MODELLING AND GENERATING COMPLEX EMERGENT BEHAVIOUR

A thesis submitted for the degree of
Doctor of Philosophy

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Abstract

Despite a general recognition of the importance of complex systems, there is a dearth of general models capable of describing their dynamics. This is attributed to a complexity scale; the models are attempting to describe systems at different parts of the scale and are hence not compatible. We require new models capable of describing complex behaviour at different points of the complexity scale. This work identifies, and proceeds to examine systems at the high end of the complexity scale, those which have not to date been well understood by our current modelling methodology. It is shown that many such models exhibit what might be termed contextual dependency, and that it is precisely this feature which is not well understood by our current modelling methodology. A particular problem is discussed; our apparent inability to generate systems which display high end complexity, exhibited by for example the general failure of strong ALife. A new model, Process Physics, that has been developed at Flinders University is discussed, and arguments are presented that it exhibits high end complexity. The features of this model that lead to its displaying such behaviour are discussed, and the generalisation of this model to a broader range of complex systems is attempted.

Themes: contextuality and complexity; reductive failure; Process Physics; quantum theories as models of complexity

I certify that this thesis does not incorporate without acknowledgment any material previously submitted for a degree or diploma in any university; and that to the best of my knowledge and belief it does not contain any material previously published or written by another person except where due reference is made in the text.

Kirsty Kitto

Acknowledgements

This work has involved a long, very interesting, but often rather torturous journey. What began as an examination of the quantum measurement problem quickly increased in scope to the foundations of quantum mechanics, then again to fundamental physics in general, before shifting its emphasis to complex systems. Before long even biology, ecology, economics and artificial life came into the scope of this project. As such, this work could not have been possible without the assistance of a large number of people from a wide range of fields, whose conversations, criticisms, witticisms and creativity has challenged and extended my knowledge.

A special thanks to my supervisor, Reg Cahill, without whose guidance and support this project would have been inconceivable and probably not allowed. His ideas have shaped more than my approach to physics.

The final thesis itself could not have been possible without the assistance of people from a variety of disciplines and fields; I would like to especially acknowledge discussions with, and often assistance from Daniel Kortschak, Susan Gunner, and Peter Wood. Also my family and friends for their patience and support.

Finally, I would like to acknowledge the internet. Without the dramatic explosion of readily accessible ideas, theories, articles, preprints and tutorials that it provides, work such as this, at the boundaries of many substantially different disciplines would be difficult if not impossible; our theories are now in a position to evolve to new levels of complexity thanks to the creation of this vital resource. May it always remain free.

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Chapter 1

Reductionism at its Limits

. . . what is proved by impossibility proofs is a lack of imagination.

Bell, p998, [60]

Reductive analysis has proven remarkably effective throughout the history of science. However, a number of systems which we seek to understand appear to be defying this analytic technique. Often, these systems have been designated *complex*, but our present understanding of complexity is mediocre at best. This chapter will briefly examine some of these issues and propose that the systems most consistently defying our techniques are those that exhibit *contextual* behaviour. It will be proposed that complexity should be thought of in terms of a scale; while a number of systems have been extensively studied and are now well understood under the rubric of complex systems theory, there are more that have defied such an analysis, and intuitively, many of these seem more complex. While contextual systems will often be seen to lie at the high end of the proposed complexity scale, such contextuality need not imply that these phenomena can only be understood in a subjective sense. Indeed, there are a number of well-defined systems which exhibit contextual behaviour but which can still be meaningfully analysed. An example of such analysis is provided by the quantum systems which will be discussed in sections 4.3.1 and chapter 5.

Before we can start to understand complexity, we must understand its origins. Specifically, the traditional reductionistic analysis which is almost equated with the scientific method.

1.1 The Scientific Method

> *Man is always reductionist in his mechanistic explanations. Indeed, when we say that we have explained a phenomenon we ordinarily mean that we have shown the phenomenon to be the consequence of interactions among system components.*
>
> O'Neill *et al.*, p61, [310]

Reductive analysis, the technique of breaking an apparently complex problem into smaller, more manageable pieces, *reduction* and then combining the solutions obtained from these smaller problems into a larger solution which represents the original system,

synthesis has been remarkably successful in science. In fact, the common conception of science is almost synonymous with reduction, leading to the often mentioned distinction between the so-called 'hard' and 'soft' sciences. It might be said that a field falls into the hard or soft categorisation depending upon whether or not it is amenable to reductive analysis [364]. Thus fields such as sociology, anthropology, biology *etc.* which examine systems that are not so amenable to the reductive approach are often classified as soft, somehow less scientific, than those such as physics and chemistry, a distinction which is not useful and can lead to some rather controversial debates.[1]

Despite its importance in the field of science, the concept of reduction is fraught with misunderstanding and many conflicting definitions [42]. It is often conflated with other related concepts such as atomism [432], materialism [293, 443], physicalism [343, 168], microreductionism [312] *etc.* Also, the tendency towards a form of eliminative reduction [127, 360], often termed 'nothing butism' [42], or 'greedy reductionism' [147], where a reduced theory is seen as nothing but its set of reducing concepts has led to a general mistrust of reductive analysis in certain areas of study and a resurgence of concepts like holism. Often, this mistrust is taken to extremes, reduction being equated in a rather slippery way with a general distrust of science [147, 341], frequently without a proper definition, or usage of the term:

> *Attempts to explain behavior in mechanistic terms are commonly denounced as "reductionist" or "determinist". The denouncers rarely know exactly what they mean by those words, but everyone knows they refer to something bad.*
>
> Pinker, p10, [341]

This thesis will not weigh into this debate, rather the emphasis shall be shifted towards those areas where reductive analysis might be seen to be consistently failing. First however, we require a more specific understanding of what this work is referring to when it uses the term reductive analysis.

1.1.1 Reductive Analysis

> *Science stands today on something of a divide. For two centuries it has been exploring systems that are either intrinsically simple or that are capable of being analyzed into simple components. The fact that such a dogma as 'vary the factors one at a time' could be accepted for a century, shows that scientists were largely concerned in investigating such systems as allowed (by) this method; for this method is fundamentally impossible in the complex systems.*
>
> Ashby, p5, [36]

[1] Consider for example the Sokal affair, where the physicist Alan Sokal wrote, and managed to publish an essentially nonsensical article in the field of social sciences [409, 408, 146]. It might be asked whether similar situations do not (admittedly unintentionally) arise in the more traditional hard sciences, the process of peer review is becoming more and more difficult to fairly implement, as the field of knowledge fragments and expands.

Physics provides a paradigmatic example of reductive analysis. Traditionally, physical analysis involves a process whereby the universe is broken into a *system* to be studied, and it's *environment*, which is everything else, *i.e.* that part of the universe not considered interesting for the current study. Within a certain set of stated[2] assumptions, the system is then considered to be *isolated* from the environment which means that it may now be studied as an independent entity. Given this separation, a theory may then be constructed tested and developed according to some scientific method [344, 253]. It is important to realize that this separation has normally included the identification of any relevant measuring apparati as also separate from the system under study. There is an often unstated assumption associated with this separation that measurement does not in any way affect in any way the results of the measurement itself, although it may change the results of any new measurements that are performed after the earlier one. While the measuring apparatus cannot be considered as separate from the system (and therefore part of the environment) it is not quite part of the system itself. This has often led to the identification of such apparatus as forming a third form of component, between system and environment [191].

Almost every field of physics has risen with the application of some form of this method. Consider for example the field of Mechanics, which involves a set of objects identified as primitive, the dynamics of which is governed by a set of well understood laws, or regularities. Often, such a complicated system can be understood in terms of its constituents, consider for example statistical mechanics. Another example can be found in the Theories of Relativity, which describe the way in which a particular frame of reference can impact upon our observations, allowing us to separate the description of a systems internal dynamics from its motion with respect to an observer.

After this brief introduction, we are now in a position to more fully define the concept of reductive analysis, and to separate it from the associated concept of reductionism. This shall be done with reference to the concept of a *modelling relation* which was defined by Rosen [363, 364]. Consider a natural system N which is modelled by some formal system F. This relationship of modelling is represented in figure 1.1, where arrow 2 depicts the process of *encoding* the natural system into the formalism, and arrow 4 the process of *decoding*, or predicting the behaviour of the system based upon the predictions of the formalism. Thus, if the model is adequate, then the causal behaviour of the system N which is represented by 1, should be equivalent in some sense to first encoding its behaviour into the model, then inferring its behaviour according to relation 3 and

[2]It must be acknowledged that there are a number of situations where assumptions are either not explicitly stated or hidden (*i.e.* not even identified as an assumption), a situation which often leads to furious debate, apparent mysteries and unexplainable behaviour *etc.* The so-called mysterious nature of quantum systems provides a notable example of this phenomenon. With its high number of competing interpretations, and the large amount of debate surrounding these, it is often the case that those supporting different interpretations appear to be talking past rather than to each other.

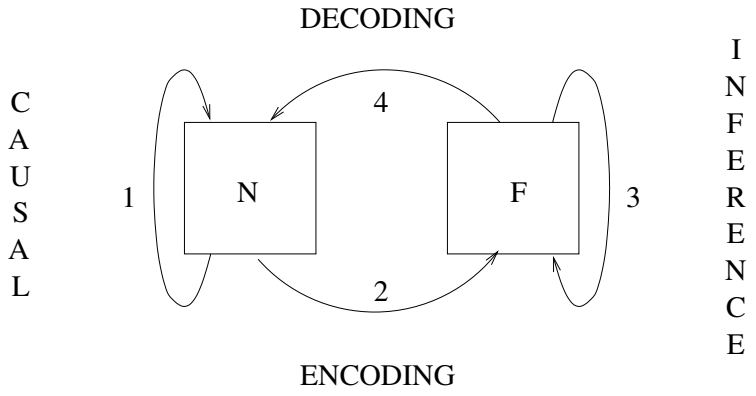
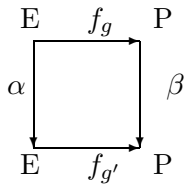


Figure 1.1: Rosen's modelling relation [364]; an explanation appears in the text.

decoding the subsequent predictions of the model back into such a form that they might be compared with the behaviour of the natural system. With reference to the modelling relation, reductionism is the belief that all systems can be so formalised *without loss*, and that there is a *largest model* which can effectively describe all natural systems. According to the current dogma, the largest model would be something like the Grand Unified Theory (GUT) of physics. Reductive analysis on the other hand need not presuppose a largest model, merely that there exists some model not necessarily commensurable with any other model, for all natural systems. Thus reductive analysis is prevalent in physics, where different theories and models, not necessarily commensurable, are used to describe different aspects of the physical world. For example, general relativity describes the way in which our models of physical systems change under motion, and quantum theory is seen as describing the very small, but these two theories while very accurate individually do not combine well; they do not appear to be commensurable, but the systems involved have shown themselves amenable to reductive analysis. Strong Reductionism requires that some larger model must exist, capable of obtaining the predictions of both relativity and quantum theory, as well as those of fields such as biology, anthropology and sociology. However, such a model may not exist.

We can formalise Rosen's Modelling Relation more fully making it simple to extend to a definition of complexity. Given a modelling relation describing some system, we can define the equivalence of two different descriptions of that system as follows:

> *Mathematically, we say that the two mappings, f_g and $f_{g'}$, are equivalent, or similar, or conjugate, if there exist appropriate transformations*
>
> $$\alpha : E \to E, \beta : P \to P, \tag{1.1}$$
>
> *such that the diagram commutes; that is if*



$$\beta[f_g(e)] = f_{g'}[\alpha(e)]. \tag{1.2}$$

Robert Rosen, p185, [361]

A system is defined by Rosen as complex if it has more than one inequivalent description, *i.e.* if it is impossible to find a mapping from one description to the other. Thus, according to Rosen, complex systems cannot be described by one formalism or model alone. If this is indeed the case, then it will have a profound impact upon ideas such as reductionism, which would *a priori* be shown to be false. Even reductive explanations of reality would be seen to be weakened, as no one explanation would be sufficient to the description of complex systems in general. This work will adopt the position that high end complexity cannot in fact be modelled by single descriptions, and that more tools are required in order that we might extend our understanding of such systems. First however, we shall continue our discussion of areas where reductive analysis has been perceived as more successful.

The similarities between physical and economic models have been attracting interest from physicists, and an increasing number of techniques from physics have been shown to have applications in economic theory [42, 415, 417]. Empirical approaches have been particularly useful at increasing our understanding of economic systems, but as our data and our modelling of such systems increase dramatically, there is a general lack of firm theoretical foundations in this field [417].

The theory of microeconomics rose in the wake of the successes of physical theory, and shares much with the field of physics [417]; again a number of isolated components are identified, each behaving in some well defined way. Unlike physics, there is no sense of direct interaction in microeconomic theory, a fact that simplifies these systems further still. Microeconomics can perhaps be most closely identified with statistical mechanics, there are many components, each assumed to be following an isolated dynamics which is dictated by the behaviour of all components.

In traditional microeconomic analysis the economy is understood as a set of constituent *firms* which produce commodities and *households* which consume them. Firms seek to maximise profit through the production of *commodities* chosen from a *production possibility set* which is the collection of all possible production plans listing the quantities of parts, raw material and finished products. Households on the other hand choose to buy commodities from a *consumption possibility set*, within the constraints of a *budget*. The preference orders of different households are often described by a *utility function*, an

analytic function which each household tries to maximise:

> *Given a household's options (the consumption possibility set), constraint (its budget), and objective (its utility function), its equilibrium state is obtained by finding the affordable commodity bundle with the highest utility value.*
>
> Auyang, p111, [42]

The *invisible hand theory* [226] gives an approximate method for solving the 'many-body' problem of finding the equilibrium state of the economy in which maximal satisfaction is obtained for all participants through the fixing of prices and quantities of commodities. However, this model of economics is almost absurdly simplistic, it treats all human activities as falling into either production or consumption, satisfaction is obtained purely through consumption, the *rationality* of individuals is defined solely in terms of their ability to optimise their utility and profit [42]. It is assumed that individuals have perfect knowledge of the qualities, and prices of all commodities, both in the present as well as in the future. There is no interaction between households, factors such as envy, imitation, and rivalry do not enter into this model, and yet our economy is apparently driven by them (in fact the very concept of advertising is founded upon concepts such as these [246]). The behaviours of firms and households are assumed to be occurring in a passive environment, unaffected by their decisions in any way. Thus concepts such as environmental sustainability, or quality of life *etc.* are very difficult to incorporate into this model. Indeed, many of the current problems confronting humanity arise from the simplistic nature of this model.

Thus, economics does not provide the simple system that traditional results and analysis appear to suggest.

One recent paper points to three varying levels of complexity in financial markets [79], with lower level phenomena better described by standard reductive analysis than higher, emergent behaviour.

1. *Price time series data* of a financial assets is nontrivial. Both short range and long range memories exist for this stochastic data; high frequency data analyses show that correlation times among data can be as short as a few minutes in highly traded stocks and indicies (in accordance with the efficient market hypothesis); however, nonlinear functions of return (such as the absolute value) are often correlated over much longer than a trading day (there are often power law correlations up to approximately 20 trading days). In addition to this odd signature, the behaviour of time series data is only asymptotically stable. Indeed the volatility of stock returns is itself stochastic.
2. *Cross correlation* between the time evolution of a set of financial entities is a well observed phenomena [111]. Understanding this cross correlation can help improve economic forecasting as each individual time series carries information about much more than just the stock itself; in particular it often reveals information about the

trade sector of the stock. So, not only are price time series in a financial market complex individually, they are also complex with respect to their synchronous interaction with other time series.

3. *Collective behaviour* exhibited by financial systems during extreme market events provides a third tier of complex behaviour. Typical and extreme days reveal statistically different ensemble return distributions, specifically, the shape of the distribution changes during extreme events such as crashes or rallies. Statistical regularities pertaining to this change of shape can be detected after very long time intervals (up to 10 years).

Thus, the modelling of financial markets exhibits a similar amenability to reductive analysis, but resistance to reductionism. In particular, the interaction between the differing levels of complexity is not at all understood; how does one level of relatively simple but complicated time series data lead to extreme collective phenomena? In order to answer this question it will be necessary to develop our understanding of complex emergent behaviour, a key aim of this work.

1.2 Complex Systems

> *A complex system cannot be reduced to a collection of its basic constituents, not because the system is not constituted by them, but because too much of the relational information gets lost in the process.*
>
> Cilliers, p10, [128]

The delineation made by reductive analysis, between system and environment is reasonable if the two are well separated, *i.e.* do not interact in a way that significantly affects the dynamics of the system. Such a clear separation between system and environment was easy to find in classical physical systems, such as mechanical and thermodynamic ones, where this technique yielded a rich set of very accurate results. However, this separation has never been quite so straightforward for all systems, a problem which often leads to the designation of these systems as *complex*.

For example, O'Neill *et al.* examine a number of examples from ecology in their attempt to understand the hierarchical[3] nature of ecological systems [310]. They understand an ecosystem as something that cannot be simply synthesised from a number of components, but must be analysed in addition to those components; ecosystems are not merely a backdrop to a number of different animal and plant species but often a system themselves.

> *Ecosystems are not simply spatially disjunct groupings of taxa (e.g., the plant community). Ecosystems cannot be arbitrarily assigned to a preconceived*

[3]Hierarchical systems fall very regularly into the general categorisation of complex behaviour. They will be discussed in chapter 3.

spatiotemporal framework (e.g., the climatic climax). Ecosystems cannot be conceptualized simply as functional entities or simply as collections of species. Instead, ecosystems must be viewed as systems in their own right.

O'Neill *et al.*, p 37, [310]

Thus, there is a very real sense in which ecosystems are not separable and could therefore be identified as complex in some sense.

Complex systems can have a number of different characteristics:

- They might consist of a large number of components (*e.g.* the large number of components inherent in economic models), but this is not enough (*e.g.* statistical mechanics is not normally considered complex).
- They often have components that exhibit a large number of different interactions (*e.g.* the interactions that occur during a process of biological development, see section 2.1.2). Interactions are generally nonlinear in these systems; small causes can have large effects, and *vice versa*. Also, both positive and negative feedback often occurs in these interactions, which often leads to effects disproportionate from their causes.
- Complex systems are generally open, that is, not separable from their environment. Often an observer has to make a choice as to where a boundary occurs.
- They generally operate far from equilibrium, which means that their survival often requires that energy must consistently flow both into and out of them (*e.g.* biological systems must constantly acquire food and excrete waste).
- The components of complex systems are themselves sometimes complex in some manner (*e.g.* organisms, themselves capable of complex behaviour, consist of a large number of different organs, and systems which themselves engage in identifiably complex behaviour, and may often consist of components that have largely defied reductive analysis).

However despite the general agreement on what characteristics should be exhibited by complex systems, complexity itself is very difficult to define [222, 128, 163].

It is common to identify complex behaviour with models such as Power Laws, Fractals and Bifurcations [383], The Renormalization Group [233, 460], Self-Organized criticality [48], Randomness [250, 264, 119], Catastrophe Theory [433], Dissipative Theory [346], Synergetics [208] *etc.*[4], however, it is likely that such simple identifications are premature.

This is because it is possible to identify two broad groups of researchers investigating complex systems, those that equate the above systems with complex behaviour and

[4]Perhaps the most valuable complexity resource is the *Hypertext Bibliography of Measures of Complexity*, created by Bruce Edmond's which while no longer maintained by him, and therefore missing references from later than 1997, contains a vast, fully crossreferenced list of articles that cover the many different ideas surrounding complex systems that had been proposed to that date. This, and a number of other resources are available at http://bruce.edmonds.name. Of particular use is Edmonds' thesis which explores the conception and measurement of complexity [163].

feel that further analysis of apparently more complex systems will yield to the same or similar analysis [275, 208], and those that feel there is some element missing from such analysis, that complexity might consist of more than just randomness, dissipation and self-organization [364, 116, 163, 128]:

> *A complicated system is composed of a large number of interacting components. Importantly, the properties of such a system can be accurately predicted from a knowledge of the properties of each of its components and a complete enumeration of their interactions. In other words, a complicated system is exactly the sum of its parts. Complex, on the other hand, is a term reserved for systems that display properties that are not predictable from a complete description of their components, and that are generally considered to be qualitatively different from the sum of their parts. A spaceship is extraordinarily complicated, but is not complex. The degree to which cells, individually or as differentiated collections called higher organisms, are simply more complicated than even spaceships, and the degree to which they are actually complex, is perhaps the most important theoretical issue facing biology today.*
>
> Nature Biotechnology Editorial, p511, [162]

This second group of researchers do not generally consider the systems described by the above theories as truly complex, or at least not *very* complex. As suggested above, systems of this form are sometimes identified as *complicated* rather than complex [365], a distinction that is possibly worth making, but our terminology cannot realistically be changed at this point in time given the weight of already existing literature. In any case, such a dipolar distinction between the systems is perhaps not the best resolution to this problem; it appears reasonable to suggest that there is a scale of complex behaviour, with simple behaviour gradually giving way to the more and more complex. This concept will arise throughout this chapter, as more concepts become available to the discussion, at present however, we can identify a number of concepts that might be used in the definition of such a scale:

- number of components
- form of interaction between components
- separation of components
- separation between system and environment
- contextual dependency upon experimental arrangements, and environmental scenarios
- dependence upon an observer.

These different concepts, and probably a number of others, will have different effects on the overall perceived complexity of a system. However, some of the potentially most important symptoms of complexity are also the least well understood. For example contextuality, or

observer dependence, is prevalent in those systems that are generally perceived as 'very' complex. The coming sections of this chapter will develop this theme in more detail.

Firstly, it must be pointed out that even in the original bastion of reductive analysis, physics, the distinction between a system and it's environment is no longer a simple one to make. With the discovery of quantum systems, physicists are now also having to develop an understanding of contextual behaviour. The contextuality of quantum systems will be discussed in detail in section 5.1. In this section we shall instead consider a different set of examples, specifically, the ways in which contextuality manifests in biological systems.

1.2.1 Contextual Complexity in Biological Systems

> *A deep reason for the difficulty in devising causal information from DNA messages is that the same "words" have different meanings in different contexts and multiple functions in a given context, as in any complex language.*
>
> Lewontin, p152, [263]

Biological systems are notoriously complex; they are often very difficult to resolve into components that are independent of one another, and they are also highly dependent upon environmental conditions (both biotic and abiotic). Thus, the traditional scientific aim of separating a system of interest from its surrounds is often not possible in biological systems. This form of dependency might be termed contextual, and in order to understand it both the genotype and the phenotype must be considered. The genotype of an organism is its genetic content, (*e.g.* it's DNA for living organisms, a Bit sequence for Artificial creatures), while an individuals phenotype arises from the interaction of the genotype with an environment during the process of *development*.

Phenotypic Plasticity

The developmental dependence upon the environment can have rather profound consequences. For example, it is well known in the biological community that organisms with the same genotype may, if placed in a different environment, reveal significantly different phenotypes, to the extent that they may even be identified as different species. This phenomena is known as *phenotypic plasticity*:

> *. . . the ability of a single genotype to produce more than one alternative form of morphology, physiological state, and/or behavior in response to environmental conditions.*
>
> West–Eberhard, p249, [452]

This phenomenon amounts to a situation where evolutionarily important characters do not have to be 'genetic' (immune to environmental effects). Thus, this phenomenon reinforces an often cited (but perhaps not truly recognized) fact that the phenotype is a product of *interaction* between the genotype *and* the environment. Phenotypic plasticity forces us

to accept that the phenotype depends in a very strong way upon the environment that surrounds it; a different environment can result in a vastly different organism.

We might understand this phenomenon as a contextual dependence of the developing organism upon the environment that it finds itself in.

Fitness

Fitness is not a characteristic that can be ascribed solely to an organism, it must be considered within an environment. *Fitness landscapes* map the fitness of the phenotypes of different organisms. It is well accepted that fitness landscapes can change quite dramatically depending upon the environment in which the organisms are found [406, 407, 83, 284]. For example, while it is beneficial that a persons skin colour be darker in colour in regions of high UV exposure, and consequent risk of melanoma, it is less beneficial to have the same skin tone in regions where skin pigmentation reduces the efficiency of vitamin D production (which is light-dependent).

Cloning

Animal cloning [304, 437] is perhaps one of the most interesting examples of contextual dependency in biology. It illustrates a number of key systemic interdependencies in biological systems. Firstly, cloning dramatically illustrates the fact that DNA alone will not replicate or form an organism, it must be placed in a cell. Also, the DNA cannot be placed in any cell, it must be placed in the correct form of cell in order to begin replication. Thus, it is not possible to separate genetic content from its surrounds and retain any meaningful sense of a functional system; DNA is contextually dependent upon not just the environment, but also the cell in which it finds itself, which can be considered a level of environment in itself.

We are left with an uneasy feeling that if contextuality is similarly prevalent in other fields, then the usual separation between system and environment may be futile; perhaps reductive analysis, while very successful historically, is not in fact the way to proceed in our attempts to extend our understanding of physical reality. This problem, while relatively new to physics is by no means new in other fields. This section will now discuss a number of perceived 'hard problems'; questions from fields that have consistently defied reductive analysis. Each shows a remarkably similar tendency to resist a separation into system and environment.

1.2.2 Defying Reduction — Very Complex Systems

> *. . . there is a sense in which complex systems are far more generic than simple, context-independent ones. Moreover, analysis and synthesis are not simple rote operations, nor are they in any sense inverses of one another. In short, the entire identification of* context-independence *with* objectivity *is itself far too special and cannot be retained in its present form as a foundation for physics itself.*
>
> Rosen, p36, [365]

This thesis shall not attempt to define complexity. Instead this work will look at a number of example systems which have thus far defied reductive analysis and are, most likely, on the high end of any complexity scale that might be identified. An attempt will be made to to develop new modelling techniques capable of dealing with these highly complex systems. It is hoped that in the attempt to construct such a technique, our understanding of such systems will be furthered. As an introduction, this section will briefly touch upon some of the current grand problems of analysis, which could in the above framework be identified as very complex systems, and which have thus far defied reductive analysis, pointing to the way in which they have stymied attempts to understand, and to model their behaviour.

The Foundations of Mathematics is a problem that has been resolved, but the implications of that resolution are still controversial [361, 118].

In the wake of the dramatic expansion of mathematics that occurred in the nineteenth century, due to the solution of many longstanding problems, and the associated development of a number of new techniques, it became generally accepted that mathematics should not concern itself with the validity of its axioms, as had been the case since the time of the ancient Greeks, but attempt to *derive theorems from some given set of axioms*. With this change in the perceived role of mathematics came a general concern over whether a given set of axioms could be shown to be *consistent*, and *complete* [295]. A complete and consistent formalisation was attempted by Russell and Whitehead in their *Principia Mathematica*, but ultimately was shown to be impossible by Gödel in two theorems published in 1931. To achieve this, Gödel constructed a metalanguage which formally represents theorems within a mathematical system, and then used this language to ask questions of the mathematical system itself about the provability of theorems that can be stated within the system. He showed that not all questions that can be stated within the system can be answered using the language of the system itself; if the system is powerful enough (complete) to ask such questions then it will necessarily be inconsistent [295, 219, 120]. But this result did not remain in the realms of mathematics alone. Alan Turing quickly proved an equivalent assertion about computer

programs, which states that there is no systematic way of testing a program and its data to say whether or not the program will ever halt when processing that data [439]. Gregory Chaitin has examined this problem from an information-theoretic perspective and found explicit examples of simple arithmetic propositions whose truth or falsity will never be known by following the deductive rules of any system of logical inference [120, 122, 119]. Casti has summarised these results eloquently:

> *Essentially, what Chaitin's results show is that such mathematical questions are simply too complex for us.*
>
> *The theorems of Gödel, Turing and Chaitin are limitations on our ability to know in the world of mathematics.*
>
> Casti, p12, [117]

However, it is possible that rather than an essential limitation, such results provide us with an opportunity.

Rosen has shed light upon the implications of Gödel's theorem by discussing it with reference to the above discussed modelling relation [362, 364]. He claims that Gödel's theorem implies that Number Theory is more complex than any of its formalisms. This is an interesting, and very suggestive result, implying that the mystery surrounding the foundations of mathematics stems from our insistence that we can analyse it reductively. With new understandings of complexity, it is possible that we would gain more insight into this problem.

The Origins of Life Problem or the problem of explaining the emergence of life from physical entities has proven to be similarly intractable. There is a long and distinguished line of researchers who have attempted to shed light on this problem, ranging from Aristotle's concept of abiogenesis, to the more recent formulation pioneered by Pasteur, modern physicists [382], and more [221, 225, 86, 281, 238, 239, 364, 404, 53], and yet it must be admitted that little progress has been made about how life might arise from a physical world. Generally, the modern explanations for the early origin of 'living' structures from 'nonliving' generally fall into gene-first hypotheses (postulating the early appearance of nucleic acids), metabolism-first (postulating the evolution of biochemical reactions and pathways first), and hybrid explanations,[5] but each of these proposals falls prey to a number of strong criticisms [238], and no one explanation has been generally accepted.

Most recently, this problem has arisen in the field of Artificial Life, or ALife, which is

> *... the study of man-made systems that exhibit behaviors characteristic of natural living systems. It complements the traditional biological sciences concerned with the* analysis *of living organisms by attempting to*

[5] See Kauffman [238] for a review of these theories as well as a list of references.

> synthesise *life-like behaviors within computers and other artificial media. By extending the empirical foundation upon which biology is based* beyond *the carbon-chain life that has evolved on Earth, Artificial Life can contribute to theoretical biology by locating* life-as-we-know-it *within the larger picture of* life-as-it-could-be*.*
>
> Langton, p1, [259]

However, this field has not tended to be as successful as might have been hoped at its inception. In chapter 2 this problem will be examined in some detail, it will be argued that this is due to a lack of complexity in the models of ALife; hidden reductionistic assumptions are colouring our models and leading to their general failure.

The Origins of Consciousness or the question of how conscious thought can arise in living systems, faces similar problems to the origins of life problem. It is closely associated with the mind-body problem which is concerned with the question of whether mental phenomena are equivalent to physical phenomena, and if not, with the question of how the two should be related. The modern form of this problem was first formulated by Rene Descartes [149] who proposed that the essence of the physical is spatial extension, and that minds are substances that are not extended in space, and therefore not physical. Descartes suggested that these two fundamentally different substances interacted in the pineal gland, something that we now know to be wrong, however, the idea of *Cartesian Dualism* remains despite its by now well known difficulties. Different solutions to the mind-body problem have been proposed, in the form of monism, parallelism, epiphenomenalism, functionalism *etc.*, but each has been rightly criticised by the proponents of other explanations. A good introduction to these issues is the collection of classical and contemporary articles edited by Dennett [123]. Materialism is at present the most popular resolution to the mind-body problem; according to this monistic view, only material (physical) entities exist. This theory owes its popularity to the perceived successes of physical theory in its explanation of much of the behaviour that we see around us. Thus the standard materialistic view is tightly bound to physical theory; if, for example, physics is shown to be inadequate in some way then materialistic explanations will be seen to have suffered a setback. This argument might be reversed however, the fact that no one explanation of the origins of mind has been forthcoming might suggest that indeed this is a complex problem and will not be resolved by recourse to reductive techniques.

The Emergence of Language and the possibility of meaning or representation, forms the basis of another set of unsolved problems [322, 404, 248, 166, 112, 272, 67, 338, 306], but is made particularly difficult due to the lack of data:

> *Language does not fossilize — for all that it was one of the great transitions in evolution, the advent of language has left no obvious equivalent to fossil teeth and bones, and seems inaccessible to enquiry. But it is not hard to imagine the emergence of a set of signals to label objects, the combinatorial nature of which allowed an infinite repertoire of sentences to be constructed from a finite set of words. An essential part of the process must have been the acknowledgement of a set of rules to combine words in such a way as to make sentences meaningful. These rules are the syntax that we all easily learn as children, but students of language evolution have a tough time explaining its origins.*
>
> Solé, p289, [410]

Although there is a general consensus that the emergence of language has biological features [404], finding those features remains a problem. As can be seen from the above references, a number of different theories have been proposed but these are hampered by this lack of data, and the general lack of a theory of language makes these problems even more evident. It is likely that many of these problems are closely related to the origin of mind or consciousness, since one of the key differences between humans and apes is due to their language processing abilities in the cortex [339]. Thus, it is likely that we will not be able to understand one set of problems without an understanding of the other; two high end complex problems are therefore likely to form part of the same problem, and neither have been solved by reductive analysis.

The continuing failure of reductionistic techniques to resolve these problems, despite the fact that some of them have existed for centuries suggests that our methodology may be flawed. We are left with the feeling that some of the most important problems in science are resistant to our main form of analysis — How are we to proceed? Perhaps reductive analysis can be saved by recourse to the concept of *emergence*.

1.2.3 Emergence

> *Emergence refers to the arising of novel and coherent structures, patterns, and properties during the process of self-organization in complex systems. Emergent phenomena are conceptualized as occurring on the macro level, in contrast to the micro-level components and processes out of which they arise.*
>
> Goldstein, p49, [194]

Emergence is a concept with a history in both science and philosophy, which is gradually gaining popularity in the field of complex systems theory [2, 290, 327, 245, 220, 363, 77, 78, 43, 252, 412, 51]. However, as can be seen from a brief reading of any selection of papers on the field (starting with the above list), the concept is far from well defined. (See [141] for a good review of the concept as well as a listing of many of its different

characterisations.) This plethora of different definitions and understandings of emergence does not rule out its usefulness as a concept, indeed if complex behaviour does fall into a scale and is exhibited by a wide range of systems then there is every reason to believe that the associated emergent behaviour will also display similar characteristics. Some of the different definitions of emergence will be discussed in this work as they become appropriate, and a new mode of emergence will be proposed in section 6.1. We shall return to this concept in section 7.1, when the ideas developed throughout this work will be placed into a larger context.

1.2.4 Complexity and Postmodernism

This section will conclude with a brief discussion of *postmodernism*, a field yet to be well understood which, I believe, has many ramifications for the field of complex systems.

Postmodernism is usually defined with respect to it's antecedent, modernism. Whereas modernism sought to place all knowledge and culture within one unifying description, postmodern arguments are usually seen as claiming such a *metadiscourse* to be impossible to a greater or lesser extent:

> *I will use the term* modern *to designate any science that legitimates itself with reference to a metadiscourse of this kind making explicit appeal to some grand narrative, such as the dialectics of the Spirit, the hermeneutics of meaning, the emancipation of the rational or working subject, or the creation of wealth. [Postmodernism is in this light an] incredulity towards metanarratives.*
>
> Lyotard, pp23–24, [271]

Thus, according to postmodernism, different narratives will be necessary in different situations, or contexts, and there is no such thing as *the* unified theory or metanarrative. But this is just what was claimed about complex systems in the discussion of Rosen's definition of complexity in section 1.1.1. It is likely that postmodernism arose from initial conflicts between our reductive techniques and complex systems, a situation made even more likely when we consider the historical roots of postmodernism in linguistics [148] and sociology [271, 227]. Postmodernism is what results when we continue blindly in our attempts to analyse complexity using our standard techniques; it actually points to the missing contextuality and complexity in our models of the world. Thus in contrast to the largely negative, or incomprehensible role that postmodernist theories are often seen to play by workers in more traditional fields of research [408, 386], they can instead be seen as pointing to the very flaws in our current methodology when we attempt to understand complex systems.

Paul Cilliers in a very interesting work examining complexity and postmodernism argues along similar lines that:

... the proliferation of discourses and meaning described in postmodern theory is not created by wilful and disruptive theorists, but that it is an inescapable effect of the complexities of our linguistic and social spaces.

Paul Cilliers, p113, [128]

These theories, rather than disrupting or destroying scientific discourse, point to an opportunity. The targets of their incredulity, static meanings, or grand metanarratives, are very similar to concepts such as objective knowledge, or systems that can be perfectly reduced to their constituents for the sake of analysis *i.e.* postmodernism rejects reductionistic explanations in a very similar manner to proponents of *observer driven* models of complexity such as Rosen (more details of this class of model will be provided in section 1.5.1). Different narratives become necessary when a contextually dependent complex system is under examination, but there is still a possibility that these different narratives can be sensibly compared, and judged in some way as to their respective validity. While it is likely that any attempt to do this will attract the ire of many postmodernists, it is possible that a number of other workers in the field will be interested in the possibility that their theories might be used in a more positive light.

1.3 Contextuality

Formally, a context means that a logical value associated with a given proposition depends on a history of the system. In particular, the order in which questions are asked is not irrelevant.

Aerts *et al.*, p1, [25]

A *contextual* system is one which depends in some way upon the behaviour of factors generally considered external to it. These may include the environment of the system, its history, its spatial or timewise extended components, the process of measuring the system, the way in which the system is examined *etc.* Many of these contextual dependencies will be discussed throughout this work, in particular sections 5.1 and 5.2 will more formally define this concept, taking quantum contextuality as the basis of this definition. For now we shall briefly examine the relationship between contextuality and complexity.

1.3.1 Complexity and Contextuality

Contextual matters are traditionally treated only in the "Material and Methods" sections of the primary experimental literature. They are not the matter sought for inclusion in textbooks, and are felt to be almost a physical embarrassment or even an impediment to the knowledge we seek for addition to the compendium.

Salthe, p85, [376]

Reductive methods have worked very well in scientific endeavors for centuries, problems only appear to be arising as we seek to apply them to systems which exhibit high

end complexity. Often, it is not possible to consider such systems as fully separable from their environment; the *context* in which we find a system will affect what we see. This contextual dependency may take a large variety of different forms, some of which have been discussed in this chapter: it may arise from the impossibility of separating a system from its environment as often occurs in biological systems: it might emerge in some sense spontaneously from the interaction of a large number of components (as in economics) where it becomes impossible to consider an individual as separate in any meaningful sense from the other individuals in the system: it might be that a system cannot be considered separately from past interactions and measurements, a series of frozen accidents might have profoundly contributed to the system as it is found in its present state (*e.g.* biological and quantum systems): the level at which a system is examined may influence what is seen as occurs in ecological systems: the response of a system to some input may depend upon other co-occurring inputs as often happens in genetic systems *etc.* Systems which exhibit such contextual dependencies are generally referred to as complex.

Such contextual dependencies will not be resolved or understood through simple refining of our current reductive techniques, they are not small problems which can be ignored or approximated away, rather they require new techniques of analysis. Concepts such as contextuality, complexity and emergence are intimately connected, they must be understood in unison. This work seeks to develop our understanding along these lines. Although it must be admitted that none of these concepts have been defined in this chapter, it is hoped that their nature has been illustrated somewhat in this discussion. Their status will be clarified more fully throughout the next four chapters.

1.4 Object Based Modelling

Our understanding of the world is *object based*; we see chairs, aeroplanes, trees, dogs *etc.* and form theories, models and predictions about their behaviour. We are often even right in our predictions. It is this object based methodology which has tended to form the basis of our reductive understanding of the world. The general idea has been that if it is possible to understand an aeroplane as composed of engines, seats wings, wiring *etc.* then it seems reasonable to expect that other aspects of the world could be understood in a similar manner. However, we have already seen that there are reasons to suspect that object based models cannot apply to all physical systems.

Even without admitting that phenomena such as complexity and contextuality are disrupting our theories we can find other reasons why object based methodologies are coming to the end of their general usefulness. Consider for example the infinite regress that is being faced by physicists attempting to define the fundamental objects of the Universe. Over a period of centuries our understanding of the world in terms of the substances we see around us has been refined, first to the atomic level, then atoms themselves were explained in terms of nuclei and electrons, and nuclei are now understood as emergent

structures formed from quarks and gluons [279]. At present, a number of models of these 'fundamental objects' have been proposed, including preons [279, 160], strings [387], branes [300] and loops [368], but we might ask even at this point what the constituents of these new objects will be. Something is obviously wrong with our methodology. This problem becomes particularly evident when we consider the nature of quarks. Considered simplistically, a nucleus is composed of three quarks undergoing a very complicated set of interactions in what is termed a colour singlet state (see section 4.3.1 for more details about QCD). Although quarks appear to be obvious contenders for the role of parts of a nucleon, the *colour confinement hypothesis* [336], suggests that they do not make sense individually, and no individual quarks have ever been discovered in nature. There is therefore a sense in which, while a quark is a reasonable modelling tool, it is a far more complex phenomenon individually than a nucleon. Thus, while it makes sense to talk about an electron and a proton, and experiments can be performed with these objects, the same cannot be said about quarks; as objects quarks exist tenuously at best. It is most likely that they are more real in our modelling then they are in reality.

This example should cause us to pause in our relentless pursuit of objects. A number of other problems with object based modelling will be discussed throughout this work, but this initial problem presents its own difficulty; how are we to construct non-object based models? Our entire modelling apparatus consists of identifying objects and their interactions, our mathematical apparatus mirrors this, if object based models are insufficient in an exploration of reality then we are left in something of a dilemma.

However, one possible resolution presents itself in the form of contextuality. A contextual system is difficult to separate into component objects, strange dependencies arise among objects which are disproportionate to their spatial separation and also to their apparent strength. With formalisms that properly incorporate contextuality we might start to develop methodologies which are not so object based. This search for an understanding of contextuality and less object-based models will form a major theme of this thesis.

