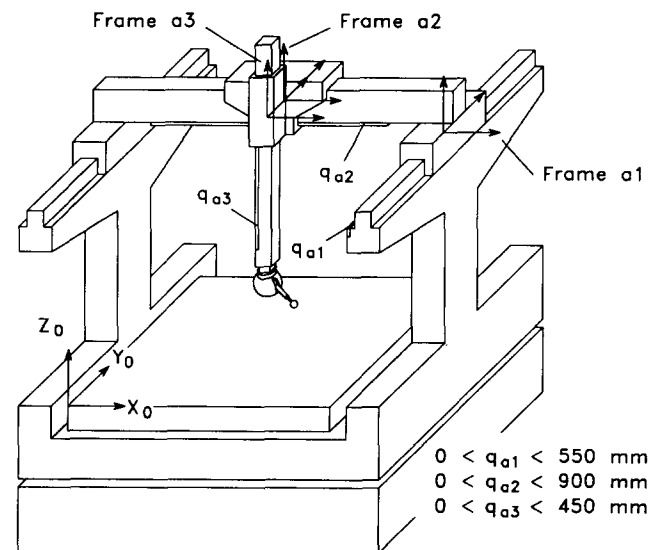


Figure 6 Gantry style CMM

prismatic elements, two with horizontal and one with vertical movement. Using the preceding rules, the finite stiffness-related errors in the location of succeeding frames can formally be expanded to

$${}^0\mathbf{E}_{a1} = {}^0\mathbf{E}_{a1}[q_{a1}, q_{a2}] \quad (41)$$

$${}^{a1}\mathbf{E}_{a2} = {}^{a1}\mathbf{E}_{a2}[q_{a2}] \quad (42)$$

$${}^{a2}\mathbf{E}_{a3} = {}^{a2}\mathbf{E}_{a3}[q_{a3}] \quad (43)$$

Due to the special air-bearing configuration in the first kinematic element, we expected a linear dependency of ${}^0\mathbf{E}_{a1}$ on the second kinematic element's position q_{a2} . This hypothesis was confirmed to a high level of confidence by the significant parameter analysis explained in the next section (Figure 7).

The position, magnitude, and direction of the forces exerted by workpiece and fixturing, in general only affect errors in the kinematic chain from workpiece to foundation. In contrast, forces exerted by the machining process can cause deformation of the total machine structure. However, in most finishing operations the effect of these forces is small due to the relative high stiffness of the machine's components. Usually their strong periodically variable components require the use of rather complex techniques to model the related variable tool deflection and its imprint on the machined surface.¹⁵

Although static finite stiffness-related errors introduced by the machine tool's bearings can be approximated analytically,¹⁴ similar analysis of the deformation of the connection elements between the various carriages is difficult. This is due to the multiple use of complex three-dimensional construction elements. Numerical methods, either finite elements or finite differences, provide an efficient tool for the computer-aided modeling of these deformations. However, at this moment, the application of such models is generally limited to qualitative analysis in the design stage of the machine tool. Therefore, we prefer the empirical method outlined in the next section to model the finite stiffness-related errors of the machine tool.

Internal heat sources and changes in the environmental temperature and thermal gradients cause alterations in the temperature profile of the machine tool. The associated time-dependent errors in the location of two succeeding frames are usually related

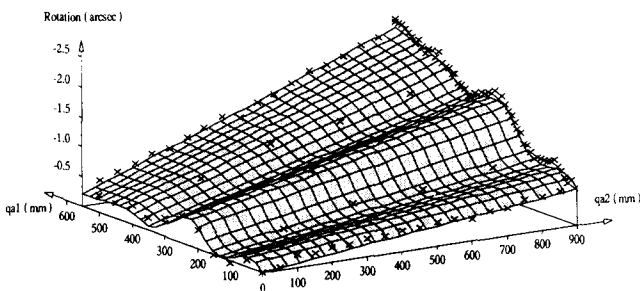


Figure 7 Angular error ${}^0\mathbf{E}_{a1x}(q_{a1}, q_{a2})$ in the location of frame $a1$ with respect to frame 0. Note the linear dependency on q_{a2}

to the temperature profile ${}_{k-1}\mathbf{TH}_k$ of the components enclosed by the frames and the intermediate carriage position q_k :

$${}_{k-1}\mathbf{E}_k = {}_{k-1}\mathbf{E}_k(q_k, {}_{k-1}\mathbf{TH}_k) \quad (44)$$

Information about the temperature profile can be obtained at three levels:

- the temperature profile of the relevant machine tool's components using suitably placed temperature sensors,
- the heat flow from the various internal and external sources, in combination with an analytical or empirical model of the machine tool's thermodynamic behavior,
- process parameters, like spindle speed, gear ratio, and slide speed, in combination with models for their influence on the heat flow from the machine's internal heat sources.

