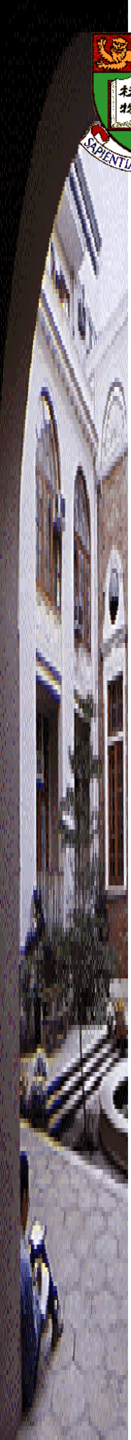

Modeling Urban Taxi Services with Multiple User Classes and Vehicle Modes

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Introduction

- Hong Kong situation
- Model assumptions
- Simultaneous optimization formulation
- Solution algorithm
- Numerical example
- Conclusions

Hong Kong Situation

- Currently 15,000 urban taxis, 2,800 New Territories taxis and 40 Lantau taxis carry 1.3 million passengers per day in Hong Kong (with a population of less than 7 million)

- In urban area, taxis currently form about 25% of the traffic stream overall, and in some critical locations, taxis form as much as 50% to 60% of the traffic stream

- Annual taxi service surveys are conducted to gather the information on passenger/taxi waiting time, taxi utilization and taxi availability

Hong Kong Situation

- Like many developing countries, the majority of taxis are cruising on the streets searching for customers
- There are several classes of taxis, such as the urban taxis which can serve anywhere in the city of Hong Kong, whereas New Territories taxis and Lantau taxis are restricted to serve the New Territories and Lantau Island, respectively; with each class of taxis having a different charging system
- The taxi industry in Hong Kong is highly regulated, with very stringent license and fare controls
- In such a high density city with great demand of taxi services, it is important to develop a tool that can model the spatial structure of the taxi market and the interaction between taxis and other traffic in the road network

Modeling Assumptions

Traffic movements in the network

P	The set of user classes
Q	The set of taxi modes
$\bar{D}_{ij}^{p}$	The travel demand of user class p from zone i to zone j