1.5 The Complexity Scale

> *Simplicity may have a unified form, but complexity has many varieties. . . . Concrete complex systems spread across a whole spectrum of complexity. For systems on the high-complexity end of the spectrum, such as brains or persons, our current sciences offer catalogues of facts but no comprehensive theory.*
>
> Auyang, p9, [42]

While simple behaviour is relatively straightforward to define there is a wide range of complex behaviour. Starting from simple reductive systems, it seems possible to define a complexity scale, which moves from simple systems such as Newtonian mechanics, through chaotic systems, fractals and power laws, into problems such as network theories and

language acquisition, and finally up to observer driven models of high end complexity. The proposed scale will not be justified here; it is not something that can be justified at present, however, it seems likely that such a scale can be constructed and that such a concept will be useful in the field of complex systems. An initial proposed ordering is illustrated in figure 1.2, which it must be stressed includes many overlaps, and is only a preliminary suggestion. Similar scale-type proposals have been made by other authors [126, 42], but never with any actual suggestion of how such a scale might look. It seems likely that before a proper understanding of any such possible scales of complexity can be obtained it will be necessary to develop new modes of analysis capable of exploring the behaviour of high end complexity. With this in mind we shall now turn to some proposed models of what might be considered observer dependent, or contextual, high end complexity.

1.5.1 High End Complexity

Although such an identification is likely to be controversial, this thesis will adopt the stance that at least one aspect of high end complexity that can be identified is some sort of *observer dependence*, or *contextuality* of the systems exhibiting such complex behaviour. This section will explore some of the definitions of complexity which have been proposed along these lines.

We have already briefly discussed Rosen's definition of complexity, which falls very clearly into this category, in section 1.1. A number of researchers have pointed to similar observer dependence, or contextuality of complex behaviour, which shall be designated hereon as *observer driven*. Consider for example:

Pattee's Epistemic Cut [325, 324, 319] which separates the *object*, the system under study, from the *subject*, the thing that is interacting with (*e.g.* measuring) the object:

> *The epistemic cut or the distinction between subject and object is normally associated with highly evolved subjects with brains and their models of the outside world, as in the case of measurement. ... The cut itself is an epistemic necessity, not an ontological condition. That is, we must make a sharp cut, a disjunction, just in order to speak of knowledge as being "about" something or "standing for" whatever it refers to. What is going on ontologically at the cut ... is a very complex process. ... in order to perform a measurement, the subject must have control of the construction of the measuring device. Only the subject side of the cut can measure or control.*
>
> *Pattee, p15, [325]*

Thus, Pattee claims that an epistemic cut is necessary, and is necessarily outside of the bounds of standard reductive analysis in the attempt to understand any system

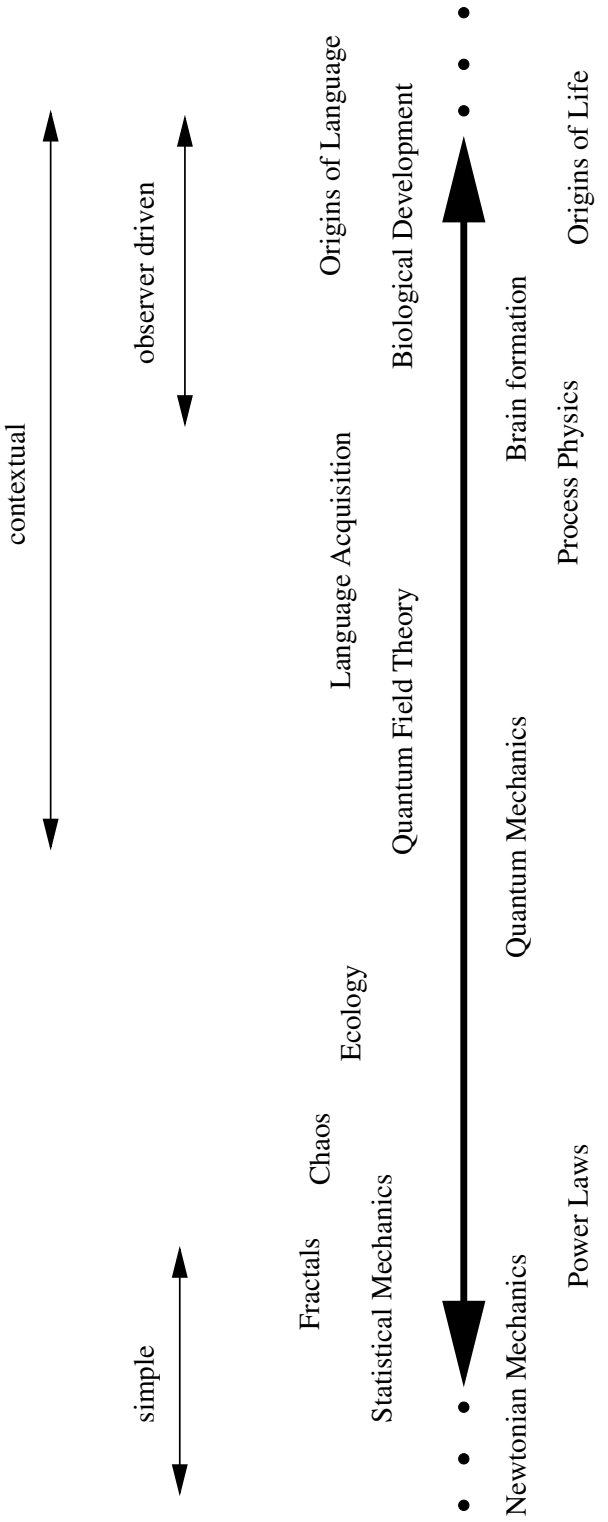


Figure 1.2: A proposed complexity scale. In this framework there are not simple and complex systems, rather all systems are understood as belonging to the scale, but some are 'more complex' than others. At the right end of the scale we see those systems and problems which might be understood as exhibiting *high end complexity*.

that involves a situation of measurement or control. In order to understand complex emergent behaviour, Pattee claims that we must be able to understand and to implement an epistemic cut in our models, and yet the process of this implementation remains a mystery. More details of this concept will be provided in section 2.1.4.

Casti built upon the conceptions of complexity forwarded by Rosen and Pattee. However, in addition to their claims, he made the stronger claim that complexity can be measured in some way; it is related to the *number* of inequivalent descriptions that an observer can generate for a system of interest [116]. This definition of complexity makes explicit reference to the observer; complexity only *makes sense* with reference to an observer, it is not a property inherent in any system itself. However, such a definition is rather difficult to implement [163].

Edmonds has formulated a syntactic definition of complexity [163] based upon inequivalent descriptions:

> *Complexity is that property of a model which makes it difficult to formulate its overall behaviour in a given language, even when given reasonably complete information about its atomic components and their interrelations.*
>
> Edmonds, p72, [163]

He defends this definition through a comprehensive examination of a number of alternative ideas surrounding complexity, such as patterns, size measures such as the number of components, size of the rule set, or the size of the minimal description in some language, processing time, or computational complexity, ignorance, variety, surprise, and improbability, the midpoint between order and disorder, logical strength, and irreducibility, claiming that these are concepts "that are frequently conflated with complexity, but which are, at best, very weak models of it." p57, [163]. With an acceptance of a complexity scale, such concepts can be seen to lie somewhere in between simplicity and complexity, explaining the weakness of such models as exemplars of complexity as well as their more complex nature when compared to simple reductive systems.

Complexity defined with respect to an observer, while not immediately appealing to the majority of researchers in complex systems does sidestep a number of issues that have plagued the general drive to find a definition of complexity.

- Complexity often seems only to make sense with respect to some level of description. For example, a process termed *coarse graining*, or the examination of a system on a larger scale [42], often turns an apparently complex system into a far simpler one [376]. A number of researchers have emphasised this point:

  > *Depending on the spatiotemporal scale or window through which one is viewing the world, a forest stand may appear (1) as a dynamic entity*

> *in its own right, (2) as a constant (i.e., nondynamic) background within which an organism operates, or (3) as inconsequential noise in major geomorphological processes. Thus, it becomes impossible to designate* the *components of* the *ecosystem. The designations will change as the spatiotemporal scale changes.*
>
> O'Neill *et al.*, p83, [310]

- As the above quotation emphasises, the time frame over which a system is examined can profoundly influence its classification as complex or simple. Organisation is often identified as *resulting* from this difference in processing rate [400, 310].
  This problem has an added difficulty that arises when we consider our current modelling paradigm. It is standard practice to model systems using differential equations, but these generally only allow for constant time intervals [137]. The behaviour exhibited by complex systems often occurs on a range of different faster and slower timescales, and this is often lost in a modelling based upon differential equations. This is a particularly relevant objection when we consider the interesting behaviour that can be generated by examining systems using differing timescales, such as was demonstrated by Turing [440]. It is likely that this characteristic can be utilized in an attempt to generate systems that can display complex behaviour, however, this possibility will not be examined in any detail in this work.
  In addition to this problem of modelling, many researchers have emphasised the way in which there appears to be a anticorrelation between the timescale over which an interaction occurs, and it's apparent strength [310], but there are few attempts to deal with this behaviour in a consistent model. Consider for example the dynamics of an ecosystem, which evolves over a timescale of centuries to millennia, where perturbations such as an increase in carbon dioxide may take 100 or more years to eventuate in changes that impact upon single organisms, which interact on a much faster timescale, and often much more strongly. (After all, a murder is a very strong interaction between two people over a potentially very short timescale, which often draws a strong reaction from a community, but the gradual warming of the earth, which has the potential to kill far more organisms, does not appear to impact upon us so strongly, at least at present). Generally an interaction over an ecosystem is far slower, and on the same timescale as organismic interactions, far weaker. A similar pattern repeats as we compare the interactions between cells in the organisms to interactions between the organisms themselves *etc.*
- As was discussed in section 1.4, it is often difficult to identify the objects that should be taken as fundamental in the modelling of complex systems, depending upon the behaviour of interest it is often necessary to make use of a different set of primitive objects as well as dynamical equations [42].

Observer driven definitions of complex behaviour sidestep these issues because different observers will see different objects as fundamental depending upon their spatiotemporal level of interest, and the dynamics that they extract from a model of a system will similarly be dependent upon their viewpoint. However, it is necessary that some sort of systematic connection between these different descriptions be possible, even if the connection does not make two different models equivalent.

Despite this advantage, the observer driven approach has not tended to attract the general attention that it deserves, especially in the field of physics. We might attribute this lack of recognition to the perceived negative implications of such an approach. In the general excitement created by the new techniques of 'complex analysis' there was a feeling of optimism that it would be possible to describe increasingly complex systems without a substantial rethink of the dominant modelling methodology. While these approaches have yielded remarkable results, the problematic phenomena listed in section 1.2.2 suggest that they will not be capable of answering all of the questions that arise in our attempts to understand complexity. For now, we shall assume that some class of observer driven theories will be necessary to understand high end complexity. More arguments will be forwarded as this work progresses to convince the reader that this is indeed the case.

It is interesting to note that in addition to the observer driven theories mentioned above, Edmonds identifies a number of other theories of complexity as special cases of his definition, notable among these special cases are computational complexity, algorithmic information complexity, and Shannon entropy. As these theories are very often identified with complex systems, the ability of Edmonds definition to incorporate them is important for its general acceptance. This also supports the identification of a complexity scale, as the possibility of describing 'less complex' behaviour within a larger definition suggests that such phenomena are at the lower end of the scale.

The focus, in the Edmonds definition upon the concept of modelling rather than upon the system under study itself is important. If complexity can be defined as a difficulty to formulate the behaviour of a system in the language of one model alone, then it is remarkable that there are so few fundamentally different models of complex systems. A major component of this thesis will consist of attempts to formulate new models which emphasise the contextuality of high end complexity.

1.6 Objectivity: Contextuality, Subjectivity and Observer Dependence

> *It must be acknowledged that the way we* look *determines what we* see*, or rather it co-determines the latter, in conjunction with what* there is*.*
>
> Kampis, p95, [236]

We might ask, at this point, how the concept of observer driven complexity affects our understanding of systems in general; is the longstanding distinction between scientific objectivity and subjective 'interpretations' to be lost? The contextuality exhibited by

nonlocal quantum systems indicates a way in which this issue might start to be resolved and will be discussed in section 5.1. For now we shall consider the implications of such results in a more general sense.

Contextuality in general raises an issue that has not been well addressed in the physical literature; the objectivity of our measurements can no longer be assumed. That is, the experimental arrangement that is used to examine some system itself appears to influence the results obtained. This is a profound result. Historically, one of the most basic tenets of scientific methodology is that the apparatus of scientific experiments is measuring some objective property of reality, it may do this in a destructive manner, or leave the system in a relatively unchanged state, but the measurement itself was assumed to be independent in some fundamental sense from the measurement result. This assumption has a long history. Traditionally, the term measurement represents the process by which humans carefully control and monitor a system in order to *determine* its state. There is, associated with this, an expectation that the process of measurement does not affect the results that are recorded. This assumption has been so strong that it has led to a distinction between the so called 'hard' and 'soft' sciences. In fact, this distinction can be attributed to the fact that traditionally, sciences such as mathematics, physics and chemistry could be discussed objectively; the results of any experiments or analysis in these fields did not depend upon the way in which they were acquired, or upon the history of the system under analysis. Consider for example Newtonian mechanics, where it is possible to measure the distance traversed by an object, and the time it took to go that distance, without in any way affecting the object itself. In contrast, fields such as sociology, biology and economics dealt with far more complex systems, many of which had contextual dependencies in even the most simple cases considered. It might quite reasonably be claimed that from the outset the concept of objectivity has been defined with respect to one of the narrowest class of systems that we might wish to consider scientifically.[6]

This problem has an even greater importance when we consider the ontology of physics; the debate about the interpretation of quantum mechanics and the associated perceived loss of realism as a viable philosophical stance in physics [305] arises because of this overly restrictive notion of objectivity in science.

It is my contension that we can identify a notion of observer dependence, or contextuality, without losing our ability to objectively analyse a system. This can be accomplished if we define these notions more formally, something that we are not yet ready to attempt. For now we can somewhat informally point to the differences[7] between these notions:

[6] Rosen raised a similar point when he identified physical systems as forming a subset of biological ones [364].

[7] Up until this point, observer dependence and subjectivity have been used interchangeably, we are now in a position to identify a slight difference between these two concepts. This notion will be discussed more in section 7.3.

Contextuality refers to a dependency of a system upon some aspect of its environment. In this work, I shall consider the environment to include anything not directly associated with the system itself; measuring apparatus, timescales, background noise, *etc.* In a biological system the environment retains its standard meaning. In some cases contextual dependency can be identified by finding a system that violates Bell-type inequalities. This will be discussed in section 5.2.

Observer Dependence is a stronger term, encompassing contextuality but requiring in addition that the contextual dependency occur with respect to measurement or observation; a system with an observational dependence will be contextual, but not all contextual systems will exhibit a direct observer dependence. An observer driven theory of such systems suggests that more than one explanation or model will be necessary to understand all of its behaviour.

Subjectivity is a stronger term again. A subjective description of a system cannot be communicated in a meaningful (*i.e.* objective) sense.

Thus, a system could be dependent on an observer in a well defined manner, in which case it would not be fair to identify it as subjective. If it was possible to formalise this dependency in some sense, then the description of the observer dependency could itself be communicated and a sense of objective description saved. We live in a contextual world, how we look can determine what we see, but if we can communicate the way in which this occurs, then we can hope to understand this contextuality.

The aim of this chapter has been to develop an understanding of the far-reaching contextuality possessed by a number of systems that are generally recognised as complex. A class of theories of high end complexity, which have been termed observer dependent, has been identified which to some extent capture this contextual nature. However, these theories are largely ignored in the complex systems area, perhaps because they are seen as limiting the extent of our possible understanding. This is not so. Rather, I have argued that such contextually dependent systems offer new opportunities for solving many of the apparently unsolvable problems besetting the scientific methodology. We require new techniques. A large part of this thesis will consist of an analysis of existing systems and their associated analytical techniques, where I will argue that in fact we have already incorporated to some extent the necessary contextual dependence into our modelling. Some of our current reductive ideas will then be extended in an attempt to incorporate contextuality more explicitly into our analytical methodology.

Chapter 2

An Example System: Artificial Life

Having identified somewhat the characteristics of high end complexity, this chapter will be devoted to a concrete example. Specifically, we shall look at the field of Artificial Life (often shortened to ALife) and its general failure to realise emergent systems which exhibit high end complexity.

Artificial Life is a diverse and sometimes disparate field, with a number of differing goals, theories, models and proposed outcomes:

- There is a drive to understand concepts such as complexity and emergence as they relate to the field.
- There are the attempts to generate emergent behavior *in vitro*, and *a machinā*[1].
- Some models, such as Avida [262, 459], in addition to their primary role as a platform for the investigation of digital life, are being used to investigate current theoretical and evolutionary problems.
- Then, there is the attempt to achieve a living organism, what might be called *Strong ALife*.

This diversity is indicative of a new vital field, with a number of promising avenues of research, however, it also indicates what might be perceived as a lack of direction; there are no obvious answers in the field, no well accepted theories that might be used to guide research. These two perceptions are in essence compatible, a newly established field will lack theories, which can acquire the status of dogma in a more developed field of knowledge.

A sign of the developing maturity of the field is the formation of the so-called Grand Challenges of Artificial Life which were identified during an open session during the 7^{th} international conference on ALife [54]. Similar to the list of mathematical challenges proposed by Hilbert at the beginning of the 20th century [218], these identify the fourteen major open problems in the field as perceived by researchers in the field. Separated into three main categories, these problems are:

[1] Although the more usual term is *in silico*, a consideration of the root noun (*L. silex*) leaves us wondering why researchers are trying to generate life in flint! This work will make use of *a machinā*, except for places where those with little regard for language are being quoted.

1. How does life arise from the nonliving?
   (a) Generate a molecular proto-organism *in vitro*.
   (b) Achieve the transition to life in an artificial chemistry *a machinā*.
   (c) Determine whether fundamentally novel living organizations can exist.
   (d) Simulate a unicellular organism over its entire lifecycle.
   (e) Explain how rules and symbols are generated from physical dynamics in living systems.
2. What are the potentials and limits of living systems?
   (f) Determine what is inevitable in the open-ended evolution of life.
   (g) Determine minimal conditions for evolutionary transitions from specific to generic response systems.
   (h) Create a formal framework for synthesizing dynamical hierarchies at all scales.
   (i) Determine the predictability of evolutionary consequences of manipulating organisms and ecosystems.
   (j) Develop a theory of information processing, information flow, and information generation for evolving systems.
3. How is life related to mind, machines and culture?
   (k) Demonstrate the emergence of intelligence and mind in an artificial living system.
   (l) Evaluate the influence of machines on the next major evolutionary transition of life.
   (m) Provide a quantitative model of the interplay between cultural and biological evolution.
   (n) Establish ethical principles for artificial life.

The article cited above discusses these problems in more detail, and contains a number of relevant references for any interested readers.

Unmentioned in the above list, but closely associated with a number of them, one of the most apparent problems in ALife is general the lack in complexity of the organisms evolved artificially. This chapter will discuss these 'lack of complexity' problems in some detail, it is believed that most of them are associated with the general problems besetting the reductive method that were discussed in the previous chapter.

2.1 ALife, Complexity and Emergence

> *A fundamental limitation for computer life is that evolution can only reflect the complexity of the artificial physical world in which organisms live. An epistemic cut affords the* potential *for efficient implementation and open-ended evolution, but in a simple world, efficient implementation will be limited and life will also remain simple.*
>
> Pattee, [323]

In order to achieve some form of artificial life, it will be necessary that our models display some sort of emergent behaviour. However, two results within the field of ALife suggest that this is not happening in a truly meaningful sense.

Firstly, it is widely accepted that ALife models do not display Open Ended Evolution (OEE) [55, 56, 403], but very little research has been conducted into the reasons why this might be the case. Rather, the tendency has been to construct ever more complicated environments, or to throw out existing environments and to almost arbitrarily develop new ones. Although there is often some explanation of the reasons behind this substitution [262], there is very little general *analysis* of this failure. We shall discuss OEE in more detail in section 2.1.3.

A second problem with ALife models their general failure to exhibit more than two levels of hierarchical structure or dynamics [68]. Without the generation of new, more complex levels there will be a highest level of complexity attainable, a point emphasised by the Ansatz for Dynamical Hierarchies (ADH), which shall be discussed in section 2.2.2.

These two problems are clearly attributed importance in the grand challenges listed above, being explicitly mentioned in challenges 6 and 8, and possibly impacting upon a number of the other challenges. For example, an understanding of the generation of rules and symbols from within the physical dynamics of living systems (challenge 5) would provide a mechanism by which OEE might be generated, as well as simplifying a mechanism by which the dynamical generation of higher levels of structure might be attained.

2.1.1 Insufficient Complexity

> *I tried several self-organizing schemes using automata models ... I eventually recognized a fundamental problem in all such rule-based self-organizing schemes, namely, that in so far as the organizing depends on internal fixed rules, the generated structures will have* limited potential complexity*, and in so far as any novel organizing arises from the outside environment, the novel structures have no possibility of reliable replication without a symbolic memory that could reconstruct the novel organization.*
>
> Pattee, p10, [325] (italics added)

The organisms that 'evolve' in ALife simulations are not very complex, particularly when we compare them to the obvious complexity exhibited by many of the organisms found in nature. It is likely that this lack of complexity can be traced to the low complexity of the simulations themselves:

- To date the true complexity of the biological environment has not been incorporated into ALife simulations. For example, experiments using environments such as Avida only tend to introduce environmental factors numbering in the tens, rather than the thousands and more, dependencies of biological systems. Also, it is rare that

simulations even attempt to model the feedback that occurs between developing organisms and their environment.

- It is rare that both the phenotype and the genotype are incorporated into models. Even when they are [140], the genotype/phenotype mapping used is simple, with little or no contextual dependence upon the environment.
- The above point means that the phenomena of phenotypic complexity discussed in section 1.2.1 are not exhibited by simulations.
- No situations resembling a dynamical ecosystem full of emergent new species adapting to new niches as they evolve have been realised; OEE is yet to be implemented.

It should be noted at this point that this problem is not evident in all fields of ALife. In particular, the current drive to create some sort of 'wet' or *in vitro* artificial life [352], where a number of physical components are combined in a 'test tube' is more likely to actually generate complex emergent behaviour. This is due to the fact that the *in vitro* approach has a well defined physics established in its behaviour; the physics of the everyday world which has already implemented epistemic cuts in a number of well defined ways. A similar point has also been taken up by Pattee:

> *Real and artificial life must have arisen and evolved in a nonliving milieu. In real life we call this the physical world. If artificial life exists in a computer, the computer milieu must define an artificial physics. . . . In other words, any form of artificial life must be able to detect events and discover laws of its artificial world.*
>
> Pattee, [323]

ALife must implement a sufficiently complex artificial physics before it can realise sufficiently lifelike behaviour. Wet ALife is effectively bootstrapping life within a physics that we already know is sufficiently complex to support life, namely our own, therefore it has neatly sidestepped one of the major obstacles confronting *a machinā* ALife. Of course this does not detract from the importance of *a machinā* methods, in order to understand the full potential of life, we must attempt to understand it in all guises and there is no reason to suspect that our physical world is the only one capable of supporting life, however, it is likely that more progress will be achieved by a sensible application of reductive analysis when it might work, and this is one such case. This is one reason for the current excitement surrounding wet ALife [352], but this field does not solve our problems. In sidestepping the issues we have not gained any understanding, rather we are racing to realise a phenomenon that we will still not understand.

Although models of phenomena are by necessity simpler than the phenomena themselves, the lack of simulation complexity in ALife is far more disturbing when considered in the context of the attempt to actually generate life *i.e.* strong ALife. If it is unlikely that high end complex behaviour will emerge from our current modelling technology, it

is far less likely that this behaviour will resemble life in any meaningful sense. We need to consider alternate technologies and models.

First however, we shall consider a specific example of real world complexity which has not been implemented in any meaningful sense in ALife.

2.1.2 The Complexity of Development

The process of biological development is one of the most complex exhibited by natural biological systems. It is also one of the least understood, and yet artificial models of this process are simplistic at best. There are a number of morphogenic models such as Dawkin's tree growing program [145], but these models do not seem to capture the true complexity of the coupling between the phenotype and the selective process. For example the tree growing program requires that an observer choose the most 'pleasing' configuration at each time step. This lack of complexity might perhaps be attributed to the problem that while the concept of phenotype is relatively easy to understand in a biological context, consisting of readily identifiable characteristics such as eye colour and height, in the field of Artificial Life, it is not a straightforward concept to identify. Indeed many of the simpler simulations performed tend to focus upon the genotype of the creatures evolved, with little consideration to how this might map to a phenotype during the process of ontogeny (or development). This is unfortunate as the influence of the environment upon an individual can be quite profound, consider the example of phenotypic plasticity discussed in section 1.2.1 a phenomenon which has not been reproduced in any ALife simulations known to the author. In ALife a genotype results in only one phenotype. The *contextuality* of genotypic behaviour is not generally recognised.

Even the complexity of genotype-genotype reactions is rarely acknowledged in ALife. Most complex organisms are diploid (*i.e.* having more than one copy of each bit of information or strand of DNA), and yet artificial organisms are generally haploid (posessing only a single strand of DNA). It is often claimed that the higher order phenomena of evolution can be captured without this added complexity, but the continuing failure of artificial environments to display phenomena such as OEE suggests that this might not be the case.

It is my contention that the contextuality of actual biological systems is missing from artificial life, and that this manifests itself in their consequent lack of complexity.

2.1.3 Measures of ALife Complexity

> *The word information has been given different meanings by various writers in the general field of information theory. It is likely that at least a number of these will prove sufficiently useful in certain applications to deserve further study and permanent recognition. It is hardly to be expected that a single concept of information would satisfactorily account for the numerous possible applications of this general field.*
>
> Shannon, p180, [388]

The problems encountered when attempting to define complexity generally crystallise when an attempt is made to define the complexity of artificially evolved organisms, and to compare this to biologically evolved ones. A wide range of different measures exist, some of which are more useful than others.

Counting Arguments

Often, the complexity of a simulation is understood in terms of simple counting arguments [351, 203]. For example, in the debate [351, 204, 350] surrounding the Ansatz for Dynamical Hierarchies (discussed below in section 2.2.2) one of the key concepts used is one of *object complexity*, which is apparently defined in those papers as a *rule set*; the number of rules necessary to define the system. This is a simple counting argument of complexity, and it is not particularly effective. Emergence is effectively ruled out from such a notion unless it is explicitly incorporated into the modelling at a higher level. See section 2.2.2 for an explicit discussion of some of the problems that can arise from such a counting approach.

Information Theoretic Measures

> *The source and function of genetic information in organisms is different from the source and function of information in physics.*
>
> Pattee, [323]

> *Because organisms are not energetically closed systems, there is no way to deduce the direction, much less the rate, of evolution from classical thermodynamic considerations. All estimates indicate that the amount of entropy, measured in physical units, involved in the formation of a one-celled biological organism is trivially small—about* $-10^{-11} cal/degree$*. The "improbability" of evolution has nothing to do with this quantity of entropy, which is produced by every bacterial cell every generation. The irrelevance of quantity of information, in this sense, to speed of evolution can also be seen from the fact that exactly as much information is required to "copy" a cell through the reproductive process as to produce the first cell through evolution.*
>
> Simon, p192, [400]

Most of the standard information theoretic measures of complexity have been used to estimate the complexity exhibited by ALife models. Although Kolmogorovorian complexity [250] or Algorithmic Information theory [122] is sometimes naively used, most researchers accept that it is not an adequate measure of information in biological systems. This is because this theory sees random sequences as having maximum complexity; by definition a random sequence can have no generating algorithm shorter than simply listing the sequence. As Gell-Mann has pointed out [178], this contradicts the notion that random sequences should contain no information.

Shannon entropy [389] is most commonly used to estimate the complexity of simulations [5, 6]. However, it falls prey to problems of identification; is the genotype or the phenotype the relevant object to be measured? While it is quite straightforward to measure the genotypic entropy, this may not in fact have very much meaning. Consider for example the contextual aspects of DNA that were discussed in section 1.2.1; in fact the genotype has very little meaning without a thourough consideration of the context in which it is found. While this is not such an issue in ALife due to the generally non-contextual nature of genotypic interactions, it is not going to remain so as the drive to artificially implement more and more complex behaviour becomes more pronounced. On the other hand, if a phenotypic analysis is attempted then isolation becomes an issue; given that the phenotype is a consequence of the interaction between the genotype and the environment, it seems likely that an accurate estimation of the complexity of a simulation could only be made with a proper incorporation of the environmental factors. In such a case, the resultant measure would most likely approach the size of the entire system of simulation, a somewhat unsatisfactory result.

A more interesting example is due to Adami [4], who attempts to move between these problems. Adopting a common simplification of the genotype/phenotype map, he uses a string to represent a phenotype. The information content of that string is given by the difference between the maximal Shannon entropy of the string and the entropy given by assuming that the string codes for some phenotype p:

$$I(g) = H(g) - H(g|p) = l - log_{32}N \tag{2.1}$$

where l is the length of the genotype in instructions, and N is the number of genotypes that give rise to the phenotype p. Again, it seems reasonable to expect that as ALife starts to approach the true complexity of biological organisms, the simplification of the genotype/phenotype map used above will become even more unrealistic than it appears at present.

The reason why such measures are even remotely useful in ALife is very closely related to the failure of those models; if ALife simulations were even remotely as complex as required then such straightforward applications of information theory, with its associated

loss of structural and dynamical information would immediately appear nonsensical. One example of this phenomenon is provided by the quotation from Simon appearing at the beginning of this section; if our measures tell us that the same amount of information is required to copy a cell as would be required to produce the first cell then intuition suggests that the measure is in some sense unsatisfactory.

Open Ended Evolution (OEE)

Bedau and Packard, along with a number of coworkers, have proposed a number of statistics which measure the evolutionary behaviour of various systems [55, 56]. These allow us to make quantitative statements about the long term characteristics of evolving systems, hence offering a way in which to classify them. The idea is to identify innovations that make a difference to the evolutionary viability of species. Such innovations are the ones that persist and get used by succeeding generations. Counters are attached to the specific components of a computational run, if the components are passed along during the reproductive process then the associated counters are passed along also. These counters are then used to define the activity counters for various species. The activity a at some time t is defined as

$$a(t) = \sum_{k \leq t} \Delta_i(k), \tag{2.2}$$

where $\Delta_i(t)$ is the activity increment for component i at time k, often defined with a delta function:

$$\Delta_i(t) = \begin{cases} 1 & \text{if component } i \text{ exists at } t \\ 0 & \text{otherwise} \end{cases} \tag{2.3}$$

Various statistics are then proposed, based upon the activity counters. For example, an obvious measure of the evolutionary behaviour of some system is obtained by counting the number of innovative components that it has. In ecosystems this often involves identifying the number of species in the system. This measure is termed diversity and denoted $D(t)$, where the t dependence is intended to specify the time dependence of diversity. Diversity can be defined in terms of the activity counters as

$$D(t) = \#\{i : a_i(t) > 0\} \tag{2.4}$$

where # denotes cardinality of the set. The *total cumulative evolutionary activity* $A_{cum}(t)$ is the sum of the evolutionary activity of all components at some time t:

$$A_{cum}(t) = \sum_i a_i(t). \tag{2.5}$$

Diversity and A_{cum} are then used to define:

Mean Cumulative Activity $\bar{A}_{cum}$, which is the cumulative evolutionary activity divided by the diversity $D(t)$

$$\bar{A}_{cum}(t) = \frac{A_{cum}(t)}{D(t)}. \tag{2.6}$$

This term is often referred to as the "mean activity".

New Evolutionary Activity A_{new} sums the evolutionary activity per component with values between a_0 and a_1, which define a strip through the component activity distribution.

$$A_{new}(t) = \frac{1}{D(t)} \sum_{i, a_0 \leq a_i(t) \leq a_1} a_i(t) \rightarrow \frac{1}{D(t)} \int_{a_0}^{a_1} C(t,a)\ da \tag{2.7}$$

where, the component activity distribution,

$$C(t,a) = \sum_i \delta(a - a_i(t)) \tag{2.8}$$

is just a sum over Dirac delta functions, (equal to one when $a = a_i(t)$ and zero otherwise), and therefore indicates the number of components with activity a at time t. This measures the number of innovations with an activity value a in the range $a_0 \leq a \leq a_1$, which are then considered the bounds at which activity values can be interpreted as having a positive adaptive significance.

These statistics have been used to identify four distinct classes of behaviour exhibited by evolving systems according to whether the above measures are unbounded[2] and positive[3], their characteristics are listed in table 2.1

It has been determined that the fossil record and patent records both exhibit class 4 behaviour, but in contrast, analysis of a number of different ALife models has shown that none of them exhibit this *open ended* behaviour [55, 56, 403]. A vital current problem is to build an ALife model that exhibits Open Ended Evolution (OEE), which is defined as behaviour in class 4.

[2]The function $f(t)$ is unbounded iff

$$t \overset{\lim}{\rightarrow} \infty \left(\frac{sup(f(t))}{t} \right) > 0. \tag{2.9}$$

[3]The function $f(t)$ is positive iff

$$t \overset{\lim}{\rightarrow} \infty \left(\frac{\int_0^t (f(t)dt}{t} \right) > 0. \tag{2.10}$$

Class	$D(t)$	$\bar{A}_{cum}(t)$	$A_{new}(t)$	Description
1	bounded	zero	zero	none
2	bounded	unbounded	none	unbounded, uncreative
3	bounded	bounded	positive	bounded, creative
4	unbounded	positive	positive	unbounded, creative

Table 2.1: The different classes of evolutionary activity, OEE is exhibited by systems in class 4, but no artificial systems have managed to realise such behaviour.

Hierarchical Complexity

One interesting attempt to understand complexity in ALife and biological complexity in general is due to Nehaniv and Rhodes [302]. This work relates complexity to the number of levels of hierarchy needed to build a computational model of, or to understand the given biological system. Postulating five axioms:[4]

1. *Bounded Emergence*: Although complexity can increase via interaction, if this interaction is one way the complexity is bounded.
2. *Noninteraction*: Complexity does not increase if one combines noninteracting components.
3. *Covering*: A part or component of the system does not have complexity exceeding that of the whole.
4. *Constructibility*: Every biological system can be emulated by a network of interacting components which individually have low complexity.
5. *Initial Condition*: Certain simple systems have complexity zero.

They discover a complexity measure $cpx : \mathcal{A} \to N$ satisfying these axioms which gives a larger natural number N for a more complex system. They also postulate that the measure should be maximal in a final sixth axiom in order to achieve some sort of uniqueness to the measure.

Within this definition, evolution is defined as open ended if it achieves unbounded increase in complexity [301]:

> *We say that an evolutionary system E exhibits open-ended evolution if for every integer N there exists a time t such that at time t the system includes an entity e whose complexity is at least N, i.e. $cpx(e) > N$.*
>
> Nehaniv, p1, [301]

While this definition is interesting, it falls prey to the same problems that beset all object-based techniques. Consider for example the covering axiom, while it might at first sight appear to be a reasonable axiom, it is not something that we can expect all systems to satisfy. In particular, if the part is inappropriately separated from the whole, then it is

[4]The original paper [302] is fully axiomatic, for the sake of clarity this discussion will be restricted to the natural language description formulated by Nehaniv in [301].

likely that it will appear to have far more complexity than the entire system. Quark colour confinement (which was discussed in section 1.4) is an example of this phenomenon.

2.1.4 ALife: Simulations versus Emergence

> *. . . attempts to formalize mathematics produced only nongeneric simulacra of mathematics. Moreover, there is no threshold of "enough" to permit the replacement of the real thing by such a simulacrum and no (syntactic, or predicative) way of pasting such simulacra together to recapture the real thing.*
>
> *. . .*
>
> *The study of such simulacra, precisely because they are simulacra, has come to be prefaced by the adjective* artificial*.*
>
> *. . .*
>
> *Today, that history is being repeated under the rubric of artificial life. Again, this finds its basis not so much in science but in mimesis, and the search for enough mimics to cross a threshold into life through software alone.*
>
> Rosen, pp41–42, [365]

Pattee has raised a number of objections to the notion that emergent artificial lifeforms are actually being created by ALife simulations. He claims that ALife, like the field of artificial intelligence, is in danger of confusing the distinction between simulations and realisations of life [324]. He distinguishes between three levels of emergence

1. *A syntactic level of emergence* which describes the new behaviour arising in systems that exhibit phenomena such as symmetry-breaking and chaos [383], catastrophe theory [433], dissipative structures [346], and fractals [276]. Pattee claims that while these phenomena provide a good starting point in attempts to understand the nature of emergence, their fundamentally predictable form of behaviour means that they do not provide examples of true emergence in all of its complexity; these processes do not show the creativity of truly emergent processes. He also asserts that a number of biological phenomena which are commonly termed 'frozen accidents'[5] fall into this category; they are not emergent in any sort of a strong sense since the frozen behaviour that emerges was simply one realised out of a set of possible outcomes, and therefore should not be classified as new behaviour.
2. *A semantic level of emergence* which is associated with high-level symbolic activity, where systems of symbols stand for some referent. Examples of this level include the emergence of DNA as a symbolic representation for some primitive phenotype, and the appearance of cognitive representative abilities, such as language, in primitive societies. Pattee also points to commonly used mathematical techniques such as

[5] Such as the choice during evolution of four bases on the DNA sequence. There is no *a priori* reason to believe that this was a dynamical necessity, it is more likely that it occurred accidentally during the course of evolution. There are many other examples of this type, see [404] for a discussion of some of the most important transitions of this type.

estimation, extrapolation, averaging and induction for further examples. He suggests that this semantic level operates on *existing* data structures which result from completed measurements or observations; no fundamentally new measurements or observations emerge at this level.

3. *Measurement* forms the basis of Pattee's third order of emergence, which he claims is the most important for evolution. A process exhibiting the full potential of emergence will actively evolve new ways of *interacting* with its environment, that is, new ways of measuring its surroundings. For example, a cell that evolves the ability to construct a new enzyme is then capable of interacting with (or measuring) a new aspect of its environment. As another example, consider an animal that discovers a new source of food and then utilizes it, or the situation where in a population of wild chimpanzees one animal learns to utilize a new tool and then teaches the rest of her pack the same trick.

Pattee claims that it is the implementation of new representations and measurements that must be understood in the search to generate new emergent behaviour:

> *Knowledge is potentially useful information* about *something. Information is commonly represented by* symbols. *Symbols* stand for *or are* about *what is represented. Knowledge may be about what we call reality, or it may be about other knowledge. It is the* implementation *of "standing for" and "about" — the process of executing the epistemic cut — that artificial life needs to explore.*
>
> Pattee, [323]

The importance of this form of emergence has also been noted by Baas in his examination of emergence within the context of hyperstructures and hierarchies [43], which shall be discussed in section 2.2.1. At present, we can note that in implementing an epistemic cut, we must make explicit reference to some form of observer; the way in which knowledge is represented will depend upon to whom it must be represented, and therefore, again we see that the epistemic cut is in essence an observer driven theory. This means that if we can incorporate a notion of measurement into our theories then there is hope to generate a notion of observer driven complexity.

Thus, according to Pattee, the most relevant form of emergent dynamics in the field of ALife is the one that leads to the creation of new processes of measurement; new ways in which a system might *interact with its environment* or *within its context.* Unfortunately, there is no theory of this process, and the inability of ALife simulations to display complex emergent behaviour suggests that the two problems may be related. Our insistence upon reduction as an analytic technique is likely to lie behind these problems; in applying reductive analysis, hence separating a system from its environment we effectively freeze our definitions of these concepts. In particular, the process of measurement, or the way

some aspect of the environment interacts with the system is static in the usual reductive techniques; there is no room for new processes of measurement to emerge in our methodology.

Pattee has proposed that an *epistemic cut* separates our knowledge from the thing that it describes; description from construction, the observer from the system, the genotype from the phenotype [323].[6] Such a cut recognises the importance of describing the way in which a world of physical systems might transform into biological evolving entities. According to Pattee, life requires such an epistemic cut, that is, a separation between *symbols* and the thing that they *stand for* (*i.e.* their referrent). He stresses that the process by which this cut is implemented is generally a mystery.

Pattee goes on to examine the differences between a physical and a biological epistemology.

> *The important point is that physical epistemology is a highly evolved and specialized form of the primitive description-construction process. The cognitive role of physical epistemology appears to be far removed from the constructive function of genes, but both define a fundamental epistemic cut. Great discoveries have been made in physics without understanding the mechanisms that actually implement the epistemic cut, because physics does not need to study the epistemic cut itself. Measurement can simply be treated as an irreducible primitive activity. That is why in most sciences the epistemic cut appears sharp — we tend to ignore the details of constructing the measurement devices and record only the results. The reality is that physical theory would remain in a primitive state without complex measuring devices, and in fact most of the financial resources in physics are spent on their construction.*
>
> *Unlike physical theory, great discoveries in the evolution of natural and artificial life are closely related to understanding how the description-construction process can be most efficiently* implemented.
>
> Pattee, [323]

Thus, Pattee draws a distinction between these two types of natural system. Physical systems can often be understood without a proper model of the mechanisms that implement the cut, which explains the early successes of physics; we do not need to understand our measuring devices in order to understand the systems described by physics. On the other hand, in any attempt to understand the evolution of natural and artificial systems it is necessary to examine the details of the cut. In particular, the *implementation* of the epistemic cut where the symbolic information contained in the genotype is mapped to a phenotype must be understood in an investigation of biological systems. ALife must

[6] Pattee traces this notion back to von Neumann's discussion of the measurement problem in quantum mechanics [446], a treatment which is not satisfactory. However, I feel that Pattee's notion of an epistemic cut is both reasonable, and useful, if founded on a theory that is less than satisfactory philosophically.

attempt to understand the epistemic cut to the extent that it can actually implement it, a far greater challenge than that faced by fields that rely upon physical modelling. We might ask to what extent this has been achieved.