By far, monitoring of temperature elevation is the easiest of all the means. The change of ambient temperature and all internal heat sources are put under full consideration, without using complex models for the machine's thermodynamic behavior with associated errors. Moreover, there is an almost nonexistent time lag between thermal displacements and temperature elevations. It must be noted, however, that there is a tendency to confuse the ability to correlate temperature measurements and deflections under steady-state conditions with an ability to predict deflections under dynamic or transient conditions.¹⁶ The latter case requires either the use of many temperature sensors to assess the nonlinear temperature field under dynamic conditions or models that relate the shape of the temperature field to changes of the process parameters, heat sources, and/or sensor readings in time. Errors associated with a nonuniformly distributed temperature (or the use of components with different expansion coefficients) are difficult to assess analytically (a temperature distribution having constant gradients is the main exception). Again an empirical estimation procedure is preferred. Recently, a research program, funded by the European Community, has been initiated between our university and other European companies and institutions to investigate methods for the software error compensation of metal-cutting machine tools, including the effects of their thermal behavior.¹⁷

In general, certain components of an error in the location of two succeeding frames can be related to the manufacturing process of the intermediate kinematic element, the used measuring scale, and the positions of the bearings. The values of these components are related to the position of the relevant kinematic element:

$${}_{k-1}\mathbf{E}_k = {}_{k-1}\mathbf{E}_k(q_k) \quad (45)$$

Identification and estimation of these errors generally requires correlation analysis among errors reported for machines of the same factory type. In some cases, also, a correlation between a kinematic element's angular and straightness errors can be identified.^{8,14}

Modeling the errors by least squares fitting

In the individual model, each not analytically modeled error in the relative location of two subsequent frames is described as a least squares estimated linear combination of known functions, defined on the position of the kinematic elements and other relevant quantified variables. The data necessary for this estimation are acquired and updated through calibrations of the machine tool. As indicated in the introduction, this study views the modeling of the various errors from a slightly different perspective than previously used. In addition to the use of the different functions as an approximation tool, i.e., a curve-fitting tool, also statistical testing procedures are applied to investigate points of structural change.

Although most mathematical functions can be used to describe the general trend of an error, their application is limited to model the frequently disjointed or disassociated nature of the remainder, i.e., its behavior in a region of the domain may be totally unrelated to the behavior in another region. Polynomials along with most other mathematical functions have just the opposite property. Namely their behavior in a small region determines their behavior everywhere. Application of the individual model for high-accuracy software error compensation required the use of a special family of functions that possesses this property to a lesser extent, the so-called piecewise polynomials. Piecewise polynomials can be described as a set of polynomials defined on limited continuous parts of the domain. The various pieces join in the so-called knots, obeying continuity restrictions with respect to the function value itself and an arbitrary number of derivatives. The number and degrees of the polynomial pieces, the nature of the continuity restrictions, and the number and positions of the knots may vary in different situations, which gives piecewise polynomials the desired flexibility.

A straightforward mathematical implementation of the continuity restrictions can be obtained by the use of truncated polynomials or “+” functions as basic elements in the piecewise polynomial models. Although not as computationally efficient as for example B-splines, their use allows the performance evaluation data to be fitted by ordinary least squares while still permitting tests of hypotheses concerning areas of structural change to be easily made. The “+” function is defined as

$$\bullet u_+ = u \text{ if } u > 0 \quad \bullet u_+ = 0 \text{ if } u \leq 0 \quad (46)$$

In general, with k knots t_1, \dots, t_k and $k + 1$ polynomial pieces each of degree n , the truncated power representation of an error E_i dependent on one variable q with no continuity restrictions, can be written as

$$E_i(q) = \sum_{j=0}^n \beta_{0j} q^j + \sum_{i=1}^k \sum_{j=0}^n \beta_{ij} (q - t_i)_+^j \quad (47)$$

Note that the presence of a term $\beta_{ij} (q - t_i)_+^j$, a discontinuity allows at t_i in the j th derivative of $E_i(q)$.