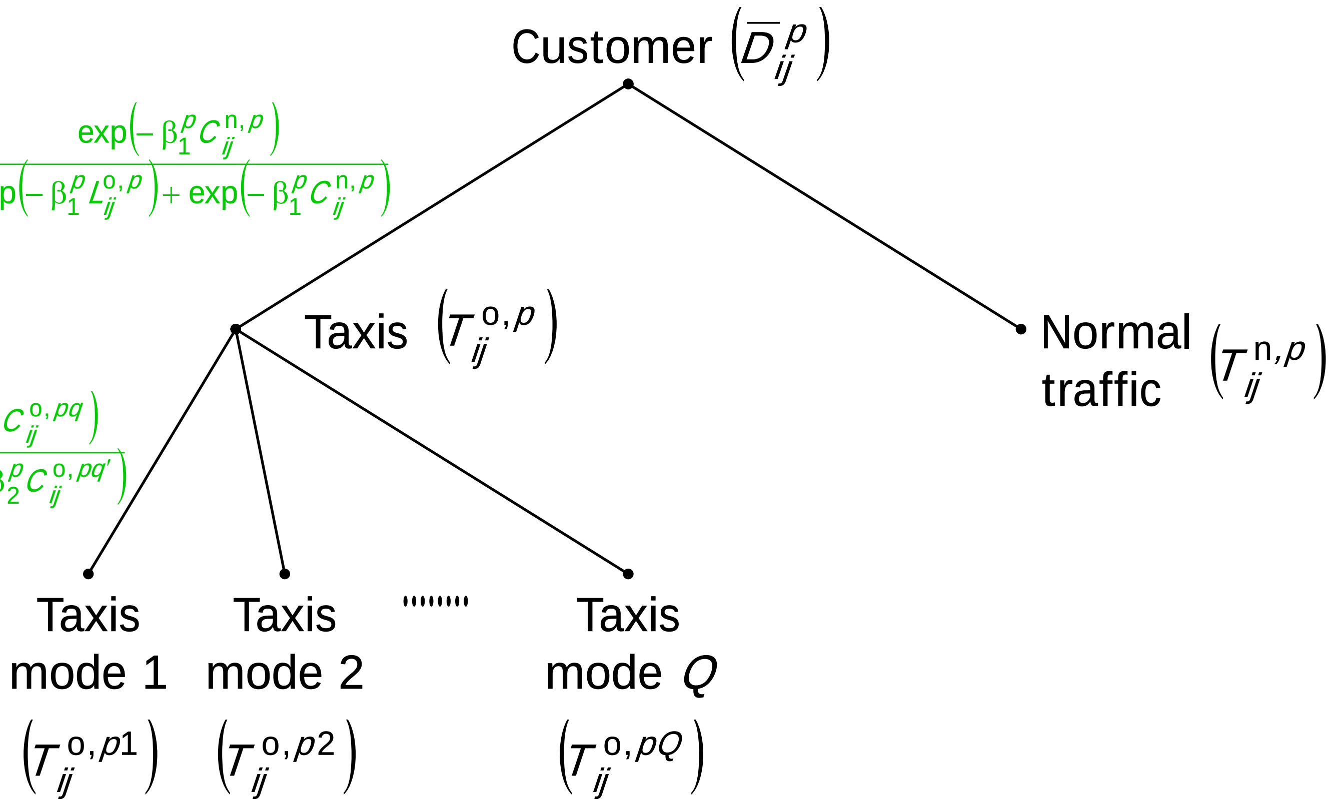
⇩

$T_{ij}^{\mathrm{n},p}$	The normal traffic (non-taxi) movements by class p users from zone i to zone j
$T_{ij}^{\mathrm{o},pq}$	The occupied taxi movements of mode q occupied by class p users from zone i to zone j
$T_{ji}^{\mathrm{v},q}$	The vacant taxi movements of mode q from zone j to zone i to search for the next customer there after setting down the last customer in zone j

All these traffic movements travel in the network in a user-optimal manner

Modeling Assumptions

Travelers' hierarchical mode choice



$$P_{ij}^{n,p} = \frac{\exp\left(-\beta_1^p C_{ij}^{n,p}\right)}{\exp\left(-\beta_1^p L_{ij}^{o,p}\right) + \exp\left(-\beta_1^p C_{ij}^{n,p}\right)}$$