In Artificial Life, implementation is achieved via computation, but the available models of computation are severely limited,[7] centering solely around achieving outcomes faster and faster. On the other hand, living organisms rely not just upon speed, but also upon efficiency and reliability during their process of replication. These concepts are not often investigated in ALife models, rather they are assumed to be a certain rate at the outset; they form a part of the high level techniques rather than being incorporated into the modelling at a fundamental level. They are not evolved. Perhaps the most problematic aspect of this is the fact that no known ALife simulations evolve new measuring techniques *i.e.* new ways of interacting with their environment. Instead, channels of measurement interaction are generally incorporated into the lowest level objects, our reductive, object based methodology has caught up with us again. This problem becomes particularly apparent when an attempt is made to artificially build up new layers of hierarchical structure.

2.2 Hierarchical Structure

Many complex systems exhibit hierarchical structure and dynamics.

For example we might consider an organism as made up of molecules, which are combined in special ways as proteins, DNA and RNA, in cells, which in turn form organs. The organism itself is part of a community or species, which may combine to form some sort of social organisation *etc.* The concept of hierarchy is general, and many different ideas and theories have been created in an attempt to incorporate it into our modelling, some of which will be discussed in the next chapter, however, at present there are no satisfactory general theories of this process.

This section will briefly discuss some of the results about hierarchies which have sprung from ALife.

2.2.1 Hyperstructures

Baas has developed a theory of hyperstructures and emergence [43] in which a hierarchy is considered to be a special case of a hyperstructure. First, Baas defines primitive objects or entities, which are termed *first-order structures*. These are denoted S, and form a family:

$$\{S_i^1\}, i \in J \text{ is some index, finite or infinite,} \tag{2.11}$$

[7] This point has also been raised by Brooks in his plenary talk at the Eighth International conference on Artificial Life in Sydney. Video footage of this talk is available at http://complexity.vub.ac.be/∼comdig/Alife8/Brooks.asf.

where the superscript denotes that the structures are first-order in nature. It is reasonable to expect that these structures would exhibit one or more *first-order properties*:

$$P = O^1(S_i) \tag{2.12}$$

dependent upon the observational mechanisms O^1 identified as relevant to the first-order structures. Note that these observational mechanisms may or may not be dynamical in nature. Subjecting the first-order structures to a family of interactions, Int, as allowed by their first-order properties we may obtain new, *second-order structures*:

$$S^2 = R(S_i^1, O^1(S_i), Int^1)_{i \in J} \tag{2.13}$$

where R stands for the result of the construction process. These second-order structures may themselves be subjected to some set of observational mechanisms, O^2 (which may be equal, overlapping or disjoint from O^1), which find a new set of second-order properties. Baas uses these concepts to propose a *definition of first-order emergence*, where P is an emergent property of S^2 iff:

$$P \in O^2(S^2), \text{ but } P \notin O^2(S_{i_1}^1) \text{ for all } i_1. \tag{2.14}$$

These ideas can be naturally extended, with a family of second-order structures forming $\{S_i^2\}$ which may then form (perhaps with the assistance of first-order structures) third-order structures

$$S^3 = R(S_{i^2}^2, O^2, Int^2), \tag{2.15}$$

and so on, with an *N-th order structure* being represented generically by

$$S^N = R(S_{i^{N-1}}^{N-1}, O^{N-1}, Int^{N-1}), \qquad i_{N-1} \in J_{N-1}. \tag{2.16}$$

Within this notion of hyperstructure, Baas understands a hierarchical system to be a sequence of hierarchy levels $X_1, X_2, \ldots, X_N$ and an ordering among the levels $X_1 \ni X_2 \ni , \cdots \ni X_N$ (*i.e.* X_1 contains $X_2 \ldots$ contains X_N). Thus according to Baas hierarchies are:

> *. . . special, but important cases of hyperstructures and occur in most* taxonomic *situations. The levels are described as mere aggregates without specifying interactions and observation mechanisms. We should also keep in mind that this gives a description of a hierarchy in existence* — not of how it was constructed or evolved — *as hyperstructures do.*
>
> Baas [italics added], p529, [43]

Clearly, Baas is defining a very restricted sense of hierarchy, dynamically generated and evolving hierarchies in the Baas understanding would fall into the hyperstructure category.

For the sake of consistency with other literature on hierarchical behaviour, we shall not adopt the Baas definition here, referring instead to hierarchies as the general phenomenon. More clarification of the general concept of hierarchy will follow in section 3.1.

Is the Baas definition of emergence sufficient? ALife VIII saw the presentation of a system that clearly pushed the boundary of what should be considered an interesting hierarchical structure [158] and yet was claimed to be hierarchical in the Baas sense.

This system consists of a triangular lattice, with each cell either occupied or not at some time step. The entire system is updated "simultaneously"[8] at each time step. This update process consists of a movement step followed by a step where the bonding of adjoining triangles is considered. During the movement phase, if the uppermost and leftmost triangle of some aggregate of triangles is selected to move into a neighbouring lattice region, then the entire aggregate is considered to move in the same direction (i.e. as a rigid body). If any movement of an aggregate would lead to the occupation of an already occupied region then it is cancelled. After this movement step, a bonding phase occurs; two separate but neighbouring triangles form a new bonded aggregate with each other with some fixed probability b, and two bonded triangles dissociate with probability d. The bonding/dissociating interactions between the primitive structures are apparently the only ones that occur, with different aggregates forming and breaking up during a run. The authors claim that emergent structures form in this system, and that it creates a nested hierarchy in the Baas sense. Triangles can form larger triangles, which can themselves form larger triangles *etc.* or they can instead form diamonds which form larger structures *etc.* This serves to illustrate the essential vacuity of the Baas definition of hierarchy; hierarchies are just nested structures, and this triangle structure forms just such a system. Since hierarchies form a subcategory of the Baas definition of hyperstructure, this system even forms some sort of hyperstructure, however, the claim of emergence made by the authors is not necessarily correct. The major basis of this claim lies in a notion of number; they claim that new properties emerge since a structure of 4 elements will experience a notion of 'move with 4 elements' whereas one of 15 will experience 'move with 15'. But given that the bonding involved is a primitive association there is no real reason to suppose that any new properties have evolved at all, and a proper consideration of the Baas definition illustrates this; movement of 4 elements involves the movement of, for example, 2 sets of 2 elements, and given the associative nature of the system, it seems reasonable to simply add these low level properties to obtain the required component number. However, there is a rather large amount of subjectivity in these alternative

[8] While the authors claim that updating is performed simultaneously, for the current system there must be an order in which triangles will be examined and allowed to move or required to remain in the same grid-point. This will lead to a time dependency for each update cycle, with those cells updated first experiencing an advantage with respect to any cells updated after them — they will be favourably allowed to move compared to those updated at a later point in time. We shall assume for the sake of discussion that this update process works from left, upper to right, lower on the grid.

definitions of the triangle system, the feeling arises that perhaps the Baas definition of emergence is not as complete as might be hoped. An alternative definition of hierarchical emergence may help to clarify these issues.

An Alternative Definition of Hierarchical Emergence

Groß and Lenaerts have presented a different definition of a dynamical hierarchy [203] which effectively rules out the triangle system discussed above (section 2.2.1). In this case a tentative definition of when a higher order object can be considered emergent in a hierarchy is:

AB is an object of order N, if

- *A, B are of order $< N$,*
- *A or B or both are of order N-1*
- *AB has a property that cannot be found at lower order objects*
- *At least one element of η or ω does not label an interaction in which subcomponents of A or B are engaging.*

Here A and B are some arbitrary agents $\eta = \{$set of all indices that label active types of input interactions between A and B$\}$, and $\omega = \{$set of all indices that label active types of output interactions between A and B$\}$. Obviously this definition is only intended to apply to the dynamical hierarchies that form in systems with well defined input and output interfaces, but the triangle hierarchy proposed by Dorin and McCormak is just such a system. This system does not display any emergent interactions from the combination of the two lower order shapes. There is just a bonding interaction. Therefore, according to the Groß-Leinarts definition, this system does not form any emergent structures above order two and should not be considered as displaying emergent hierarchical behaviour.

While it is relatively easy to examine natural systems and see hierarchies everywhere, an adequate model of these phenomena is not so easy to discover. The next chapter will turn to a more in-depth examination of some existing theories of hierarchical systems, showing that in general they have not captured the true complexity and associated emergent behaviour of these systems. The remaining chapters will start to develop a new theory of emergent, high end complexity.

2.2.2 The Ansatz for Dynamical Hierarchies (ADH)

ALife models do not tend to exhibit more than two levels of hierarchical development [68]. Without the generation of new, more complex levels there will be a highest level of complexity attainable, a point emphasised by the Ansatz for Dynamical Hierarchies (ADH):

> *Given an appropriate simulation framework, an appropriate increase of the object complexity of the primitives is necessary and sufficient for the generation of successively higher-order emergent properties through aggregation.*
>
> Rasmussen *et al.*, p347, [351]

This statement has some rather profound consequences. It limits the overall complexity that might be obtained from any system formed as part of a dynamical hierarchy and thus violates what Rasmussen *et al.* call the 'complex systems dogma' which claims that we can use simple rules and states to generate complex behaviour. If the ansatz is true then our current ALife modelling methodology may not be able to produce dynamical structures of unlimited complexity.

Before we can consider the truth or falsity of the ansatz, we must consider the core concepts of simulation framework and object complexity that it utilises.

Object Complexity: This is the key term of the ansatz, specifying the complexity of the individual objects in the hierarchy. It is important that we are able to define the complexity of the objects contained in a hierarchy in order to make use of, or even to test the ansatz. Unfortunately there is no formalised definition of object complexity.

Simulation Framework: The above rather *ad hoc* definition of object complexity in the ansatz is made possible by the requirement that the simulation framework be given. A simulation framework is provided by the specification of the rule set of the system. Thus the requirement that a simulation framework be given forces the rule set of the system to be fully specified, and to remain constant, throughout an application of the ansatz to some system or set of systems.

Perhaps the most interesting aspect of the ADH comes from the observation that the rule set defining the simulation framework is commonly also used to define object complexity. With this point in mind, we might rephrase the ansatz as:

> *Given an appropriate* rule set*, an appropriate increase of the* rule set *of the primitives is necessary and sufficient for the generation of successively higher-order emergent properties through aggregation.*

This makes the ansatz appear to be rather trivial, a point independently raised by Rasmussen *et al.*:

> *The* ansatz *is in some sense trivial: Assume that we have a minimal rule set that generates a particular dynamical hierarchy but only up to order N. If we stay within this simulation framework, it is necessary to add new rules to generate an additional order $(N + 1)$ of emergence. How can the system generate a new, higher level of behaviour unless something new is added to the elements?*
>
> Rasmussen *et al.*, pp367–368, [350]

It appears there is some sense in which the complexity of any system satisfying the ansatz is built into the lowest level objects [203]. In an agent based model, this means that the most complex objects are those defined as primitive objects, the available interactions are effectively 'used up' as the zeroth order objects interact and form bonds with each other to form new higher order objects. This appears to be rather counter-intuitive. In the natural world, higher order objects often seem to have a higher complexity.

The ansatz has been subject to some debate [204, 350, 158]. One key point of disagreement concerns the possibility of comparing the object complexity of two different hierarchies. The simulation framework restriction reduces the applicability of the ansatz rather dramatically, limiting the result to individual systems. No comparison between different systems is allowed as they are automatically considered to be part of a different simulation framework.

These problems suggest that the ansatz may be the result of some fundamental misconception, which while apparently straightforward is in fact wrong. Our discussion from section 1.4 suggests the nature of this assumption; in fact, the ADH is a reformulation of the problems besetting our object based, reductive modelling methodologies. With an insistence upon primitive objects, themselves relatively simple, we find that there is no room in which new interactions or behaviour *can* emerge. Without new modelling techniques capable of displaying complex forms of emergence it is to be expected that there will be a limit to the amount of complexity that can arise in an artificial system. We see that the ADH is symptomatic of the missing complexity of current ALife models. The interactions of the standard agent based methodology provide a direct example of this problem.

This problem, while less obvious in some of the other ALife environments, still exists. Consider for example Avida [262, 459], one of the most comprehensively developed environments. Avidans evolve within an environment that rewards them for their ability to solve a set of defined problems, but the initial set of problems does not generally change, or increase during this process. The possible interactions between Avidans and their environment is defined *a priori* with the specification of the set of possible problems, and as these are solved no new possible interactions spring into being; the niches of the environment are used up one by one as Avidans evolve to fill them. Certainly, the environment of individual Avidans is also affected by the (biotic) interactions between the Avidans themselves, however, the (abiotic) environment of problems is not likely to be sufficiently complex for either OEE behaviour to occur, or levels upon levels of hierarchical structure to form. Attempts to increase the complexity of the simulation, such as the current drive to introduce diploidy will certainly increase the amount of behaviour displayed by Avida, but until a much higher amount of both biotic and abiotic interaction becomes possible Avida will not display behaviour even approaching the complexity of the natural world.

2.3 Towards the Generation of Interesting Emergent Behaviour

> *Can the emergence of real new properties in complex systems really be explained? If the sciences of complexity offer important new insights, theories, and methodologies for dealing with complex, higher-order phenomena (as we think they do), and if the traditional view of explanation cannot account for the explanatory strategies we find here, we should look for other accounts of scientific explanation. Perhaps the very idea of scientific explanation as a strictly deductive argument should be reinterpreted and explanations seen in a more dynamic and context-dependent setting, eventually themselves being emergent structures, "emergent explanations".*
>
> Baas *et al.*, [44]

The general failure of ALife models to exhibit high end complex behaviour is a direct consequence of our reductive techniques of analysis. With the assumption that the complex behaviour of systems of interest can be broken into a set of components and their interactions comes the corollary that components interacting in some way can generate complex emergent behaviour, but this is not necessarily the case. If, as was argued in the previous chapter, high end complexity is not well understood by reductive analysis then there is every reason to suspect that such a methodology will fail in the field of ALife. A new class of theories, models and simulations is required. The next chapter will discuss hierarchical systems in more detail, finding a similar general failure in our understanding and modelling of dynamically emergent behaviour. Chapter 4 will turn to an examination of a system that has been developed at Flinders University which shows signs of exhibiting a more interesting class of emergent complex behaviour. The remainder of this work will be devoted to discussing the reasons behind this success and attempting to generalise the modelling to other systems.

Chapter 3

Hierarchical Systems

> *In application to the architecture of complex systems, "hierarchy" simply means a set of Chinese boxes of a particular kind. A set of Chinese boxes usually consists of a box enclosing a second box, which, in turn, encloses a third—the recursion continuing as long as the patience of the craftsman holds out.*
>
> Simon, p5, [399]

Hierarchical structure appears to be a fundamental characteristic of complex systems. This chapter will be devoted to an examination of hierarchies, as well as a number of theories about their generation and behaviour. Many of the problems surrounding object based, reductive methodologies, which have been discussed in the previous chapters arise in new guises when we attempt to understand dynamically generated, evolving hierarchies. This chapter will examine these problems in some detail. In the next chapter the discussion will turn to a system which has been developed at Flinders University which appears to side step many of the issues that have been identified in these introductory chapters, thus providing a candidate for a new modelling methodology.

The concept of hierarchy is by no means new to science [398, 310, 456, 320, 31, 376, 372]. Simon is responsible for some of the most widely known discussions of hierarchical systems [400], particularly for his arguments as to why hierarchical structure is more stable and therefore more likely to evolve than a flat structure with a similar number of components. Simon considered two different watchmakers each attempting to assemble complete watches consisting of 10000 components, but faced with a constant stream of interruptions in the guise of phone calls occurring on average at the point where 150 components have been assembled. An interruption causes any set of components that do not form a stable system to fall apart. In order to deal with this problem, the first watchmaker develops an assembly procedure in which each watch is constructed from a collection of 100 stable subassemblies (or modules) each consisting of 100 elements. None of these stable modules will fall apart during a phone call. This modularity of assembly allows the first watchmaker to finish his watch after answering around 11 phone calls, however the second watchmaker has developed no such system and will almost never finish a watch. The collection of modules involved in the more stable assembly procedure

forms what might be termed a second order hierarchy; where a watch consists of 100 stable modules each in turn composed of 100 components, thus forming two levels. From this metaphor Simon suggests that "Among possible complex forms hierarchies are the only ones that have time to evolve," p197, [400].

Thus, hierarchies offer a potential time saving in the generation of complex behaviour which makes the evolution of the complex structures that we see around us more likely; complexity might be generated by the gradual building of stable modules which themselves gradually start to interact and produce more complex modules themselves *etc.* In order for such a process to occur in an open ended way new modes of interaction must be provided by modules at each level in a forming hierarchy, otherwise the existing possibilities for interactions between modules will gradually be used up, and the formation of new structure halted. In such a case the ADH (see section 2.2.2) will apply and new levels of complexity will only be achieved by adding to the complexity of the base objects. However, the way in which such new modes of interaction might emerge is unclear, again as a result of our object based modelling methodology (see section 1.4).

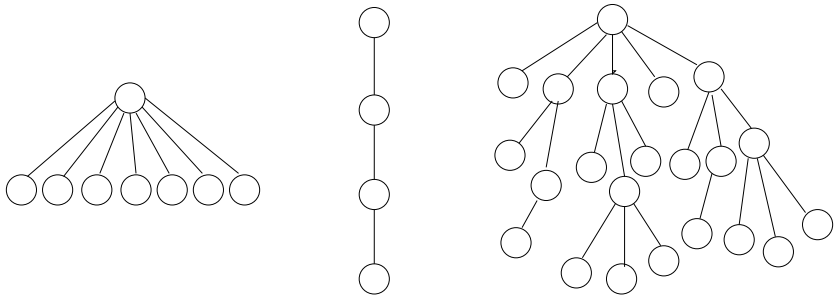
In addition to Simon, a number of other researchers have developed concepts relevant to the description of hierarchies, both structurally and dynamically.

3.1 Hierarchical Concepts

A number of concepts are helpful in the identification, and description of hierarchies. In this section we shall examine some of the more commonly used and foundational concepts in hierarchical systems. The following section will briefly discuss some of the theories of hierarchical systems.

3.1.1 The Alphabet of a Hierarchy

Hierarchical systems commonly have a set of foundational elements, or *primitive objects*, often termed an *alphabet*, which forms a natural basis of the hierarchy. This set usually consists of a smaller number of elements than any other level in the hierarchy. For example, the Roman alphabet consists of (including punctuation) around 30 letters, and yet at the next level up, we see that it can form a vast number of words, which can be used in an almost infinite number of legal combinations, in many different languages. The periodic table provides another example; under one hundred atoms can be used to form innumerable molecules, and even these atoms can be explained in terms of their constituent electrons, neutrons and protons *etc.* A similar phenomenon occurs in biology, we know that a set of four nucleic acids can form around twenty amino acids which can in turn form an enormous number of proteins. Simon has examined the importance of alphabet in biology [399], making the observation that proteins are far too specific to their functions in particular organisms to be satisfactorily exchanged between organisms. Amino acids on the other hand, can come from many different sources and can be used



(a) (b) (c)

Figure 3.1: The span of a hierarchy, (a) illustrates a system with a very high span, whereas (b) illustrates one with a very low one. Although both systems could be regarded as hierarchical in nature, consisting of components which themselves consist of subcomponents, it is not usual to consider such structures hierarchical. In contrast, (c) illustrates a hierarchical structure which contains a number of elements at all levels, each level consisting of a moderate span.

in many different ways. We see a similar increase in flexibility for each of the hierarchical systems mentioned above as alphabetical components are used in the exchange of resources. This greatly assists in the transfer of what we might term *information* from one system to another — a necessity when we consider the openness of living systems.

3.1.2 Span

A hierarchy is divided into a number of components, or subsystems, each of which may themselves be subdivided. The way in which this subdivision occurs can vary and plays a part in whether or not the system is considered hierarchical. Consider for example two different situations, one of a solid crystal (the structure of which is illustrated in figure 3.1(a)) and the other a set of Russian dolls (figure 3.1(b)). A crystal consists of a number of atoms, each of which consists of protons, neutrons and electrons. At the crystal level, there can be almost any number of constituent atoms; at this level, the crystal may have a very large *span*. Alternatively, we might consider the Russian dolls, one inside the other *etc.* which has a very shallow span, each component itself contains only one element, and hence it would not normally be considered as forming a hierarchy.[1] While such systems *could* be considered hierarchical, each consisting of subsystems embedded within a system, we would not generally *choose* to do so. Instead, we tend to consider systems which have a reasonable number of levels, each containing a moderate span, hierarchical. Consider for example the traditional structure of a company, where at each level of control, a number of of subordinates report to a higher ranking superior.

[1]Despite Simon's obvious identification as Russian dolls as forming a hierarchical structure in the quotation at the beginning of this chapter.

3.1.3 Dynamical Rates and Strength of Interaction

There is a correlation between hierarchical levels, their dynamical rates, and the strength at which their components interact [310, 376]. Components at a lower level in a hierarchy tend to experience stronger interactions on a faster timescale than those at a higher level. Some authors actually prefer differences in rate as a definition of hierarchical structure, with the relevant components of some hierarchy being identified with reference to rates associated with them:

> *. . . tangible boundaries are only a special case of boundaries defined on a strict rate criterion. Consider, for example, a thermistor moved about in the body of a homeothermic mammal. We can use the temperature as an indicator of the rates at which metabolic processes are occurring. As we move the thermistor from the core out to the skin, the rate processes change gradually. At the surface of the skin, there is a rapid change in temperature (i.e., a steep gradient in the rate processes). We can use this gradient to define the surface of the organism.*
>
> *. . .*
>
> *If we maintain that components in an ecological system must be tangible in their own right, we have difficulty solving some functional problems of interest in ecosystem analysis. But, if we use differences in rate as our criterion, then we can include both tangible and intangible components.*
>
> O'Neill *et al.*, pp88–89, [310]

This is an interesting response to the problems with object based modelling that were discussed in section 1.4.

3.1.4 Nearly-Decomposable Systems

A *nearly-decomposable* system is one for which only aggregate properties of the parts (or modules) are relevant to the description of how those parts interact [400]. If such behaviour is evident, then it becomes possible to identify as modules those components that are largely independent of the other parts of the system, and to examine both their short-run behaviour (intermodule), and their long-run (intramodule) behaviour. This will become clearer with an example provided by Simon [400], who considers the thermal behaviour of a building where outside walls provide perfect thermal insulation from the external environment. Inside, the building is divided into rooms whose walls are good, but not perfect insulators, with each room being divided into cubicles by partitions that are poor insulators. Inside each cubicle is a thermometer. Supposing that initially there is a high variation in temperature from cubicle to cubicle, room to room, what can we surmise about the temperature variations of the different parts of this system during the next few days? After a number of hours, we will find that the temperature variations between cubicles are minimal, but that cubicles in different rooms vary in their temperature.

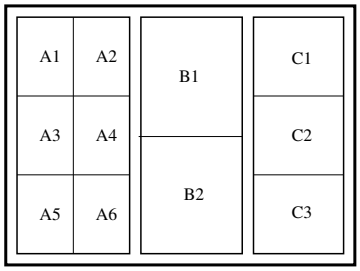


Figure 3.2: An example of a nearly-decomposable system. A building contains a set of rooms, denoted alphabetically, and cubicles within those rooms, denoted numerically. The thermal transfer between the rooms and cubicles is described in the text.

Eventually, the system will reach an equilibrium with the temperature differences from room to room, and cubicle to cubicle, having become minimal. We construct a matrix representation of its dynamics as follows. Describing the process in which the system comes to equilibrium using a standard set of heat flow equations, we would denote the i-th cubicle i and the j-th cubicle j, with the rate of heat flow between these two cubicles, r_{ij}. Referring to figure 3.2 as an example, we see that if the two cubicles are separated only by a partition then the heat transfer coefficient r_{ij} will be large, if the two cubicles are not in the same room, and are not in two adjoining rooms then r_{ij} will be zero. Finally, if i and j are not in the same room, but are in rooms that adjoin each other then r_{ij} will be small, but non zero. Representing the pattern of heat flow coefficients in a matrix, we see that the system takes a nearly block diagonal form, with a set of matrices with large entries on the diagonal. For the model system illustrated in figure 3.2 we get a matrix of the following form:[2]

	$A1$	$A2$	$A3$	$A4$	$A5$	$A6$	$B1$	$B2$	$C1$	$C2$	$C3$
$A1$	–	1	1	1	0	0	0	0	0	0	0
$A2$	1	–	1	1	0	0	.1	0	0	0	0
$A3$	1	1	–	1	1	1	0	0	0	0	0
$A4$	1	1	1	–	1	1	.05	.05	0	0	0
$A5$	0	0	1	1	–	1	0	0	0	0	0
$A6$	0	0	1	1	1	–	0	.1	0	0	0
$B1$	0	.1	0	.05	0	0	–	1	.1	.05	0
$B2$	0	0	0	.05	0	.1	1	–	0	.05	.1
$C1$	0	0	0	0	0	0	.1	.05	–	1	0
$C2$	0	0	0	0	0	0	.05	.05	1	–	1
$C3$	0	0	0	0	0	0	0	.1	0	1	–

(3.1)

[2]The numbers used in this equation are intended to represent some form of percentage of heat flow, their actual values are not relevant, rather their relative sizes.

We thus see a concrete way in which a near-decomposable system can be identified, if it can be represented in matrix form.[3] As Simon has pointed out, if there exists some small number ϵ, as the upper bound for the off-diagonal elements, then a matrix that can be written in this manner represents a nearly-decomposable system. Thus, the concept of nearly-decomposable systems is nearly equivalent with a well understood mathematical concept; block diagonal systems. Within this definition, the modules of the system are clearly identifiable as those subsystems that are responsible for the submatrices weakly interacting via the off-diagonal components. While this definition is far clearer than the earlier (and more widely discussed in the literature) concept of aggregate properties, its somewhat limited scope of application makes it inadequate (block diagonal representations cannot always be identified in hierarchical structures). We might consider it as indicating a direction in which future work on a theory of hierarchies might proceed, but it will be necessary to consider some alternative representations and definitions of both modularity and near-decomposability in order to develop such a theory.

We might consider a second example. In Simon's watchmaker metaphor, the subcomponents used in the assembly of the watch only interact with each other via the process of assembly, *once they are assembled*. Prior to the process of assembly each component has a separate existence, and once the submodules of the watch are assembled they too have a separate, stable existence. This is a very simple form of interaction. Very few hierarchical systems exhibit such a level by level independence. Consider for example the subcomponents of a biological hierarchy, with DNA, cells, tissues, organs, organisms, and even communities and species forming ecosystems. Only at the descriptive level of the organisms in the hierarchy do we arrive at a sense of components with some sort of stable existence, and yet not even these components can function in isolation. We might ask if the modules in the watchmaker example are perhaps more separable than Simon supposes. This problem is highlighted if we return to the definition of a near-decomposable system as one for which only aggregate properties of the parts (or modules) are relevant to the description of how those parts interact. Here we see an interesting contrast between the cubicle example and the watchmaker example. In Simon's heat transfer example the warmth of a room in the short run depends only upon the warmth of the individual cubicles but not of the adjoining rooms, and in the long run the warmth of a room depends upon the aggregate warmth of all rooms. Such behaviour is not exhibited by Simon's watchmaker example. By his own definition the watches do not appear to be near-decomposable.

Watson has raised similar concerns, claiming that the watchmaker example is in fact

[3]Not all hierarchical systems have an obvious representation in matrix form, the system discussed above is very simple, with only one form of interaction. Consider as a counter-example an attempt to represent the hierarchical interactions occurring in the human body, something which is not likely to be possible, and would be difficult to interpret if it were.

a fully decomposable, rather than a near-decomposable system [447, 448]. He claims that "the problem of finding the correct assembly for each module is entirely separable from the configuration of every other module," p85, [448]. Thus, the organisation of the watches is entirely separable. We see here one of the principle problems with near-decomposable systems; they are very difficult to correctly identify. Simon himself seems to make a mistake in formulating his watchmaker example. Watson has provided a formalism that highlights this, and makes the identification of nearly-decomposable systems simpler, at least within a certain subset of behaviour. He defines a measure that makes it possible to extract quantitative information about the independence of a module given some property of interest [447]. The measure works as follows, given some set of possible configurations C for a module in some system of interest, we might be interested in finding out which ones are the most stable. Denoting the number of configurations satisfying the property as C', a set of relationships between these two numbers exists:

If $C' = C$, the system is *non-decomposable*.

If $C' < C$, the system is *decomposable*.

If $C > C' = 1$, a special case of decomposable behaviour occurs, the configuration of interest C' is always the same regardless of the number of configurations of the system itself, it is *separable*.

If $1 < C' < C$, the system is decomposable but not separable, it exhibits *modular interdependency*.

This is an interesting classification, which exhibits the usefulness of the concept, certainly it helps to understand the idea of near-decomposability, as a concept occurring where $C' \lesssim C$. However, it must be asked whether the necessary configurations can be identified in any realistic setting. While Watson's definition is interesting, and clarifies the status of separability as opposed near-decomposability it appears to be more of a toy definition, suitable for the investigation of very special cases, rather than real world near-decomposability. All of the systems he examines using this concept are essentially toy models. It is likely that more realistic systems will have a number of relevant configurations, all interacting within the system in a highly nontrivial manner.

3.1.5 Scalar and Specification Hierarchies

Salthe has intensively investigated hierarchies over the years and developed a number of what might be termed epistemological theories about their formation [376, 371, 372, 373, 374, 375]. Of particular interest, he claims that any natural system can be analysed from the point of view of both a *scalar hierarchy*, and a *specification hierarchy*, with different understandings of the system resulting from each perspective [372].

Scalar hierarchies [376] are extensional. In this understanding of a system, complexity results from the structure of the system, in the form of dynamical changes in constraint relations between levels. Each level is represented in the structural form

[higher level [lower level]], that is, the more specialised structures appear further to the outside of the representation as in *e.g.* [organism [cell [macromolecule]]]. This structure describes a relationship of parts and wholes. Salthe has quite reasonably claimed that three adjacent levels of description, a *triadic system*, are necessary and sufficient for the description of a nontrivial scalar hierarchy [376]. He points to the necessity that a adequate description of complex behaviour consist of the system, its environment and its components, and that the constraints between these levels must be well understood. According to Salthe, scalar hierarchies also exhibit different rates of interaction (as was discussed in section 3.1.3), and grow by interpolating new levels between existing ones (as in the growth of a organism from a fertilised egg). The different rates of interaction imply that the constraints are indirect, the dynamics between levels are effectively 'screened off' by the differing time scales.

In contrast, specification hierarchies involve an intensional understanding of complexity, with higher levels in the hierarchy forming a more specialised role. Such a representation is found in the common description of the sciences as forming specialised, emergent descriptions of more specific phenomena [371]. Thus, the lower level descriptions are seen to encompass (and apply at least in principle) to the higher levels *e.g.* {physical theory {chemical theory {biological theory {social theory {*etc.*}}}}}. This representation generalises as {lower level {higher level}}, with the more internal levels representing the increasing specialisation of the system of interest. Instead of the parts and wholes of scalar hierarchies, specification hierarchies describe classes and subclasses at the different levels of the hierarchy. New levels are added at the top of specification hierarchies, as more organised behaviour evolves, it becomes possible to add new constraints and elicit new behaviour from the system. Interestingly, what is true at the most general level of a specification hierarchy continues to be true at the more specialised level, which means that these hierarchies have a transitive form; biological systems must still satisfy the laws of physics. However, this is not the case with scalar hierarchies, parts and wholes do not all share the 'most general' characteristic; an organism is not a macromolecule, rather it is made from collections of them organised in some form.

More details of these two different representations can be found in Salthe's more recent book [372], where general rules for each form of hierarchy are defined, and [371], which examines specification hierarchies in some detail. The differences according to Salthe between the two forms of hierarchy are summarised in table 3.1 which can be found in [371].

As Salthe himself points out, "any natural system could be analyzed from either scalar or specification standpoints." p36 [372], which suggests that, while each framework can prove useful in different settings as a different mode of analysis, there is a deeper ontological significance that causes this appearance of hierarchical structure around us; such

Scalar Hierarchy	Specification Hierarchy
Parts nested within wholes	Nested classes representing emergent orders
System of extensive constraint relations	System of semiotic/logical orders parallels system of intensive constraint relations
Higher and lower levels based on scale	Inner and outer integrative levels based on degree of specification
Essentially synchronic	Diachronic in spirit, implying developmental emergence of innermost levels
Process, events	Canonical sequence of development from general to specific
Constitutive relationships across levels non-transitive, transitive perturbations cause disruptions	Change epigenetic, always irreversible
Intensity of transitive perturbations between levels drops off with distance between levels	Relationship between levels fully transitive
System description requires minimally three levels	System description essentially two leveled
Unbounded top and bottom	Truncated at both ends
Objective	Subjective relation of observer to innermost levels

Table 3.1: The differences between a scalar and a specification hierarchy as defined by Salthe.

structures actually exist.[4] This work is concerned more with the ontology of the natural world, that is, with questions of the form: how can complex hierarchical structures arise? rather than questions of how best to model particular structures. While Salthe's work certainly sheds much light on the nature of such structures it is felt that this is something of a blind alley. Consider for example an attempt to describe an organism hierarchically. Either of Salthe's frameworks can be utilised effectively in understand the system; a specification approach describing the physical dynamics of its atoms, their chemical reactions and the resulting biological processes is possible, but so is a scalar approach, where the organism is described as consisting of organs, cells and macromolecules. This work adopts the stance that this is due to the actual hierarchical nature of the organism, and attempts to understand how this structure could dynamically evolve. Similar approaches can be taken for all natural systems, rather than getting 'bogged down' in the classification of such systems we must attempt to understand their dynamical nature.

We shall return to this point shortly, but should first look at another set of classifications which has largely been developed by Pattee.

3.1.6 Structural and Control Hierarchies

Structural hierarchies are those that can be identified by the imposition of a set of part-whole relationships upon some system; they arise from a partial ordering of some system which makes it identifiable as a hierarchy. This is an essentially static separation, the modules and submodules formed by this imposition do not interact in a dynamical way to form new structures. The structure of the hierarchy is identified as part of an externally imposed organisational technique rather than as part of a natural process. In Simon's Chinese boxes example, quoted at the beginning of this chapter, we can identify a set of boxes, each one placed within the other in a partial ordering (the property of being inside); the boxes form a structural hierarchy.

A more interesting example can be found in an examination of the physical structure and formation of matter, which as has already been described, claims that matter is composed of a number of different molecules in a variety of different arrangements, which are in turn formed from a set of atoms, each made up from a number of different elementary particles, as well as composite particles themselves formed from differing combinations of quarks and gluons.

Pattee has noted that at each level of a structural hierarchy we often use a set of approximations

[4] While it is philosophically impossible to assert that a world exists 'out there' [369], it is highly likely that such a realistic stance is correct, and as such, it seems fair to suppose that hierarchical structures actually exist in that world. As a physics thesis, this work shall not enter into such a debate, rather we work from the hypothesis a physical world exists 'out there', even if it is not necessarily independent of our actions (*i.e.* complete objectivity is lost).

> *... that one particle is typical or representative of the collection, that the fast motions one level down are averaged out, and that the slow motions one level up are constant. ... The simplicity and solvability of most physical equations depend on making these approximations.*
>
> Pattee, p77, [320]

Such a situation is well exemplified by statistical mechanics, ecological hierarchies, biological hierarchies *etc.* In fact, it is a reapplication of the principle, discussed in section 3.1.3, that the rate of interaction between hierarchical components at a lower level is often faster than that at a high level. Pattee notes that this approximate treatment of the dynamics amounts to what Simon calls near-decomposability, and that it is usually possible to ignore the interface between levels in such a structural hierarchy.[5] He then identifies a subset of hierarchies that do not have this property, *control hierarchies*, in which upper levels of the hierarchy have an authority relation over the elements in the lower levels [320].

In control hierarchies these approximations are not appropriate, instead, Pattee claims that it is not possible to ignore the specifics of the lower levels due to the dynamic constraints exerted by the upper levels upon their dynamics. He points to the development of multicellular organisms as an example, where

> *... the cells do not simply aggregate to form the individual, as atoms aggregate to form crystals. There are chemical messages from the collections of cells that constrain the detailed genetic expression of individual cells that make up the collection. Although each cell began as an autonomous, "typical" unit with its own rules of replication and growth, in the collection each cell finds additional selective rules imposed on it by the collection, which causes its differentiation.*
>
> Pattee, p77, [320]

He notes from this, that the interesting problem of understanding hierarchical control consists of explaining how this new extraordinary authority arises in ordinary molecules. With this problem we find that instead of being satisfied with a set of theories describing each hierarchical level, we must now pay more attention to the way in which the interface between these levels behaves.

3.1.7 Dynamical Hierarchies

Both structural and control hierarchies provide a snapshot of the behaviour of some system at some given point in time. As such, theories of these phenomena do not attempt to describe the way in which such systems arise, rather their dynamics and constraints given

[5] Clearly, the notions of structural and scalar hierarchies are essentially identical.

their existence. A dynamical description of the way in which a hierarchy arises offers a possibility to develop an understanding of the interface between hierarchical levels.

An example will help to clarify these concepts. Consider the current developmental understanding of the way in which an insect grows dynamically from the complex interactions of an initially small number of elements whose interactions relay and expand upon small asymmetries within the egg cell (oocyte). Within the *Drosophila melanogaster* (vinegar fly) oocyte, and subsequently the embryo, four genetic cascades (*i.e.* processes which act in a dynamical way to generate a hierarchy of positional information) act to subdivide the embryonic anlage into specific tissue and organ progenitor regions. Initiation of the cascades results directly from the mechanics of oocyte production and the asymmetries involved in this process, together with the laying down of only four gene products in a spatially defined manner. These define anterior (head), posterior (tail), terminal (distance in from the anterior and posterior ends) and dorso-ventral (back-front) position within the oocyte and subsequent embryo. Through gene regulatory interactions between members of genetic cascades, the four initial products help establish tight localisations of regulatory molecules which specify cell identities. Strong interactions within and weaker interactions between cascades thus result directly in the specification of tissue types and subsequent organogenic development leading to a complete functioning body. The above considerations are dynamical, they describe the process by which the hierarchical structure of an organism composed of organs *etc.* arises. Once it has formed, we may examine the body of the fly and determine that it is composed of a set of organs, in turn composed of cells and subcomponents, such an examination is structural — we are imposing a set of part-whole relationships upon the system and examining their form. Also, in this particular case the regulatory interactions and their cascades of genetic responses might be considered to form a control hierarchy. The system itself can be examined in any one of these three different modes, but with a key difference. The dynamical mode provides an indication of the way in which the system is generated and hence the interactions between different stages of the process of development and with this the hierarchical interfaces of the system. In contrast, the structural modes of examination tend to focus upon existing systems, and then a set of disjointed theories are used to explain the origin of the components of the system.

There is no exact definition of a dynamical hierarchy [261], or of hierarchical structure in general. However, there are a number of well-accepted concepts associated with hierarchy (such as those discussed in the present section), and the extension to dynamical should not be too difficult; a dynamical hierarchy is simply one that arises spontaneously from an apparently simple structure, and evolves in time according to some set of principles.

In contrast to the essentially static hierarchical systems proposed by Pattee, the scalar and specification hierarchies discussed in section 3.1.5 can encompass the notion of constraint deployment, and it it is consequently expected that these notions are far more

closely aligned to the concept of dynamical hierarchy. However, as discussed in that section, it is believed that some form of more ontologically committed theory of hierarchical growth is necessary. Once such a theory exists, it will be interesting to compare it with the concepts developed by Salthe.

3.1.8 Downward Causation

In contrast to the reductionistic claim that the behaviour of higher levels in a hierarchy can be explained entirely with reference to the dynamics of the lowest level elements, downward causation claims that the upper levels of some hierarchical structures can directly affect the dynamics of the lowest level elements [109, 110, 33, 71, 167]. Many different forms of downward causation have been identified [167], depending upon how strongly their proponent rejects reductionism. Even in its weakest form downward causation implies that the higher level is conceived as an organizational level, it characterises the pattern or structure or form into which the constituents are arranged. Therefore, downward causation directly suggests that causes can propagate both up and down in a hierarchical structure. Concrete examples of this phenomenon will be discussed in the next section.

3.1.9 The Context of a Hierarchy

> *. . . every event, every statement about such an event, requires, in particular, other events at a higher level of organization, another statement referring to a higher level of organization, to make it complete or to frame it — to give it a context that will allow us to understand it or to judge its truth.*
>
> Salthe, p*viii*, [376]

Hierarchical systems provide an intrinsic context for their lower level components. Consider for example the hierarchical structure presented by a human body. Each of us is composed of body wide systems (such as circulatory, nervous, and immune), themselves composed of organs which are made from cells, which can again be thought of as composed of subunits. The body provides a context for all of these components; a person experiencing an attack of HIV AIDS and therefore a compromised immune system, will gradually experience a wide range of problems associated with different systems, organs *etc.* Similarly, an ecosystem provides a context for all of the plants, animals, bacteria *etc.* that comprise it. The health of the ecosystem can profoundly affect the associated health of its member organisms; consider the current problems besetting the world's oceans, such as the problems currently being experienced by coral reefs which have been bleaching and dyeing due to increased oceanic temperatures, and the associated loss of fish species as they lose the niches provided by the coral. Conversely, keystone species are those upon which ecosystems depend more than might be expected from their relative abundance

[314]; if they are lost, then all species associated with that ecosystem are very much in danger of extinction.