Thus, different continuity restrictions can be imposed at different knots simply by deleting the appropriate terms. For example, if the software error compensation is actively used in controlling the position of the tool, these restrictions must be applied to ensure that there is no discontinuity in the force applied to the drives. Normally, however, it is sufficient to ensure that each model is continuous with respect to the function value and its first derivative.

An inherent problem in constructing the individual model is the unknown nature of the errors to be described. The model's potential to accommodate irregular errors is to an extensive degree determined by the number and position of the knots. If the position of these knots is considered variable, i.e., parameters to be estimated, they enter into the regression problem in a nonlinear fashion, and all the problems arising in nonlinear regression are present. The use of variable knot positions also carries the practical danger of overfitting the data and makes testing of hypotheses considering areas of structural change virtually impossible. Unless prior information is available, we use a basic model that contains enough polynomial pieces with a fixed length and a specified maximum allowable degree to accommodate the most complex error expected. The respective knots are usually spaced at equidistant intervals along the domain of the error's independent variable(s). For relatively complex errors, adequate models have been obtained using quadratic piecewise polynomials with five knots, which allow a discontinuity only in the function's second derivative (seven unknown parameters). The smooth character of many observed errors allows them to be accurately described by similar models having only one or two knots (three to four unknown parameters). In the parameter estimation process, a stepwise regression procedure is implemented to remove statistically insignificant parameters from the model. The reason for this removal is twofold:

- Including insignificant parameters hardly improves the model's quality of fit but increases the variance of the estimated parameters and response.
- Identification of structural parameters enhances the diagnostic properties of the individual model and can be used in the design of the (periodic) performance evaluation.

In assessing the individual model's validity, the detection and analysis of outliers in the calibration data plays an important role:

- Due to the inherent flexibility of the proposed model, in combination with the nature of the least squares estimation process, outliers have a large potential influence on the estimated model.
- Outliers can indicate areas in which the proposed model is inadequate to describe a particular error or in which the nature of the random error in the various observations is incompatible with the use of the least squares estimation method.

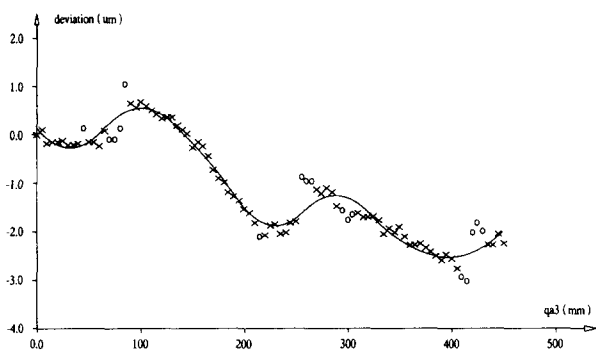


Figure 8 Measured scale error and its model (the identified outliers are represented by the symbol ○)

Observations are detected as possible outliers, in case their removal from the performance evaluation's data results in a significant reduction of the residual sum of squares. As an example, the measured scale error of a CMM at our laboratory is presented in *Figure 8*. The cluster of identified outliers at positions 100, 260, and 420 indicate areas in which the proposed model is inadequate to describe the measured errors. Further analysis showed that this effect was caused by a repeatable loss of counts every 160 mm.

The efficiency of procedures to calibrate machine tools

In this section an attempt is made to use the functions associated with the parameters to be estimated in the machine tool's error model to evaluate and optimize the efficiency of the calibration or performance evaluation. The efficiency of such a procedure, with respect to the choice of the points measured, can be assessed by two criteria:

- the assembly of parameters that can be estimated from the calibration's observations,
- the accuracy of these estimators, given a certain repeatability of the calibration's observations.

The proposed model for the machine tool's systematic errors affects these criteria in a structural manner. Hence, every calibration is a compromise between its efficiency in the estimation of modeled errors and its ability to detect errors not implemented in the proposed model. As an example of this concept, consider the problem of estimating the relation between an independent variable x and a dependent variable y . Suppose the experimentist is allowed to take ten observations of y at certain discrete values of x (assumed to be achieved without errors). If the relation between x and y is known to be exactly linear, an experimental design of five observations at x_{min} and five observations at x_{max} will result in the most accurate estimation of this relation, given a certain constant variance in the observations of y . Note, however, that a departure of the actual relation between x and y from the proposed model cannot be detected from these observations. In case there is no a priori information

available about the possible relation between x and y , an experimental setup with the ten observations uniformly distributed over the domain of x is preferred. However, such a design has a low efficiency in the estimation of this relation, if it is indeed linear.