$$P_{ij}^{o,pq} = \frac{\exp\left(-\beta_2^p C_{ij}^{o,pq}\right)}{\sum_{q' \in Q} \exp\left(-\beta_2^p C_{ij}^{o,pq'}\right)}$$

Modeling Assumptions

Travelers' hierarchical mode choice

$$T_{ij}^{o,pq} = T_{ij}^{o,pq}\left(W_i^q, F_{ij}^q, h_{ij}\right)$$

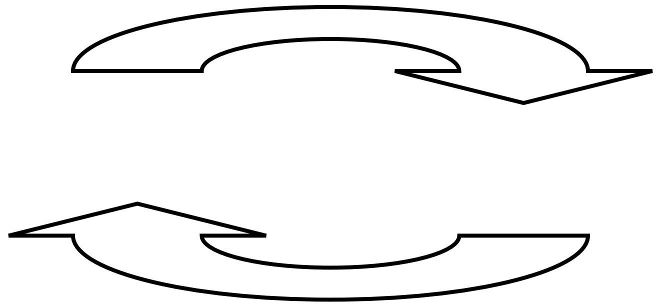
The customer demand is affected by :

- W_i^q customer's waiting time for a taxi
- F_{ij}^q taxi fare (distance-based charge + time-based (congestion) charge)
- h_{ij} travel time

Modeling Assumptions

Operational behavior

Customer waits for a taxi, W_i^q



Taxi searches for a customer, w_i^q

Relationship of customer's and taxi's waiting times

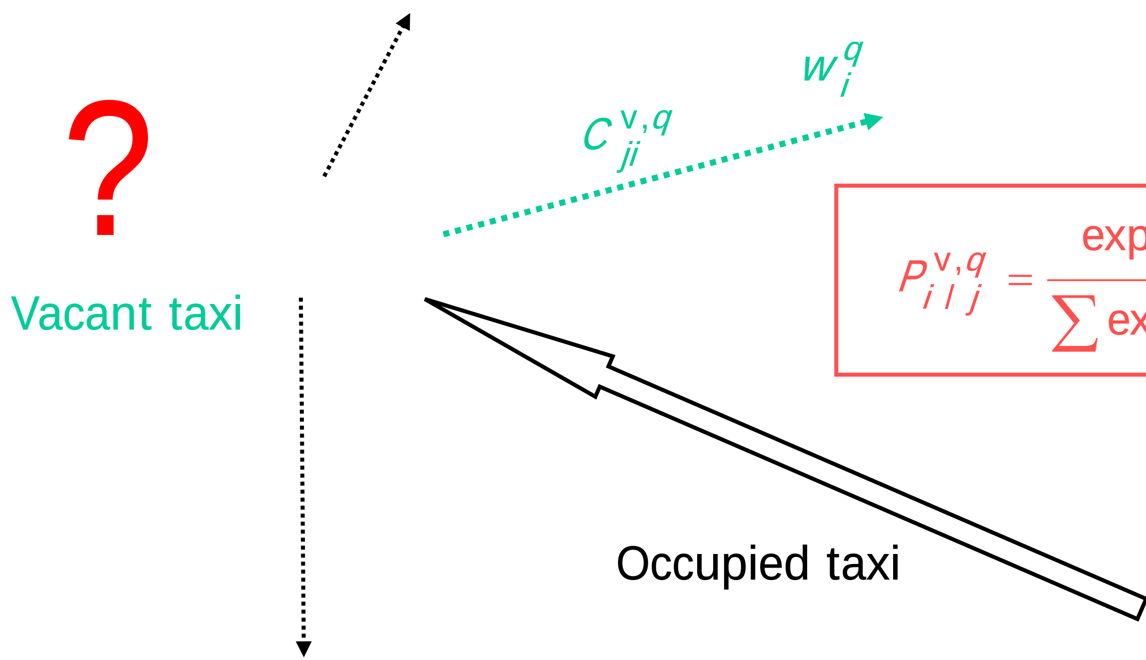
$$W_i^q = W_i^q\left(O_i^q, w_i^q\right)$$