Thus, the lower levels of a hierarchical system can be strongly affected by the upper ones (their context), and can in turn affect those upper levels in a feedback-type situation. If we consider the examples of contextual complexity that have been discussed so far in this work then the contextual interdependency between levels of a hierarchy is made stronger still:

Phenotypic plasticity provides a good example of the fact that it is possible to represent contextual systems hierarchically. Simplistically, the environment can be seen to contain the resultant phenotype, which itself contains the genes that were relevant to the realisation of the phenotype. However, the causal relationships between the components of the hierarchy are much more interesting. Clearly, the phenotype depends upon the genotype, but it also depends upon the environment; we see examples of both upward and downward causation respectively. There is also a feedback relationship between the phenotype and the environment, as the phenotype contributes to the biotic component of the environment which can cause a change in the context of all phenotypes over time.

Fitness applies to an organism within the context of an environment, hence fitness can be represented by a 2-tier hierarchical structure. Since fitness itself is set by both biotic and abiotic factors, we see that relationships again take both upward and downward forms in the hierarchical structure.

Cloning exhibits the same hierarchical contextuality; DNA is placed within a cell, but again we see causal relationships extending both up and down through this hierarchy. If the DNA is placed in the wrong form of cell then the system will not be viable, but in the appropriate cell type the DNA will strongly affect the organism that is obtained.

Similar considerations can be applied to all apparently contextual systems, at the very least a two tier hierarchical structure can be created with the context containing the system of interest. One interesting point arises from consideration of the above examples, in each case there is both upward and downward causation at work, that is there is an inherent feedback in the creation of the context itself which depends partly upon the lower level components of the system. Not all hierarchical systems have this level of complexity in their relationships. For example, the control hierarchy of an army is very much top down, in principle no causal mechanisms can propagate from the soldiers up to the general.[6] This control structure is far simpler than the examples considered above, and would not normally be considered contextual. Is it possible that as with complexity, there is some sort of hierarchy scale?

[6] Although this is not always the case in practice, consider reviews, mutinies *etc.*

At present, it seems likely that hierarchies provide a way in which we might start to understand contextual systems, if we can understand the hierarchical structures themselves. However, given our poor general theoretical understanding of hierarchical systems is perhaps more likely that an understanding of contextuality will be developed concurrently with our understanding of hierarchical systems.

3.1.10 Evolutionary Transitions

In biological evolution we see a number of situations that appear to challenge the Darwinian explanation of evolution as consisting of the gradual accumulation of small changes. Consider for example the origin of eukaryotic from the original prokaryotic cellular structure. Prokaryotic cells have a relatively simple internal structure; a cell wall that encloses a plasma membrane which itself encloses a cytoplasmic compartment that contains DNA, RNA, proteins and a number of small molecules. In contrast, eukaryotic cells have a far more complex structure. By definition they consist of a nucleus which contains most of the cell's DNA enclosed by a membrane, this membrane keeps the DNA separate from the cytoplasm (the rest of the cell) where most of the metabolic reactions of the cell occur. The cytoplasm contains a number of structures, or organelles such as the chloroplasts and mitochondria.

A number of authors have drawn attention to the gradual increase of complexity that accompanies evolution [404, 405, 216, 438, 429, 337, 138], pointing to a concept of *evolutionary transitions*, where more complex behaviour, function or structure emerges during an evolutionary process.

Maynard Smith and Szathmáry have extended the general concept of evolutionary transitions to the stronger one of *major transitions* [404, 405], which are defined as events where previously reproductively independent entities get organized during a transition into an encompassing system which can only replicate as a whole. Within this framework they consider problems such as the origin of chromosomes, eukaryotes, sex, multicellular organisms and social groups, explaining each as a major evolutionary transition. If complexity is defined with respect to the independence that is lost during the transition, then a major increase in the complexity of the system occurs at each one of these transitions. Returning to the example of the evolution of eukaryote from prokaryotes, this transition involves the union of a number of previously free entities into a new whole. The originally independent ancestors of chloroplasts and mitochondria at some point became incapable of replication outside of their host cells, as they are today.

The concept of major evolutionary transitions is analogous to hierarchy formation; a set of simpler entities are interacting in some way to create new higher-level complexes. And indeed, after a transition, originally independent components will have formed a nearly-decomposable system (see section 3.1.4) due to their new found reproductive dependence. Thus, major evolutionary transitions provide a very interesting mechanism

by which concepts such as near decomposability might be created; before the transition the components of interest are separate, however, after the transition has occurred they cannot be so simply analysed. Unfortunately the analysis of these transitions is in general very specific, and has not been generalised. While this concept will undoubtedly remain useful, it requires more work to become a theory of hierarchical development and behaviour.

3.2 Theories of Hierarchical Behaviour

Although many different theories have been proposed which have some relevance to the generation of, or the explanation of the behaviour resulting from hierarchical structure [270, 356, 320], few have any general applicability. This section will consider some of the most relevant theories that have caught the attention of the author. By no means are the theories discussed here the only ones relevant to hierarchies, many different resources can be found by means of a thourough literature search.

Pattee has been instrumental in developing the idea of *constraints* in hierarchies [321]. A constraint is a forcible limitation of freedom, thus, the way in which an upper level of a hierarchy affects the behaviour of its sublevel can be understood as just such a constraint. Pattee claims that a hierarchical constraint requires an alternative description from the lower level elements that it governs; the same concept can be seen in many physical models. For example the move to statistical mechanics from a standard mechanical description involves the application of a constraint:

> *. . . the physicist's idea of constraint is not a microscopic concept. The forces of constraint to a physicist are unavoidably associated with a new hierarchical level of* description*. Whenever a physicist adds an equation of constraint to the equations of motion, he is really writing in two languages at the same time. . . . forces of constraint are not the detailed forces of individual particles, but forces from collections of particles or in some cases from single units averaged over time.*
>
> Pattee, pp85–86, [321]

This is very closely related to the idea of downward causation that was discussed in section 3.1.8, however, Pattee himself points out that the way in which constraints arise is not understood (even in physics) [321]. Thus, while constraints and downward causation are definitely appropriate concepts for the discussion and modelling of hierarchies, more work must be carried out before they can be understood in a general theory. Again we note that this is a very similar concept to that provided by Rosen; in adding an equation of constraint and writing in two languages we are effectively ruling out the possibility of describing some system using one model alone. We may not have made explicit reference to the higher level model but it exists, and is necessary for a proper description of the dynamics of the system.

Collier has claimed that there must be a relation between the levels of a dynamical hierarchy which is both transitive and asymmetric [134], however, his definition is too general; according to his definition any two hierarchies with the same relational structure are identical even if they are dynamically distinct. This leaves a suspicion that in fact Collier is talking about a structural hierarchy as was discussed in section 3.1.6. It would be preferable to incorporate the dynamical evolution of a hierarchy into its very definition.

The Renormalisation Group (RG) is often used in the description of allegedly complex hierarchical structure [415]. Low level constituents of the hierarchy are progressively grouped (normally spatially), their collective behaviour is set to one value for the new cell, and then the lattice of behaviour is shrunk down to the original size. This procedure can be repeated for as many scales of behaviour as can be identified, however, at each level the structural information between levels is lost, and this is precisely the characteristic in which we are interested in our attempt to understand complex hierarchical structures. It is unlikely that the RG can be applied in any situation but scale free networks, which generally consist of the same elements (*e.g.* avalanches) at all levels. Therefore, despite the current interest in this technique, it is unlikely to be of interest in our search for a theory of complex hierarchical behaviour. Some of the more likely candidates for such a theory will now be discussed.

3.2.1 Hierarchies and Open Dissipative Systems

O'Neill *et al.* have presented the case that

> *... all complex systems, including ecosystems, appear to be hierarchically structured as a natural consequence of evolutionary processes operating on thermodynamically open, dissipative systems.*
>
> O'Neill *et al.*, p101, [310]

As has been shown by the work of Onsager [311] and Prigogine [347, 348], dissipative systems satisfy the *minimum dissipation principle* which claims that at steady state the entropy production of a dissipative system is a local minimum; this state can be maintained by a smaller input of energy than neighbouring states. This local decrease of entropy production has been recognised as essential for the existence of living systems [382, 291, 444] which require a constant flow of energy to maintain their high level of internal order.

O'Neill *et al.* claim that the process of increase in organization necessary to eventually generate living structures requires a constant creation of quasi-stable structures which can in turn be used as building blocks for higher levels of organization. They point to the concept of *stratified stability* introduced by Bronowski [85] as an example of this behaviour, which is really a process of hierarchy generation. This generation of hierarchical structure can be understood within the minimum dissipation principle. During the process of evolution, if a new structure emerges which utilizes its constituents more efficiently, then

it is likely that it would replace its competitors, thus increasing the total energy dissipation of the system. Each new more structured and therefore more complex state, in its turn would have made the system vulnerable to new fluctuations which as new structures are developed to take advantage of them lead to the generation of new fluctuation induced instabilities. This is the fluctuation-dissipation sequence proposed by Prigogine *et al.* [347, 348], a process of feedback which, it has been suggested, is necessary for the eventual generation of life.

3.2.2 Hierarchies and Emergence

A number of researchers have drawn a connection between the concepts of hierarchy and emergence, and developed theories of complexity and emergence based upon the concept of hierarchy [376, 43, 302], but none of these develop understanding of the actual generation of hierarchical structure. The question of how levels of structure might spontaneously emerge is a largely unanswered one. We shall return to this in section 6.1, where a new methodology will be proposed which, it will be claimed, is more capable of dynamically generating such emergent structure.

Hyperstructures

The Baas definition of a hyperstructure [43] has already been discussed in section 2.2.1. Despite the difficulties in the application of the Baas definition of hierarchy (which forms a subcategory of hyperstructure) that were mentioned in that section, the hyperstructure concept remains one of the most comprehensive theories of hierarchies/hyperstructures developed to date.

Baas has defined two conceptions of emergence with respect to his formalism (which was discussed in section 2.2.1):

Deducible, or Computational Emergence occurs for systems for which there is a deducible or computational process or theory which determines the emergent properties $P \in Obs^2(S^2)$ from the first order set $(S_i^1, O^1(S_i), Int^1)$.

Observational Emergence occurs when the emergent properties P of a system cannot be deduced computationally.

This is a very useful specification, if some property is computationally emergent then it is possible to reduce it to the lower level description, however, observational emergence gives rise to a new description, one that is not reducible. This is very similar to Pattee's notion of constraint as leading to a new level of description discussed above, and the Rosen definition and other observer driven definitions of complex systems as those that need more than one description that was discussed in sections 1.1 and 1.5.1. A system which exhibits observational emergence will display high end complexity.

Hierarchical Information Systems and Cohesion

In a recent paper [134], Collier has introduced the concept of a *Hierarchical Dynamical Information System*, which he claims is of interest for the study of how functional information might be embodied physically within some system. This concept is a culmination of a number of years of research [135, 136].

Fundamental to these systems is a concept of *cohesion* [135, 134]. This is defined as the closure of the causal relations among the dynamical parts of a system that determines its resistance to fluctuations (both internal and external) which might cause it to lose its integrity. If objects are cohesive then they are to a certain extent causally bound, which in turn means that they will act in a coherent manner. However, cohesion is not constant, neither in a given object over its lifetime (no object is indestructible) nor between different objects; a rock is probably considered more cohesive than a swarm of bees, but both are considered objects by Collier *et al.* due to the causal interactions that give rise to their organic unity [135]. Objects might in turn exhibit *cohesive properties i.e.* properties that are insensitive to fluctuations and therefore stable to a certain extent. For example, a kite does not disintegrate despite its constant bombardment by air molecules and might therefore be considered a cohesive object. The lift of a kite which results from the net effect of these molecules hitting the kite on both sides, can be considered a cohesive property of the kite. The effects of the individual molecules are averaged by the cohesive forces that hold the kite together. Collier argues that the insensitivity of cohesive objects to fluctuations makes a reductive explanation of the effect inappropriate. This is because such a reduction would have to include information that was irrelevant to the object that was being explained (the individual trajectories of the particles striking the kite do not add to the description of the kite in any way). While Collier acknowledges that these irrelevant factors might be avoided by reducing the emergent object or property (*i.e.* the lift of the kite) to some set of lower level objects, he claims that such a reduction is guilty of either a category error (making the lift an abstract object, which it is not), or else makes use of an equivalent property that exists only at the level that is currently being reduced [135]. With the emergence of coherent properties, the possibility of a new level of objects arises, and thus a hierarchical model of a number of phenomena starts to appear.

Clearly the cohesion of the objects within a hierarchy is important. Objects must have some sort of stability with respect to their surrounding environment. This cohesion of objects is closely related to their near-decomposability; the very concept emerges from this same partial insensitivity to environmental fluctuations. We can conclude that the two concepts are closely related, if not essentially equivalent.

3.3 Towards the Dynamical Generation of Emergent Hierarchical Structure

The concepts of emergence and hierarchy have been generally accepted as important, and many researchers have attempted in some way to clarify, and to provide theories about

them. However, the problem of dynamically generating emergent hierarchical structures, and understanding their evolution in time, is eluding our current models and theories. How can such behaviour be understood?

At this point, a set of closely related concepts has presented itself, all of which will prove useful in the analysis and generation of complex emergent behaviour. Firstly, we have seen the dangers of adopting an entirely reductive, or object driven approach; in doing so it is likely that we sacrifice our ability to generate new emergent interactions between the subcomponents of some system of interest. In attempting to move beyond this object driven methodology, concepts such as context, observational emergence and observer driven complexity appear to be systematically occurring. Many different researchers seem to be utilising such concepts, even if they are not explicitly recognising this fact. This will become more apparent throughout this work. Complex systems often appear to exhibit hierarchical structure, and those at the high end of the complexity scale appear to exhibit some sort of contextual hierarchical structure, with interdependent relationships working both upward and downward through the hierarchy.

The next chapter will turn to the examination of a system which has been developed at Flinders University which appears to generate emergent hierarchical behaviour. The modelling of this system does not fall into the category of traditional reductive techniques, which suggests that this model may in fact be a realisation of the new modelling technology that we appear to require. The remainder of this work will be devoted to developing this new technology in such a way that it can be more generally applied.

Chapter 4

An Example System: Process Physics

The preceding chapters have shown that complex emergent behaviour is very difficult to model and to generate. It has been argued that the most complex emergent behaviour is intrinsically contextual, and that the lack of this property in models of emergent behaviour such as ALife simulations has led to the simplistic nature of the generated behaviour.

This chapter will examine a system that has been developed at Flinders University which, it will be claimed, shows good indications of exhibiting a more interesting variety of emergent phenomena. This system forms the basis of a set of theories which will be discussed in this work, all of which fall into a new modelling methodology, termed *Process Physics*, which seeks to replace the standard object driven, reductive methodologies of traditional physics with a more dynamic set of models capable of exhibiting phenomena such as observational emergence and high end complexity. It is expected that Process Physics will explain more phenomena than can be understood by the traditional physical methodology, and in fact, a number of unexplained phenomena have been incorporated into the Process Physics modelling (see section 6.4.5). The remainder of this work will be devoted to an examination of the reasons behind the success of the simple relational model presented in this section, and its generalisation to a new theory of emergent behaviour which falls into the Process Physics methodology.

4.1 Modelling Reality

> *There is a theory which states that if ever anyone discovers exactly what the Universe is for and why it is here, it will instantly disappear and be replaced by something even more bizarrely inexplicable.*
>
> *There is another theory which states that this has already happened.*
>
> Adams, p130, [7]

As was discussed in section 1.1, since the time of Newton the aim of physics has been to capture a context free, formal model of reality. That is, we aim to describe the Universe in terms that are independent of an observer. This method has been remarkably successful, yielding rich results, many of which have been experimentally verified to a high level of accuracy. However, there are strong arguments suggesting that such a research paradigm may be fundamentally flawed. One example of this problem is provided by

Gödel's theorem [295], which as was discussed in section 1.5.1, suggests that this reductionistic programme, if it were complex enough to result in an adequate description of the Universe, must lead to a mathematical model which is either incomplete or inconsistent. While the potential impact of this result in the field of physics is still being debated [118], Chaitin has extended this result, showing that Gödel's unprovable truths are essentially random in character — they are incompressible [119, 120, 121]. This result has new ramifications for physics — can these random, incompressible truths be captured by physical models?

Even if these foundational issues are not considered important, a number of apparently unsolvable problems have begun to emerge in the field of physics during the last century. For example, a successful theory of quantum gravity has yet to be discovered. Our model of time as a simple one-dimensional manifold has led to some very well known problems such as the missing arrow of time and the inadequate description of the present moment effect [345]. Even the interpretation of quantum mechanics remains mysterious, with a number of competing alternative interpretations [87, 150, 191, 309, 328] each leading to a profoundly different understanding of reality.

Section 1.4 discussed the way in which the very methodology of the modelling used in fundamental physics is also problematic. Our description of reality has been steadily increasing in its detail, from atoms to protons, electrons and neutrons to the current family of 'fundamental particles'; quarks, leptons, mesons etc. However this progression of theories has yet to end. A number of new contenders for the fundamental entities of physics are starting to emerge; strings [387], loops [368] and preons [279]. Is fundamental physics heading towards a regress?[1] At some stage an explanation of how a particle can be understood as truly fundamental must be proposed, but in postulating a particle as fundamental we have lost our ability to explain that particle. A new methodology is required.

The traditional methodology of the physicist has been identified as object based, it centres around some set of fundamental objects and then attempts to describe their dynamics. One of the most widely cited counterexamples to this methodology arises in the dispute between Leibniz and Newton. It is usually claimed, in the spacetime or physics oriented literature, that Leibniz attempted to construct a *relational* model of space but encountered insurmountable problems in this attempt. This claim is somewhat simplistic; it is generally agreed in the philosophical literature that Leibniz was constructing a theory of relations, but that this does not constitute a relational theory [41]. However, this spacetime interpretation of Leibniz is very suggestive (if somewhat naive), and formed an invaluable starting point to the research conducted at Flinders University, leading to the question of whether there might be some way in which to construct a successful relational

[1] This problem has been independently recognised by a number of different researchers, such as Wheeler [453].

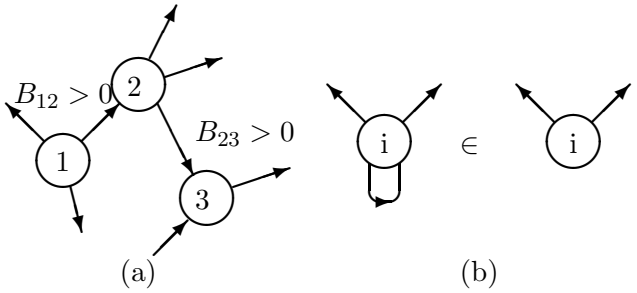


Figure 4.1: (a) Nodes i and j are considered connected if they have a non-zero B value. Arrows indicate the sign of the B_{ij} value. (b) Self-links are considered internal to a node so $B_{ii} = 0$.

model of the Universe. A subtle bootstrap mechanism has been proposed to solve the above problem of object driven methodologies. In this section we shall first construct the new relational model, and then discuss the bootstrap mechanism, and finally the hierarchical characteristics of the system.

4.1.1 The Process Model of Fundamental Physics

Considering a set of N nodes, we assume that they are connected in some way, with a connection strength between node i and node j given by the real value B_{ij} (see figure 4.1). We shall represent a set of these relational values as a square antisymmetric[2] matrix B. Notice that while the pitfalls of object driven methodologies have been explicitly recognised above, we are still driven to talk in terms of nodes. It is very difficult to leave an object methodology behind. As discussed above however, any system positing such *a priori* objects cannot be considered fundamental when modelling the Universe, as the explanation of these objects must lie outside the system being modelled. A solution to this dilemma arises if we recognise that the nodes can in turn be defined for the purposes of the model as a system of nodes, connected with weaker B_{ij} values.[3] It becomes apparent that all nodes can be thought of as composed of collections of nodes in turn, and in particular that the start up nodes can be viewed as names for subnetworks of relations. This result is ensured if the constructed system exhibits self-organised criticality (SOC), which enforces a fractal structure on the system [48, 47]. Thus, the dissipative nature of this system suggests that it has a hierarchical structure, in line with the discussion of section 3.2.1. This supposed structure will be investigated more fully in the next section.

The SOC requirement is intrinsically linked with a new processing notion of time in this system. This is because the relational fractal structure is generated by a noisy non-linear iterative map displaying SOC behaviour. Thus in attempting to construct a model

[2] Antisymmetry ensures that $B_{ii} = 0$ thus avoiding explicit node self connection. The internal structure of nodes will be incorporated shortly.

[3] B is assumed to be a very large ($\to \infty$) matrix.

that does not postulate *a priori* fundamental objects, we find the need to introduce a time-like process. In contrast, standard physics with it's use of *a priori* objects is linked with the standard geometrical model of time and the associated problems mentioned above.

The particular map used was suggested by the Global Colour Model of quark physics [97]. Details of the history of this map will be discussed in section 4.3, but for now we note that stripping away the space time and quantum number indices [94], and adding a noise term suggested by the stochastic quantisation procedure of Parisi and Wu [316] leads to the following iterative process [108]

$$B_{ij} \to B_{ij} - \alpha(B + B^{-1})_{ij} + \omega_{ij}, \tag{4.1}$$

where

$$i, j = 1, 2 \ldots, N \quad \text{and} \quad N \to \infty. \tag{4.2}$$

α is an arbitrary parameter which tunes the behaviour of the system; too large and the system over-connects too rapidly, too small and the rate of connection is drowned out by the noise term (see [247] for more details about the behaviour of this term), The term ω_{ij} represents an additive noise term, which provides a sense of openness in the system. At each iteration, the noise term creates new B_{ij} links, incorporating a sense of innovation and contingency into the system. The noise term, when used iteratively in equation (4.1) is responsible for the notion of time that arises in the model. The dynamics are irreversible, with one particular past, which can be recorded as a history, but not relived. Future states of the system cannot be known, however certain sets of ensemble predictions can be made. In this sense a processing notion of time is captured by the system, with a markedly different ontology from the static four dimensional spacetime of standard physics.[4] This leads to the identification of this system and any associated modelling techniques as *Process Physics*.

The nonlinear matrix inversion term also performs a critical role in the system. It causes separate structures brought into existence by the noise term to link up, modelling a process of self-assembly. It is interesting to examine the dynamics of this process in detail.

The system can be started with $B \approx 0$ which represents the absence of any significant relational information. Under successive iterations of equation (4.1) the B matrix assumes a sparse structure that can be organised into a block diagonal form.

Assuming that the large ω_{ij} arise with fixed but very small probability p, the geometry of the structures formed can be revealed by studying the probability distribution of minimal spanning graphs with D_k nodes and k links from an arbitrary node i where

[4]For the purposes of the following chapters, we note that this modelling of time is far more appropriate in the context of living systems, providing a sense of contingency and dynamism.

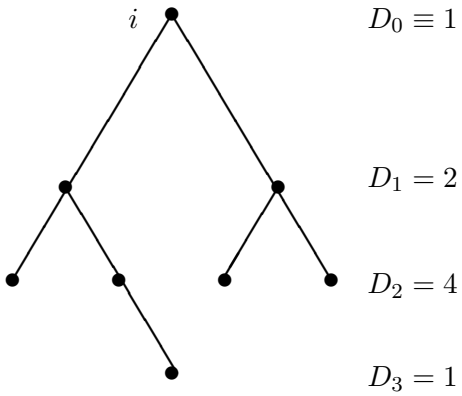


Figure 4.2: A $N = 8$ spanning tree for a random graph (not shown) with $L = 3$. The distance distribution D_k is indicated for node i.

$D_0 \equiv 1$ (see figure 4.2). This probability distribution is given by [297]

$$\mathcal{P}[D, L, N] \propto \frac{p^{D_1}}{D_1! D_2! \dots D_L!} \prod_{i=1}^{L-1} (q^{\sum_{j=0}^{i=1} D_j})^{D_{i+1}} (1 - q^{D_i})^{D_{i+1}} \tag{4.3}$$

where $q = 1 - p$, N is the number of nodes and L is the maximum depth from node i. The most likely pattern can be found by numerically maximising $\mathcal{P}[D, L, N]$ for fixed N with respect to L and D_k. This procedure has been performed [108], and the following results are from that analysis.

Figure 4.3 shows the set of D_k (distance distribution) values obtained from one of these numerical experiments, where the log of the probability of a large noise value is set at $\log_{10} p = -6$, and the number of nodes is fixed at $N = 5000$. Also shown in the figure is a curve

$$D_k \propto \sin^{d-1}(\pi k / L)\,. \tag{4.4}$$

with best fit to the data when $L = 40$ and when the dimensionality of the fit, $d = 3.16$. This same curve is obtained from the surface area of a n-dimensional sphere. Figure 4.4 shows the range in d for fixed $N = 5000$ and varying p values. We see that for p below some critical value $\log_{10} p < -5$, $d \approx 3$.

This indicates that the connected nodes have a natural embedding in a S^3 hypersphere, which is very suggestive of the 3-dimensionality of space. Thus this model goes some way towards predicting the observed three dimensional structure of space, as an emergent phenomenon, a feature that is usually assumed in physics. Notice that the nodes are not exactly embeddable (which would require $d = 3$), there is a proportion of extra links. This is a key observation which forms the basis of a theory of matter as a quantum foam that will be discussed in section 6.4.5.

While they are not the only structures generated by the system, their maximum

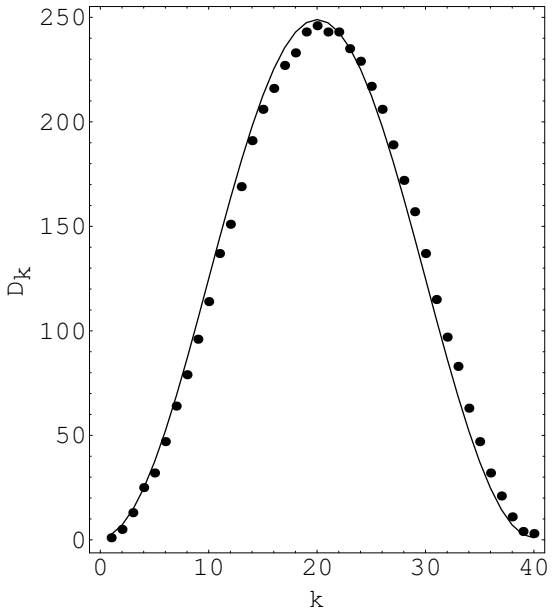


Figure 4.3: The distance distribution D_k between nodes for a typical emergent structure under equation (4.1). This is obtained by numerically maximising $\mathcal{P}[D, L, N]$ where $N = 5000$ and $\log_{10} p = -6$. A curve is also drawn showing that $D_k \propto \sin^{d-1}(\pi k/L)$, with best fit $d = 3.16$ and $L = 40$.

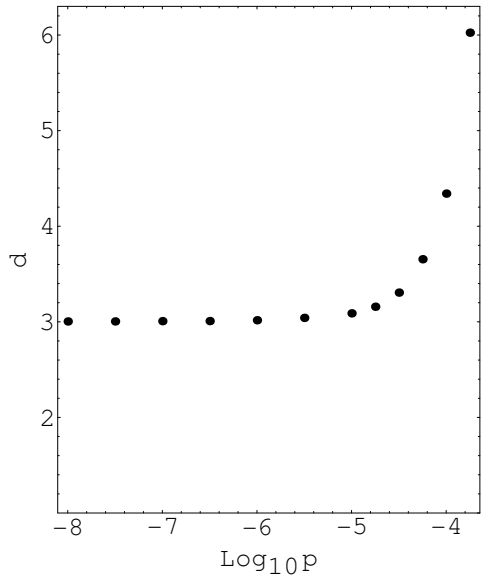


Figure 4.4: Range of d values for $N = 5000$ as a function of the probability p of large noise values.

likelihood makes these structures the most common. Splitting the large B matrix into its constituent independent submatrices B_{sub} we realise that each is almost singular ($\det(B_{tree}) \approx 0$) but that the noise term ensures extra B_{ij} terms which lead to a small valued determinant. Upon inversion of the B matrix, this small valued determinant implies that the next iterative step of the system will lead to new large valued B_{ij} entries (depending upon the specific ω_{ij} noise terms). Hence tree structures are sticky, at each iteration cross-links form between the structures which act to join them, and to produce larger structures. This behaviour has been examined in detail in a recent PhD thesis [247].

Thus, under the influence of the iterator (4.1) the system can be seen to 'grow' with a steady increase in relational structure. As the tree structures stick together, they become less easily embeddable in a S^3 structure. This phenomenon can be seen in figure 4.4, where beyond the critical value of $d \approx 3$ the dimension of the structures quickly becomes very high; they are no longer embeddable in S^3 and should instead be thought as *defects* in the system. These defect graphs gradually lose the ability to form new links, which tend to form only at the level of leaves, thus, they become unable to sustain themselves and eventually they fade away from the system, or 'die'. However, the system itself generally grows faster than it loses structures (see [247] for more details of these dynamics). The openness of the system provided by the noise term is essential here, as otherwise the system could be seen as violating the 2^{nd} law of thermodynamics.

The nonlinear term is self-referencing; all elements of B are required in order to compute the next value of each B_{ij} element from the previous iteration. Thus, this term can be seen to incorporate a weak notion of internal self-observation into the system. In particular, any node has the capacity to profoundly affect the rest of the system if it randomly receives a large ω_{ij} value at the next iteration of the map. Thus the system is in a sense holistic; the fact that one node is not 'close' to another does not mean that it cannot be affected strongly by it. This leads to a very strong form of contextual behaviour in the system, no one element can be considered as isolated, in fact, at the next iterative step it may become strongly linked to a node which was previously not considered important to its dynamics. The noise term ω_{ij} limits this self-referencing of the system, providing the system with a sense of innovation.

As mentioned above, there is every reason to suspect that this system displays some sort of hierarchical structure. This was initially shown to be the case in [243], but a more in-depth understanding of the hierarchical nature of this system has since been obtained. The next section will start to analyse this idea from a number of the different viewpoints developed in chapter 3.

4.2 A Hierarchical Analysis of the System

There is a sense of emergence in this system. Structures appear to be forming which are not explicitly present in either the update equation (4.1) or the original very simple relational structure of nodes and connections. An examination of this claim within the framework of hierarchy theory helps to clarify, in less technical terms than were used in the previous sections, this aspect of the behaviour of the system.

4.2.1 Near-decomposability

The first characteristic of this system relevant to a hierarchical analysis is the block diagonal form of the matrix of B_{ij} values, that arises under the update equation (4.1). This form also arose in equation (3.1), which occurred during the examination of Simon's understanding of near-decomposable systems performed in section 3.1.4. Thus, it is possible to form an understanding of this system as nearly-decomposable. This result makes sense. If nodes are seen as themselves made up of structures of nodes, then clearly some sort of hierarchical system has been created.

At this point we start to see that Simon's analysis is not particularly helpful beyond its ability to identify structures that have a nearly-decomposable nature and are therefore to some extent hierarchical. No understanding of the dynamics of the structure has been gained, merely an indication of that structure.

4.2.2 Baas hierarchy structure

An alternative view of hierarchies proposed by Baas, was discussed in section 2.2.1. It is possible to understand the Process Physics model as forming at least a $2nd$ order hyperstructure in the Baas sense.

Recall that this framework depends fundamentally upon the notion of observation used in the identification of new, higher order structures [43]. Thus, in order to register the emergence of new structures or entities within a system it is necessary to identify a mechanism capable of observing those entities, which often takes the form of their context. Given that this system is aimed at modelling the Universe, we might ask what its context could be. How could an observational mechanism external to the Universe be identified? Such a mechanism can be found through the adoption of what might be regarded as an *external perspective*. The majority of analytical tools appear to be of an external nature, but this does not imply that an entity exists external to the Universe, merely that these external modes of analysis are simpler to implement. This point has been recognised by a number of different researchers [236, 235, 442, 441], many of whom claim that it is necessary to develop *internal perspectives* in order to understand complex phenomena such as the mind and life. The term endophysics [366, 169, 236] has been coined to refer to the study of systems which have enclosed observers in them. Such a view is participatory, how we look determines what we see, thus this idea bears many similarities with the necessity of accounting for context in complex systems. Through

a proper treatment of context we loose much of the confusion that often surrounds an examination of the Universe; an observer of the Universe can exist *within* that Universe.

To construct a Baas hierarchy, we consider the original nodes used in the construction of the relational structure to be a very simple family of N first-order structures $\Xi^1 = \{\Xi^1_r : r = 1, 2, \ldots N\}$,[5] where the first-order observational mechanism O^1 is defined as membership in the set Ξ^1 . Now, under the influence of equation (4.1) which we consider to be an update functional R, we find that these nodes join up, forming a second order structure:

$$\Xi^2 = R(\Xi^1_i, O^1, B_{ij}) \qquad i, j = 1, 2, \ldots, N \tag{4.5}$$

where B_{ij} is the connection strength between nodes and Ξ^1_j designates the structure that appears in the B matrix in the previous sections. The family of second order structures consists of the maximum likelihood structures discussed above that are embeddable in the three-dimensional hypersphere (E_{S^3}), along with a variety of other *defect* structures that have not as yet been classified. However, considering the space-type structures alone, we can define the property of being embeddable in S^3, (E_{S^3}) as a second order observationally emergent property,

$$P_{E_{S^3}} \in O^2(\Xi^2). \tag{4.6}$$

That is, this property is not present in the simple set of nodes that we started with,

$$P_{E_{S^3}} \notin O^2(\Xi^1). \tag{4.7}$$

Only under the influence of the update functional (4.1) did this extra aspect of the behaviour of the nodes become apparent. This property is claimed to be observationally emergent since external techniques must be utilised in order to extract the notion of S^3 embeddability, and it is only approximate at best, due to the influence of the defect structures.

The analysis of the remaining structure of this system is very complex and only in the preliminary stages, but it is expected that stable patterns identifiable as third order structures will emerge in the system. The full theory of this process is currently under development, but the general argument can be sketched out. More detail will be provided in section 6.4 when a higher level model, a *Quantum Homotopic Field Theory* (QHFT) of the simple iterative system discussed above will be introduced.

We can somewhat artificially classify the structures in the system as being either exactly embeddable in S^3 (the E_{S^3} structures), or non-embeddable structures which we will term defects (D). It is expected that the system contains stable defects, which we shall term Topological Defects (TD). The stability of these structures will be structural,

[5]Note the change in notation, with Ξ describing the structures instead of Baas' original S which was used in section 3.2.2, but will not be used here in order to avoid confusion with the hypersphere notation.

and constitutes a new third order observation mechanism within the system O^3. Thus it is expected that the stable TD's will form a set of K new emergent structures $\Xi^3 = \{\Xi^3_v : v = 1, 2, \ldots K\}$ within the system. Equation (4.1) ensures a constant updating of their components — rather like the units of an organism which are being constantly regenerated. These TD's (if rigorously identified) would be third order structures in the hierarchy. That is

$$P_{TD} \in O^3(\Xi^3_v), \text{ and } P_{TD} \notin O^3(\Xi^2_r) \tag{4.8}$$

because their emergent stability within the system is defined by their structural stability, i.e. it constitutes a new observation mechanism. It is expected that TD's made up of aggregations of E_{S^3} structures will occur in this system, not themselves having the E_{S^3} property.

Thus there is a sense of non-computational, or rather observational emergence (as defined by Baas, see section 3.2.2) in this system. More than one mode of analysis is required in order to understand its behaviour. This suggests that this system is truly exhibiting some sort of high end complex behaviour. We shall return to this claim at the end of this work in section 7.2.

So far a third order hyperstructure has been identified (but not rigorously at the third order).

$$nodes \quad > \underbrace{S^3 + D}_{O^2(\Xi^2)} > \quad \underbrace{stable\ TD}_{O^3(\Xi^3)} > \ldots \tag{4.9}$$

It is expected that more structure will emerge from the system. This is because the identified stable structures of this model have a deep connection with the more standard object driven methodologies of particle physics which will be discussed section 4.3. In [91] a link from the QHFT to the theory of preons[6] was discovered, and therefore it appears that these systems can recover much of the behaviour of the Standard Model.

The system presented is rather minimal when compared to the complex task of modelling living systems. This is to be expected. While a surprising amount of structure appears to emerge from this system, the lack of objects in the model plays a key role in limiting the storable information of the system. For example, consider biological modelling where complex information can be stored in DNA molecules and their enveloping cells. This provides an *a priori* fundamental set of information the interactions of which must be incorporated into any model. It also allows for the creation of extremely complex structures, as the system can use this prestored information to generate new structures. We expect that with a strong argument for the stability of the TD's the Process Physics system may become capable of storing such complex information, and hence generating a very rich set of structures undergoing complex interactions. This is clearly a prior-

[6] Preons are one of the proposed replacement 'fundamental particles' which in bound states form quarks [279, 160].

ity for future research, however, the transition to stored information is no simple one, a point explicitly recognised by Pattee in his discussion of the need to understand the implementation of the epistemic cut in such theories (discussed in section 1.5.1).

Finally, we recall the ADH discussed in section 2.2.2. While the system presented here appears to be very minimal, with very simple first order structures, the update equation (4.1), being derived from Quantum Chromodynamics presumably has a large amount of intrinsic information. In addition to this, it appears that the information originally stripped away from equation (4.1),[7] is in some sense re-emerging within this dynamical system. This behaviour is presumably not allowed by the ADH, as a number of higher-order properties are emerging from this system without a complex set of object primitives. Thus we see that the ADH is in fact flawed as was claimed in section 2.2.2. This Process Physics system appears to be providing a direct counterexample, and it does so through a shift in emphasis from the standard reductive, object driven methodologies to a more open, contextual, processing system of relations.

4.3 The Historical Roots of Process Physics

The fact that this very minimal system exhibits both observational emergence and a contextually dependent hierarchical structure suggests that it may have some sort of fundamental characteristic that could be used in a more general modelling of emergent behaviour. In order to fully explore this possibility we will look at the historical roots of the model.

As mentioned in section 4.1 the map used to update the Process Physics system was suggested by the Global Colour Model (GCM) of quark physics [90, 97, 205]. This model uses the functional integral method, and approximates low energy hadronic behaviour from the underlying quark-gluon quantum field theory. In this section we shall briefly discuss the fundamental nature of quantum theories before looking at the way in which an equation from the GCM led to the development of Process Physics. Chapter 5 will discuss the structure of quantum theories in more detail, arguing that they can be applied to a more general set of systems than is assumed to be the case at present.

4.3.1 The Nature of Quantum Theories

All quantum theories have the same fundamental structure, which is illustrated in figure 4.5. In order to implement some quantum theory of a system, the following set of three steps is followed:

1. First, a map from a classical *state space*, S to a complex number $\mathbf{C}$ is found. This map is often written in the form of the symbol ψ.

[7] The Minkowski measure of space-time, and the quantum number indices.

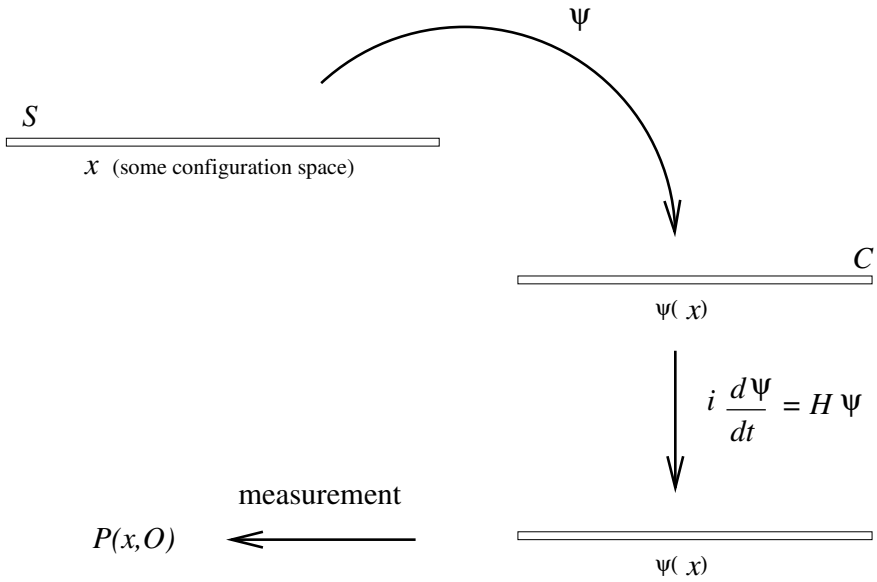


Figure 4.5: The fundamental structure of any quantum theory involves a map ψ from a state space to a complex number, which evolves according to some time evolution equation.

2. Depending upon the system under examination some time evolution equation is chosen from a set of possibilities including the Schrödinger equation, the Klein-Gordon equation, the Dirac equation *etc.* each of which map to another set of complex numbers.
3. Steps 1 and 2 are mathematically well defined and understood, however, complex numbers are not revealed when a measurement is performed on a quantum system. Instead, the system is found to be in some classical state which is closely related to the configuration of the experiment performed (see section 1.3). The dynamics of this process are not understood, and there are a number of competing theories of quantum measurement, each of which lead to a different interpretation of quantum mechanics [63]. Mathematically, this process is carried out by mapping the inner product of ψ at the point of time in which we are interested to some sort of probability space from which a set of predictions about the system are obtained.