With respect to the calibration of machine tools, there is a priori information available regarding the machine's general model. Also the machine's type-dependent model usually provides information concerning an error's independent variables and, in some cases, a class of suitable functions to be used in its model. Further specification of the proposed model for each error in the general model is dependent on the goal of the performance evaluation or calibration. For example, if an error's value has to be determined at certain discrete points, as is the case with most current performance evaluations, a first-order piecewise polynomial might be chosen with knots at these points. An impression of the various errors can be obtained when using low order piecewise polynomials whose complexity, i.e., the number of unknown parameters β , is restricted by the allowed performance evaluation's effort. In order to check the estimated software error correction, the model can be chosen in accordance with the functions associated with the identified significant parameters in the various models.

Application of a calibration to estimate the machine tool's error structure is founded on the comparison of the measured artefact's features with their respective nominal (calibrated) values. The chosen approach^{8,18} to this estimation is essentially different from the usual direct trigonometric calculation. As discussed, a model is built of the machine's error structure, including a suitable description of the component errors ${}_{k-1}E_k$ identified in the machine's general model. According to the nature of the measured feature(s), an algebraic relationship is next established between the observed errors in their measurement and the unknown parameters β , which describe the machine's component errors. Finally, these parameters are estimated by standard regression techniques, such that the residual sum of squares between the modeled and observed errors in the measured feature(s) is minimized.

Since the proposed model for the machine tool's error structure is linear in the unknown parameters β , the relation between these parameters and the observed errors in the artefact's measurement can usually be described or approximated by a linear model:

$$y_i = \mathbf{f}^T(\mathbf{x}_i) \boldsymbol{\beta} + \gamma_i \quad i = 1, 2, \dots, n \quad (48)$$

which we express in matrix notation as

$$\mathbf{y} = \mathbf{X}\boldsymbol{\beta} + \boldsymbol{\gamma} \quad (49)$$

The $n \times 1$ vector \mathbf{y} contains the difference between the measured and nominal features of the artefact at different locations. The $p \times 1$ vectors $\boldsymbol{\beta}$ and \mathbf{f} are the (significant) parameters that describe the machine tool's error structure and the associated known functions that describe their effect on the measured artefact's feature. The $q \times 1$ vector \mathbf{x}_i contains the variables that describe the status of the machine tool

and its environment during the i^{th} observation (e.g., the position of the machine's axes when measuring the artefact). X is an $n \times p$ matrix, with row i containing $\mathbf{f}^T(\mathbf{x}_i)$. \mathbf{y} is the vector of random errors, assumed to be independent and identically distributed with mean zero and variance σ^2 . In the case of calibrations that use artefacts with unknown constant dimensions, the vector $\boldsymbol{\beta}$ with unknown parameters has to be expanded to include these dimensions. The least squares estimates of the parameters $\boldsymbol{\beta}$ are given by¹⁹

$$\hat{\boldsymbol{\beta}} = (X^T X)^{-1} X^T \mathbf{y} \quad (50)$$

The calibration's potential to estimate the unknown parameters $\boldsymbol{\beta}$ can be assessed by the following rule¹⁹:

- a linear combination $\boldsymbol{\lambda}^T \boldsymbol{\beta}$ of the parameters $\boldsymbol{\beta}$ can be estimated from the calibration's observations if $\boldsymbol{\lambda}^T$ is part of the vectorspace spanned by the rows of X .

In case all the parameters $\boldsymbol{\beta}$ can be estimated, the calibration's efficiency can be further assessed by

- The variance-covariance matrix of the estimated parameters $\hat{\boldsymbol{\beta}}$:

$$\text{var}(\hat{\boldsymbol{\beta}}) = \sigma^2 (X^T X)^{-1} \quad (51)$$

- The variance of the predicted machine tool's error \hat{y} , when measuring an artefact's feature:

$$\text{var}(\hat{y} | \mathbf{x} = \mathbf{x}_i) = \sigma^2 \mathbf{f}^T(\mathbf{x}_i) (X^T X)^{-1} \mathbf{f}(\mathbf{x}_i) \quad (52)$$

Both Equations 51 and 52 depend on the design of the calibration only through the $p \times p$ matrix $(X^T X)^{-1}$, and suggest that a good experimental design will be one that makes this matrix small in some sense. Since there is no unique size ordering of $p \times p$ matrices, various real-valued functions have been suggested as measures of "smallness." Such criteria²⁰ either assess the quality of the parameters to be estimated (minimization of the determinant, trace, or maximum eigenvalue of $(X^T X)^{-1}$) or the quality of the estimated model's response surface for the errors in the artefact's measurements (minimization maximum or average variance-predicted errors).