At equilibrium, the customer waiting time is affected by :

- O_i^q total customer's demand
- w_i^q taxi searching (waiting) time

Modeling Assumptions

Operational behavior



$$P_{i|j}^{v,q} = \frac{\exp(\theta^q (C_{ji}^{v,q} + b_0^q w_i^q))}{\sum \exp(\theta^q (C_{jm}^{v,q} + b_0^q w_m^q))}$$

Modeling Assumptions

Conservation of flows and time

$$\sum_i T_{ji}^{v,q} = D_j^q$$

At equilibrium, the total vacant taxi movements originating from a zone *j* should be equal to the total demand traveling to that zone

$$\sum_j T_{ji}^{v,q} = O_i^q$$

At equilibrium, the total vacant taxi movements attracting to a zone *i* should be equal to the total taxi demand from that zone

For each taxi mode, the total taxi service available in one hour is equal to the sum of occupied taxi time and vacant taxi time (including traveling and searching)

$$\sum_{i \in I} \sum_{j \in J} \sum_{p \in P} T_{ij}^{o,pq} h_{ij}^{o,pq} + \sum_{j \in J} \sum_{i \in I} T_{ji}^{v,q} \left\{ h_{ji}^{v,q} + w_i^q \right\} = N^q$$

(N^q = fleet size of taxi mode q)

Simultaneous Optimization Formulation

Combined network equilibrium model (CNEM)

Most (but not all) of the functional relationships can be incorporated into the following variational inequality problem (CNEM) :

$$\sum_{i\in I}\sum_{j\in J}\Bigg(\sum_{\psi\in\Psi}\sum_{r\in R_{ij}^{\psi}} C_r^{\psi}(\mathbf{f}^*)(\mathbf{f}-\mathbf{f}^*)+\sum_{p\in P}\Bigg(\frac{1}{\beta_1^p}\ln T_{ij}^{\mathrm{n},p*}\left(T_{ij}^{\mathrm{n},p}-T_{ij}^{\mathrm{n},p*}\right)+\frac{1}{\beta_1^p}\ln T_{ij}^{\mathrm{o},p*}\left(T_{ij}^{\mathrm{o},p}-T_{ij}^{\mathrm{o},p*}\right)$$

$$+\frac{1}{\beta_2^p}\sum_{q\in Q}\left(\ln\left(T_{ij}^{\mathrm{o},pq*}/T_{ij}^{\mathrm{o},p*}\right)\left(T_{ij}^{\mathrm{o},pq}-T_{ij}^{\mathrm{o},pq*}\right)\right)\Bigg)+\sum_{q\in Q}\left(\frac{1}{\theta^q}\ln T_{ji}^{\mathrm{v},q*}\left(T_{ji}^{\mathrm{v},q}-T_{ji}^{\mathrm{v},q*}\right)\right)\Bigg)\ge 0,\ \forall(\mathbf{f},\mathbf{T})\in\Omega$$

Except for 4 sets of constraints :

1. Customer's and taxi's waiting times relationship
2. Conservation of demand at origins
3. Conservation of demand at destinations
4. Conservation of taxi service time (which is related to the Dual Variables of the CNEM)

Simultaneous Optimization Formulation

A set of linear and nonlinear equations (SLNE)

To ensure the satisfaction of the remaining 4 sets of constraints, the following set of linear and nonlinear equations are set (SLNE) :

$$R_{1i,q}\left(\mathbf{W},\mathbf{O},\overline{\mathbf{D}},\mathbf{c}\right)=\left(\overline{\alpha}_i^q\left(\mathbf{W},\mathbf{O},\overline{\mathbf{D}}\right)+c^q\right)W_i^q\sum_{j\in J}\sum_{p\in P}T_{ij}^{o,pq}\left(\mathbf{W},\mathbf{O},\overline{\mathbf{D}}\right)-\eta\Omega_i=0,\ i\in I,q\in Q$$