Only a few example theories with this structure follow, but all quantum theories can be placed within this general framework.

Non-Relativistic Quantum Mechanics

The simplest example of this general structure is standard (non-relativistic) quantum theory. It consists of a mapping ψ from space and time coordinates $(t, x_1, x_2, x_3) = (\mathbf{x})$ to the complex numbers, $\psi(\mathbf{x}) \rightarrow c \in C$. The time evolution of this system is given by

the *Schrödinger equation*

$$i\hbar\frac{d\psi}{dt} = H\psi \tag{4.10}$$

Usually, a requirement is added that the time evolution of the relevant field (represented by ψ) is norm preserving,

$$\frac{d}{dt}||\psi|| = 0 \tag{4.11}$$

which amounts to the requirement that no creation or annihilation of matter occurs. This occurs if H is a hermitian operator, $H = H^\dagger$, which means that the time evolution of the system will be unitary.

The act of measuring the action of some physical observable O represented by the operator $\hat{O}$ upon the system is predicted by calculating the *expectation value*:

$$\begin{aligned} \langle\psi(\mathbf{x})|\hat{O}|\psi(\mathbf{x})\rangle &= \int d\mathbf{x}\psi^\dagger(\mathbf{x})\hat{O}\psi(\mathbf{x}) \\ &= \sum_j |c_j|^2 o_j, \end{aligned} \tag{4.12}$$

where the o_j are the real eigenvalues of the (hermitian) operators $\hat{O}$ such that $\hat{O}\psi_j = o_j\psi_j$, where ψ has been expanded in terms of the complete set of states ψ_j weighted by the values c_j; $\psi = \sum_j c_j\psi_j$. The interpretation of equation (4.12) usually states that $|c_j|^2$ is the *probability* that a measurement of O will give an *eigenvalue* o_j.

Quantum Field Theory — The Functional Integral approach

Quantum field theory unifies quantum theory with special relativity. It is one of the most successful theories in physics, yielding results that are accurate to better than one part in one million. In chapter 5 we shall discuss some of the key features of quantum field theories in more depth, for now we shall be content with a brief discussion of the structure of quantum field theory as it arises in the *functional integral method* [234, 336, 193].

The functional integral method is a widely used technique for constructing quantum field theories. It consists of the familiar quantum mapping, but the domain of the map ψ has changed from the simple space and time coordinates of quantum theory to that of a set of fields ϕ over these same space and time coordinates. Again the function maps to the complex numbers; $\psi(\phi(\mathbf{x})) = \psi[\phi] \rightarrow c \in C$, and an equation in the Schrödinger form can be used to find the time evolution of the fields, although this is rarely done. A *correlation function* $\mathcal{G}$ is calculated to find the relevant information that will be compared with experimental outcomes. For example,

$$\mathcal{G}(\mathbf{x}_1, \mathbf{x}_2) = \langle\psi_0, \hat{\phi}(\mathbf{x}_1)\hat{\phi}(\mathbf{x}_2)\psi_0\rangle, \tag{4.13}$$

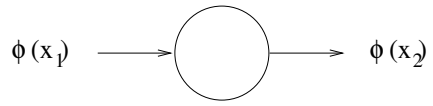


Figure 4.6: A two point correlation function, calculated between two fields $\phi(\mathbf{x}_1)$, and $\phi(\mathbf{x}_2)$.

where ψ_0 is the *vacuum state*[8] which describes what is termed a two point correlation function between the spacetime points $\mathbf{x}_1$ and $\mathbf{x}_2$. It describes the way in which an event at point $\mathbf{x}_1$ affects an event at $\mathbf{x}_2$ and vice versa. The operators $\hat{\phi}(\mathbf{x}_1)\hat{\phi}(\mathbf{x}_2)$ play a similar role to the operator $\hat{O}$ in quantum mechanics, in this case they describe a perturbation of the system at the positions $\mathbf{x}_1$ and $\mathbf{x}_2$. Correlation functions can be represented using Feynman diagrams along with a set of associated rules [234] which simplifies the calculational procedure, but this is in no way representative of reality. For example the two point correlator in equation (4.13) is represented by figure 4.6 which simplifies the extraction of the correlation functions. Despite the usefulness of this calculational tool, it is important not to make the mistake of thinking that these images describe localised particles interacting like billiard balls. The objects of quantum field theories are fields spread over all of space, partaking in highly complex interactions [41].

Thus, the fundamental structure of quantum field theory is the same as that of quantum mechanics. Despite this equivalence of structure, the process of calculating the different values of $\mathcal{G}$ is far more complex due to the intricate set of interactions involved between the different fields.

In order to calculate the value of $\mathcal{G}$ for the desired correlation function, the functional integral method makes use of what is termed the *generating function*, which is, in the Euclidean metric

$$Z = \int \mathcal{D}\phi e^{-S[\phi]}. \tag{4.14}$$

$\mathcal{D}\phi$ indicates an integration over all of the possible fields ϕ, in which we are interested, and $S[\phi]$ is a classical action which is generated from taking the relevant Lagrangian[9] describing those fields and integrating it over time, or more usually, the Lagrangian density $\mathcal{L}$ is integrated over all spacetime:

$$S[\phi] = \int dt L[\phi] = \int d^4\mathbf{x}\mathcal{L}(\phi, \partial_\mu\phi). \tag{4.15}$$

[8]Which will be discussed in more detail in section 5.5.

[9]The Lagrangian describes the difference between the kinetic and potential energies of the fields. All realistic Lagrangians satisfy a conservation law of some type. *Noether's theorem* links conservation laws with *symmetries*. It states that if the Lagrangian is invariant under some set of transformations of spacetime and field variables that can be expressed by a finite parameter then there is a conserved quantity associated with that symmetry [41, 234, 336, 81]. We shall discuss the importance of the concept of symmetry in more detail in section 5.3.

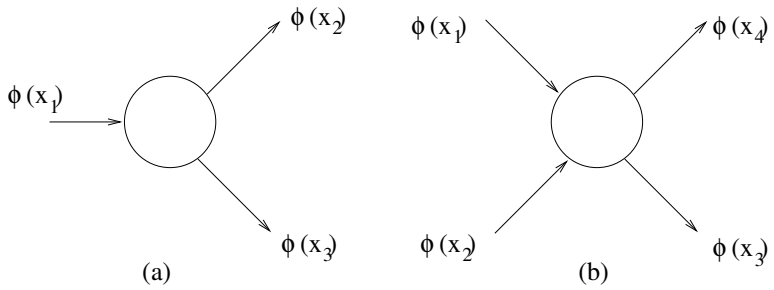


Figure 4.7: (a) A three point correlation function for the field ϕ, described by the correlator $\mathcal{G}(\mathbf{x_1}, \mathbf{x_2}, \mathbf{x_3}) = \frac{\int \mathcal{D}\phi\phi(x_1)\phi(x_2)\phi(x_3)e^{-S[\phi]}}{\int \mathcal{D}\phi e^{-S[\phi]}}$ and a (b) four point correlation function with the associated correlator $\mathcal{G}(\mathbf{x_1}, \mathbf{x_2}, \mathbf{x_3}, \mathbf{x_4}) = \frac{\int \mathcal{D}\phi\phi(x_1)\phi(x_2)\phi(x_3)\phi(x_4)e^{-S[\phi]}}{\int \mathcal{D}\phi e^{-S[\phi]}}$.

The generating function is then used to determine the relevant correlation function. For example the two point correlator of equation (4.13) using the following identity:

$$\begin{aligned} \mathcal{G}(\mathbf{x_1}, \mathbf{x_2}) &= \langle\psi_0|\phi(\mathbf{x_1})\phi(\mathbf{x_2})|\psi_0\rangle & (4.16) \\ &= \frac{\int \mathcal{D}\phi\phi(x_1)\phi(x_2)e^{-S[\phi]}}{\int \mathcal{D}\phi e^{-S[\phi]}}. & (4.17) \end{aligned}$$

This relation can be generalised by either increasing the fields of interest (in the action), or by increasing the number of points of interest on the top line of equation (4.16), giving more interesting three point, four point *etc.* correlation functions (see figure 4.7).

Depending upon the field theory to be constructed, the fields of interest, as well as the Lagrangian describing their dynamics change.

Quantum Electrodynamics (QED) is the theory of photons and electrons. The behaviour of these fields is described by the Lagrangian density:

$$\mathcal{L} = \bar{\psi}(x)\left(i\gamma^{\mu}\partial_{\mu} - m\right)\psi(x) - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} - g\bar{\psi}\gamma^{\mu}\psi A_{\mu} \tag{4.18}$$

where

$\bar{\psi}(x)$ is the **antielectron (positron) field**

$\psi(x)$ is the **electron field**

$A_{\mu}(x)$ is the electromagnetic vector potential, representing the **photon field**.

$F_{\mu\nu}$ is the electromagnetic field tensor, $= \partial_{\mu}A_{\nu} - \partial_{\nu}A_{\mu}$.

g is the electron charge.

γ^{μ} is the set of Dirac matrices, satisfying the Lorentz group $U(1)$.

The term $-g\bar{\psi}\gamma^{\mu}\psi A_{\mu}$ comes from fixing the gauge (*i.e.* the localizing the symmetry group). This makes the Lagrangian *gauge invariant*, or invariant under local transformations. However it has an important physical consequence of coupling the electron fields with the photon fields. Thus the requirement for gauge invariance is what makes interaction between the different fields of QED possible. In fact, this is so for all gauge theories, see section 5.3 for a more detailed discussion of the importance of, and connection between, symmetry, invariance and interaction.

Quantum Chromodynamics (QCD) describes the interactions between quarks and gluons. It is defined by the quantization of six quark fields satisfying a $SU(3)$ flavour symmetry, and eight gluon fields satisfying a $SU(3)$ colour symmetry. The dynamics of these fields is given by the Lagrangian density (in the Euclidean metric):

$$\mathcal{L} = \frac{1}{4}F_{\alpha\beta}F^{\alpha\beta} + \frac{1}{2\xi}(\partial_\mu A^a_\mu)^2 + \bar{q}\left(\gamma_\mu(\partial_\mu - ig\frac{\lambda^a}{2}A^a_\mu) + \mathcal{M}\right)q \tag{4.19}$$

where

$\bar{q}(x)$ is the set of 6 **antiquark fields**

$q(x)$ is the set of 6 **quark fields**

$A^a_\mu(x)$ is the set of 8 **gluon fields**

$F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + g f^{abc} A^b_\mu A^c_\nu$
describes the self-interaction of the gluons
where f^{abc} are the structure constants,
and g is the colour charge.

$\mathcal{M}$ is the free mass term for each quark,

γ_μ are the Dirac matrices,

$\frac{\lambda^a}{2}$ are the eight $SU(3)$ colour generators in the Gell–Mann representation.

We immediately see that QCD is a far more complicated theory than QED. It contains far more fields each obeying a far more complicated symmetry group. Even the dynamics of the Lagrangian itself is more complex. Note the self-interaction term in the gluon dynamics, a term that leads to one of the main differences between the two theories.

Generally, given some action that describes the classical dynamics of a set of fields in which we are interested, it is in principle possible to quantize those dynamics and then extract correlation information for some set of time and spatial coordinates, using equations such as (4.13). However, this is not usually a simple process. This is because of the infinitely long expansion characterised by the exponential term in equation (4.14), and hence the correlation functions. While some of these series converge (*e.g.* QED, and QFD[10]) not all do (*e.g.* QCD) which means that there is no immediate guarantee that the higher order terms will not contribute to the generating function in a non-trivial way. At this point the concept of symmetry comes into its own, where principles such as gauge invariance are used to guarantee that the system is well behaved [279].

We have not discussed QED or QCD in any great detail, the reader is referred to

[10] Quantum Flavour Dynamics, the theory of the electroweak interaction.

any good book on the subject for further details, see for example [234, 279, 336, 450]. For now, we shall move onto discuss a model of QCD which formed the original basis of the iterator equation (4.1). With the understanding of the history of Process Physics so provided, we shall then move in chapter 6 to a more detailed discussion of some general characteristics of quantum theories which, it will be argued, can be used in the discussion of high end complexity.

4.3.2 The Global Colour Model (GCM) and the Functional Integral Calculus (FIC)

The Global Colour Model (GCM) [97, 205] is a low energy approximation to QCD which describes the behaviour of hadrons.[11] At low energy (or long wavelength) we observe only those degrees of freedom associated with hadron behaviour; individual quarks and gluons are not directly observable. This suggests that a reasonable approximation to low energy behaviour could be achieved by choosing a set of variables which reflect only the behaviour that we can observe in an experimental setting. This choice of new variables is not arbitrary, it is achieved using what are termed Functional Integral Calculus techniques [96, 90, 95] which change variables in a dynamically determined way. In fact, these techniques amount to analogues of the various tricks used in ordinary integral calculus. This process changes the variables of the QCD generating functional as follows:

$$
\begin{aligned}
Z &= \int \mathcal{D}\bar{q}\mathcal{D}q\mathcal{D}A exp\left(-S_{QCD}[A,\bar{q},q]\right) && (4.20)\\
&\approx \int \mathcal{D}\bar{q}\mathcal{D}q\mathcal{D}A exp\left(-S_{GCM}[A,\bar{q},q]\right) \qquad \text{(GCM)} && (4.21)\\
&= \int \mathcal{D}\mathcal{B}\mathcal{D}D\mathcal{D}D^{*} exp\left(-S_{bl}[\mathcal{B},D,D^{*}]\right) \qquad \text{(bilocal fields)} && (4.22)\\
&= \int \mathcal{D}\bar{N}\mathcal{D}N\ldots\mathcal{D}\pi\mathcal{D}\rho\mathcal{D}\omega..exp\left(-S_{had}[\bar{N},N,..,\pi,\rho,\omega,..]\right) && (4.23)
\end{aligned}
$$

This process has been termed *action sequencing* [94]. In section 5.3.5 this concept will be examined generally, in particular the term will be broadened to signify any application of dynamically determined variable changes in the extraction of high level, or emergent, behaviour from a low level model. Thus we might consider FIC as the first documented implementation of action sequencing.

The details of this procedure are very complicated, for the purposes of illustration, we shall briefly consider the change of variables that takes the GCM action to one that describes bilocal fields.[12]

The GCM action is an approximation of QCD [97]:

$$
S_{GCM}[A,\bar{q},q] = \int d^{4}x\left(\bar{q}(-\gamma\partial + \mathcal{M} + iA^{a}_{\mu}\frac{\lambda^{a}}{2}\gamma_{\mu})q + \frac{1}{2}A^{a}_{\mu}D^{-1}_{\mu\nu}(i\partial)A^{a}_{\nu}\right) \qquad (4.24)
$$

[11] Hadrons (*i.e.* nucleons) are bound states of quarks.

[12] A simplified bilocal action will be the starting point for the extraction of equation (4.1).

where the new symbol $D_{\mu\nu}$ is the effective quark-quark coupling correlator which can be determined from experimental data. First, an integration over the gluon fields is implemented, leaving the action dependent upon quark behaviour only. This procedure results in a generating functional describing the quark fields (which follow a Grassman algebra):

$$Z = \int D\bar{q}Dq exp\left(-S\left[\bar{q},q\right]\right) \tag{4.25}$$

where the action is now in the form

$$S_{[}\bar{q},q] = \int d^4x d^4y \Bigg(\bar{q}(x)\left(\gamma\partial_x + \mathcal{M}\right)\delta^4(x-y)q(y) - \frac{1}{2}\bar{q}(x)\frac{\lambda^a}{2}\gamma_\mu q(x) D^{ab}_{\mu\nu}\bar{q}(y)\frac{\lambda^b}{2}\gamma_\nu q(y)\Bigg). \tag{4.26}$$

From this action, it is in principle possible to perform a direct integration over the Grassman variables ($\bar{q}$ and q) in order to extract the quark correlation functions *etc.*, however this integration is difficult. Instead the quark variables, $\bar{q}q$ are paired. This pairing of the variables removes their Grassman nature; the quark pairs attain the status of ordinary numbers which makes the integration far simpler to perform. Definitionally $\bar{q}q$ pairs are bosons, therefore this process is referred to as *bosonisation*.

The quartic term in equation (4.26) is reorganized, making use of appropriate Fierz identities [95],[13] to obtain a new action term:

$$\begin{aligned} S[\bar{q},q] = \int d^4x d^4y \Bigg[& \bar{q}(x)\gamma\partial\delta^4(x-y)q(y) - \frac{1}{2}\bar{q}(x)\frac{M^\theta_m}{2}q(y)D(x-y)\bar{q}(y)\frac{M^\theta_m}{2}q(x) \\ & - \frac{1}{2}\bar{q}(x)\frac{M^\phi_d}{2}\bar{q}(y)^{cT}D(x-y)q(y)^{cT}\frac{M^\phi_d}{2}q(x)\Bigg], \end{aligned} \tag{4.27}$$

where $\bar{q}(y)M^\theta_m q(x)$ describes $\mathbf{1}_c$ bilocal $\bar{q}q$ fields with flavour $\mathbf{1}_f$ or $\mathbf{8}_f$ generated by the flavour terms in M^θ_m, and that $q(y)^{cT}M^\phi_d q(x)$ are $\bar{\mathbf{3}}_\mathbf{c}$ bilocal qq fields with flavour $\bar{\mathbf{3}}_f$ or $\mathbf{6}_f$ generated by terms in M^ϕ_d. Making use of these relationships to perform a FIC change of variables results in the generating functional

$$\begin{aligned} Z = \int \mathcal{D}\bar{q}\mathcal{D}q\mathcal{D}\mathcal{B}\mathcal{D}D\mathcal{D}D^* \exp\Bigg(\int \Bigg[& -\bar{q}(x)(\gamma\partial + \mathcal{M})\delta^4(x-y)q(y) \\ & - \frac{\mathcal{B}^\theta(x,y)\mathcal{B}^\theta(y,x)}{2D(x-y)} - \frac{D^\phi(x,y)D^\phi(x,y)^*}{2D(x-y)} - \bar{q}(x)\frac{M^\theta_m}{2}q(y)\mathcal{B}^\theta(x,y) \\ & - \frac{1}{2}\bar{q}(x)\frac{M^\phi_d}{2}\bar{q}(y)^{cT}D^\phi(x,y)^* - \frac{1}{2}D^\theta(x,y)q(y)^{cT}\frac{M^\theta_d}{2}q(x)\Bigg] \end{aligned} \tag{4.28}$$

where $B^\theta(x,y) = B^\theta(y,x)^*$ are hermitean bilocal fields (which will be discussed somewhat in section 5.3.5), $\mathcal{B}(x,y) = \mathcal{B}(x,y)\frac{M^\theta_m}{2}$, and the D^ϕ, D^θ are the associated higher level

[13] See either of [205, 97] for the full details of this process.

diquark fields. Now an integration over quark fields is performed to complete the FIC change of variables to bilocal meson and diquark fields,

$$Z[\bar{\eta},\eta] = \int \mathcal{D}B\mathcal{D}D\mathcal{D}D^{*}(Det\mathcal{F}^{-1}[\mathcal{B},D,\bar{D}])^{\frac{1}{2}} \exp\Bigg(\int -\frac{\mathcal{B}^{\theta}(x,y)\mathcal{B}^{\theta}(y,x)}{2D(x-y)} - \int \frac{D^{\phi}(x,y)D^{\phi}(x,y)^{*}}{2D(x-y)}. \quad (4.29)$$

Finally the determinant identity $Det\mathcal{F}^{-1} = (Det(G^{-1}))^{2}Det(\mathbf{1}+\bar{D}G^{T}DG)$ proved in [95], is used which results in the bilocal action:

$$S_{bl}[B,D^{*},D] = -TrLn\left(G[B]^{-1}\right) + \frac{1}{2}TrLn\left(\mathbf{1}+\bar{D}G[B]^{T}DG[B]\right) + \int \frac{B^{\theta}(x,y)B^{\theta}(y,x)}{2D(x-y)} - \int \frac{D^{\theta}(x,y)D^{\theta\dagger}(x,y)}{2D(x-y)}. \quad (4.30)$$

At the end of the complete action sequencing process the following hadronic action has been derived (to low order),

$$\begin{aligned} S_{had}[\bar{N},N,\ldots,\pi,\rho,\omega,\ldots] = & \int d^{4}x Tr\left\{\bar{N}\left(\gamma.\partial + m_{0} + \Delta m_{0} - m_{0}\sqrt{2}i\gamma_{5}\pi^{a}T^{a} + \ldots\right)N\right\} \\ & + \int d^{4}x\Bigg[\frac{f_{\pi}^{2}}{2}[(\partial_{\mu}\pi)^{2} + m_{\pi}^{2}\pi^{2}] + \frac{f_{\rho}^{2}}{2}[-\rho_{\mu}(-\partial^{2})\rho_{\mu} + (\partial_{\mu}\rho_{\mu})^{2} + m_{\rho}^{2}\rho_{\mu}^{2}] \\ & + \frac{f_{\omega}^{2}}{2}[\rho\rightarrow\omega] - f_{\rho}f_{\pi}^{2}g_{\rho\pi\pi}\rho_{\mu}.\pi\times\partial_{\mu}\pi - if_{\omega}f_{\pi}^{3}\epsilon_{\mu\nu\sigma\tau}\omega_{\mu}\partial_{\nu}\pi.\partial_{\sigma}\pi\times\partial_{\tau}\pi \\ & - if_{\omega}f_{\rho}f_{\pi}G_{\omega\rho\pi}\epsilon_{\mu\nu\sigma\tau}\omega_{\mu}\partial_{\nu}\rho_{\sigma}.\partial_{\tau}\pi \\ & + \frac{\lambda i}{80\pi^{2}}\epsilon_{\mu\nu\sigma\tau}Tr\left(\pi.F\partial_{\mu}\pi.F\partial_{\nu}\pi.F\partial_{\sigma}\pi.F\partial_{\tau}\pi.F\right) + \ldots\Bigg]. \end{aligned} \quad (4.31)$$

this equation is reproduced here for the sake of completeness, for this reason we shall not list the new notation used in this action, details can be found in the review article [97] or PhD thesis [205].

However, even without the details of the notation, when we compare this expression to that of the QCD action discussed above we see a vast difference in the complexity of the two equations. Equation (4.31), which is is only a low order expansion suggests that there is a very rich set of behaviour evident in the behaviour of the nucleon; it is an extremely complex system, but its description is well approximated by this theory. Thus, some very complicated emergent behaviour has been extracted using this technique. Is it possible that this technique could be generalised in some way? We shall explore this possibility in more detail in section 5.3.5.

4.3.3 The Derivation of the Iterator Equation

The richness of the behaviour exhibited by this model of hadrons led to the hypothesis that it may be possible to regain much or the dynamics of this model, if it had not been already incorporated into the model axiomatically. This derivation takes as its starting point an action that was derived for a bilocal field theory in an early GCM paper [96][14]

$$S[B] = -TrLn\left[\partial\!\!\!/\delta(x-y) + \frac{M^{\theta}}{2}B(x-y)\right] + \int d^4x d^4y \frac{B(x,y)B(y,x)}{2g^2 D(x-y)}. \tag{4.32}$$

Setting any variables that are deemed irrelevant equal to 1, and dropping the gluon interaction term $D(x-y)$ and setting the variables of the B terms in the integral term as equivalent (*i.e.* assuming no significant interaction between the fields at this level of the description) results in a simpler action:

$$S[B] = TrLn\left[\mathbf{1} + B\right] + \int d^4x d^4y B(x,y)^2. \tag{4.33}$$

Now, making a transition to a lattice representation in place of the continuous representation above, we obtain

$$S[B] = B^2 + TrLn\left[\mathbf{1} + B\right]. \tag{4.34}$$

The next step makes use of stochastic quantization, we shall briefly outline this procedure and then continue with our derivation of equation (4.1).

Stochastic Quantization

The stochastic quantisation procedure of Parisi and Wu [316, 199] provides the final step in the derivation of (4.1). This procedure is based upon the observation that the quantum correlation functions $\mathcal{G}$ obtained from equation (4.16) can be obtained in the following alternative manner.[15]

1. Introduce a 5th time τ in addition to the usual 4 space-time points x_μ. Postulate that the dynamics of the field ϕ in this extra time τ is given by the Langevin equation:

$$\frac{\partial\phi(x,\tau)}{\partial\tau} = -\frac{\delta S[\phi]}{\delta\phi} + \eta(x,\tau), \tag{4.35}$$

[14] This paper was an early attempt at using FIC to bosonize QCD but made use of a less appropriate change of variables than the later work utilizing this technique. The change of variables used here was to $\mathbf{1}_c$ and $\mathbf{8}_c$ bilocal $\bar{q}q$ variables which worked well in the extraction of meson observables but was less physical, due to the $\mathbf{8}_c$ fields which are repulsive for for $\bar{q}q$ states. The later work was based upon $\mathbf{1}_c$ meson variables coupled with $\bar{\mathbf{3}}_c$ and $\mathbf{3}_c$ diquark variables [90]. The less physical equation was used as a basis for the derivation because it is slightly simpler than its equivalent in the later work (equation (4.32)) but still results in rich behaviour.

[15] For the sake of simplicity the following review of stochastic quantization shall examine situations in which we are interested in the dynamics of only one simple field ϕ, the reader is referred to the references for information regarding more complicated scenarios.

where η is a Gaussian random variable satisfying,[16]

$$\langle \eta(x,\tau)\rangle_\eta = 0, \tag{4.36}$$

$$\langle \eta(x,\tau)\eta(x',\tau')\rangle_\eta = 2\delta(x-x')\delta(\tau-\tau'), \tag{4.37}$$

$$\langle \eta\cdots\eta\rangle_\eta = 0. \tag{4.38}$$

2. Next the stochastic average of all fields ϕ_n satisfying equation (4.35) is evaluated:

$$\langle \phi_\eta(x_1,\tau_1)\phi_\eta(x_2,\tau_2)\cdots\phi_\eta(x_l,\tau_l)\rangle_\eta. \tag{4.39}$$

3. Finally we set $\tau_1 = \tau_2 = \cdots = \tau_l$ in equation (4.39), and take the limit $\tau_1 \to \infty$. Parisi and Wu proved perturbatively that this limit of the average is equal to the correlation function of the field of interest, *i.e.* a l-point correlation can be defined:

$$\lim_{\tau_1\to\infty}\langle \phi_\eta(x_1,\tau_1)\phi_\eta(x_2,\tau_1)\cdots\phi_\eta(x_l,\tau_1)\rangle_\eta = \frac{\int \mathcal{D}\phi\ \phi(x_1)\phi(x_2)\ldots\phi(x_l)e^{-S[\phi]}}{\int \mathcal{D}\phi e^{-S[\phi]}}. \tag{4.40}$$

We make use of this procedure to derive equation (4.1), substituting the action in equation (4.32) into the Langevin equation (4.35) to obtain

$$\frac{\partial B(x^\mu,\tau)}{\partial\tau} = -\frac{\delta S[B]}{\delta B} + \eta(x^\mu,\tau). \tag{4.41}$$

So under the external time τ parameter, and changing the symbol η to ω, an equivalent noise term which operates over the matrix representation of the B field will update:

$$B_{ij} \longrightarrow B_{ij} - \frac{\delta S[B_{ij}]}{\delta B_{ij}} + \omega_{ij}. \tag{4.42}$$

Referencing the action in equation (4.34), we make use of the calculus identities $\delta B^2 \to B$ and $\delta lnB \to B^{-1}$ to obtain the desired equation (4.1). ■

It is worth emphasising that the use of the stochastic quantization procedure in this process suggests that a system emergent from this equation should exhibit quantum behaviour in the limit of sufficient numerical experiments that run for a long enough time. Note also that the derivation is not exact, (extraction might perhaps be a more appropriate choice of term) the iterative equation (4.1) is not computationally equivalent to the Bilocal action 4.22 in the GCM, rather there is almost a sense of observation in the derivation itself; we choose which variables to consider relevant and which to ignore.

It is remarkable that despite the dramatic loss of structure that has been achieved in this derivation, much richness of behaviour is retained. In this case, it appears that a set of key characteristics are behind this behaviour, namely the nonlinear aspect of

[16] Angled brackets are intended to denote connected averages with respect to the variable η

the equation, its grossly 'nonlocal' and holistic form, as well as the noise that drives its behaviour. Most important perhaps is the structure of the general equation, it is close enough to the bilocal action that it incorporates its key physical properties in some way. It is expected that there will be a class of equations all of which exhibit behaviour of this form, but at this point in time (4.1) is the only one known.

Chapter 5

Generalising Quantum Theories

This chapter will begin with a short summary of the arguments presented so far:

- Chapter one briefly examined the state of the technique of reductive analysis as it is generally applied to what might be termed 'complex systems'. It postulated a scale of complexity, defined in terms of the success of reduction in the analysis of the system under consideration, and discussed a number of highly complex systems that have so far defied such an analysis. The contextual dependence of these systems upon their environment was identified as one of the contributing factors to this failure, as was an insistence upon object based methods.
- Chapter two discussed a specific example of this failure, the attempt to create some form of artificial life. It was claimed that the failure of work performed within the field of ALife to generate complex emergent behaviour is due to the general lack of complexity of the simulations themselves.
- Complex systems often manifest in a hierarchical form, and yet ALife simulations do not exhibit emergent hierarchical behaviour, therefore, chapter three turned to an examination of hierarchical systems, concepts and theories. These are often an aspect of complexity which is poorly understood and not well modelled by present day techniques. A number of ideas that have been developed in order to discuss these systems, but theoretically this field is not well developed. In particular, the dynamical generation and evolution of emergent hierarchical structures appears to be a problem that has eluded theoretical understanding.
- Chapter four started to discuss a new modelling methodology, Process Physics, which displays some very interesting emergent behaviour. The origins of this methodology lie with an example system that is characterised dynamically by equation (4.1). Although the analysis of this system is still in its early stages, and is both computationally and analytically very difficult, there are a number of promising results already suggesting that this system is capable of dynamically generating the emergent hierarchical structures missing in our current modelling of complex systems. The origins of this system were examined in an attempt to understand the characteristics that are contributing to this emergent behaviour. Its historical connection with the Global Color Model, a model of QCD which generates hadronic behaviour

was discussed. This historical basis, as well as the fact that stochastic quantisation was used in the extraction of (4.1), suggests that even though the system is not obviously quantum mechanical, the roots of its interesting emergent behaviour might lie in its quantum background. The suspicion arises that quantum models are perhaps more general than has traditionally been assumed to be the case.

Thus, we have reached a turning point in this thesis. Having clarified some of the reasons why complex emergent systems have not been well modelled to date, and also having identified one model that appears capable of exhibiting complex emergent behaviour, we are now in a position to start a more detailed examination of the reasons behind the success of this model, and to extend its general methodology to other systems.

This chapter will examine quantum models. Since there is a strong connection between quantum models and the iterative system discussed in section 4.1, it is possible that quantum theories are more generally applicable than has generally been assumed to be the case; that they may be well equipped to deal with high end complexity. Some key characteristics of these theories will be discussed in more detail with the objective of extracting a description of a general class of complex behaviour.

5.1 Bell's Theorem and Contextuality

> *In view of the locality theorems as well as their violation by the modern experimental results, which were not available when the orthodox interpretation of quantum mechanics was invented, some physicists conclude triumphantly: Bohr was right!, while others will claim with the same enthusiasm, Bohr was wrong! Both these opinions make sense, depending on what aspect of the debate one wishes to favor.*
>
> Laloë, p672, [255]

The essential difference between quantum and classical systems is one of contextuality:

- classical systems can generally be completely separated from their environment whereas quantum systems cannot,
- the behaviour of classical systems during measurement does not depend upon the measurement setting; there are *elements of reality* that can be objectively measured, but this is not necessarily the case with quantum systems.

Much of the so-called mystery surrounding quantum behaviour [395, 202, 430, 397, 267] stems from an inappropriate expectation that classical concepts will apply just as readily in the quantum world. This section will examine some of the core differences between classical and quantum behaviour.

5.1.1 The Assumptions Behind Classical Ontologies

> *That so much follows from such apparently innocent assumptions leads us to question their innocence.*
>
> Bell, [58], p8 in [64]

A number of assumptions always lie behind any theory that we formulate. Often these assumptions are hidden, and we do not even identify their existence until a new, unexplainable phenomenon mandates their extension or even replacement. This was indeed the case with the classical mechanics of Newton which was so successful that a number of the assumptions of that theory acquired the status of in-controvertable truths about the nature of reality. With the discovery of quantum mechanical systems it became necessary to rethink many of these assumptions. Yet, it is likely that in the case of the nonlocal phenomena exhibited by entangled quantum systems this process of rethinking and reanalysis has not yet been carried far enough.

Elements of reality

Inherent within our understanding of classical systems is the notion of pre-existing *elements of reality*. This aspect is even reflected by the terminology used; measurements *measure* the status of a system, they do not influence the result of the measurement itself.[1] When looking at a classical system, measurements reveal a pre-existing reality. This idea was so fundamental to physics that it was unquestioned, acquiring the status of an assumption about the nature of all physical systems. The status of this notion can be seen within the debate that emerged surrounding the completeness of quantum mechanics as a physical theory.

This debate started with the now famous EPR (Einstein, Podolsky and Rosen) paper [164], which provides a good illustration of this assumption. EPR considered a *gedanken-experiment*, involving a quantum state entangled over position/momentum variables. Instead of making use of these continuous variables, most discussions of the EPR argument now make use of the discrete spin variables first introduced by Bohm [75], a conceptual simplification which makes the experiment easier to treat mathematically. The modified experiment is illustrated in figure 5.1. Consider a molecule of total spin-0 which is allowed to decay, ejecting two atoms of opposite spin-$\frac{1}{2}$. These are then allowed to separate far enough apart that they might be considered to have ceased their interaction *i.e.* they are spacelike separated. If each atom is sent through a Stern–Gerlach magnet, then depending upon the spin of that atom, it will travel up or down to one of the two detectors illustrated on each side of the figure.

EPR explicitly formulated a number of assumptions about the nature of reality that had until the invention of quantum mechanics been uncontested, and then used the above

[1] Although they may influence the state of the system that would be found were another measurement performed after the initial measurement.

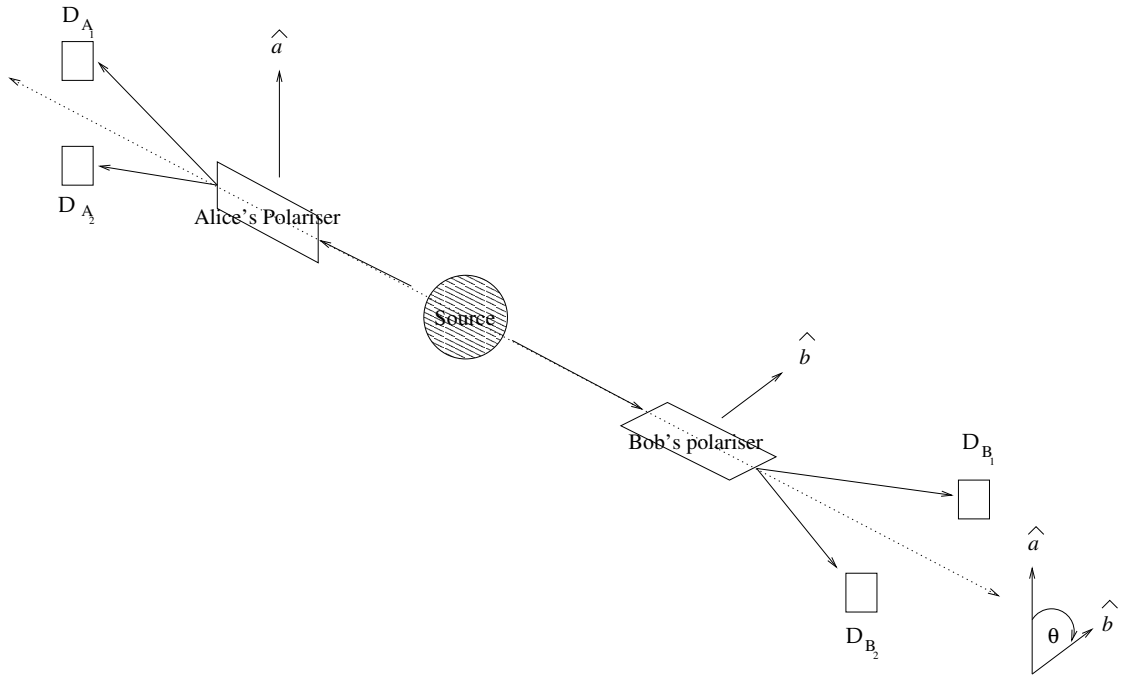


Figure 5.1: The EPR experiment. Two entangled spin-$\frac{1}{2}$ atoms separate, and then propagate through two Stern–Gerlach polarizers, oriented in the directions $\hat{a}$ and $\hat{b}$, chosen by the experimentalists Alice and Bob respectively. Depending upon which detector records a 'click', Alice and Bob can make statements about the spin of their particle, and because of entanglement, the associated spin of their partners particle. Each experimentalist is positioned at a spacelike separated distance.

experimental scenario to show that these assumptions led to an apparent contradiction if quantum mechanics was considered to be a fundamental theory.

First, EPR defined a well accepted notion of realism with the introduction of what they termed an element of physical reality:[2]

> *If, without in any way disturbing a system, we can predict with certainty (i.e., with probability equal to unity) the value of a physical quantity, then there exists an element of physical reality corresponding to this physical quantity.*

and they made an explicit assumption about the completeness of a physical theory:

> *every element of the physical reality must have a counterpart in the physical theory.*

EPR suggested that the success of a physical theory should depend upon this completeness, as well as its correctness; a successful theory should not predict phenomena that do not occur. Both of these assumptions were made as a result of the successes of classical theory; a theory so successful that the assumptions at its base were specified in the EPR

[2]It is important to realize that EPR considered the following conditions to be necessary but not sufficient definitions of realism and completeness.

paper as those that *must* be satisfied by any physical theory before it can be considered satisfactory.

These assumptions allowed EPR to formulate, in terms of the experiment in figure 5.1, the following argument.

Suppose the experimentalist on the left hand side (call her Alice) decided to measure the the spin in the $\hat{z}$ direction, σ_z. She would orient the polarizer in the $\hat{z}$ direction, and then wait for one of the two detectors to register a reading, perhaps she measures spin up. Now, because of the original correlation between the two atoms, Alice would be able to say that if the experimentalist on the right hand side (traditionally called Bob) were to measure σ_z also, then he would record an opposite value to Alice, in this case spin down. Bob does not even need to actually carry out this measurement. Alice can reason *counterfactually* [57, 424] about the effect of Bob making measurements as follows. Since the two particles are no longer interacting, and Alice knows the value of the spin in the $\hat{z}$ direction, then according to the EPR definition she is able to claim that there is an element of reality corresponding to Bob's particle on the right hand side of the experiment in the $\hat{z}$ direction. EPR reasoned that this element of reality must have existed *before* Alice's measurement took place, since the two particles are no longer interacting; the process of measurement could not have *affected* the particle on Bob's side in any way due to the spatial separation of the two arms of the experiment; Alice was simply finding the value of a pre-existing element of reality. This means that were Alice to choose to re-orient her polarizer and obtain a reading in a different arbitrary direction, then she would know more about the system than is permitted by quantum mechanics. For example, she could know the spin of Bob's particles in both the $\hat{x}$ and the $\hat{z}$ directions, something which is not permitted in quantum mechanics due to the uncertainty relation. EPR concluded that the wavefunction does not describe all of physical reality, and postulated the existence of *hidden variables* which would be understood and described by the new, complete theory.

Separability

The EPR argument makes use of another assumption, that of separability.

If two parts of a classical system interact and then separate, then a measurement performed upon one part at a later point in time will not affect the other part of the system. The two parts are entirely *separable* after their interaction. While their behaviour might be correlated due to the interaction, any subsequent interactions will not affect the distant member of the pair. In fact this behaviour is so accepted that one set of classical theories, statistical mechanics and thermodynamics, is founded upon it.

As a simple example, consider two billiard balls, one red and one white, which collide and then move away from each other. If the white ball collides with a black ball at a later point in time, this will not affect the behaviour of the red ball as it moves around the billiard table. The two balls, after their brief interaction have separated.

This *separability* of classical systems has a mathematical form. Their state space is generally a Euclidean phase space which represents all possible momentum and position information. A classical system with N point particles will have a state space of size $6N$.[3] In addition, classically there is no difference between the state space and the list of possible measurement outcomes; if a configuration is listed in a state space, then it will be possible to find the system in that configuration under a measurement. This is not so with quantum theories, a state space that consists of just the system itself, and ignores the states of the relevant measurement apparatus, does not provide a sufficient description of quantum systems. The separable aspect of classical systems makes their description simpler; allowing us to make approximations that simplify it. For example, when describing a system of many components, separability allows us to treat only the interactions of current interest, neglecting others as not relevant.

This notion of separability features heavily in the EPR argument but was not emphasised in the original paper. Bohr highlighted the problems with this assumption of separability in his response to EPR [76]. Pointing to what he claimed were a number of mistakes in the EPR argument, Bohr claimed that it was impossible to consider the orientation of the measurement apparatus in isolation from the system being analysed. Thus, Bohr pointed to what might now be termed the *contextuality* of quantum systems, but did not analyze this feature of quantum systems in any detail. For a number of years the general response to EPR fell into one of following two categories. The majority of physicists accepted Bohr's reply. They generally acknowledged that while the debate surrounding the EPR argument was philosophically important, it was not particularly testable. However, a small but significant minority of researchers remained dissatisfied. While a number of attempts were made to show that hidden variable interpretations of quantum mechanics were not possible [446, 231, 192], these did not completely rule out simple hidden variable interpretations [58, 287]. It was not until almost 30 years later that the EPR argument was suitably developed, by John Bell, into a new form which moved the discussion of hidden variables, locality and separability out of philosophy and into a situation where experimental predictions could be formulated.