As an example, we consider minimizing the determinant of $(X^T X)^{-1}$. One of the attractive features of this so-called D-criterion is that designs that are optimal with respect to it are invariably "good" in many respects (e.g., low variances for the parameters, low correlations among the parameters, low maximum variance in the predicted errors). Essentially the criterion tries to minimize the contents of the p -dimensional confidence region associated with the estimated parameters. Although the comparison of existing calibrations using this or other criteria is quite straightforward, optimization is a very complicated process. The most frequently encountered difficulties are

- the large number of parameters that must be handled and the associated large size of the matrix $X^T X$,

- the complex nature of object function $\det(X^T X)$, especially the existence of several local minima,
- the large number of design variables that describe the configuration of the artefact(s)'s measurable points, its location(s) in the machine tool's workspace, and the probe extension(s) used to measure it,
- the complex boundaries of the design space and the often discrete character of certain design variables (e.g., available probe extensions and artefacts).

Here attention is restricted to the optimization of the location (s) of existing artefact (s). As an example, we confine the assembly of possible artefacts to gauge blocks (or calibrated ball bars) with different lengths. In the chosen approach²¹ to the optimization problem, a discrete design space X is defined of r so-called candidates x_1, \dots, x_r . Each candidate represents a gauge block at a certain location in the machine tool's workspace. The design problem can now be defined as the choice of n not necessarily distinct observations $X_{(1)}, \dots, X_{(n)}$ from X . The adapted (exchange) algorithm commences with an initial random design of n points (observations) and makes a number of excursions whereby the design is improved until no further progress can be made. Each excursion is a series of additions or subtractions of one or more candidates to the current collection of design points, eventually returning to a possibly new and better n -point design.

Example 3

As an example of this procedure we consider the simplified 2D coordinate measuring machine depicted in Figure 9. Its errors are limited to linear scale errors in the position of both axes and a constant squareness error in their respective orientation. Disregarding the probe offset, errors in the measured position can be expressed as

$$\begin{bmatrix} e_x \\ e_y \end{bmatrix} = \begin{bmatrix} {}_0e_{1x} - {}_0e_{1z}q_2 \\ {}_1e_{2y} \end{bmatrix} \quad (53)$$

$$= \begin{bmatrix} q_1 & 0 & -q_2 \\ 0 & q_2 & 0 \end{bmatrix} \begin{bmatrix} \beta_1 \\ \beta_2 \\ \beta_3 \end{bmatrix} \quad (54)$$

where

${}_0e_{1x}$	Scale error axis 1	$({}_0e_{1x} = \beta_1 q_1)$
${}_1e_{2y}$	Scale error axis 2	$({}_1e_{2y} = \beta_2 q_2)$
${}_0e_{1z}$	Squareness error	$({}_0e_{1z} = \beta_3)$
	between axes 1 and 2	
$\boldsymbol{\beta}$	Parameters to be estimated	

The difference ΔL between the measured and nominal length L of a ball bar, whose orientation is determined by the angle α with the machine's first axis, can be approximated as a linear combination of the unknown

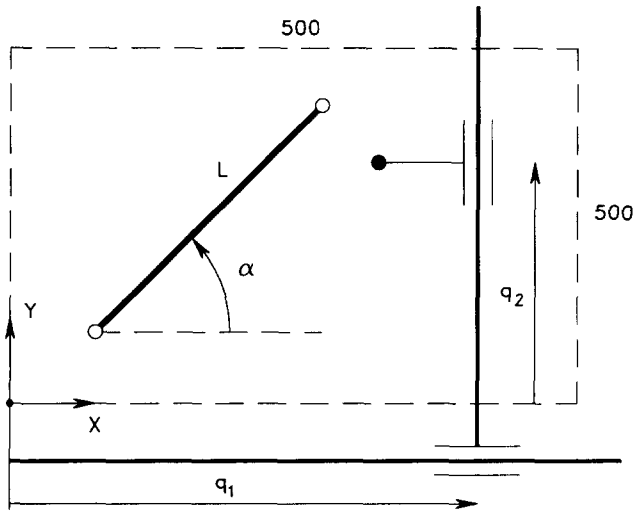


Figure 9 2D CMM

parameters β :

$$\Delta L \approx [\cos \alpha \quad \sin \alpha]$$

$$\begin{bmatrix} L \cos \alpha & 0 & -L \sin \alpha \\ 0 & L \sin \alpha & 0 \end{bmatrix} \begin{bmatrix} \beta_1 \\ \beta_2 \\ \beta_3 \end{bmatrix} + \gamma$$