Waiting times relationships

$$R_{2i,q}\left(\mathbf{W},\mathbf{O},\overline{\mathbf{D}},\mathbf{c}\right)=O_i^q-\sum_{j\in J}\sum_{p\in P}T_{ij}^{o,pq}\left(\mathbf{W},\mathbf{O},\overline{\mathbf{D}}\right)=0,\ i\in I,q\in Q$$

Conservation of demand at origins

$$R_{3j,q}\left(\mathbf{W},\mathbf{O},\overline{\mathbf{D}},\mathbf{c}\right)=D_j^q-\sum_{i\in I}\sum_{p\in P}T_{ij}^{o,pq}\left(\mathbf{W},\mathbf{O},\overline{\mathbf{D}}\right)=0,\ j\in\{J-z\},q\in Q$$

Conservation of demand at destinations

$$R_{4,q}\left(\mathbf{W},\mathbf{O},\overline{\mathbf{D}},\mathbf{c}\right)=\sum_{i\in I}\sum_{j\in J}\sum_{p\in P}T_{ij}^{o,pq}\left(\mathbf{W},\mathbf{O},\overline{\mathbf{D}}\right)h_{ij}^{o,pq}\left(\mathbf{W},\mathbf{O},\overline{\mathbf{D}}\right)$$

Taxi service time

$$+\sum_{j\in J}\sum_{i\in I}T_{ji}^{v,q}\left(\mathbf{W},\mathbf{O},\overline{\mathbf{D}}\right)\left\{h_{ji}^{v,q}\left(\mathbf{W},\mathbf{O},\overline{\mathbf{D}}\right)+\overline{\alpha}_i^q\left(\mathbf{W},\mathbf{O},\overline{\mathbf{D}}\right)+c^q\right\}-N^q=0,\quad q\in Q.$$

Solution Algorithm

Newtonian approach

Stackelberg game

$$\text{SLNE}(\ X, \text{CNEM}(X)\ ) = 0$$

Let $X^{(k)}$ be the current solution and $[J^{(k)}] = \nabla_X \text{SLNE}(X^{(k)})$. The improved solution can be obtained by

$$X^{(k+1)} = X^{(k)} - \alpha^{(k)} [J^{(k)}]^{-1} \text{SLNE}(X^{(k)})$$

The question is how to evaluate $[J^{(k)}]^{-1}$

Solution Algorithm

At each iteration, perform a sensitivity analysis on the CNEM at the current solution, in which we adopt a diagonalization approach to approximate the Jacobian matrix $[J^{(k)}]$ with constant link travel times

The core of the sensitivity analysis is to derive the changes in

$$\left(T_{ij}^{o,pq}, T_{ji}^{v,q}, \forall i \in I, j \in J, p \in P, q \in Q\right)$$

with respect to the changes in perturbation parameters

$$\underline{\varepsilon} = \left(W_i^q, O_i^q, D_i^q, \forall i \in I, q \in Q\right)$$

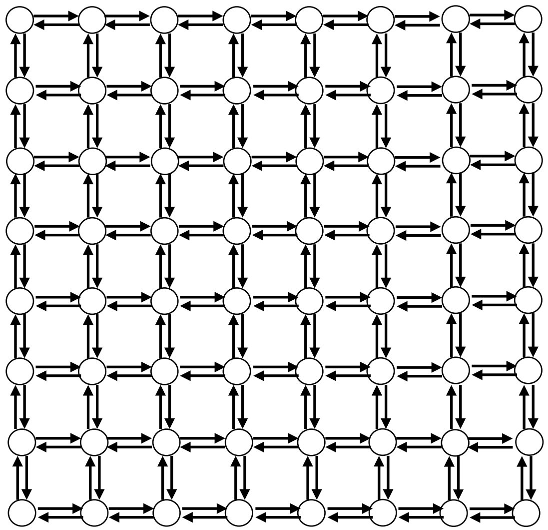
The diagonalization approach, which assumes constant link travel times at the perturbed solution, obviates the need of inverting a very large matrix that represents all active paths. Once the Jacobian matrix $[J^{(k)}]$ is evaluated from the sensitivity analysis, the descent direction can be obtained by solving

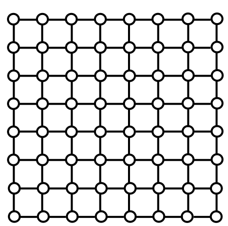
$$\left[\mathbf{J}^{(k)}\right]\mathbf{d}^{(k)} = \mathbf{SLNE}^{(k)}$$