Bell explicitly represented EPR's hidden variables with a parameter λ, and introduced a mathematical identity to represent the separability assumption that the result of measuring the spin in some direction $\hat{a}$, in Alice's region $\mathcal{A}$ should not depend on the setting of the analyser at Bob's region $\mathcal{B}$ and vice versa. Thus Bell assumed that the expectation value P, of the outcomes of the experiments in the two spatially separated regions should be *factorisable*, that is,

$$P(\hat{a},\hat{b}) = \int d\lambda\ \rho(\lambda)\ A(\hat{a},\lambda)B(\hat{b},\lambda). \tag{5.1}$$

[3]Each particle can be completely represented at a given point in time by 3 space coordinates and 3 momentum coordinates.

Where $\rho(\lambda)$ is the probability distribution of the hidden variables λ, $A(\hat{a}, \lambda)$ represents the result of a measurement at $\mathcal{A}$ where the analyser is set to measure the $\hat{a}$ component of the spin of the atom at A and similarly for $B(\hat{b}, \lambda)$. This factorization condition amounts to an explicit representation of the separability of Alice and Bob's experiment. Bell used this condition in an analysis of the EPR experimental arrangement, and obtained what is now termed *Bell's inequality*, which is a condition that must be satisfied by any system satisfying (5.1), *i.e.* any separable system. He then showed that the expectation value for the EPR experiment obtained from a quantum mechanical calculation does not fall within the bounds set by his inequality.

5.1.2 Bell's Theorem and Nonlocality[4]

Bell's theorem defines a relationship between the statistical outcomes of three individual experimental arrangements. Experiments are again conducted by our two experimentalists Alice and Bob (recall figure 5.1), but considering any possible orientation rather than just the mutually perpendicular ones necessary for the application of the uncertainty relation in EPR-type arguments. In the first experiment, Alice orients her analyzer along direction $\hat{a}$, Bob lines his up along $\hat{b}$, and together they obtain an outcome $O_{\mathrm{EXP}}(\hat{a}, \hat{b})$ (say Alice obtains a click in detector D_{A_1} and Bob finds that D_{B_1} clicks). In the next experiment, Alice leaves her analyzer oriented along the same direction, Bob rotates his analyzer to a new direction $\hat{c}$ and they find the value for $O_{\mathrm{EXP}}(\hat{a}, \hat{c})$. In the last experiment, Alice rotates her analyzer along $\hat{b}$ and they measure $O_{\mathrm{EXP}}(\hat{b}, \hat{c})$. This set of experiments is repeated many times, and a statistical description of the results obtained, giving the expectation values $P_{\mathrm{EXP}}(\hat{a}, \hat{b})$, $P_{\mathrm{EXP}}(\hat{a}, \hat{c})$ and $P_{\mathrm{EXP}}(\hat{b}, \hat{c})$.

Bell's theorem is a restriction on the theoretical expectation values that can be exhibited by any hidden variable theories satisfying the separability assumption (5.1) for this scenario:

$$|P_{\mathrm{HV}}(\hat{a}, \hat{b}) - P_{\mathrm{HV}}(\hat{a}, \hat{c})| \leq 1 + P_{\mathrm{HV}}(\hat{b}, \hat{c}). \tag{5.2}$$

There are numerous variations of this basic theorem,[5] this section will discuss first the proof of Bell's theorem itself, and then conclude with a short summary of the current status of experimental tests of Bell-type inequalities, as well as some more recently derived Bell-type theorems. We shall see that a number of limits have been placed upon the form that can be taken by any physical hidden-variable type theory.

[4] In what follows we shall avoid confusion by explicitly specifying expectation values with a subscript. This will identify between hidden variable (HV), quantum mechanical (QM) and experimentally observed (EXP) values.

[5] An excellent modern review of both the history and current experimental status of Bell-type inequalities, as well as their consequences for many interpretations of quantum mechanics is provided by the review article [255].

Proof of Bell's theorem

Expressing the responses of Alice's analyser by

$$A = \begin{cases} +1 \text{ if Alice's analyser determines 'spin up'} \\ -1 \text{ if Alice's analyser determines 'spin down'} \end{cases} \tag{5.3}$$

and similarly, the responses of Bill's analyser by

$$B = \begin{cases} +1 \text{ if Bill's analyser determines 'spin up'} \\ -1 \text{ if Bill's analyser determines 'spin down'.} \end{cases} \tag{5.4}$$

The assumption made by EPR that Alice's measurements cannot affect the outcomes of measurements performed by Bob, that of *locality* is mathematically expressible in terms of the results A and B as

$$\begin{aligned} A = A(\hat{a}, \lambda) \text{ and not } A = A(\hat{a}, \hat{b}, \lambda) \\ B = B(\hat{a}, \lambda) \text{ and not } B = B(\hat{a}, \hat{b}, \lambda), \end{aligned} \tag{5.5}$$

that is, Alice's result has no dependency upon results in Bob's area, and *vice versa*.

If $\rho(\lambda)$ is the normalised probability distribution of the hidden variable, λ, that is

$$\int \rho(\lambda) d\lambda = 1, \tag{5.6}$$

then our ignorance about the actual values taken by the hidden variables forces us to integrate over all possible λ's,

$$P_{\mathrm{HV}}(\hat{a}, \hat{b}) = \int d\lambda \rho(\lambda) A(\hat{a}, \lambda) B(\hat{b}, \lambda). \tag{5.7}$$

In the EPR scenario, when the two analysers are parallel Alice and Bob will receive opposite readings

$$A(\hat{a}, \lambda) = -B(\hat{a}, \lambda). \tag{5.8}$$

This allows us to consider three arbitrary orientations of the analysers, $\hat{a}$, $\hat{b}$ and $\hat{c}$, and to write down the relationship

$$P_{\mathrm{HV}}(\hat{a}, \hat{b}) - P_{\mathrm{HV}}(\hat{a}, \hat{c}) = \int \left[A(\hat{a}, \lambda) B(\hat{b}, \lambda) - A(\hat{a}, \lambda) B(\hat{c}, \lambda) \right] \rho(\lambda) d\lambda \tag{5.9}$$

$$= -\int \left[A(\hat{a}, \lambda) A(\hat{b}, \lambda) - A(\hat{a}, \lambda) A(\hat{c}, \lambda) \right] \rho(\lambda) d\lambda \tag{5.10}$$

using (5.8) in the substitution. Now, since from (5.3) and (5.4) the results A and B can only be ± 1,

$$\left[A(\hat{b}, \lambda) \right]^2 = 1. \tag{5.11}$$

Which makes it possible to factor (5.10),

$$P_{\mathrm{HV}}(\hat{a},\hat{b}) - P_{\mathrm{HV}}(\hat{a},\hat{c}) = \int A(\hat{a},\lambda)A(\hat{b},\lambda)\left[1 - A(\hat{b},\lambda)A(\hat{c},\lambda)\right]\rho(\lambda)d\lambda. \tag{5.12}$$

Finally, we note that the outcome of any of the above two measurements can be only ± 1, therefore in particular the factor $A(\hat{a},\lambda)A(\hat{b},\lambda) \leq +1$, which allows it to be dropped from equation (5.12), giving

$$|P_{\mathrm{HV}}(\hat{a},\hat{b}) - P_{\mathrm{HV}}(\hat{a},\hat{c})| \leq \left|\int \left[1 - A(\hat{b},\lambda)A(\hat{c},\lambda)\right]\rho(\lambda)\right| d\lambda. \tag{5.13}$$

Now, applying the normalisation condition (5.6), and making use of the EPR result (5.8) leads to

$$\begin{aligned} |P_{\mathrm{HV}}(\hat{a},\hat{b}) - P_{\mathrm{HV}}(\hat{a},\hat{c})| &\leq 1 + \int A(\hat{b},\lambda)B(\hat{c},\lambda)\rho(\lambda)d\lambda \\ &\leq 1 + P_{HV}(\hat{b},\hat{c}). \end{aligned} \tag{5.14}$$

Which, on comparison with (5.2), is seen to be Bell's theorem. ■

It is important to realise that while they are closely associated concepts, locality and separability are not quite the same. This was apparent in the above proof, where locality is represented by condition (5.5) and separability by (5.1). Separability is the more general condition. If a system is not separable then it exhibits a contextual dependency upon some aspect of its environment (which may include measurement apparatus, other parts of the official system, or even a part of the universe which was officially designated as not of interest). Nonlocal systems form a subset of this aspect of inseparability; a nonlocal system is one which exhibits nonseparable behaviour over spacelike distances.

Testing Bell's inequality

The quantum mechanical prediction is simple to obtain [202] the result alone will be reproduced here.

If two spin-$\frac{1}{2}$ particles are emitted from a source in a singlet spin state and propagate in opposite directions, then the wavefunction of the entangled spin state is

$$|\psi\rangle = \frac{1}{\sqrt{2}}\left(|+-\rangle - |-+\rangle\right). \tag{5.15}$$

When they reach distant locations, they are then submitted to spin measurements, as per the EPR arrangement in figure 5.1, with orientations pointing in the $\hat{a}$ and $\hat{b}$ directions. For an angle θ between the orientations of the polarisers in the two regions $\hat{a}$ and $\hat{b}$, quantum mechanics predicts that the probability for a double detection of results $+1,+1$ or $-1,-1$ is

$$P_{QM_{++}} = P_{QM_{--}} = \sin^2\theta \tag{5.16}$$

while the probability of two opposite results is

$$P_{QM_{+-}} = P_{QM_{-+}} = \cos^2\theta. \tag{5.17}$$

Early tests of Bell-type inequalities were hampered by the search for a suitable source of entangled particles. Many were inadequate due to the high energies of the entangled states that were used (e.g. rather than using photons one experiment used gamma rays), which were not efficiently transmitted through the polarisers being used at the time. It was necessary to correct the results obtained using the very quantum theory that the experiments were intended to test [237, 257]. What are now termed the Aspect experiments [39, 40, 38] are generally seen as the first direct tests of Bell-type inequalities.

These experiments made use of a new source, a calcium cascade, which emitted entangled photons rather than the molecules discussed above [172]. Individual calcium atoms were excited by lasers to an unstable state, which then decayed emitting two entangled photons. The photons were then sent through what was in essence the same experimental arrangement as illustrated in figure 5.1, (albeit with the necessary modifications to analysers and detectors in order to cater for photons) in a set of experiments that gradually increased in their sophistication, in order to eliminate a series of local hidden variable theories that made use of loopholes in the earlier experiments [255, 202].

Coincidence counters were used to ensure that counted photons were incident upon a detector within a window of another photon hitting the opposite detector and were therefore emitted by the source rather than being noisy random events, but this made the arrangement of the experiment more complex. The first experiment [39] simplified this problem through the use of a single-channel analyser *i.e.* only photons parallel to the transmission axis of the analyser were counted, those that were perpendicular were ignored. The second experiment [40] used a two-channel analyser which allowed for direct comparison with the Bell inequalities. Finally, a loophole concerning the possibility that signals at or below the speed of light were being sent between the analysers and/or source was closed through the use of time-varying analysers [38].

In each experiment, very clear plots were obtained revealing that as the angles between the analysers were varied, coincidence counts were found that were in very good agreement with quantum mechanics, and in violation of the predictions of Bell's theorem.[6] These results have been extended by a recent experiment [434] which has demonstrated violations of Bell-type inequalities at distances of more than 10 kilometres.

Thus there is some sense in which quantum mechanical systems exhibit both *nonseparable* and *nonlocal* behaviour; it is not always possible to consider a spatially separated, but entangled pair of photons to be separated. In particular, it is not appropriate to consider

[6] There are a number of very good introductions to these experimental results [202, 397, 430], as well as the original papers themselves. This work will not rehash these details, rather, we shall move onto a discussion of the implications of this result.

two separate particles from an entangled state to have their own individual properties, the context in which one particle is found can affect the other.

It is important to appreciate that hidden variable interpretations of quantum mechanics are still possible (one such interpretation will be discussed in section 5.6.1), but that the nature of such theories has been substantially changed by these results. Bell-type inequalities and their violation by experiment have drastically altered our possibilities when we attempt to understand the nature of physical reality.

There is one other group of Bell-type theorems which deserves mention; the Greenberger–Horne–Zeilinger (GHZ) [201, 200] and Hardy [209, 210, 211] *gedankenexperiments*, which demonstrate Bell-type results without the direct use of inequalities for entangled states of three and two particles respectively. These two results hinge on the construction of situations where instead of mathematical constraints on expectation values, individual events are predicted by quantum mechanics which are impossible according to local realism. The GHZ result is particularly strong, it obtains a direct contradiction between quantum mechanics and local realism. If the contradictory case occurs then either one or the other of the two theories will have been validated depending upon the outcome. Neither of these experiments have been realised as yet, but experimental technology is improving to the point where they will be in the near future [254, 268, 370], and very few researchers doubt that quantum theory will provide the correct prediction.

5.1.3 Nonlocality and Relativity

The nonlocality of quantum mechanics poses potentially serious problems when considered in conjunction with the theories of relativity but there is a wide range of views on this subject.

Most researchers adopt a stance claiming that although nonlocality is a very real phenomenon, it does not violate relativity in any meaningful sense. Usually, the claim is made that it is impossible to use this phenomenon to send *signals* between Alice and Bob [395, 394, 393, 283, 187, 179, 180, 202]. This is because while a subtle quantum mechanism is linking their results, this mechanism does not allow either Alice or Bob to *control* the results. Each experimentalist will record a series of perfectly random results regardless of how they try to affect the outcomes of the other. Shimony coined the rather notorious term *passion at a distance* [391], in contrast to the more straightforward and unallowed by relativity, action at a distance, to describe this weaker sense of interaction between entangled particles.

While this conclusion is not necessarily wrong, it is possibly premature. This point has been taken up by Maudlin who has written a comprehensive work [283] examining the implications that quantum non-locality may have on the theories of relativity. He makes the pertinent observation that while relativity is generally understood as restricting something from going faster than light, there are many different candidates for what

that something actually is. Maudlin identifies four general possibilities: Matter or energy, Signals, Causal Processes, and Information, and discusses the way in which each interacts with the phenomenon of quantum nonlocality. He finds that while the violation of Bell's inequality does not require neither superluminal matter or energy transport nor the transmission of superluminal signals, the phenomenon of nonlocality requires superluminal effects behind both the causal processes and the transmission of information. Thus, according to Maudlin there is room for a peaceful coexistence [393, 390] between quantum mechanics and relativity. For example, a perfectly consistent theory could be constructed if relativity was taken as forbidding superluminal signalling.

Maudlin also makes the observation that the most conservative interpretation of relativity, that theories must be Lorentz invariant, is compatible with every one of the above four processes occurring faster than light. He undertakes a detailed examination of Lorentz invariance, during which he finds that a number of different mechanisms are available by which Lorentz invariant theories can consistently violate the Bell inequalities. Specific examples of such theories include ones that entail explicit backwards causation [143], hyperplane dependence [170, 185, 27, 153], even many minds interpretations [29, 28] which maintain their Lorentz invariance by denying a violation of the Bell inequalities occurs. Maudlin also shows that it is possible to formulate a Lorentz invariant and yet nonlocal theory that makes use of a preferred reference frame undetectable by any means. One of these, the Quantum State Diffusion (QSD) model [328], shall be discussed in section 5.6.1. This model has been extended and applied within Process Physics in the high level QHFT which will be discussed in section 6.4.1.

In a work spanning decades, Stapp has been attempting to prove that quantum theory *is* nonlocal, rather than the more commonly accepted interpretation that any hidden variable type replacement of quantum mechanics must be [418, 419, 420, 421, 422, 130, 131]. This attempt has more recently attained some notoriety as the *nonlocal character of quantum theory* debate [423, 288, 424, 392, 425, 426, 396]. While opinion is still rather divided over whether Stapp has achieved this goal, this debate has served to clarify both terminology and concepts often used in debates about the meaning of quantum theory. For example, the meaning of counterfactuals has been extensively debated in this series of papers. The most recent formulation of this attempt has taken the form of the following theorem:

> *Suppose a theory or model is compatible with the premises:*
>
> - *Free Choices: This premise asserts that the choice made in each region as to which experiment will be performed in that region can be treated as a localized free variable.*
> - *No Backward in Time Influence: This premise asserts that experimental outcomes that have already occurred in an earlier region can be considered*

to be fixed and settled independently of which experiment will be chosen and performed later in a region spacelike separated from the first.

- *Validity of Predictions of quantum theory (QT): Certain predictions of quantum theory in a Hardy-type experiment are valid.*

Then this theory or model violates the following Locality Condition: The free choice made in one region as to which measurement will be performed there has, within the theory, no influence in a second region that is spacelike separated from the first.

Stapp, p300, [426]

This is a very similar result to that obtained by Hardy himself [209], although it has been directly criticised by Shimony [396]. The ongoing debate surrounding Stapp's work involves some very delicate distinctions between logical statements, and differing definitions of vital concepts such as counterfactuals, and for this reason will not be discussed here (consult the above references for details). Instead, we shall mention two different approaches towards this debate, one stronger than the other, but both of which are in keeping with the current work. Firstly, one could adopt the stance that Stapp has adequately illustrated the nonlocal nature of quantum theory. A second, weaker claim is also possible; while the nonlocality of quantum mechanics has not yet been proven, any hidden variable theory wishing to reproduce the results of quantum theory must contain nonlocal aspects. In the end, the above debate misses the point, there are nonlocal elements in the *formalism* of quantum mechanics, even if it is still possible to claim that at a fundamental level, quantum systems are not nonlocal. Both the wave packet reduction postulate, and the very state vector itself exhibit highly nonlocal properties; they extend over all available states, and changes to both of them must occur in some instantaneous[7] sense This point has also been expressed by Laloë:

. . . even if one can discuss whether or not quantum mechanics is local or not at a fundamental level, it is perfectly clear that its formalism is not. . .

Laloë, p675, [255]

and by Goldstein:

. . . in recent years it has been common to find physicists. . . failing to appreciate that what Bell demonstrated with his theorem was not the impossibility of Bohmian mechanics, but rather a more radical implication — namely nonlocality — that is intrinsic to quantum theory itself.

Goldstein, [195]

[7]The Tittle *et al.* experiment [434] shows that instantaneous effects must occur over intervals of at least 10km and very few researchers doubt that if cohesion can be maintained over longer distances then the same results will be found.

In aiming to extend the quantum formalism to the modelling of complex systems, we are applying the formalism, and must accept that there is some sense in which that formalism has nonlocal aspects, and, at the very least is in violation of the spirit of relativity, if not its experimental predictions.

This is by no means an unusual conclusion, even if it is in the minority and rather unpopular. Researchers such as Einstein with his notion of "spooky action at a distance", Bohm [73, 74], Bell [61], Hardy [209], Stapp [426] and Percival [332] have argued along these lines to a greater or lesser extent. However, some [209, 332] have drawn the much stronger conclusion that quantum mechanics implies a preferred reference frame (PRF), and is therefore in direct contradiction with one of the most inbuilt assumptions of relativity, even if it does not violate the predictions of that theory. In section 6.4.4 this result will be seen to spring naturally from a realistic interpretation of quantum mechanics which models the simple iterative Process Physics map (4.1) as a quantum foam. There, one of the often raised objections to realistic interpretations of quantum mechanics, that they imply a PRF, will be seen in a different light, as a natural consequence of Process Physics and explicable within that methodology.

5.1.4 Contextuality

> *The tacit assumption that a hidden-variables theory has to assign to an observable A the same value whether A is measured as as part of the mutually commuting set $A, B, C, \ldots$ or a second mutually commuting set $A, L, M, \ldots$ even when some of the $L, M, \ldots$ fail to commute with some of the $B, C, \ldots$, is called "noncontextuality".*
>
> Mermin, p811, [287]

A *contextual* system is one that cannot be separated from its surroundings. These might include the experimental arrangement itself, factors external to the apparatus (*i.e.* the environment surrounding the system and experimental apparatus), the history of other experiments performed upon the system, *etc.*. Such contextuality is often perceived as negative, leading to the loss of realism, but this is a far stronger claim than is justified. As was discussed above, contextual systems *do not have* pre-existing elements of reality. This means that if such a system is examined under two different contexts then a very different set of results may be obtained. Results are not merely discovering reality, there is a very real sense in which they might be creating some aspect of that reality. Quantum systems provide an example of this phenomenon, the different results obtained from different experimental arrangements can even be conflicting; one set of variables may not coexist with the other.

This quantum result has been demonstrated unequivocably in two independent proofs, a discrete one due to Kochen and Specker [249], and a continuous argument made by Bell

[58] one year earlier. (This work will therefore refer to the two general results as the Bell-KS theorem.) These two papers followed in, and generalised, a tradition of proofs started by von Neumann [446] who was broadly considered to have shown that hidden variable theories were impossible, but who made an inappropriate and too restrictive assumption about the additivity of expectation values in his proof [215, 58, 287], which over the years was rectified in more general proofs [231, 192]. All Bell-KS-type theorems rely upon showing that there exists a set of observables for which it is impossible to consistently assign an eigenvalue. This is despite the fact that the sets of observables are commuting. These proofs have historically involved "a moderately elaborate exercise in geometry" p804 [287], which has led to disinterest on the side of the general physics community, however, Mermin has discovered a number of simpler proofs, which "are so simple that even those physicists who regard such efforts as pointless can grasp the argument with negligible waste of time and mental energy." p804, [287].

Mermin's argument makes use of the Pauli matrices for two independent spin-$\frac{1}{2}$ particles σ^1_μ and σ^2_ν, where ν and μ represent any of the three different coordinate axes x, y and z. These matrices provide us with the well known set of relations:[8]

$$(\sigma^i_\mu)^2 = 1 \Rightarrow |\sigma^i_\mu|^2 = \pm 1 \tag{5.18}$$

$$\left[\sigma^1_\mu, \sigma^2_\nu\right] = 0 \text{ when } \mu \perp \nu \tag{5.19}$$

$$\{\sigma^i_\mu, \sigma^i_\nu\} = 0 \tag{5.20}$$

$$\sigma^i_x \sigma^i_y = i\sigma^i_z \tag{5.21}$$

for $i = 1, 2$. If we arrange the following set of nine observables in the following manner:

$$\begin{array}{ccc} \sigma^1_x & \sigma^2_x & \sigma^1_x\sigma^2_x \\ \sigma^2_y & \sigma^1_y & \sigma^1_y\sigma^2_y \\ \sigma^1_x\sigma^2_y & \sigma^2_x\sigma^1_y & \sigma^1_z\sigma^2_z \end{array} \tag{5.22}$$

then using the relations in (5.18)–(5.21) we can easily see that it is impossible to consistently assign values to all nine of these observables at once:

(a) Firstly, we note that the observables of each of the three rows and columns are mutually commuting. This is straightforward for the top two rows and first two columns, and follows for the bottom row, and rightmost column because in every case there is a pair of anticommutations. Thus for example, $\sigma^1_x\sigma^2_x - \sigma^1_y\sigma^2_y = \sigma^1_x\sigma^2_x - \sigma^1_x\sigma^2_x = 0$ from a simple double application of the anticommutation relation (5.20).

[8] See any good quantum mechanics textbook for a more detailed discussion of the Pauli matrices, for example [75], is a good source.

(b) It is also straightforward to show that the product of the three observables on the right hand column is equal to -1:

$$\begin{aligned}
\sigma_x^1\sigma_x^2\sigma_y^1\sigma_y^2\sigma_z^1\sigma_z^2 &= \sigma_x^1\sigma_y^1\sigma_x^2\sigma_y^2\sigma_z^1\sigma_z^2 && (5.23)\\
&= i\sigma_z^1 i\sigma_z^2\sigma_z^1\sigma_z^2 && (5.24)\\
&= -(\sigma_z^1)^2(\sigma_z^2)^2 && (5.25)\\
&= -1. && (5.26)
\end{aligned}$$

(c) Similarly, the products of the observables in the other two columns and all three rows is $+1$.

(d) Now, the values assigned to mutually commuting observables must satisfy any identities obeyed by the observables themselves, hence from (a) the product of the values assigned to each row and the first two columns must be $+1$ (from (b)), while that of the last column must be -1.

However (d) is impossible to satisfy, as the row identities require the product of all nine values to be $+1$, while the column identities require that it be -1. We have a contradiction, which means that the measurement of an observable depends upon other measurements made simultaneously; observables are not independent even if they commute. ■

This means that the requirement for commuting sets of observables is not strong enough to allow for a consistent assignment of eigenvalues. A direct result of this theorem it that it is impossible to completely describe a quantum system independently of an experimental arrangement. This result closely mirrors Bohr's reply to the initial EPR argument:

> *. . . the wording of the above mentioned criterion of physical reality proposed by Einstein, Podolsky and Rosen contains an ambiguity as regards the expression 'without in any way disturbing a system." Of course there is in a case like that considered no question of a mechanical disturbance of the system under investigation during the last critical stage of the measuring procedure. But even at this stage there is essentially the question of an* influence of the very conditions which define the possible types of predictions regarding the future behavior of the system. *Since these conditions constitute an inherent element of the description of any phenomenon to which the term "physical reality" can be properly attached, we see that the argumentation of the mentioned authors does not justify their conclusion that quantum-mechanical description is essentially incomplete. On the contrary this description, as appears from the preceding discussion, may be characterised as a rational utilization of all possibilities of unambiguous interpretation of measurements, compatible with the finite and uncontrollable interaction between the objects and the measuring in-*

struments in the field of quantum theory. In fact it is only the mutual exclusion of any two experimental procedures, permitting the unambiguous definition of complementary physical quantities which provides room for new physical laws, the coexistence of which might at first sight appear irreconcilable with the basic principles of science. It is just this entirely new situation as regards the description of physical phenomena, that the notion of complementarity *aims at characterizing.*

Bohr, p700, [76]

Many researchers have found Bohr's reply very difficult to comprehend. Indeed, Bell after a direct quote of much of the above passage went on to say

I have very little idea what this means. I do not understand in what sense the word 'mechanical' is used, in characterizing the disturbances which Bohr does not contemplate, as distinct from those which he does. I do not know what the italicized passage means — 'an influence on the very conditions...'. Could it mean just that different experiments on the first system give different kinds of information about the second? But this was just one of the main points of EPR, who observed that one could learn either *the position* or *the momentum of the second system. And then I do not understand the final reference to 'uncontrollable interactions between measuring instruments and objects', it seems just to ignore the essential point of EPR that in the absence of action at a distance, only the first system could be supposed disturbed by the first measurement and yet definite predictions become possible for the second system. Is Bohr just rejecting the premise — 'no action at a distance' — rather than refuting the argument?*

Bell, pp155–156, [64]

This is perhaps a little unfair of Bell, I think that Bohr was quite clearly pointing to the contextuality of quantum systems; claiming that it was impossible to compare the two experimental arrangements counterfactually, and that only one *or* the other might be considered. But whereas Bohr's reply was longwinded and difficult to understand,[9] we now have a straightforward contradiction. The understanding of quantum systems was advanced not by a insistence upon complementarity, but by a detailed analysis of alternative theories.

5.1.5 The Relationship Between Nonlocality and Contextuality

The Bell–KS theorems establish that in a hidden-variables theory the values assigned even to a set of mutually commuting observables must depend on the manner in which they are measured — a fact that Bohr could have told

[9]Indeed much of the response consists of a rehashing of the earlier debate between himself and Einstein, rather than a proper examination of the new result.

us long ago (although he would have disapproved of the whole undertaking). And Bell's Theorem establishes that the value assigned to an observable must depend on the complete experimental arrangement under which it is measured, even when two arrangements differ only far from the region in which the value is ascertained ...

Mermin, p814, [287]

In the same paper as the above quotation [287], Mermin presents another alternative proof of the Bell–KS theorem using ten observables of three particles in a state space of eight or more dimensions. He shows that this system provides a direct link between the Bell–KS theorem and the GHZ result [201] mentioned above. Thus, there is some sort of an equivalence between the two theorems, in particular, they both demonstrate that the assignment of pre-existing elements of reality to quantum mechanical systems is impossible. However, Shimony points out that Bell's inequality provides a stronger result

... one of these theories, that of Bohm, postulated a peculiar kind of nonlocality or action at a distance in order to recover the quantum mechanical predictions concerning correlated, spatially separated systems... Bell asked whether such nonlocality is a necessary condition for the recovery of the statistical predictions of quantum mechanics. His positive answer to this question is called 'Bell's Theorem'. ... The factorisability condition is reasonable when the two systems are separated by typical laboratory distances (a few metres), and especially reasonable if the operations performed upon the two systems are events with space-like separation. Bell concluded that no physically acceptable contextual hidden variables theory could agree completely with the statistical predictions of quantum mechanics. Since the experiments inspired by his work overwhelmingly support quantum mechanics, it follows that no contextual hidden variables theory is viable unless it postulates a kind of nonlocality*.*

Shimony, p26, [394], (italics added)

Bell's Theorem therefore forms a subset of the more general Bell-KS type theorems, and both nonlocality and contextuality must be exhibited by any satisfactory hidden variables theory. However, there is a very real sense in which both theorems are describing the same phenomenon, that of *contextuality*. Nonlocality is simply contextuality over a distance.

5.1.6 The Meaning of Nonlocal and Contextual Behaviour

... in the absence of inefficiency, Bell's inequalities can reliably be violated only when the response of one of the particles depends (at least sometimes) on the question asked its partner.

Maudlin, p182, [283]

Nonlocality and contextuality provide two strong results illustrating the way in which quantum behaviour violates what might be regarded as a traditional reductionistic stance. In addition, these two results are intimately related, a nonlocal system will exhibit contextual behaviour.

The contextual dependence of quantum systems is strikingly similar to that discussed in section 1.3. In fact, contextuality is a general phenomenon, exhibited by many different 'classical' systems, not all quantum mechanical. In section 5.2 we shall discuss a number of systems for which it is possible to generate Bell-type inequalities, and to show that they are violated in certain situations. However, while the modelling of quantum systems is quite advanced, the contextual systems of other fields are traditionally not well understood. This is the source of much of the confusion surrounding systems such as those discussed in section 1.2.2; there are few clear, well established and general models of such phenomena. However, if their contextuality can be shown to be similar to that of quantum systems, then this might lead to the possibility of generalising the quantum description to a broader class of contextual system. The success of quantum modelling thus might lead to more successful models of contextuality, and hence complexity, in general. This proposal will be persued throughout the remainder of this work. First however, we shall briefly discuss some attempts that have been made to analyse the meaning of nonlocality in greater detail, and to extract specific results about what the violations of Bell-type inequalities might entail.

As was discussed in the previous section, nonlocality falls within a broad class of contextual behaviour. The dependence in Bell-type experiments upon both measurement settings and upon previous experiments performed upon the entangled system at a spacelike distance reveals this contextuality explicitly. Often, attempts are made to separate nonlocality into subsets of nonclassical phenomena, with an associated insistence upon one or the other of these mechanisms as being the ones that led to the resultant behaviour.

For example, both Jarrett [230, 50] and Shimony [395] have attempted to analyse Bell's factorization condition (5.1) as a conjunction of two different physically significant principles. Jarrett used the terms simple locality and completeness and Shimony Parameter Independence (PI) and Outcome Independence (OI). We will follow Shimony's terminology in order to avoid overloading the already generally overused term locality, the two conditions are mathematically equivalent even if they are given different interpretations by the two authors.

PI is the condition stating that the result of a given measurement is statistically independent of the setting of the distant detector. In order to understand this concept, it is necessary to extend the notation used in section 5.1.1 in order to incorporate not just the way in which the result of a measurement depends upon the direction of the polarizer and upon some hidden variable, $A(\hat{a}, \lambda)$, but also the premeasurement states of the two detectors before the measurement, D_A and D_B, as well as the orientation of the

apparatus in the opposite region. For the EPR–type experiment illustrated in figure 5.1, PI states that the expectation values found in region $\mathcal{A}$, are equal to[10]

$$P_A(A|\hat{a}, D_A, D_B^0, \lambda) = \sum_B P(A, B|\hat{a}, \hat{b}, D_A, D_B, \lambda), \tag{5.27}$$

where D_B^0 denotes that the device in region $\mathcal{B}$ performs no measurement. Similarly, the expectation values in region $\mathcal{B}$ are, according to PI,

$$P_B(B|\hat{b}, D_A^0, D_B, \lambda) = \sum_A P(A, B|\hat{a}, \hat{b}, D_A, D_B, \lambda). \tag{5.28}$$

OI on the other hand, states that the probability of getting a result on one side of the experiment is independent of the result on a distant wing if both this result and the distant setting are given, that is

$$P(A, B|\hat{a}, \hat{b}, D_A, D_B, \lambda) = \sum_{B'} P(A, B'|\hat{a}, \hat{b}, D_A, D_B, \lambda) \times \sum_{A'} P(A', B|\hat{a}, \hat{b}, D_A, D_B, \lambda). \tag{5.29}$$

Jarrett defines OI as describing that the results of measurements other than some measurement of interest, M, should provide no information that can be used to predict the result of M which is not already contained in the state description of the system [50].

It is trivially possible to re-derive Bell's factorization condition (5.1) from the combination of these two conditions:

$$\begin{aligned}
&P(A, B|\hat{a}, \hat{b}, D_A, D_B, \lambda) \\
&= \sum_{B'} P(A, B'|\hat{a}, \hat{b}, D_A, D_B, \lambda) \times \sum_{A'} P(A', B|\hat{a}, \hat{b}, D_A, D_B, \lambda) \qquad \textit{(by outcome independence)} \\
&= P_A(A|\hat{a}, D_A, D_B^0\lambda) \times P_B(B|\hat{b}, D_A^0, D_B, \lambda) \qquad \textit{(by parameter independence)} \\
&= P_A(A|\hat{a}, D_A, \lambda) \times \ P_B(B|\hat{b}, D_B, \lambda),
\end{aligned} \tag{5.30}$$

which is the familiar factorization condition. ■

Thus, the violation of Bell's inequality might entail either a violation of PI or OI, or both. In fact, Jarrett claims to prove that a violation of PI (simple locality in Jarrett's terminology) allows for the possibility of superluminal signals. He discards this outcome claiming that it violates relativity, and thus concludes that the violation of Bell's inequalities is due to it's lack of OI (or predictive completeness).

A number of questions can be raised about this procedure:

1. Can experiment tell which of the two conditions is violated?
2. Is the interpretation of these mathematical identities correct?

[10] Both Shimony and Jarrett use different notation across papers, the current choice of notation aims to keep consistentency with the earlier discussion, as well as the common notation used by Bell and others.

3. Is the physical significance attributed to PI and OI justified?

Firstly, we can say that experiment does not reveal one or the other condition as being violated, and this problem is compounded by the fact that some theories that recover quantum predictions violate PI, while others violate OI. For example a Bohmian pilot wave type interpretation [73, 74] violates PI, but not OI, whereas standard quantum mechanics violates OI but not PI, and the QSD spontaneous localisation model which will be discussed in section 5.6.1 violates both of these conditions.

Given that there is no experimental proof that one or the other of these conditions is important, we might now ask whether the splitting of the factorization condition that was carried out to extract them is justified. It is likely that this is not the case, and that factorization is the only relevant condition when it comes to describing such systems. If this is the case then physical significance is being attributed to conditions which are irrelevant to the phenomena that they are allegedly describing. Maudlin has discussed a number of these questions [283]. He illustrates the arbitrary nature of the above separation of the factorization condition by performing a different separation which he claims to be equally valid. His first principle (P1):

$$P_A(A|\hat{a}, D_A, \lambda) = P_A(A, B|\hat{a}, D_A, \lambda) \tag{5.31}$$

$$P_B(B|\hat{b}, D_B, \lambda) = P_B(A, B|\hat{b}, D_B, \lambda) \tag{5.32}$$

can be interpreted as saying that the probability assigned to a given outcome on one side is not changed if one knows the result, but not the setting of the apparatus on the other side. This can then be coupled with a second principle (P2):

$$P_A(A, B|\hat{a}, D_A, \lambda) = P_A(A, B|\hat{a}, \hat{b}, D_A, \lambda) \tag{5.33}$$

$$P_B(A, B|\hat{b}, D_B, \lambda) = P_B(A, B|\hat{a}, \hat{b}, D_B, \lambda) \tag{5.34}$$

with which he associates the interpretation that the probability of a given result on one side, assuming that the result on the other side is already known, does not change if one also knows the setting of the distant analyser. Clearly these two conditions can be used to derive the factorization condition (5.1), but different interpretations can be assigned to them; Maudlin claims, I think correctly, that it is possible to call (P1) Outcome Independence$'$ (OI$'$) as it conditionalises on the distant outcome, and (P2) Parameter Independence$'$ (PI$'$) because of its conditionalisation on the distant setting.

It is apparent that a simple mathematical trick is being credited with more physical significance than is appropriate.[11] However, the fact that there are two forms of inequal-

[11]It should be pointed out that there is at least one other author who has performed this separation of the factorization condition, Howard has attributed separability and locality respectively to PI and OI [223, 224]. His arguments are less well known than those of Shimony and Jarrett and will not be discussed

ity, the Bell-type and the Bell-KS type suggests that some sort of separation is justified. Perhaps the biggest problem associated with finding such a separation is the general tendency to ascribe a negative status to nonlocality and contextuality. Researchers have sought to rectify this status through the identification of one violated characteristic as being that behind the violations of these inequalities, hence generally the strange new behaviour of quantum mechanics, and the other characteristic as relatively straightforward. Such a separation is probably not possible. Instead we can perceive factorisation as a starting condition for contextual behaviour, with nonlocal behaviour being apparent when the contextual system is spread over spacelike distances.

A contextual system will exhibit some sort of dependency upon other parts generally seen as separate; hence it will defy, to some extent, reductionistic explanation.

A nonlocal system will exhibit contextual behaviour, some of which might be spread over spacelike distances. Nonlocality forms a subset of the more general class of contextual behaviour.

The Bell and Bell–KS-type inequalities thus provide us with an invaluable toolset for the identification of contextual behaviour. It is possible that these conditions could therefore be used in the identification of contextual, and hence high-end complex systems; if a system violates Bell's theorem, or shows signs of incompatible sets of observations then it is contextual. We might ask if there are nonquantum systems that exhibit such violations. In section 5.2 we shall find that this is in fact the case.

5.1.7 Observers in Quantum Theory

The contextuality of quantum systems has led many researchers to suggest that there is an 'observer dependence' in their behaviour [173, 213, 367].

This idea began with researchers such as Bohr [76] and Heisenberg [213], but is perhaps most famously attributed von Neumann [446]. He discovered that the unitary time evolution of quantum mechanical systems represented by the Schrödinger equation (4.10) do not behave properly under measurement. This phenomenon is simple to illustrate. If we consider a simple superposition $\psi = \alpha|\uparrow\rangle + \beta|\downarrow\rangle$, where $|\alpha|^2 + |\beta|^2 = 1$, and suppose that this system is incident upon an experimental apparatus where a detector initially represented by $|\psi\rangle$, 'click' only when a particle in the $|\uparrow\rangle$ state is incident upon it. Such a detection event is represented by:

$$|\uparrow\rangle|d\rangle \rightarrow |\uparrow\rangle|d_{click}\rangle. \tag{5.35}$$

Conversely, if a particle in a spin down state is incident upon the detector then there will be no detection event:

$$|\downarrow\rangle|d\rangle \rightarrow |\downarrow\rangle|d_{miss}\rangle. \tag{5.36}$$

here, they fall prey to the same criticism raised above. Maudlin has discussed Howard's work along with that of Shimony's and Jarrett's [283].

This description is adequate when we simple consider pure states such as those above, but a problem emerges if we return to the superposition state above. The Schrödinger equation predicts an entangled state:

$$\Big(\alpha|\uparrow\rangle + \beta|\downarrow\rangle\Big)|d\rangle \Longrightarrow \alpha|\uparrow\rangle|d_{click}\rangle + \beta|\downarrow\rangle|d_{miss}\rangle = |\Phi^E\rangle, \tag{5.37}$$

rather than what is actually observed. In fact, only one of the alternatives from equation (5.37) are observed; the detector either clicks or does not, and if a number of experiments are run where a state prepared in the same way is incident upon the same detector state, then a trend will develop whereby a 'click' or a 'miss' is recorded with some probability related to the constants in equation (5.37). This allows us to make the statement that

$$|\Phi^M\rangle = \begin{cases} |\uparrow\rangle|d_{click}\rangle \text{ with probability } |\alpha|^2 \text{ or} \\ |\downarrow\rangle|d_{miss}\rangle \text{ with probability } |\beta|^2. \end{cases} \tag{5.38}$$

Introducing a new state for some postulated observer $\Big\{|O_\uparrow\rangle, |O_\downarrow\rangle\Big\}$ say, does not cause this reduction from what is predicted by the Schrödinger equation (5.8) to what is actually observed (5.38) does not help either, it leads to the new, even more entangled state

$$|\Phi^E\rangle = \alpha|\uparrow\rangle|d_{click}\rangle|O_\uparrow\rangle + \beta|\downarrow\rangle|d_{miss}\rangle|O_\downarrow\rangle \tag{5.39}$$

where the same problem is still apparent. The (even more entangled) system is still in a superposition of states, and such a state is not observed during the process of detection. Thus, if only Schrödinger dynamics is applied to a system then an infinite regress results; we can introduce as many extra states as we like but will never obtain the 'collapse' of the correlated state into a final outcome. It is necessary to add an additional postulate in order to accomplish the transition $|\Phi^E\rangle \Rightarrow |\Phi^M\rangle$. To terminate this regress, von Neumann postulated what we now recognize as the familiar nonunitary *reduction of the state vector*, which makes the outcomes independent of each other and thus breaks the superposition [446]. This means that according to von Neumann, and the standard formulation of quantum theory, there are two distinct stages to the evolution of the state vector.