(55)

$$= L [\cos^2 \alpha, \sin^2 \alpha, -\cos \alpha \sin \alpha] \beta + \gamma$$

(56)

Note that this error is unrelated to the ball bar's position in the machine's workspace. In this analysis, the set of candidates is confined to ball bars with a length of 400 mm. Their position and orientation are separated by intervals of 50 mm and 15°, respectively. As an example, the number of ball bar measurements to calibrate the machine is limited to six. In the computer output presented in Table 1, the properties of the proposed experimental designs 1, 2, and the calculated D-optimal design 3 (Figure 10) are shown.

Note that the calculated D-optimal design is superior to the proposed designs with respect to all criteria mentioned. In order to analyze this difference, the variance-covariance matrices of designs 1 and 3 are calculated:

$$\text{Design 1: } \text{var}(\hat{\beta}) = \frac{\sigma^2}{2} L^{-2} \begin{bmatrix} 1.00 & 0.00 & 1.00 \\ & 1.00 & 1.00 \\ & & 6.00 \end{bmatrix}$$

(57)

$$\text{Design 3: } \text{var}(\hat{\beta}) = \frac{\sigma^2}{2} L^{-2} \begin{bmatrix} 1.00 & -0.33 & 0.00 \\ & 1.00 & 0.00 \\ & & 2.67 \end{bmatrix}$$

(58)

Although the first design exhibits no covariance between the scale parameters β_1 and β_2 (matrix

element [1,2]), the variance of its squareness parameter β_3 is quite high (matrix element [3,3]) and shows significant covariance with both scale parameters (matrix elements [1,3] and [2,3]). This explains the higher efficiency of the calculated D-optimal design. The D-optimality of this design was proved by evaluating all possible designs based on six ball bar measurements whose orientation is limited to the previously mentioned discrete angles. In Figure 11 values of the inverted optimality criterion (i.e., $\det(X^T X)$) are shown for a subset of these designs.

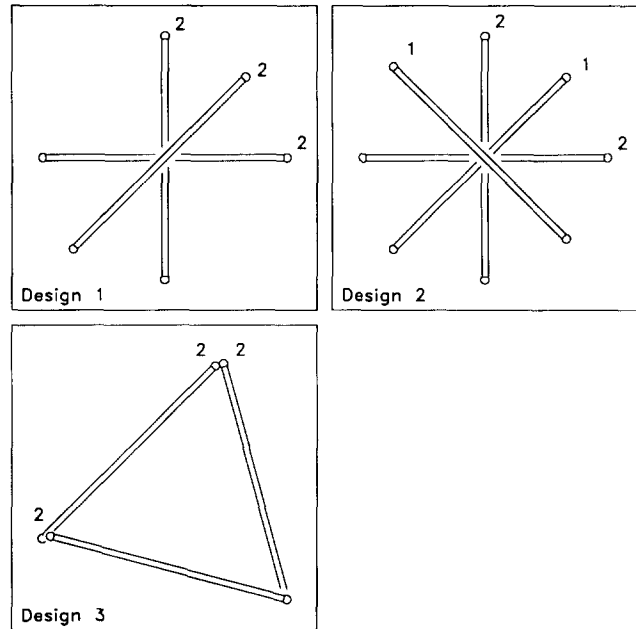


Figure 10 Proposed experimental designs 1, 2, and the calculated D-optimal design 3