This avoids the inversion of a matrix, and is much efficient

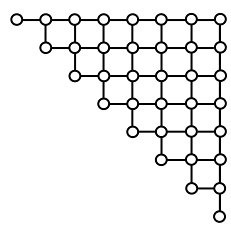
Numerical Example

Example network





Service area for normal taxis and luxury taxis



Service area for restricted area taxis

There are two classes of users

- high income
- low income

There are three modes of taxis

- normal taxis
- luxury taxis
- restricted area taxis

BPR link impedance functions are assumed in the network

$$t_a = t_a^0 \left(1 + 0.5 \left(v_a / s_a\right)^2\right)$$

$t_a^0 = 0.04$ h

$s_a = 3000$ veh/h

$d_a = 3$ km

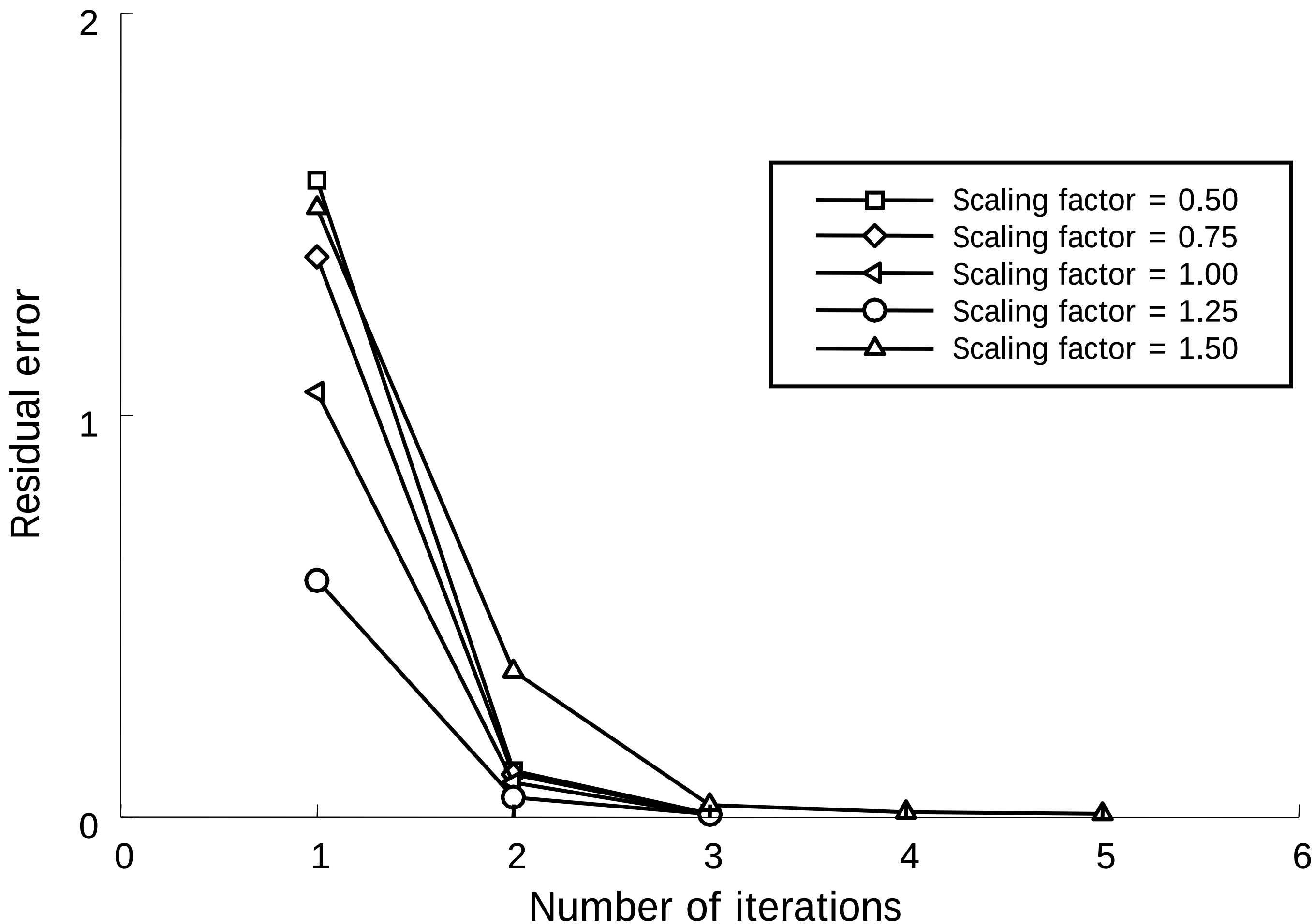
Numerical Example

Example network

Input parameters	Values
Users' values of time	$(b_0^p, p \in P) = (100, 50)$ (\$/h)
Values of customer waiting time for taxis	$(b_1^p, p \in P) = (200, 100)$ (\$/h)
Mileage cost that is charged to a user in normal traffic	$(b^{\mathrm{n}}) = 3$ (\$/km)
Mileage cost that is charged to a taxi customer	$(b_1^{\mathrm{o},q}, q \in Q) = (3, 4, 2)$ (\$/km)
Time based cost that is charged to a taxi customer	$(b_2^{\mathrm{o},q}, q \in Q) = (60, 80, 40)$ (\$/h)
Bias coefficients for the preference of taking taxis	$(\rho^{pq}, p = 1, q \in Q) = (0, 40, 0)$ (\$) $(\rho^{pq}, p = 2, q \in Q) = (20, 0, 20)$ (\$)
Hourly operating costs of taxis	$(b_0^q, q \in Q) = (80, 100, 80)$ (\$/h)
Mileage operating costs of taxis	$(b^{\mathrm{v},q}, q \in Q) = (0.5, 0.5, 0.5)$ (\$/km)
Dispersion coefficients for the upper-level logit mode choice	$(\beta_1^p, p \in P) = (0.01, 0.03)$ (1/\$)
Dispersion coefficients for the lower-level logit mode choice	$(\beta_2^p, p \in P) = (0.02, 0.06)$ (1/\$)
Dispersion coefficients for vacant taxi search behavior	$(\theta^q, q \in Q) = (0.2, 0.2, 0.2)$ (1/\$)
Taxi fleet sizes	$(N^q, q \in Q) = (10000, 5000, 5000)$ (veh)
Parameter for the relationship of customer and taxi waiting times	$(\eta\Omega_i, i \in I) = 2$ $(\mathrm{veh} \cdot \mathrm{h})$