1. The continuous, and well understood time evolution described by the Schrödinger equation (4.10). This continuous evolution only leads to an entangled state, which is not observable in reality, hence it is necessary to invoke the second stage process.
2. An abrupt and discontinuous jump or 'collapse' (sometimes referred to as the 'reduction of the state vector') caused by a measurement. The wave collapses into the or case. For our simple example,

$$|\Phi^E\rangle \overset{\text{collapses}}{\Longrightarrow} |\Phi^M\rangle \tag{5.40}$$

Despite its place in the postulates of quantum mechanics, this rule is of a very ad-hoc nature — what causes the collapse? Where does it occur? Von Neumann was very concerned by these questions, and systematically examined a chain of measurement, (like the one we have begun to construct above) broken up into small steps (representing increasing levels of entanglement) from the quantum system right up to the mind of the observer. He showed that we could break the chain with a 'collapse' at any stage of this measurement chain, thus the collapse is to some extent arbitrary. A choice had to be made as to where the collapse postulate should be applied, and this choice was in no sense predicted by quantum mechanics. Von Neumann concluded that it was the mind of the observer that caused the collapse of the wave function.

Thus, von Neumann adopted a form of observer created reality — observables cannot exist until some sort of an observer is there to observe them. But where should we draw the distinction between observers and the rest of the quantum world? What is an observer? The Schrödinger cat [381], and its extension Wigner's friend [457, 458] test such interpretations, but this has not generally led to their rejection. Perhaps the most concerning objection to such an interpretation is the status of observers in the theory; they occupy a special place in the theory and hence, it cannot explain them. However, despite these objections, the common naive interpretation of quantum mechanics adopted by most physicists can be seen to fall in this category; quantum theory predicts the correct behaviour, even if the formalism itself is inconsistent and prone to some interpretation in its application. Where and when a measurement is seen to have occurred is normally viewed as self-evident and the measurement rule can be applied, although this boundary is becoming increasingly difficult to identify as experiment improves [66, 212]. Such a naive interpretation may not remain satisfactory indefinitely.

However, the adoption of this naive interpretation is not necessary. Clearly, with an understanding of the way in which the context of the experimental situation can affect its outcome we might begin to extricate the observer from quantum systems; how the observer chooses to orient experimental apparatus will affect what is seen, but dynamically, the mechanism behind this lies in the interaction between the quantum system and the apparatus itself. With the adoption of such an interpretation, it becomes necessary to provide a dynamical mechanism behind this dependence of quantum systems upon their context. A number of possibilities exist, such as Bohmian mechanics [73, 74, 72], and theories that postulate additional variables [303]. Another possibility (which will be discussed in some detail in this work) consists of the spontaneous localisation models. These modify the Schrödinger equation with terms that dynamically drive the system to localise when it undergoes interactions with 'macroscopic' systems of a suitable form [188, 189, 190, 328, 183, 181, 154]. This interpretation of quantum mechanics will be discussed more thoroughly in section 5.6.

Thus, although the reasons to prefer one interpretation of quantum mechanics to

another are largely a matter of philosophical taste, a properly dynamical model of measurement starts to provide the required difference between subjectivity and contextuality that was mentioned in section 1.6. An interpretation that places the contextuality of quantum systems with the observer of the system is, in addition to being contextual, adopting a stronger subjective description of reality. Dynamical theories of measurement suggest that this may not necessary.

5.2 Contextuality is Generic

> *There is no living unit which can be considered 'living' without reference to the external environment . . . Biologists should emphasise over and over that 'living' is unavoidably a* total ecosystem *property and not the property of an isolated collection of macromolecules. It seems to me that the central question of the origin of life is* not*, 'Which comes first, DNA or proteins?', but rather 'What is the simplest possible ecosystem?'*
>
> Pattee, p219, [318]

The contextuality apparent in quantum systems is not restricted to them. As has been discussed in section 1.3, there are a variety of systems not regarded as quantum mechanical which exhibit strongly contextual behaviour. It is possible that many of the lessons learned from the behaviour of quantum systems can be used in the analysis of this broader class of systems. As was also discussed in section 1.3.1, many of these contextual systems are generally considered to be complex. Is it possible that there might be a connection between the two types of system? This is an interesting possibility because in contrast to a number of complex systems, the analytical nature of quantum theory is well developed. It may be possible to utilize some of these quantum results in the analysis of complex systems. This section will investigate the notion that quantum theory is in fact more widely applicable than is generally thought to be the case.

This claim has been most strongly developed by those associated with the CLEA[12], in particular Diederik Aerts. In early work [9] Aerts made use of a simple example to illustrate that contextuality is not limited solely to quantum systems, but might also be found in purely classical systems.

Aerts begins by assuming that four yes-no experiments $\alpha, \beta, \gamma, \delta$ can be performed upon a system of interest, and associating with each experiment μ a variable signifying its outcome X_μ:

$$X_\mu = \begin{cases} +1 \text{ if a yes answer is returned,} \\ -1 \text{ if a no answer is returned.} \end{cases} \tag{5.41}$$

He points out that it may be possible to combine experiments, obtaining answers to both experiments at the same time. In this case he defines the coincidental outcomes as

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follows:[13]

$$X_{\mu\nu} = \begin{cases} +1 \text{ if the answers are correlated,} \\ -1 \text{ if the answers are anticorrelated.} \end{cases} \tag{5.42}$$

It is important to note that the coincidence experiment $X_{\mu\nu}$ is a new experiment and must be considered separate from the two individual experiments X_μ and X_ν; there is no reason to suppose that there will be a correlation between $X_{\mu\nu}$ and some legitimate combination $X_\mu \oplus X_\nu$. This amounts to a recognition of the possibly contextual nature of the system.

In order to construct a Bell inequality, Aerts supposes that it is possible to perform compatible coincidence experiments $\alpha\beta$, $\alpha\gamma$, $\delta\beta$ and $\delta\gamma$ on a system consisting of two parts S_1 and S_2 which are localised in two different regions of space. Experiments α and δ are performed on S_1 and experiments β and γ are performed on S_2, see figure 5.2. Assuming that the system is separable (or noncontextal):

$$X_{\alpha\beta} = X_\alpha X_\beta \tag{5.43}$$

$$X_{\alpha\gamma} = X_\alpha X_\gamma \tag{5.44}$$

$$X_{\gamma\beta} = X_\gamma X_\beta \tag{5.45}$$

$$X_{\delta\gamma} = X_\delta X_\gamma, \tag{5.46}$$

the Bell inequality takes the following form [9]:

$$|X_{\alpha\beta} - X_{\alpha\gamma}| + |X_{\delta\beta} + X_{\delta\gamma}| \leq 2. \tag{5.47}$$

Aerts then chooses the experimental apparatus illustrated in figure 5.2, and formulates four specific experiments that can be performed upon this apparatus as follows:

Experiment α tests whether the volume of water contained in the vessels is more than 10 litres. The experiment is performed by siphoning off water from one of the vessels into a reference vessel with volume of 10 litres. If the reference vessel overflows then we answer yes, and if it does not overflow by the time that the first vessel is empty then we answer no.

Experiment β tests whether the depth of the water in the reference vessels of interest (defined by the observer) is more than 15cm. The experiment is performed by placing a ruler vertically in the vessel, and reading off the height of the water on the ruler. If the water is higher than 15cm then we answer yes, if not then we answer no.

Experiment γ tests whether the water is drinkable. The experiment is performed by taking a spoonful of water from the vessel and drinking it. After five minutes, if we

[13]The answers are correlated if they are of the form, {yes,yes} or {no,no}, and anticorrelated if they are of the form, {yes,no} or {no,yes}.

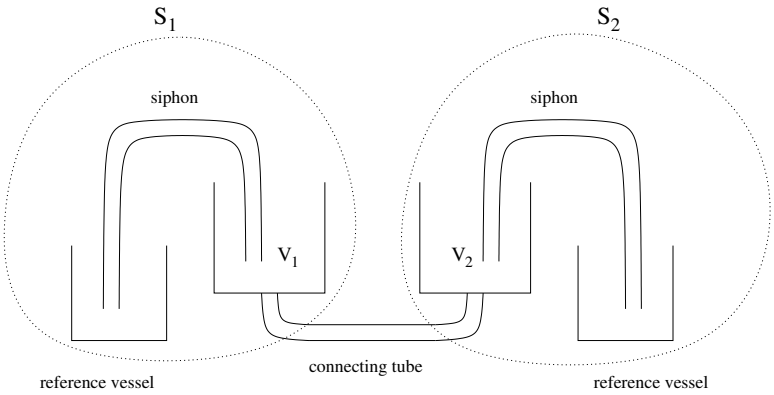


Figure 5.2: A system consisting of two cubic vessels (which have sides of length 20cm and can each hold 16 litres of water) connected via a tube (which can hold 16 litres) is filled with 32 litres of water. Depending upon the experiments performed upon this system, it can exhibit violations of Bell-type inequalities.

are feeling good then we answer yes, if we are ill then we answer no.

Experiment δ tests whether the water is transparent. The experiment is performed by taking a spoonful of water and placing it in a glass which is then held against a light source. If the light gets through the water then we answer yes, if not then we answer no.

He then considers the experimental arrangement in figure 5.2, and shows that when combined these experiments do not satisfy (5.47).

The coincidence experiment $X_{\alpha\beta}$ returns a value -1, as the two experiments are anticorrelated. That is, the siphoning of the water into the reference vessel for the purpose of experiment α drops the water level in both V_1 and V_2[14]. From the geometry of the vessels, we ascertain that if there is more than 10 litres in V_1 then there must be less than 15cm of water in each of the vessels when we measure the depth of the water; if experiment α returns a yes then β must return a no. In a similar manner we can determine that it is quite reasonable to find a situation where

- $X_{\alpha\gamma} = +1$. That is, more than 10 litres of water gets emptied into the reference vessel, and the water is drinkable.
- $X_{\delta\beta} = +1$. That is, taking a spoonful of water to determine the transparency of the water (which we take to be transparent) does not lead to the depth of the water changing to less than 15cm.
- $X_{\delta\gamma} = +1$ That is, the water is both transparent and drinkable.

Given these outcomes we find that:

$$|X_{\alpha\beta} - X_{\alpha\gamma}| + |X_{\delta\beta} + X_{\delta\gamma}| = +4 \tag{5.48}$$

[14]By the same amount due to the tube connecting them.

in clear violation of equation (5.47).

It is worth noting that this system is classical, and that there are no unknown factors affecting the outcomes; the situation is entirely transparent. In particular there are no hidden variables. We should consider the mechanism in the experiment that leads to the violation of Bell's inequality. This takes the form of the tube connecting the two containers. This is almost self-evident, the two systems can no longer be regarded as separated once the tube connects them, even if the vessels are spatially separated. While this might appear obvious with the current arrangement, were the experiment to be concealed inside a black box, with only the tops of the two vessels showing then the tube would acquire the status of a hidden variable, and the mystery of the correlation obtained from the boxes, a spooky status similar to that of quantum systems. This spookiness arises because of the contextual dependency of one box upon the other; the two boxes cannot be considered as separate. In the system of figure 5.2 this dependency is well-understood, but this is not the case in quantum mechanics, and there is no reason to suppose that the mechanism will be similar. We might conclude that it is the contextual dependence of one box upon the other that drives the strange outcomes of quantum mechanics, not the specific quantum mechanism itself.

With this realization comes the possibility that Bell-KS type inequalities are exposing more than just quantum behaviour, they are exposing a *more general form of contextual behaviour*.

This idea has been explored by a number of different researchers. It has been proven that it is not always appropriate to utilize a Kolmogorovorian probability model to describe our lack of knowledge about some systems [3, 139, 206, 342, 349, 462], in particular quantum systems. However, this impossibility becomes even more explicit in the case of EPR-type correlations, the separation condition (5.1) used to derive Bell's theorem provides a necessary condition for the very existence of a classical (Kolmogorovorian) probability model.

In a sequence of papers [21, 20], Aerts has examined the origin of quantum probabilities in great detail. He has found that

> *. . . we obtain quantum-like probability structures, if the measurements needed to test the properties of the system are such that:*
>
> 1. *The measurements are not just observations but provoke a real change of the state of the system.*
> 2. *There exists a lack of knowledge on what precisely happens during the measurement process.*
>
> Aerts, p2, [17]

The containers of water example discussed above clearly illustrates the way in which classical measurements can lead to a real change of the system, termed an *aspect of*

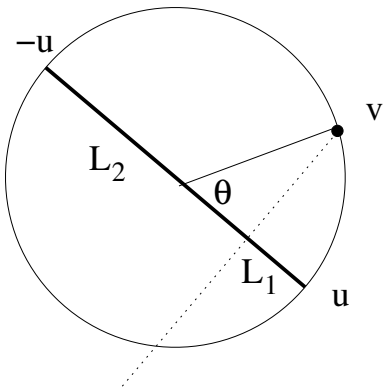


Figure 5.3: A quantum machine capable of exhibiting the random behaviour of a spin-$\frac{1}{2}$ quantum entity.

creation by Aerts *et al.* [23], and the nonseperability of this scenario led to the violation of Bell-type inequalities, but there was no lack of knowledge about what happens during an act of measurement and therefore a probabilistic model was not necessary [21, 20]. A different example has been provided to illustrate the way in which lack of knowledge, coupled with a process whereby measurement to some extent creates what is observed, leads to quantum-type probabilities.

This scenario consists of a 'quantum machine' which provides a classical model of a spin-$\frac{1}{2}$ quantum entity [20, 11, 22, 17]. Illustrated in figure 5.3, it consists of a point particle which is free to move on the surface of a sphere of radius 1. Measuring the position of this particle consists of stretching a piece of elastic inside the sphere between two antipodal points $[-u, +u]$ upon its surface, and then projecting the particle directly onto the elastic. As soon as the particle hits the elastic, it sticks to it, and the elastic then breaks randomly at some point in the interval $[-u, +u]$. A measurement then consists of finding the particle at either of the points $-u$ or $+u$, in much the same way that a Stern–Gerlach analyser separates spin-$\frac{1}{2}$ systems into two streams that go to two separate detectors. In the case of this thought experiment, there is a lack of knowledge about where the elastic breaks, which results in a quantum type probability as follows. If we decide to perform a measurement in the $\hat{u}$ direction, upon a particle that starts at position v on the unit sphere, then the probabilities for the transition of the particle to either u or $-u$ are given by:

$$P[v \to u] = \frac{1 + \cos\theta}{2} = \cos^2\frac{\theta}{2} = 1 - P[v \to -u] \tag{5.49}$$

where θ is the angle between the unit vectors $\hat{u}$ and $\hat{v}$ defined from the centre of the sphere to the relevant points upon the surface. This is clearly the same probability as that obtained for Stern-Gerlach measurement on spin-$\frac{1}{2}$ quantum systems. This is to be expected, as the quantum machine is simply an elaboration of the Bloch/Poincaré model

of spin-$\frac{1}{2}$ quantum particles.

It is possible to extend this simple system to a two sphere linked system that exactly mirrors the results of EPRB–type experiments [15], but this extension is carried out by making an explicit connection between the two spheres, in the form of a rigid but extensible rod that connects the two point particles, and leads to the correlation between their results; similarly to the tube connecting the two vessels in figure 5.2, the rod makes the nonseparability between the two spin-$\frac{1}{2}$ mechanical entities explicit. It can be proved that it is possible to construct a similar sphere model to mimic any arbitrary quantum entity [133, 132].

The acts of creation framework suggests that quantum measurement is a process of 'creation-discovery'; measurements do not simply record some elements of reality relevant to a system, they actively influence what is seen. Thus, this framework has the potential to describe contextual systems in general. The group at CLEA has been working towards precisely this goal; they have been examining an interesting set of contextual processes, all seen within the framework of *context-driven actualization of potentials* (CAP) where a specific instance is actively chosen from a set of alternative scenarios. Consider some form of entity in a state $p(t_i)$ at time t_i. Under the influence of some context $e(t_i)$ (which may be zero), the state will change to state $p(t_{i+1})$ at time t_{i+1} if it is deterministic, or to one of the set of new states $\{p_1(t_{i+1}), p_2(t_{i+1}) \ldots, p_n(t_{i+1}), \ldots\}$ if it is nondeterministic. Depending upon the system under consideration, different levels of contextual dependence will be evident, classical Newtonian systems exhibit almost no contextuality, whereas quantum systems usually show a far greater dependence upon their environment.

This framework has been used to unify the description of a number of different processes in addition to quantum and classical evolution. Phenomena that have been described within this framework include:

The determination of opinion where for example, a survey of whether people are in favour of, or opposed to, nuclear energy can be significantly influenced by the interviewer during the process of data acquisition [17]. This is because data is not merely acquired, rather, it is often created during the process of surveying; opinions are not necessarily passive, pre-existing elements of reality, they can be actively created during collection.

The formation of concepts and creative thought provides an example of CAP similar to that of the determination of opinion. In particular, this theory appears to be able to cater for the guppy effect [313], where the concept of 'guppy', which is not a good example of either the concept 'pet' or that of 'fish' singly, is actually very typical of those concepts when combined [19, 174, 16, 175].

An alternative formulation for the democratic voting process has been proposed, where the decision making process itself is not predetermined, but actualised as this

becomes necessary [13].

A theory of cultural evolution is being developed, as a part of a drive towards a generalised theory of all evolutive processes [176, 177].

A general theory of evolution has been proposed within this CAP framework, where concepts such as fitness are explicitly recognised as being dependent upon both the phenotype and the environment. This theory is in its early stages, but its explicit recognition of the contextuality of biological systems is important, mirroring the claims that were made in chapter 2 that the true complexity of biological evolution could not be modelled by ALife without incorporating the true complexity of the environment.

Clearly, the CAP framework is a very rich formulation, capable of describing in a unified manner a wide variety of systems and processes traditionally thought to be complex, and nonunifiable. In section 5.6.2 we shall discuss a proposal to unify quantum mechanics and relativity that arises within this framework.

Thus, the work of the group at CLEA shows that contextuality occurs far more widely than in just quantum systems, something that we have been led to suspect throughout this work, but which has now been formalised in a number of interesting results. There is at least one aspect of quantum behaviour that is more generic than is traditionally thought to be the case. Quantum-type models can be used in attempts to understand systems that exhibit a contextual dependence upon their environment.

Many of the systems examined by the group at CLEA fall into the previously discussed framework of 'complex systems', and, as was discussed in section 1.5.1, a often occurring notion of complexity concerns the necessity of using multiple descriptions to understand one complex system. An alternative framework had been developed independently at Flinders University, under the rubric of Process Physics, before the work at CLEA was discovered and the connection between these two frameworks understood. This work uses quantum field theory as its starting point, rather than quantum logic, and has already been partially discussed in chapter 4. The next chapter will return to this set of theories, and a number of similarities between the two different frameworks will become apparent. For example, the emphasis upon aspects of creation in the CLEA framework is realised in Process Physics by use of a spontaneous localisation approach where measurement is understood to actively influence a system rather than just passively measure. This same approach makes measurement a contextually dependent process in Process Physics; spontaneous localisation offers a measurement mechanism which will serve to cause a system to localise differently depending upon how it is measured. The CLEA framework also appears to emphasise evolutive, process-driven models [176] where rather than the static 4-dimensional model of the Universe suggested by relativity theory, the observers of a system can be understood to be actively participating with its dynamics; different

behaviour will result in a different Universe, *i.e.* different 'runs' could lead to a very different reality. It might be suggested that similarities such as these will be necessary in any set of theories that *can* describe high-end complexity; although the frameworks might be different in their details, they share many key features, and each will have different strengths and weaknesses. The remainder of this work will focus upon the Process Physics framework. While it is expected that more interesting correlations can be drawn between the two frameworks this will be reserved for future work.

5.3 Symmetry

> *Spaces that are symmetrical have a fundamental importance in modern physics. Why is this? It might be thought that completely exact symmetry is something that could arise only exceptionally, or perhaps just as some convenient approximation. ... Yet, remarkably, according to the highly successful physical theories of the 20th century, all physical interactions (including gravity) act in accordance with an idea which, strictly speaking, depends crucially upon certain physical structures possessing a symmetry that, at a fundamental level of description, is indeed necessarily exact!*
>
> Penrose, p247, [326]

Symmetry plays a vital role in attempts to understand quantum behaviour, from the simplest non-relativistic system to the most complex field-theoretic calculation of nucleonic properties in QCD. In fact, symmetry has been applied in almost every area of physical modelling, in normative, explanatory, classificatory and unificatory roles. Perhaps the widest use of symmetry is that of the application of *symmetry principles* to physical situations, phenomena and laws. There are a number of different symmetry principles in physics, each of which have been utilized in different quantum field theories:

The Principle of Relativity asserts simply that:

> *The laws by which the states of physical systems undergo changes are independent of whether these changes of states are referred to one or the other of two coordinate systems moving relatively to each other in uniform translational motion.*
>
> Einstein, [165]

Depending upon whether this principle is taken to be global or local in nature, this principle turns into either The Principle of Special Relativity, or The Principle of General Relativity respectively.

Permutation Symmetry was the first non-spatiotemporal symmetry introduced into physics. It concerns the indistinguishability of 'identical' particles in some system. This principle states that if an ensemble of particles is invariant under a permutation of those particles then the permutations which merely exchange indistinguishable

particles should not be counted; the exchanged state is in some sense symmetrical to the original state.

CPT symmetry describes the conservation of dynamical laws under combined changes of Charge conjugation, Parity[15] and Time reversal. The laws governing gravity, electromagnetism, and the strong interaction are invariant with respect to independent changes of C, P and T, however, β-decay which occurs in the weak interaction does not respect independent applications of these discrete reversals. The discrete symmetries C, P and T are connected by the so-called CPT theorem [450], which states that the combination of C, P, and T is a general symmetry of physical laws.

Gauge Symmetry describes the local versions of the various internal symmetries that are associated with invariance under phase changes of quantum states. Such symmetries are usually described by the special unitary groups $SU(N)$.

There are quantum theories that make use of each of these principles, even the Principle of General Relativity which is well known to be in some sense incompatible with quantum field theories is being used as a criterion of success (*i.e.* in a normative role) for any extentions to these theories, particularly in the dream of unifying gravity with the strong, weak and electromagnetic forces.

The standard notion of symmetry utilised in physics is that of a *symmetry group*. Mathematically, a group G is an algebraic structure comprised of a set of elements g_i satisfying the following set of requirements:

$$\text{composition is satisfied:} \qquad g_i, g_j \in G \Rightarrow g_i g_j \in G \tag{5.50}$$

$$\text{composition is associative:} \qquad (g_i g_j) g_k = g_i (g_j g_k) \tag{5.51}$$

$$\text{there is an identity element:} \qquad \exists I \in G \; st \; \forall g \in G, gI = g \tag{5.52}$$

$$\text{there is an inverse element:} \qquad \forall g \in G, \exists g^{-1} \in G \; st \; gg^{-1} = g^{-1}g = I \tag{5.53}$$

Thus, a symmetry group induces a partition into equivalence classes, the different symmetrical arrangements of the system.

It is possible to classify each of the principles above into two general classes of symmetry[16]:

Space-time symmetries describe the behaviour of some field in space and time. These are generally represented by the Lorentz and the Poincaré groups.

Internal symmetries mix particles among one another. These symmetries rotate fields and particles in isotopic space, rather than real space-time. An example of such an internal symmetry is the often used $SU(3)$ group of QCD which describes the mixing of three quark varieties.

[15] That is, spatial reflection.

[16] Supersymmetry can be described as the dream of finding a nontrivial combination of the relevant space-time and internal symmetries of physics.

It is worth noting that while internal symmetries are compact, space-time symmetries are not, that is, the range of their parameters does not include the endpoints (*e.g.* massive particles can only approach c) [234]. Both classes of symmetry can be either global or local. *Global symmetries* are independent of space-time, whereas *local symmetries* vary with each point in space and time. For example, Special Relativity is a global theory, but General Relativity is local. A local internal symmetry has a very special status in physics, it is defined as a *gauge symmetry*. These are particularly important in QFT a point which will be discussed in section 5.3.2.

There are far more possible symmetry groups than those that are realised in physical systems. We might ask what leads to a symmetry group being physically realised. In fact there is a close connection between symmetries and *conservation laws*.

5.3.1 Symmetry and Conservation Laws

> *Transformations in physics are not arbitrary.* They are generated by dynamical variables*, through which symmetries are associated with* conservation laws*.*
>
> Auyang, p35, [41]

There is a deep connection between exact symmetries and conservation laws, first formalised by Noether [82, 81].

Given a system described by a set of fields $\phi = (\phi_1, \ldots, \phi_n)$, and their derivatives $\partial_\mu \phi$ we can define an Action describing the dynamics of those fields in terms of either the Lagrangian L or the Lagrangian density $\mathcal{L}$

$$S[\phi] = \int dt L[\phi(x)] = \int d^4x \mathcal{L}(\phi(x), \partial_\mu \phi(x)). \tag{5.54}$$

The principle of least action can be used to derive classical equations of motion as follows

$$\begin{aligned} 0 &= \delta S = \int d^4x \left(\frac{\delta \mathcal{L}}{\delta \phi} \delta\phi + \frac{\delta \mathcal{L}}{\delta \partial_\mu \phi} \delta \partial_\mu \phi \right) && (5.55) \\ &= \int d^4x \left[\left(\frac{\delta \mathcal{L}}{\delta \phi} - \partial_\mu \frac{\delta \mathcal{L}}{\delta \partial_\mu \phi} \right) \delta\phi + \partial_\mu \left(\frac{\delta \mathcal{L}}{\delta \partial_\mu \phi} \delta\phi \right) \right] && (5.56) \end{aligned}$$

The last term vanishes at the endpoint of the integration, leaving the *Euler–Lagrange equations of motion*:

$$\partial_\mu \left(\frac{\delta \mathcal{L}}{\delta(\partial_\mu \phi)} \right) - \frac{\delta \mathcal{L}}{\delta \phi} = 0. \tag{5.57}$$

These equations are intimately related to both symmetries and conservation laws as follows. If the fields ϕ^α vary globally with some small parameter $\delta\epsilon^\alpha$, then (5.55) can be

rewritten:

$$\delta S = \int d^4x \left(\partial_\mu \frac{\delta \mathcal{L}}{\delta \partial_\mu \phi^\alpha} \delta \phi^\alpha + \frac{\delta \mathcal{L}}{\delta \partial_\mu \phi^\alpha} \partial_\mu \delta \phi^\alpha \right) \tag{5.58}$$

$$= \int d^4x \partial_\mu \left(\frac{\delta \mathcal{L}}{\delta \partial_\mu \phi^\alpha} \delta \phi_\alpha \right) \tag{5.59}$$

a relationship that can be used to define the *current*:

$$J^\mu_\alpha = \frac{\delta \mathcal{L}}{\delta(\partial_\mu \phi^\beta)} \frac{\delta \phi_\beta}{\delta \epsilon^\alpha}. \tag{5.60}$$

If the action is invariant under this transformation of the field, then the current is conserved,

$$\partial_\mu J^\mu_\alpha = 0. \tag{5.61}$$

Integrating this conservation relation over space:

$$0 = \int d^3x \partial_\mu J^\mu_\alpha \tag{5.62}$$

$$= \int d^3x \partial_0 J^0_\alpha + \int d^3x \partial_i J^i_\alpha \tag{5.63}$$

$$= \frac{d}{dt} \int d^3x J^0_\alpha + \underbrace{\int dS_i J^i_\alpha}_{\text{surface term}}. \tag{5.64}$$

Assuming that the fields appearing in the surface term vanish sufficiently rapidly far from the area of interest, we can neglect the surface term. Defining the *charge* of this system as

$$Q_\alpha \equiv \int d^3x J^0_\alpha, \tag{5.65}$$

allows us to determine that there is also a conserved charge

$$\frac{d}{dt} Q_\alpha = 0, \tag{5.66}$$

which corresponds to a conserved characteristic of this system. Thus, the symmetry of the action under continuous transformations of the field implies the conservation of a current, and hence a conservation principle.

Noether originally proved two theorems, the result above is a simplification of her first result, which relates continuous global symmetries to conserved characteristics. Any transformation that exhibits a global symmetry will have parameters that remain constant regardless of their spacetime position. As an example, consider the principle of Special Relativity, which is derived from such a global symmetry group; the ten parameters of the Poincaré transformations of Special Relativity are position invariant. However, not

all of the symmetries of physics have this global characteristic. In fact, the shift from Special to General Relativity is accomplished by localising the Poincaré group, which makes the parameters of the transformation vary with spatiotemporal position. When this localisation is applied to an internal symmetry it is referred to as *gauging the symmetry.*

Noether's more complicated second theorem indicates that there is an inherent underdetermination (*i.e.* more unknowns than independent equations of motion) in any theory with a local symmetry. This result will not be discussed here, the paper by Brading and Brown [82] contains an excellent summary of both of Noether's results as well as their implications.

5.3.2 Symmetry and Interaction

> *Local phase invariance is not possible for a free particle wave equation, so in order to locally change the phase of a charged particle's wave function we need to introduce a field in which the particle moves. Or, in slightly less physical terms, local phase invariance is possible only if we have an* interacting *theory.*
>
> Morrison, p350, [292]

Local internal symmetries, *gauge symmetries*, are intimately linked with the concept of interaction in physical theories. Through the application of gauge symmetry principles what generally starts as two sets of noninteracting fields, a matter[17] field and an interaction[18] field are coupled. Thus, there is a deep sense in which symmetry is fundamentally linked with the concept of interaction.

Examples of this interdependence abound in physical systems. For example, the full Lagrangian of QED, equation (4.18), is derived by properly incorporating the phase of the electron wavefunction into the free electron Lagrangian density $\mathcal{L} = \bar{\psi}(x)(i\gamma^\mu\partial_\mu - m)\psi(x)$. This equation is invariant under the global transformation $\psi(x) \rightarrow \psi'(x) = e^{i\phi}\psi(x)$ of the space-time $U(1)$ group, but is not when this transformation is localised *i.e.* $\phi = \phi(x)$. The local dependence of the Lagrangian upon the phase of the electron field is an experimentally verified phenomenon. For example, if the electron field is propagated through a double slit, then an interference pattern results, but this interference pattern is changed if the phase of the electron field that propagates through one slit is changed. However, a solenoid with a small current can be placed such that the original phase change is reversed, which results in the original interference pattern. This phenomenon is known as the Aharanov-Bohm effect after its discoverers [26], and has been experimentally verified on a number of occasions. Thus the locality of the phase transformation must be

[17] That is, a spin $\frac{1}{2}$, or fermion field, *e.g.* electron and positrons in QED, quarks and antiquarks in QCD.

[18] That is, a spin-1, or boson field *e.g.* photons in QED, gluons in QCD.

incorporated into a description of the electron field when it is exposed to an electromagnetic influence. However, in order to regain the invariance of the Lagrangian density under local transformations of the phase ϕ it is necessary to incorporate not just an electromagnetic term to the free electron Lagrangian density, but to add an extra term $-\bar{\psi}(x)g\gamma^{\mu}A_{\mu}(x)\psi(x)$ to the free particle equation. This new term represents the interaction between the electron field and the electromagnetic field (*i.e.* between electrons and photons). Thus, properly localising the internal symmetry of this system results in an interaction between its two primary fields.

A similar effect occurs whenever the internal symmetries of some quantum field theory are localised. Auyang has discussed the logic of gauge field theories in her book [41]. She claims that in all local field theories (which in her analysis includes classical theories such as general relativity and electromagnetism) what starts as a free matter field is turned into an interacting field system in three steps:

1. *The localization of symmetry transformations.* The localization of the parameters involved in symmetry transformations (*i.e.* making them functions of their spatiotemporal-temporal position) results in their localization to *each point* in the matter field, the points in different positions undergo different phase transformations. Thus an individual state space is created for the local field, for each point in space-time.
2. *The enforcement of global invariance and the derivation of the term for reconciliation.* It is required that the Lagrangian of the whole field is invariant under local transformations. Ensuring that there is consistency between the now localized fields, this process involves finding some term that preserves the global invariance that was jeopardised in the previous step. Thus the different phase factors are reconciled at their various points in the field.
3. *The introduction of an interaction field.* Local symmetry endows this extra term with a full physical significance by interpreting it as a minimal coupling between the phase of the matter field and the potential of the interaction field. This coupling is universal (*i.e.* the same for any matter field with the same symmetry properties). The characteristics of this interaction field are then found by examining the structure of the coupling term. At the end of this step, we have a fully interacting matter/interaction field system.

Thus, according to Auyang, local symmetries are intimately related to the concept of interaction in physical modelling. Through an understanding of the spatial dependence of internal symmetries describing some set of phenomena, we also gain an understanding of how such phenomena couple in a system. This spatial dependence can be regarded as a weak form of contextual dependency in a field; the effect of the field changes depending upon its location, and the location of other fields relevant to the system. Is it possible that

there is a deep principle here that can be used to describe contextual complex systems in general? We shall return to this question in chapter 6.

5.3.3 Symmetry Breaking

> *. . . in a situation characterized by an absolute symmetry, nothing definite could exist, since absolute symmetry means total lack of differentiation. For the presence of some structure, a lower symmetry than the absolute one is needed: in this sense, symmetry breaking is essential for the existence of structured "things".*
>
> Castellani, pp322–3 [115]

Exact conservation laws arise in the context of exact symmetries, a situation that arises only if:

1. The Lagrangian density is invariant under the symmetry, $\delta\mathcal{L} = 0$, and,
2. The physical vacuum, that is, the lowest energy state of the system, is invariant under the same symmetry transformations.

If either one of these conditions is not satisfied, then an approximate conservation law may still result. Many of the internal symmetries that arise in physical systems are of this form.

The first case is relatively straightforward. If the Lagrangian density is imperfectly symmetric, then it is often the case that any symmetry breaking term arising in the Lagrangian density is small in some sense, which makes it possible to write the Lagrangian density as

$$\mathcal{L} = \mathcal{L}_{\text{symmetric}} + \epsilon\mathcal{L}_{\text{symmetry breaking}}, \tag{5.67}$$

and treat the arising situation as a perturbation on the symmetric term. This is precisely the technique used in the view of the strong, electromagnetic and weak forces as a hierarchy where the dominant interaction respects the largest group of symmetries. Consider for example the situation where $\mathcal{L} = \mathcal{L}_{strong} + \epsilon\mathcal{L}_{EM}$, where the strong-interaction is isospin-invariant, but the electromagnetic term $\mathcal{L}_{EM}$, violates this symmetry. There is no reason to expect that such a technique could not be used more generally, a point that we shall return to in section 6.1.2.

Spontaneous Symmetry Breaking

> *We do not have to look far for examples of spontaneous symmetry breaking. Consider a chair. The equations governing the atoms of the chair are rotationally symmetric, but a solution of these equations, the actual chair, has a definite orientation in space.*
>
> Weinberg, p163, [451]

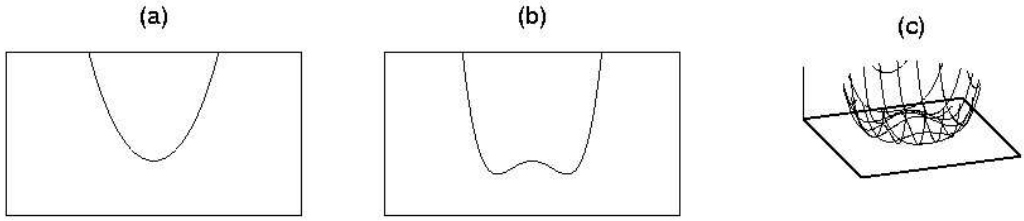


Figure 5.4: The potential $m^2\phi^2/2 + \lambda\phi^4/4!$ for (a) $\mu^2 > 0$, and (b) $\mu^2 < 0$. The potential in (c) corresponds to the 'Mexican hat' continuous set of minima that arises in the system discussed in section 5.3.4.

If on the other hand, the Lagrangian is perfectly invariant under the action of some symmetry, but the the dynamics determined by $\mathcal{L}$ give rise to a degenerate set of vacuum states which are not invariant under the same symmetry then the system undergoes *Spontaneous Symmetry Breaking* (SSB).[19] We shall start with a simple example.

Consider the following Lagrangian density, describing a single self-interacting scalar field ϕ:

$$\mathcal{L} = \frac{1}{2}\left(\partial_\mu\phi\right)\left(\partial^\mu\phi\right) - V(\phi). \tag{5.68}$$

If the potential is an even functional of the field ϕ

$$V(\phi) = V(-\phi) \tag{5.69}$$

then equation (5.68) is invariant under the parity transform $\phi = -\phi$. The specific potential

$$V(\phi) = \frac{1}{2}m^2\phi^2 + \frac{1}{4!}\lambda\phi^4 \tag{5.70}$$

is one such even functional, however, it has interesting properties depending upon the value of the mass term m. If $m^2 = \mu^2 > 0$, then the potential function illustrated in figure 5.4(a) results. This function has a unique minimum, occurring at $\mu = 0$, and the associated lowest energy state of the system, defined as $\langle\phi\rangle_0 = \langle 0|S|0\rangle$ and termed the *vacuum expectation value*, uniquely occurs at this value. This situation is not so simple if $m^2 = -\mu^2 < 0$, a scenario illustrated in figure 5.4(b). Here we see that there are two minima corresponding to this, occurring where $\frac{\delta V}{\delta\phi} = 0$, a situation that arises when

$$\langle\phi\rangle_0 = \pm\sqrt{\frac{6}{\lambda}}\,\mu \equiv \pm v \tag{5.71}$$

[19]It is worth noting that both SSB and the approximate symmetry breaking of the Lagrangian discussed above often combine in physical systems. For example, the Standard Model of elementary particle physics involves both forms.

Thus, when $\mu^2 < 0$, the lowest energy state is degenerate. Suppose that the system is near one of the minima, $+v$ say. It is likely that this state will be realised as the ground state for this particular situation. We can therefore define a new *shifted field*, $\sigma(x)$, in terms of the physical situation:

$$\sigma(x) \equiv \phi(x) - v. \tag{5.72}$$

This corresponds to arbitrarily choosing one of the two possible vacuum states as the realised lowest energy state. Using this new field, the minimum of the potential now occurs when $\sigma = 0$. Taking this into account, and dropping the constant term, we find that in terms of the new shifted field the Lagrangian density becomes

$$\mathcal{L} = \frac{1}{2}(\partial_\mu\sigma)(\partial^\mu\sigma) - \frac{1}{2}(2\mu^2)\sigma^2 - \sqrt{\frac{\lambda}{6}}\,\mu\sigma^3 - \frac{\lambda}{4!}\sigma^4 \tag{5.73}$$

which has no obvious symmetry properties unless the relationships among the coefficients are explicitly considered, the obvious parity symmetry of the original Lagrangian density has been *hidden*; a situation that is referred to as *spontaneous symmetry breaking* (SSB).

5.3.4 Goldstone's Theorem

Goldstone's Theorem [196, 197, 279, 451] states that for every spontaneously broken continuous symmetry exhibited by a theory, there must exist a massless particle associated with it, termed a Nambu–Goldstone (NG) boson. This theorem has been used extensively in the development of the Standard Model of Elementary Particle Physics [279], but also has a wide range of applications in condensed matter, and solid state physics [37].

The system in the previous section, although it undergoes SSB, does not generate NG-bosons due to the fact that the symmetry in question, the parity transform, is not continuous. However, it is simple to make the leap to a system that does generate NG-bosons. We consider a simple extension of the above model, that of a complex scalar field $\phi(x)$ which can be written in terms of two real (Hermitian) fields $\sigma(x)$ and $\pi(x)$, $\phi(x) = \frac{1}{\sqrt{2}}(\sigma + i\pi)$. The dynamics of this field takes almost the same form as the above example,

$$\begin{aligned} \mathcal{L} &= (\partial_\mu\phi^*\partial_\mu\phi) - \mu^2|\phi|^2 - \lambda|\phi|^4 && (5.74)\\ &= \frac{1}{2}(\partial_\mu\sigma)(\partial^\mu\sigma) + \frac{1}{2}(\partial_\mu\pi)(\partial^\mu\pi) - V(\sigma^2+\pi^2), && (5.75) \end{aligned}$$

where $\lambda > 0$ (since there is no lower bound on the energy otherwise), and,

$$V(\sigma^2+\pi^2) = \frac{\mu^2}{2}(\sigma^2+\pi^2) + \frac{\lambda}{4}(\sigma^2+\pi^2)^2. \tag{5.76}$$

This Lagrangian density is invariant under the global phase transformation $\phi(x) \rightarrow e^{i\alpha}\phi(x)$, and thus satisfies the $U(1)$ symmetry group. Again, this system exhibits a unique

vacuum expectation when $\mu^2 > 0$, which becomes degenerate if $\mu^2 < 0$. The potential in this case takes the form of the Mexican hat illustrated in figure 5.4(c), $V(\sigma^2 + \pi^2)$ exhibits minima occurring around the circle $(\sigma^2 + \pi^2) = v^2 = \frac{-\mu^2}{\lambda}$, one of which must be chosen arbitrarily. Again, it becomes necessary to arbitrarily choose (via a change of coordinates) one of this set of minima, as the one that gives rise to the physical vacuum state, we choose

$$\langle 0|\sigma|0\rangle = v, \qquad \langle 0|\pi|0\rangle = 0 \tag{5.77}$$

as the arbitrary minimum value. Again, this results in a shifted field,

$$\sigma'(x) \equiv \sigma(x) - v \tag{5.78}$$

which has a local minimum at zero, and a new Lagrangian density which hides the original symmetry of the system

$$\mathcal{L} = \frac{1}{2}(\partial_\mu\sigma)(\partial^\mu\sigma) + \frac{1}{2}(\partial_\mu\pi)(\partial^\mu\pi) - \mu^2\sigma'^2 - \lambda v\sigma'(\sigma'^2 + \pi^2) + \frac{\lambda}{4}(\sigma'^2 + \pi^2)^2. \tag{5.79}$$

This equation shows that the shifted field σ', acquires a mass $\sqrt{2}|\mu|$, but no terms contain a π term with a μ term; the π field is massless and termed a *Nambu–Goldstone boson*, or NG-mode.