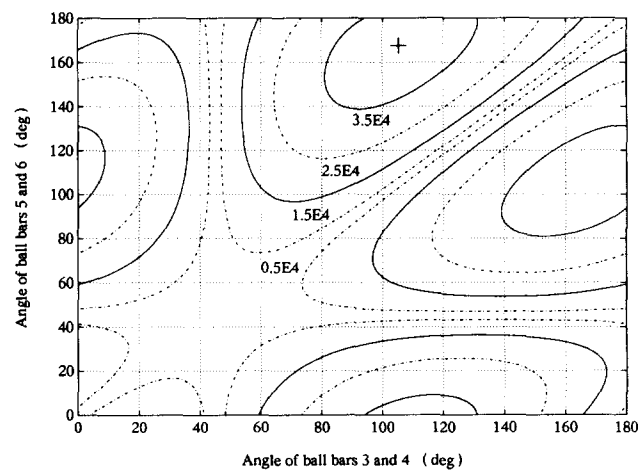


Figure 11 Contour lines of constant $\det(X^T X)$ of experimental designs having two ball bars at a 45° angle with the machine's X-axis. The numerical values for this inverted optimality criterion are scaled for the number of parameters (three) and observations (six). The calculated D-optimal design is represented by the symbol +

Table 1 Properties proposed experimental designs 1, 2, and the calculated D-optimal design 3

Design 1: design points							
Point	Replications	Coordinates first sphere			A-angle	B-angle	Length ball bar
		X	Y	Z			
1	2	50.0	250.0	0.0	0.0	0.0	400.0
2	2	250.0	50.0	0.0	90.0	0.0	400.0
3	2	100.0	100.0	0.0	45.0	0.0	400.0
Design 2: design points							
Point	Replications	Coordinates first sphere			A-angle	B-angle	Length ball bar
		X	Y	Z			
1	2	50.0	250.0	0.0	0.0	0.0	400.0
2	2	250.0	50.0	0.0	90.0	0.0	400.0
3	1	100.0	100.0	0.0	45.0	0.0	400.0
4	1	100.0	400.0	0.0	315.0	0.0	400.0
Design 3: design points							
Point	Replications	Coordinates first sphere			A-angle	B-angle	Length ball bar
		X	Y	Z			
41	2	50.0	150.0	0.0	45.0	0.0	400.0
97	2	450.0	50.0	0.0	105.0	0.0	400.0
101	2	450.0	50.0	0.0	165.0	0.0	400.0
Design properties							
Design	Iterations	Det ^{^(1/3)} covariance matrix	Max E-val covariance matrix	Trace covariance matrix	Average variance	Maximum variance	
1	1	2.9764 E-5	1.1948 E-4	1.5000 E-4	4.3540	9.0000	
2	1	2.6001 E-5	7.5000 E-5	1.0625 E-4	3.1974	4.0000	
3	31	2.5000 E-5	5.0000 E-5	8.7500 E-5	3.0000	3.0000	

All results scaled for number of observations and error variance.

Example 4

In order to evaluate the proposed artefact-based calibration and the associated optimization of its efficiency, the thus estimated error structure of the CMM depicted in *Figure 6* is compared with results obtained using conventional laserinterferometer-based techniques. As an example, we restricted the problem to the estimation of those errors that mainly determine the CMM's accuracy in its upper X-Y plane. Again we chose not to optimize the artefact(s) used but tried to define an efficient experimental design using existing artefact(s), in this case calibrated ball bars of 300, 400, 500, and 600 mm length. Since we only wish to

determine the trend of the various errors, low order piecewise polynomials were used in their proposed model:

$$\bullet \quad {}_0e_{a1z} = \beta_1 + \beta_2 q_{a1} + \beta_3 q_{a1}^2 + \beta_4 (q_{a1} - 225)_+^2 \quad (59)$$

$$\bullet \quad {}_0e_{a1x} = \beta_5 (\frac{1}{6}550^2 - 550q_{a1} + q_{a1}^2) \quad (60)$$

$$\bullet \quad {}_0e_{a1y} = \beta_6 q_{a1} + \beta_7 q_{a1}^2 + \beta_8 (q_{a1} - 225)_+^2 \quad (61)$$

$$\bullet \quad {}_{a1}e_{a2x} = \beta_9 q_{a2} + \beta_{10} q_{a2}^2 + \beta_{11} (q_{a2} - 450)_+^2 \quad (62)$$

$$\bullet \quad {}_{a1}e_{a2y} = \beta_{12} (\frac{1}{6}900^2 - 900q_{a2} + q_{a2}^2) \quad (63)$$

In order to estimate the total of 12 unknown parameters in the previous models, an optimal experimental design was calculated using 18 observations (Figure 12). In the actual calibration, five measurements of the ball bar's length were taken at each of these observations. In Figures 13 and 14 the thus estimated scale error $a_1 e_{a2x}$ and straightness error ${}_0 e_{a1x}$ are compared with