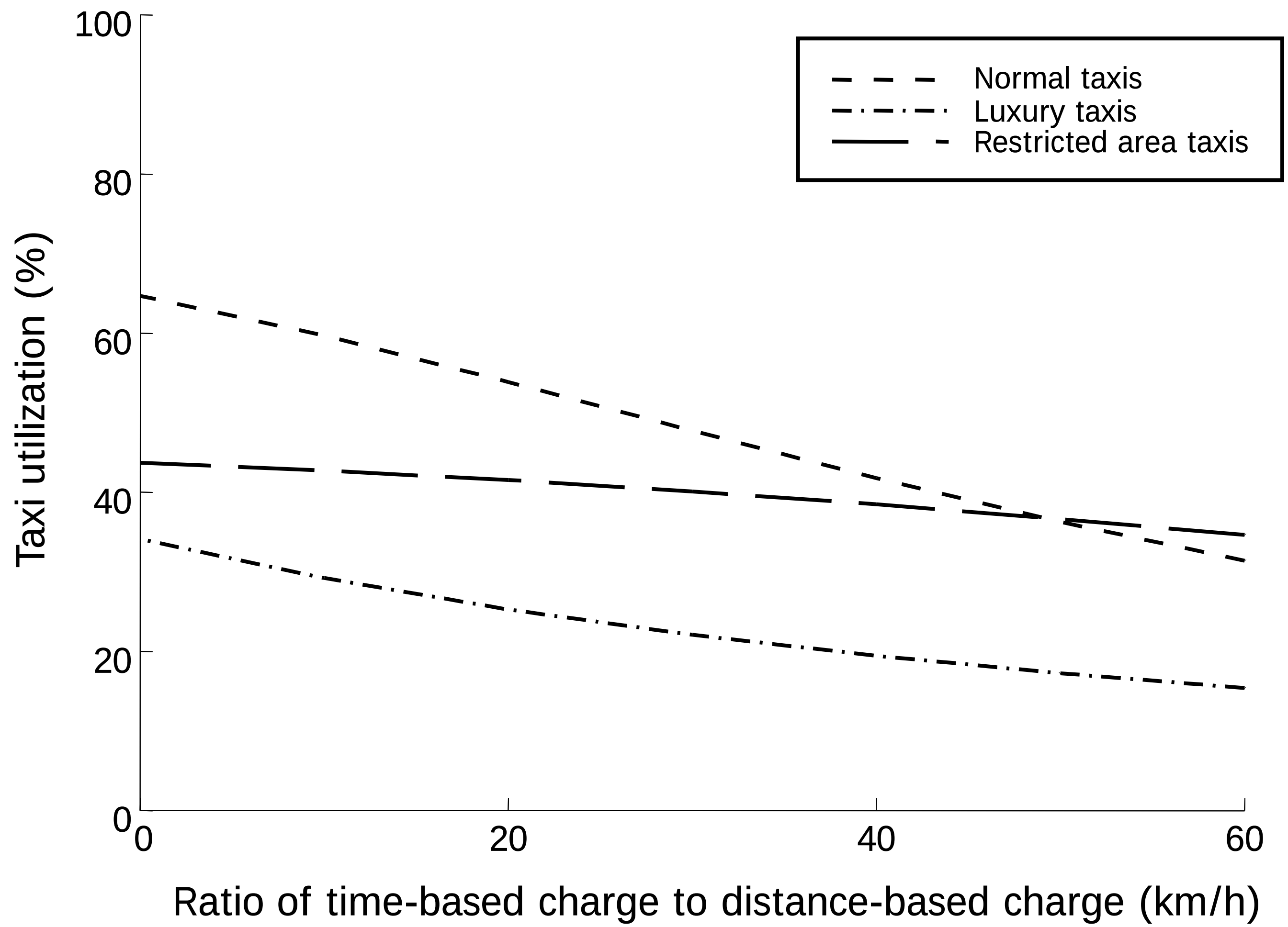
Numerical Example

Convergence characteristics



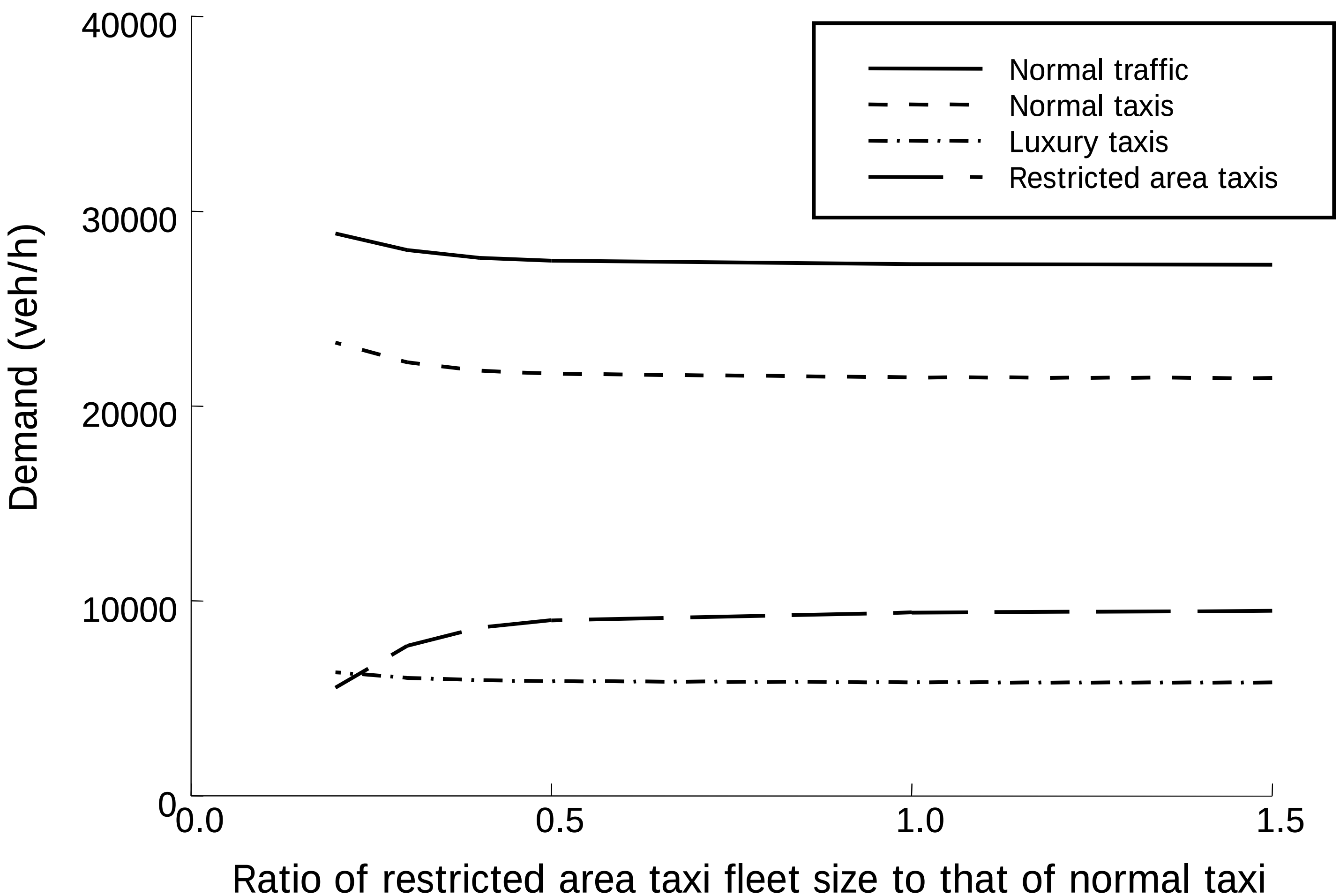
Numerical Example

Effect of time-based charge on taxi utilization



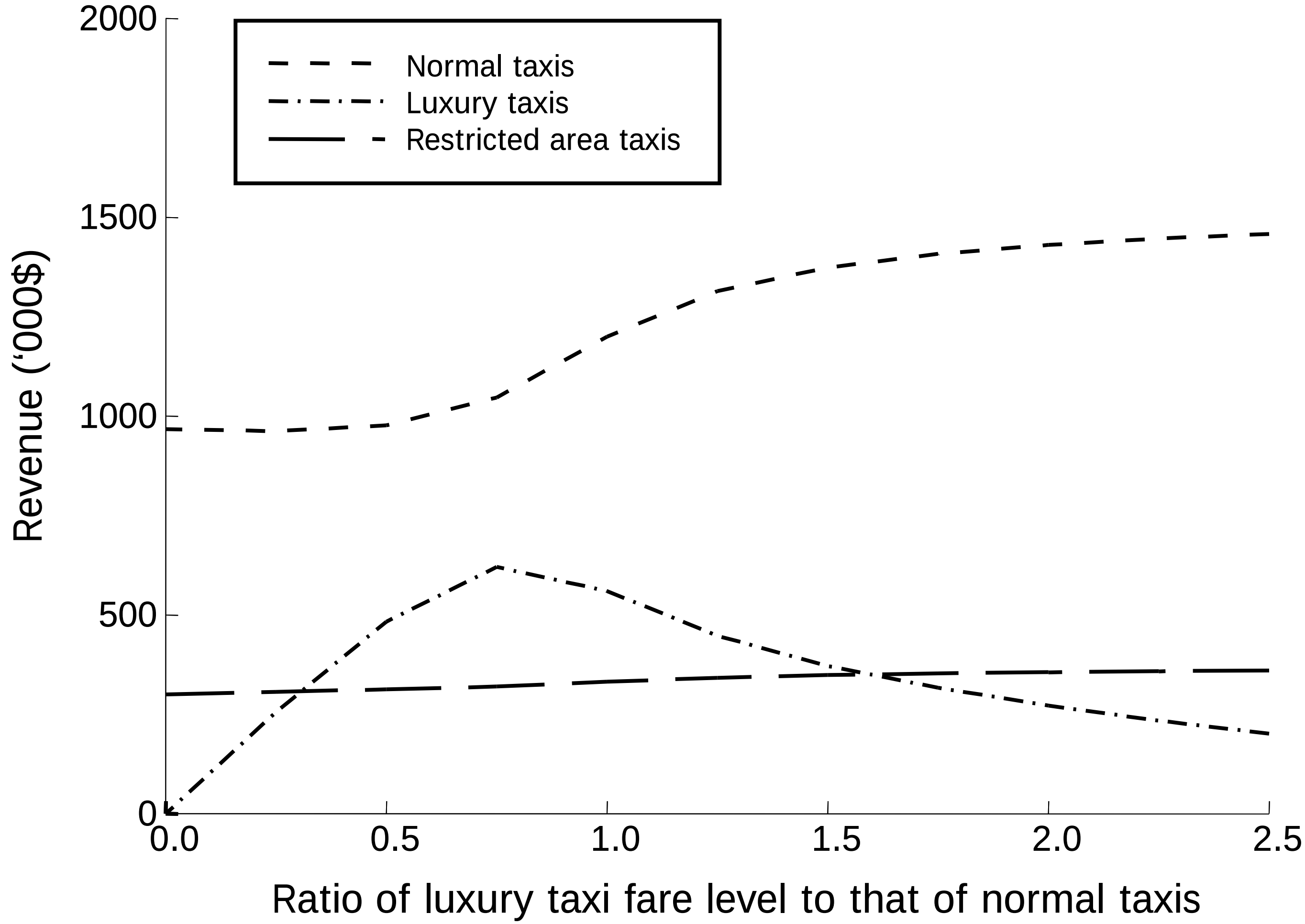
Numerical Example

Effect of fleet size on travel demand



Numerical Example

Effect of fare level on taxi revenue



Conclusions

- The modeling of urban taxi services with multiple user classes and vehicle modes in a congested network has been introduced
- The customer's choice of travel mode is governed by a hierarchical logit mode choice model, whereas the vacant taxi tries to minimize his/her individual expected search cost that is required to meet the next customer
- Both distance-based and time-based charges have been considered in the generalized taxi fare structure
- A simultaneous optimization formulation has been discussed, which takes advantages of the state-of-the-art advances in combined models and sensitivity analyses in network modeling
- A Newtonian algorithm has been proposed to solve the resultant problem
- A Numerical example has been given to demonstrate the effectiveness of the proposed methodology