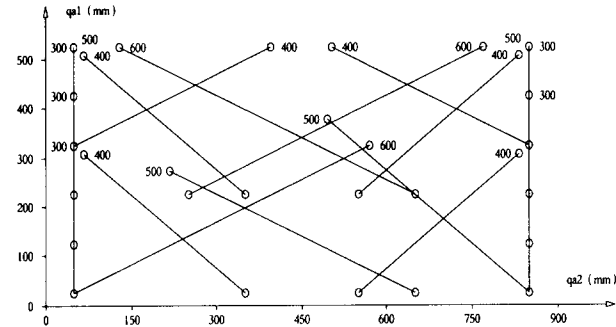


Figure 12 Calculated optimal experimental design in the CMM's upper X-Y plane

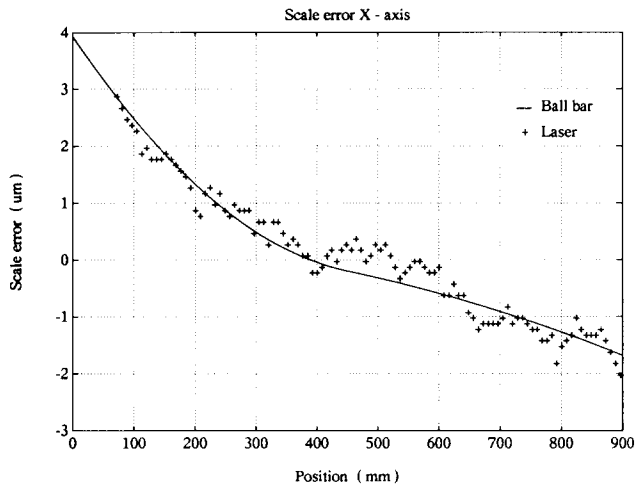


Figure 13 Scale error $a_1 e_{a2x}$

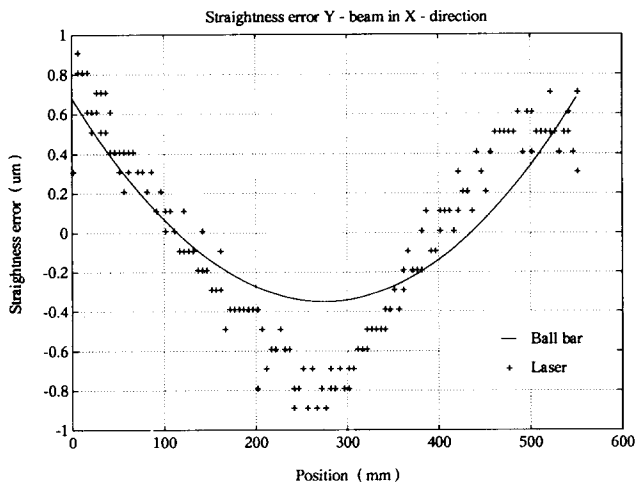


Figure 14 Straightness error ${}_0 e_{a1x}$

the associated laser measurements. The difference between the straightness results from the ball bar measurements and laser measurements is mainly due to the fact that the laser measurements cannot be accurately modeled using the second-order polynomial proposed.

Although evaluation and especially optimization of a calibration's efficiency using the method presented in this section is not without problems, the first results look promising. Further research is required on the following topics:

- evaluation and optimization of calibrations to estimate subsets of parameters,
- methods for the sequential update of calibrations using calibration data already obtained (e.g., in order to find artefact location (s) where maximum errors are obtained), and
- planning and evaluation of experiments in the face of model uncertainty (model-robust designs that yield reasonable results for the model proposed even though it is known to be inexact, and model-sensitive designs that highlight suspected inadequacies).

Conclusions

Applicable to multi-axis machines of arbitrary configuration, a general methodology has been developed to obtain a model that relates quasistatic errors in the relative location between tool and workpiece to the machine's status and relevant quantified environmental conditions. A so-called type-dependent model is developed that contains the common properties of error structures belonging to machines of the same factory type. The use of piecewise polynomials in combination with the selection of significant parameters in the least squares estimation procedure results in a powerful model for each of the errors between succeeding frames. Finally, a method has been presented to use components in the various models to evaluate and optimize the calibration's efficiency.

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