Goldstone's theorem holds for both global and local continuous internal spontaneously broken symmetries. However, gauge (*i.e.* local) symmetries lead to an interesting extra effect, called the Higgs mechanism [279] which serves to dispose of the surplus massless NG modes through an interesting combination with initially massless gauge fields. This mechanism will not be discussed here, instead we shall now relate the number of emergent NG bosons to the number of broken generators of a given symmetry.

Consider a set of fields ϕ_i invariant under some representation of a group G, which has N generators. The dynamics of this system are described by the now familiar Lagrangian density:

$$\mathcal{L} = \frac{1}{2}(\partial_\mu\phi)(\partial^\mu\phi) - V(\phi) \tag{5.80}$$

for some potential of the fields $V(\phi)$. Assuming that a nontrivial vacuum exists, it can be found in the standard way by calculating the minimum of the potential:

$$\left.\frac{\delta V}{\delta\phi_i}\right|_{\phi=v} = 0. \tag{5.81}$$

As with the above cases, a minimum of the potential is arbitrarily chosen to occur at some point v_i on the minimum, that is

$$\langle\phi_i\rangle_0 = \langle 0|\phi_i|0\rangle = v_i. \tag{5.82}$$

Thus the degeneracy of the vacuum can again be broken by arbitrarily singling out some direction in the state space

$$\langle\phi_i\rangle_0 = \begin{pmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ v_{M+1} \\ \vdots \\ v_N \end{pmatrix} = (v_i) \tag{5.83}$$

This arbitrarily chosen vacuum state is invariant under some subgroup, $H \subset G$, which has M generators, $\bar{L}^a_{ij}$, which by construction leave the vacuum state v_i unchanged, $\bar{L}^a_{ij}v_j = 0$. There are also the $N - M$ remaining generators of the group G, for which $\tilde{L}^a_{ij}v_j \neq 0$, and which do not therefore annihilate on the vacuum v_i.

Taylor expanding the potential around the new shifted minimum $\phi_i = v_i$, up to second order we obtain

$$V(\phi) = V(v) + \frac{1}{2}(\phi - v)_i(\phi - v)_j \left(\frac{\partial^2}{\partial\phi_i\partial\phi_j}V\right)_v + \dots . \tag{5.84}$$

The coefficient of the quadratic term is known as the *mass matrix*,

$$\left(\frac{\partial^2}{\partial\phi_i\partial\phi_j}V\right)_v \equiv M^2_{ij}, \tag{5.85}$$

this is a symmetric matrix whose eigenvalues give the masses of the relevant fields. The eigenvalues cannot be less than the minimum value v.

Goldstone's theorem implies that every continuous symmetry of the Lagrangian density which is not a symmetry of the vacuum solution $\langle\phi\rangle_0$ generates a zero eigenvalue of the mass matrix (5.85).

A general continuous symmetry transformation takes the form

$$\phi^a \longrightarrow \phi^a - i\theta^a L^a\phi \tag{5.86}$$

where θ^a is a set of infinitesimal parameters, and $L^i \equiv i\epsilon^{ijk}x^j\partial^k$ is a generator of the fields ϕ_i. The potential $V(\phi)$ must be invariant under this transformation, $V(\phi^a) = V(\phi^a + \theta L^a\phi)$, which can be written

$$\frac{\delta V}{\delta\phi_i}\delta\phi_i = -i\frac{\delta V}{\delta\phi_i}\theta^a L^a_{ij}\phi_j = 0, \tag{5.87}$$

however, since the θ^a are arbitrary parameters, equation (5.87) decouples to N equations

$$\frac{\delta V}{\delta \phi_i} L^a_{ij} \phi_j = 0. \tag{5.88}$$

Differentiating in order to discover the minimum value of this set of equations, we obtain

$$\frac{\delta^2 V}{\delta \phi_i \delta \phi_k} L^a_{ij} \phi_j + \frac{\delta V}{\delta \phi_i} L^a_{ik} = 0, \tag{5.89}$$

which is simplified further upon substitution of the minimum of ϕ as the second term vanishes ($\frac{\delta V}{\delta \phi_i} = 0$ here). We are left with

$$\left. \frac{\delta^2 V}{\delta \phi_i \delta \phi_k} \right|_{\phi = v} L^a_{ij} v_j = 0 \tag{5.90}$$

Inserting the mass matrix (5.85) into this relation we obtain a set of eigenvalue equations governing the possible masses realised by modes satisfying these dynamics

$$(M^2)_{ij} L^a_{jk} v_k = 0. \tag{5.91}$$

If L^a is a generator of the subgroup H for which the vacuum is invariant (*i.e.* $\bar{L}^a_{ij} v_j = 0$) then this equation is trivially satisfied. However, when L^a is one of the $N - M$ generators which do not annihilate the vacuum (*i.e.* $\tilde{L}^a_{ij} v_j = 0$) a less trivial solution must be found. Equation (5.91) asserts that for each of these generators, there must be a zero eigenvalue of the M^2 matrix. There are therefore $N - M$ massless bosons in the theory, one for each generator which does not annihilate the vacuum. ■

This result is very general, there is no reason to assume that it cannot be applied to any set of fields exhibiting the necessary properties. A point that will be persued in the next section, and then used in the following chapter, specifically in sections 6.4.3 and 6.5.

Implications of Goldstone's Theorem

Goldstone's theorem implies that (see the excellent review [89] for proofs of the following statements):

- The NG-mode must be *gapless*, *i.e.* its energy must vanish in the limit that its (three-) momentum vanishes.
- The NG-mode for any exact symmetry must completely decouple from all of its interactions in the limit that its momentum vanishes. This is because in the zero-momentum limit the NG-state literally is a symmetry transformation of the ground state, and is therefore completely indistinguishable from the vacuum in this limit.

These properties particularly affect the low-energy behaviour of any system to which Goldstone's theorem applies. The first guarantees that the NG-mode must itself be one

of the 'light' states of the theory, and so it must be included in any effective Lagrangian analysis of this low energy behaviour; it provides a natural description of the system. The second property ensures that the NG-mode must be weakly coupled in the low-energy limit, and strongly limits the possible form its interactions can take.

These properties of gaplessness and low-energy decoupling can apply even if the spontaneously broken symmetry in question is really not an exact symmetry. If the symmetry-breaking terms of the system's Hamiltonian are small, then the symmetry may be regarded as being approximate. In this case the violation of the gapless and decoupling properties can be treated perturbatively. The resulting *pseudo-Goldstone modes* for any such approximate symmetry are then systematically light and weakly coupled at low energies, instead of being strictly massless or exactly decoupled. An example of this, the pion, will be discussed shortly in section 5.3.5.

NG-modes can be thought of as obtained from the ground state by performing a general symmetry transformation, [89]

$$\phi(x) = \chi(x)e^{i\theta(x)} \tag{5.92}$$

where both θ and χ are defined as real. Since this transformation involves a local transformation parameter θ, we find that although the Lagrangian vanishes when it is evaluated at $\phi = v$, the configuration $ve^{i\theta(x)}$ is only related to $\phi = v$ by the symmetry when θ is a constant. This fact that θ parametrizes a symmetry direction only when it is restricted to constant field configurations guarantees that any θ-dependence of L must involve at least one derivative of θ, thereby dropping out of the problem in the limit of small derivatives *i.e.* small momenta, or long wavelengths.

All of this suggests that θ would make a good representation for the NG-mode, since this is precisely what a Goldstone mode is supposed to do: decouple from the problem in the limit of small momenta. We are led to the suggestion of using polar coordinates in order to better exhibit the NG-mode properties.

Thus, the masslessness of the NG-mode can be understood as follows. The NG-field describes oscillations tangential to the potential, and hence encounters no resistance from it, remaining massless. It can be thought of as free to move within this low energy state without acquiring any extra mass, or energy. Thus it is a long wavelength mode. The remaining fields are radial to to the lowest energy state of the potential and encounter resistance from it hence they interfere with the potential and acquire mass.

It is possible to derive the relevant NG-modes purely on the grounds of the above symmetry transformation (5.92), *i.e.* ignoring the details of the underlying model (the following has been taken from [89], more details can be found there). This allows us to extract a 'most general' Lagrangian form which will lead to the generation of NG-modes. Given that a NG-mode emerges from the description, we can rewrite the symmetry

transformation in terms of the expected mode and the arbitrary choice of coordinate that must be made, this gives the *normalised symmetry transformation*

$$\chi = v + \frac{1}{\sqrt{2}}\chi' \qquad \theta = \frac{1}{v\sqrt{2}}\varphi \tag{5.93}$$

then the canonically normalised field φ becomes

$$\varphi \to \varphi\sqrt{2}v\alpha \tag{5.94}$$

under the general symmetry transformation $\theta \to \theta + \alpha$ and, for simplicity, Poincaré invariance. The most general Lagrangian which is invariant under this transformation is an arbitrary function of the derivatives, $\partial_\mu\phi$ of the field. An expansion in interactions of successively higher dimension then gives:

$$L^{eff}(\varphi) = -\frac{1}{2}\partial_\mu\varphi\partial^\mu\varphi - \frac{a}{4v^4}\partial_\mu\varphi\partial^\mu\varphi\partial_\nu\varphi\partial^\nu\varphi + \ldots \tag{5.95}$$

where we have inserted a power of v as appropriate to ensure that the parameter a is dimensionless. This accords with the expectation that it is the symmetry-breaking scale, v, which sets the natural scale relative to which the low energy limit is to be taken.

5.3.5 Action Sequencing

Action sequencing provides a new example QFT modelling, which is to some extent separate from that of standard QFT. It is effectively a high level model of QFT, and as such should be treated somewhat separately from that modelling. The process by which the hadronic action (4.31) is derived from QCD:

$$S_{QCD}[A, \bar{q}, q] \to S_{GCM}[A, \bar{q}, q] \to S_{bl}[\mathcal{B}, D, D^*] \to S_{had}[\bar{N}, N, .., \pi, \rho, \omega, ..], \tag{5.96}$$

is not exact, and results have only been obtained to low order, but there is a clear, well defined methodology which allows for the extraction of very complex, high level behaviour from QCD. As such, it illustrates a way in which an appropriate modelling of quantum behaviour could be used to develop an understanding of complex behaviour in general.

This full process will not be discussed in detail here (see either the recent PhD thesis [205], or review article [97] for more details about the actual process of extraction), instead, this section will discuss the implications of a key intermediate step in this process, and show its relationship with Goldstone's theorem.

This step involves the determination of the dominant configuration taken by the system of bilocal fields. This is found by minimising the bilocal action from equation (4.30)

with respect to the relevant fields, $\mathcal{B}$, D and D^*:

$$\frac{\delta S_{bl}}{\delta \mathcal{B}} = \frac{\delta S_{bl}}{\delta D} = \frac{\delta S_{bl}}{\delta D^*} = 0. \tag{5.97}$$

The bilocal action has a minimum at $D^* = D = 0$ and $\mathcal{B} \neq 0$, which means that the action only has to be minimised with respect to $\mathcal{B}$:

$$\frac{\delta S_{bl}[\mathcal{B}, D, D^*]}{\delta \mathcal{B}(x, y)} = 0. \tag{5.98}$$

The dominant configuration is given by (the details of the derivation can be found in [205])

$$B^{\theta}_{CQ}(x, y) = D(x - y) \left[Tr \left(G(x, y, [B_{CQ}] \frac{M^{\theta}_m}{2} \right) + \dots \right] \tag{5.99}$$

which takes the form of a Schwinger–Dyson equation (where the diquark and baryon parts have been ignored). This is a nonlinear equation describing the extensive self-energy of quarks due to dressing by gluons. Equation (5.99) has degenerate solutions which take the form

$$G(q) = \left[iA(q) q.\gamma + \mathcal{M} + B(q) \right]^{-1}. \tag{5.100}$$

As was discussed in sections 5.3.3 and 5.3.4, the fact that equation (5.98) has nonzero solutions implies that the potential of the bilocal action has a minimum away from zero; the vacuum solution is therefore degenerate and symmetry breaking occurs. The relevant symmetry in this case is the chiral symmetry of QCD.

According to the standard understanding of QCD, there are two quark fields, u and d, which have relatively small masses. In the approximation that they are indeed massless, the Lagrangian density of QCD becomes

$$\mathcal{L} = -\bar{u} \gamma^{\mu} D_{\mu} u - \bar{d} \gamma^{\mu} D_{\mu} d - \cdots, \tag{5.101}$$

where $D_{\mu} \phi = \partial_{\mu} \phi - i A_{\mu} \phi$ is the covariant derivative. This new Lagrangian density is invariant under transformation

$$\begin{pmatrix} u \\ u \end{pmatrix} \rightarrow exp \left(i \theta^V \cdot t + i \gamma_5 \theta^A \cdot t \right) \begin{pmatrix} u \\ u \end{pmatrix}, \tag{5.102}$$

where θ^V and θ^A are independent real three-vectors and t is the three-vector of isospin matrices

$$t_1 = \frac{1}{2} \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}, \quad t_2 = \frac{1}{2} \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \quad t_3 = \frac{1}{2} \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}. \tag{5.103}$$

This Lie algebra can be written in terms of two $SU(2)$ subalgebras, which act only on the left and right-handed parts of the quark fields. However, since the u and d quarks do have small nonzero masses, the $SU(2) \times SU(2)$ symmetry is not exact. Therefore there must exist a massless NG-boson associated with this broken symmetry. This is indeed the case, the NG-boson corresponds to the pion [451].

The same symmetry can also be directly obtained from (5.100) in the approximation that quark current masses, $\mathcal{M} \to 0$. Chiral symmetry is a global chiral symmetry, $U_L(N_f) \times U_R(N_f)$, which occurs because of a Mexican hat configuration in the relevant action [97]. As per Goldstone's theorem the existence of this broken symmetry implies emergent NG-bosons. In order to find their form, we must explore the degenerate vacuum solutions in more detail. Equation (5.100) is specialised in the approximation $\mathcal{M} \to 0$ to the form [96]

$$G(q;V) = [iA(q)q.\gamma + VB(q)]^{-1} = \zeta^\dagger G(q;\mathbf{1})\zeta^\dagger, \tag{5.104}$$

where $\zeta = \sqrt{V}$, $\quad V = exp\ (i\sqrt{2}\gamma_5\pi^a F^a)$ and $\{\pi^a\}$ are arbitrary real constants $|\pi| \in [0, 2\pi]$. The NG-bosons form homogeneous Riemann coordinates for the dominant configuration manifold, again, they are the fields that are directed tangentially to the minimum of the potential. As occurred in section 5.3.4, they can be found by making an appropriate choice of new fields adapted to this manifold, in place of some of the $\mathcal{B}^\theta(x,y)$. In this case, the angles $\{\pi\}$ are chosen as new NG-mode field variables $\{\pi(x)\}$ rather than the simple coordinates above [357, 358]. Performing a derivative expansion in $\partial_\mu V(x)$, and making use of the Dirac algebra results in a matrix

$$U(x) = \exp\left(i\sqrt{2}\pi^a(x)F^a\right) \tag{5.105}$$

where $V(x) = P_L U(x)^\dagger + P_R U(x) = \exp\left(i\sqrt{2}\gamma_5\pi^a(x)F^a\right)$.

NG-modes are therefore intimately related to the structure of hadrons; not only do they occur in the hadronic action (4.31), but they are essential to its derivation.

5.4 Symmetry is Generic

> *The concept of symmetry contains a concept for difference, another for identity, and a third relating the two.*
>
> Auyang, p33, [41]

The concept of symmetry is one that has been theoretically utilized in the construction of models since the the time of the ancient Greeks. Symmetries occur somewhat prolifically in natural systems. This section will discuss a number of fields which have made use of concepts such as symmetry and symmetry breaking.

5.4.1 Economics

Economics has always had a strong connection with mathematics, but it lacks the emphasis upon geometry which drove much of the work on symmetry in physics. Despite this, there are a number of symmetry principles used in this field:

- It is possible in economics to pose questions about subspaces defined by a preference relation, *indifference curves*, and their invariance. The collection of articles in [377], and in particular the introduction by Sato and Ramachandran [378] has a good introduction to the uses, as well as the development of, the techniques of symmetry and conservation laws, or *geometric methods* in economics.
- The collective behaviour exhibited by stock time series data during extreme market events [265, 79, 415], reveals a symmetry breaking event. The shape of the ensemble return distribution changes from its symmetrical shape on typical trading days to either a positive or negative skew depending upon whether the event is a rally or a crash. This suggests that on extreme days the behaviour of the market cannot be described in the same way as during normal trading days.
- Sornette and Malevergne have created a model of rational expectation bubbles which is based upon the identification of the bubbles with alleged NG-modes of the fundamental rational pricing equation which emerge due to spontaneous breaking of price-parity symmetry [413, 415]. We shall now examine this model in more detail.

Rational expectation (RE) bubbles were introduced by Blanchard and Watson [69, 70] in an attempt to account for the empirical fact that observed prices can deviate significantly, often for extended time periods from fundamental prices. The *fundamental price*, p^f, of a security can be thought of as the present value of all its future cash flows. Rational expectation theory [294] bases this on two hypotheses; the rationality of agents (rational expectation condition), and the 'no-free lunch' condition.

Under the rational expectation condition, the best estimate of the price of an asset at a time $t+1$, p_{t+1}, can be obtained by conditionalising its expectation, $E[p_{t+1}]$, upon all possible information accumulated up to time t, which is represented by the filtration $\{F_t\}$:

$$E[p_{t+1}|F_t]. \tag{5.106}$$

The 'no-free lunch' condition imposes equality on the expected returns of every asset under a given risk neutral probability measure $\mathcal{Q}$. Combining these two conditions results in the valuation for the price:

$$p_t = \delta E_{\mathcal{Q}}[p_{t+1}|F_t] + d_t, \qquad \forall \{p_t\}_{t\geq 0}, \tag{5.107}$$

where d_t is some external dividend, and $\delta = \frac{1}{1+r}$ is the discount factor. Equation (5.107) expresses both the fact that something valuable today will be less valuable tomorrow due

to the action of the discount factor, and that any dividend that is paid out will decrease the expected price tomorrow as this is incorporated into the new pricing of the asset.

The fundmental price is a well known forward solution to equation (5.107) [294, 413]:

$$p_t^f = \sum_{i=0}^{+\infty} \delta^i E_{\mathcal{Q}}[d_{t+i}|F_t]. \tag{5.108}$$

Another solution of (5.107), in fact the general solution [198], can be found by adding an arbitrary component X_t obeying an arbitrary martingale condition: $X_t = \delta E_{\mathcal{Q}}[X_{t+1}|F_t]$, to the fundamental price

$$p_t = p_t^f + X_t, \tag{5.109}$$

an equation that nicely illustrates the way in which the valuation of an asset can deviate from the fundamental price and still satisfy all of the conditions of rational expections theory.

RE bubbles allow for just such a deviation, a rational bubble can arise when the actual market price depends, for example, upon an expected rate of change in a stock or asset. Blanchard's model consists of bubbles, X_t, which exponentially grow (through multiplication of the price by a factor $a_t = \bar{a} > 0$ with probability π) and then periodically collapse to zero ($a_t = 0$ with probability $1 - \pi$), thus returning to the fundamental price. This time dynamics of the bubbles are modelled by the iterative equation

$$X_{t+1} = a_t X_t + b_t, \tag{5.110}$$

where

$$a_t = \begin{cases} \bar{a} & \text{with probability } \pi \\ 0 & \text{with probability } 1 - \pi \end{cases} \tag{5.111}$$

and b_t is an arbitrary constant. This equation allows for the creation new bubbles after a collapse through the application of a nonzero b_t term, and these can become very large through successive multiplication of a nonzero a_t term. RE bubbles allow for a deviation from the fundamental price while keeping it as a fundamental anchor point of modelling.

Sornette and Malevergne have observed that the component X_t in (5.109) plays a role analogous to a Goldstone mode [413]. This work stems from Sornette's identification of a *price parity symmetry*:

$$p \to -p \tag{5.112}$$

between the positive and negative prices that are associated with the rational expectation condition [414, 415]. A positive or negative price quantifies our liking or disliking for some commodity; we pay a positive price for a commodity that we like or desire, and a negative price for those that we would rather not have. Consider for example a situation that arose recently in Australian politics, when it was proposed that large amounts of money could

be made if the Australian government would store nuclear waste for other countries.[20] Effectively Australia would be buying this waste for a negative price. (Conversely, Japan say, would pay Australia money to store their nuclear waste for them.)

A company is made valuable in an economy by its earnings and subsequent dividend payments. In the absence of dividends and speculation the price of a share is zero, as its earnings are nil; no share is intrinsically more or less desirable [415]. However, in the presence of earnings, and dividend payments this symmetry is broken, since a positive dividend, and its associated capital gain makes a share desirable, leading it to develop a positive price.

Sornette and Malevergne claim that the bubble terms X_t in equation (5.109) play a role analogous to NG modes [413]. This is because X_t acts to restore the broken symmetry; bubble prices can wander up or down, and in the limit where the absolute value of bubble prices become very large they will dominate over the fundamental price, which restores the independence of the share with respect to dividend; the share price will depend only infinitesimally on its dividends, and the largely random bubbles will dominate.

While this conclusion is certainly interesting, it is not actually correct. Of particular concern is the nature of parity symmetries in general; they are not continuous but rather discrete symmetries, and therefore the relevance of Goldstone's theorem is debatable in this case. In fact, the authors do not actually apply the techniques discussed in section 5.3.4 and derive NG-modes, rather they present an analogy. In this case, the analogy is flawed, NG-modes do not arise in a system without at least two fields, before such modes can arise, the potential must exhibit some form of continuous minimum (consider for example the Mexican hat in figure 5.4(c)). This is because NG modes must be free to move within the low energy state.

However, this work leads us to suggest a hypothesis that NG-modes are more prevalent than is traditionally assumed to be the case in physics. If a situation can be identified where a continuous symmetry is broken then NG-modes will offer one possible description of the system.

[20]The former Australian Prime Minister, Bob Hawke was recently quoted as saying "What Australia should do, in my judgement, as an act of economic sanity and environmental responsibility, say we will take the world's nuclear waste.... If we were to do that we would have a source of income," http://www.abc.net.au/news/newsitems/200509/s1468931.htm

5.4.2 Self-Organised Criticality and NG-modes

> *. . . the past decade has witnessed a clear acknowledgement that many natural phenomena must be described by power law statistics. . . . This has led in particular to the concept of 'self-organized criticality' (SOC), according to which certain dynamically driven spatially extended systems evolve spontaneously towards a* critical *globally stationary dynamical state with no characteristic time or length scales.*
>
> Sornette *et al.*, p2, [416]

The concept of *self-organised criticality*, (SOC), has gained wide recognition since its proposal by Bak *et al.* [46, 47]. This work presented a cellular automata model of a sandpile, where sand was slowly dropped onto a horizontal plane. Gradually, the piles of sand at each position grow larger and larger, until they become unstable (*i.e.* too high) with respect to their neighbours, this precipitates an avalanche, which itself might lead to further avalanches as the sand on top of the original 'pile' cascades onto other piles thus possibly increasing their height beyond the critical, unstable value. Similar SOC models have been presented for earthquakes [45, 228], fractal growth [32, 49, 229], forest fires [159, 129], *etc.* For more information about SOC, the reader is referred to any of the general reviews on this subject, such as [232, 48]. A certain familiarity with the concept will be assumed here.

Sornette *et al.* have presented a conceptual framework which bases emergent SOC behaviour upon the recognition that it is actually an unfolding of the parameter space representing an unstable dynamical critical point [416].

The authors consider an extension to the simple sandpile model [269], where the simple plane of the sandpile is placed into a cylinder which rotates with angular velocity $\frac{d\phi}{dt}$, brought about by the constant application of a torque T, caused by a torsion spring attached to the cylinder (see figure 5.5). Starting the as per figure 5.5(a) with the torsion force at zero ($T = 0$), we can define the rotation angle $\theta = 0$. The surface of the sand is horizontal. Then, as per figure 5.5(b), starting to exert a torsion the cylinder rotates by some angle θ, the plane of the sand is now at this angle, and the sand itself is now subjected to the same torque. At some critical point θ_c, the torque on the sand exceeds the friction holding it in place and a sand flow J starts, the magnitude of which increases if $T > T_c$. This system exhibits what is termed a *critical sliding transition*, from repose ($J = 0$ when $T < T_c$) to an active slide ($J > 0$ when $T > T_c$) the speed of which corresponds to the average rotation of the cylinder at nonzero average angular velocity $\langle \frac{d\theta}{dt} \rangle$. Suppose that the angular velocity is brought to a vanishingly small positive value ($\frac{d\theta}{dt} = 0^+$), then the response of the sandpile will be equivalent to that of the standard sandpile model; the avalanches caused will satisfy a power law distribution.

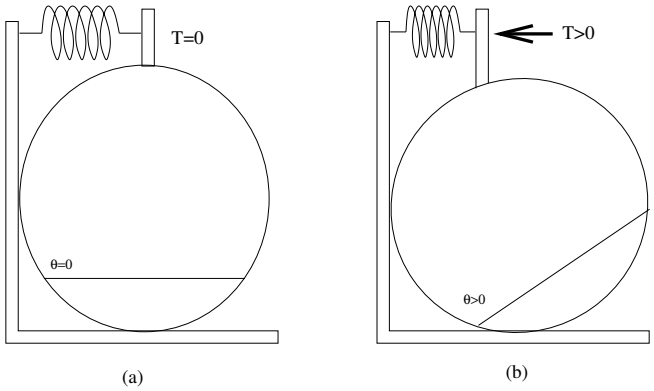


Figure 5.5: A modified SOC sandpile, sand in a cylinder is rotated at speed $\frac{d\phi}{dt}$ by the constant application of a torsion force T.

The authors claim that SOC fundamentally relies upon the existence of such an underlying sliding critical point. The mapping of SOC onto unstable critical points is controlled by driving the relevant order parameter at an infinitesimal value. This work does not explain the appearance of the corresponding unstable critical points, however, it does allow for the unification of a number of separate theories and phenomena. In particular, the authors claim that this framework clarifies a number of concepts associated with SOC,[21] and is therefore a reasonable advance upon current understanding. Of particular interest to the current discussion, this paper is claimed to clarify another work which draws a very interesting connection has been drawn between *self-organised criticality*, (SOC), and NG-modes [307]. Here, the claim is made by Obukhov that the phenomenon of SOC arises as a natural property from NG-modes in a many-body degenerate state.

Since Goldstone modes are the long wavelength modes of the system, in this limit they can be thought of as a homogeneous displacement, or rotation, of the the system of interest, that is, a translational invariance.

Obukhov claims that for any ordered, or correlated many-body state, gapless modes can be introduced which are associated with the translational invariance (*i.e.* degeneracy) of the order parameter. He claims to show that even if the system has no such degeneracy a more general system can be found which in the limit becomes the system of interest, and for which appropriate NG-modes can be found. The nonlinear interaction of the Goldstone modes causes the nontrivial critical exponents of SOC behaviour. Thus, Obukhov claims that SOC stems from the *interaction* of NG gapless modes.

[21] Such as supercritical bifurcations, the slow driving rate commonly exhibited by SOC systems, the renormalisation group analysis of SOC, the fact that these systems often evolve according to a diffusion equation (on the large scale) which itself satisfies a global conservation law but sometimes exhibit diffusion type behaviour without obeying a global conservation law, the existence of feedback mechanisms which attract the dynamics onto a critical state *etc.* refer to the paper [416] for details.

Instead, Sornette *et al.* claim that the avalanches are *nothing but* the NG fluctuations attempting to restore the broken symmetry. They claim that instead the gapless modes result from the underlying unstable critical point, stabilised by the special driving condition of the system under consideration. However, we have already seen in the previous section that the understanding of NG-modes exhibited by these authors is of a generic and undefined sense, and the same problem is exhibited by this paper; no specific applications of Goldstone's theorem are presented, and the formalism necessary for this presentation is not developed, instead NG-modes are identified by drawing a analogy between the properties of NG-modes (discussed at the end of section 5.3.4) and those of SOC systems. Again, without a proper understanding of the fields behind this system, and their dynamics, these claims are problematic at best.

Despite this inadequacy, these arguments are very suggestive. It is likely that SOC *is* closely related to NG-modes, as noted by the above authors, the scale free nature of SOC behaviour means that it can occur over scales proportional to the system of interest, and NG-modes have the same characteristic. However, until explicit models which clearly show the link between these phenomena are presented these ideas will not be useful.

One possibility presents itself in the models of Process Physics. Clear connections are made with both SOC and NG-modes in these models (see sections4.1.1 and 6.4.3). However, no rigorous link has been found within this system between these two concepts. This is a topic for future investigation.

5.4.3 Biological Modelling

Symmetry arguments are often found in biological models. For example, the body plans of most multicellular organisms exhibit some form of symmetry, either radial or bilateral. It is commonly accepted that symmetry provides a reasonable indicator of the fitness of an organism, and this has even been linked with a tendency (across four taxa) for choice of sexual partners to be affected by symmetry considerations; more symmetrical organisms are chosen as sexual partners [289].

Biological development itself consists of a process whereby symmetries are gradually broken in the creation of internal axes and the associated organ growth [315], yet very few mathematical models of this process exist. There are however a number of models of pattern formation which utilize symmetry techniques [440, 184, 274]. Some authors have argued that the genetic code itself may be a product of symmetry breaking [34, 171].

Ecology encompasses a number of fields that benefit from symmetry techniques. For example, arguments have been presented that symmetry breaking is linked with the creation of boundaries, or the severing of symmetrical structures [113], and sympatric speciation has been modelled as a symmetry breaking bifurcation [427, 428].

However, there are few formal models of symmetry considerations in the field of biology. This is not necessarily because they are impossible, and it is likely that more

understanding could result from a more consistent attempt to discuss biological systems in terms of symmetry driven models. In section 6.5 a new model of sympatric speciation will be presented which makes use of NG-modes to dynamically link species formation to niche formation.

5.4.4 Arts, Crafts and Aesthetics

Symmetry has played a strong role in a number of artistic considerations, for example, architecture has often featured been dominated by highly symmetrical structures, patterns used in Navajo and Oriental rugs, Chinese and Persian pottery, and quilting. Music often features symmetry considerations, composers such as Bartók, Tenney and Reich use symmetry as a constraint (in particular the arch form, which consists of the notes ABCBA) [411], and Bach used symmetry concepts of permutation and invariance heavily in his music [219].

This tendency towards symmetrical forms might be understood within the biological preference for symmetrical mates discussed above, but it should be understood that often a highly symmetrical piece of art is considered boring or too confusing. It is likely that much could be learned about the brain with some formal investigations of how humans relate to symmetrical structures [461, 463].

5.4.5 A Generalised Model of Emergent Behaviour?

Symmetry is a generally observable phenomenon across many apparently different systems, and symmetry breaking is already a widely applied concept in a number of areas, although not well formalised outside of physics. Even the concept of NG-modes has been developed for some systems, and used to explain (although not formally) emergent long range collective behaviour. Especially interesting is the postulated connection between NG-modes and SOC behaviour which was discussed above. Might it be possible to construct a generalised model of emergence using these already well developed theories? We shall return to this question in chapter 6, where I shall show that this is indeed the case.

5.5 The Structure of the Vacuum

> *. . . spontaneous symmetry breaking underscores the point that the vacuum in QFT is not a formless nothingness but rather a state with an intricate structure.*
>
> Earman, p341, [161]

The quantum vacuum, or ground state, is defined as the state with the lowest energy, but this does not mean that it is empty, or featureless. Rather it is an inherent part of the theory, playing a number of very important roles in different QFT's.

Firstly, as was discussed in section 5.3.3, the vacuum plays an essential role in SSB. The shape of the potential function is the instigating factor in this case; if its minimum

occurs in more than one position then the ground state becomes degenerate, and symmetry breaking occurs.

The vacuum also plays a role in procedures such as the *dressing* of *bare* properties. For example, the mass parameters used in QFT Lagrangians[22] are not physically observed, they are separate idealised mass terms, hence termed bare. Only when the interaction with the vacuum is taken into full account is a physical mass term extracted, *i.e.* dressed [336]. In order to relate the parameters of QFT with physically observable quantities the vacuum must be considered; it provides a sense of context, which is very poorly understood in current modelling.

The vacuum acquires a particularly important status in QCD; Hadrons can be viewed as collective excitations of the vacuum, so the properties of the ground state must be taken into account when studying their properties [379]. However it is not a simple state but rather a highly non-trivial state including both quark and gluon condensates; the vacuum can be thought of as a very dense state of matter, composed of quarks and gauge fields that are interacting in a very complex way. Thus, in order to give a full solution of QCD, a full description of not just hadrons but the vacuum from which they emerge must be provided. The most successful models of hadron structure all take account of some form of vacuum structure in their calculations [97, 379, 298, 125, 124].

Thus the ground state is not "empty" or uninteresting, it is rather a key feature of QFT's. At the very least it is bubbling with quantum energy fluctuations, many of which lead to observable phenomena [256, 80, 258, 214]. However, it is far more than that, it provides a context to the objects of any QFT; a different potential, or different fields obeying new symmetries can result in a system which exhibits vastly different dynamics. This idea will form a theme in the remaining chapters of this work.

5.6 Modelling Quantum Behaviour

While QFT provides one modelling of a number of quantum phenomena, there are other methodologies, particularly when one considers standard quantum theory. As this work adopts the stance that quantum behaviour is ontological, epistemological interpretations of quantum theory, or those that do not adequately describe the process of measurement are rejected; if we are to apply quantum modelling to complex, contextual and emergent systems then a consistent methodology must be adopted. This section shall discuss two interesting quantum models, before the next chapter turns this discussion to a generalisation of complex, emergent, hierarchical behaviour within the framework of QFT.

[22] Such as (4.18) and (4.19).

5.6.1 *Quantum State Diffusion and Spontaneous Localisation Theories*

> *A* quantum measurement *is an interaction between a quantum system and its environment in which the state of the quantum system significantly influences a classical dynamical variable of the environment.*
>
> Percival, p72, [328]

An interesting family of theories has been proposed, which modify the Schrödinger equation in such a way that systems become localised under certain conditions. In particular the process of measurement is incorporated into these theories. There are two general formulations of these theories; The GRW theory that was developed by Ghirardi, Rimini and Weber [183] which is based upon stochastic 'quantum jumps', and the quantum state diffusion (QSD) theory that has been suggested by Gisin and Percival [333, 188, 189, 190], which is based upon a process of continuous diffusion. Diósi has reformulated the GRW jump theory as a diffusion theory [154], so these two approaches are essentially the same.

Both of these formulations have their origin in a computational technique that was developed as an alternative way of solving the master equations that are often derived in a description of quantum systems [207]. Master equations describe the time evolution of every possible member of an ensemble of possible states that a system $\mathcal{S}$ can be found in, and the (to some extent unknowable) interaction of each of these states with their surroundings. As such, they can quickly become very complicated, and virtually useless as computational tools for all but quite simple systems, if the attempt is made to solve them directly.

One explicit generalized master equation is the Bloch Equation in Lindblad form (BELF), [266], which describes the time evolution of a density matrix $\rho = |\psi\rangle\langle\psi|$ which is in its reduced form, ρ_r, due to the tracing out of 'irrelevant' environmental factors (see appendix C for a brief introduction to generalised master equations)

$$\dot{\rho_r} = -\frac{i}{\hbar}\left[H, \rho_r\right] + \sum_j \left(L_j \rho_r L_j^\dagger - \frac{1}{2} L_j^\dagger L_j \rho_r - \frac{1}{2} \rho_r L_j^\dagger L_j \right), \tag{5.113}$$

This equation represents the interaction of the smaller quantum system ρ_r, (evolving according to a Hamiltonian H), with a set of 'detectors' the effects of which, are represented by the *Lindblad operators* L_j [266]. However, the predictions obtained are statistical in nature, phenomena such as the 'click' heard when an extended quantum system is incident upon a detector cannot be obtained from this formalism. In order to describe individual cases, this equation must be 'unravelled' into a noisy evolution equation.

The Quantum State Diffusion equation (QSD) is just such an unravelling of equation (5.113),

$$|d\psi\rangle = -\frac{i}{\hbar}|\psi\rangle dt - \frac{1}{2}\sum_j (L_j^\dagger L_j + l_j^* l_j - 2l_j^* L_j)|\psi\rangle dt + \sum_j (L_j - l_j)|\psi\rangle d\xi_j, \tag{5.114}$$

which describes the differential evolution of the state vector $|\psi\rangle$. (See appendix C for the derivation of this unravelling.) The l_j are defined

$$l_j \equiv \langle\psi|L_j|\psi\rangle. \tag{5.115}$$

and the $d\xi_j$ are independent, complex differential random variables which represent a complex normalized Wiener process,[23] hence, if M represents a mean taken over the relevant probability distribution,

$$\begin{aligned} &\mathrm{M}\left(\mathrm{Re}\left(d\xi_j\right)\mathrm{Re}\left(d\xi_k\right)\right) = \mathrm{M}\left(\mathrm{Im}\left(d\xi_j\right)\mathrm{Im}\left(d\xi_k\right)\right) = \delta_{jk}dt \\ &\mathrm{M}\left(\mathrm{Re}(d\xi_j)\mathrm{Im}(d\xi_k)\right) = 0 \\ &\mathrm{M}(d\xi_j) = 0. \end{aligned} \tag{5.116}$$

This formalism should be compared with the ad-hoc expectation value that is invoked in the standard formulation of quantum mechanics. The L_j in the QSD map have an explicit physical significance; they are indicative of the way in which the individual Lindblad operators representing the environment act upon the state $|\psi\rangle$ and hence cannot be arbitrarily chosen. In (5.114) they act upon the time evolution of the system in two different ways. The first sum in equation (5.114) represents what could be termed the 'drift' of the state vector (if an analogy with Brownian motion is considered), while the second sum represents random fluctuations of the state vector due to the interaction of the (open) system with its environment. While the Hamiltonian term serves to delocalise $|\psi\rangle$, dispersing it and entangling it with any states it comes into contact with (including those represented by the Lindblad operators), the noise term serves to localise $|\psi\rangle$, forcing it into one of the set of alternative channels [328]. The time evolution of $|\psi\rangle$ is normalised under the dynamics of this equation due to the action of the drift term (which is essentially chosen to respect this normalisation), hence this equation is unitary despite the extra terms. This can be shown as follows:

[23] See appendix B for more details.

Because we are using Itô calculus, differentials up to the second power must be evaluated, so calculating the norm involves finding

$$||\langle\psi|\psi\rangle|| = M\int dx \left(|\psi\rangle^\dagger + d|\psi\rangle^\dagger\right)\left(|\psi\rangle + d|\psi\rangle\right) \tag{5.117}$$

$$=M\int dx\, |\psi\rangle^\dagger|\psi\rangle + M\int dx\, d|\psi\rangle^\dagger|\psi\rangle + M\int dx\, |\psi\rangle^* d|\psi\rangle + M\int dx\, d|\psi\rangle^\dagger d|\psi\rangle. \tag{5.118}$$

H is hermitian, $dt^2 = 0$, and $M(d\xi_j) = M(d\xi_j^*) = 0$, so after an amount of straightforward manipulation this reduces to

$$|||\langle|\psi|\psi\rangle|| = M\int dx\, |\psi\rangle^\dagger|\psi\rangle + M\int dx\, |\psi\rangle^\dagger \sum_j \left(\langle L_j\rangle L_j^\dagger - \frac{1}{2}L_jL_j^\dagger - \frac{1}{2}\langle L_j\rangle\langle L_j^\dagger\rangle\right)|\psi\rangle\, dt +$$

$$M\int dx\, |\psi\rangle^\dagger \sum_j \left(\langle L_j^\dagger\rangle L_j - \frac{1}{2}L_j^\dagger L_j - \frac{1}{2}\langle L_j^\dagger\rangle\langle L_j\rangle\right)|\psi\rangle\, dt +$$

$$M\int dx\, |\psi\rangle^\dagger \sum_{j,k} \left(L_j^\dagger - \langle L_j^\dagger\rangle\right)\left(L_j - \langle L_j\rangle\right)|\psi\rangle\, dt. \leftarrow \text{because } M(d\xi_j d\xi_k^*) = \delta_{j,k}dt \tag{5.119}$$

Now, if $L_j^\dagger L_j = L_jL_j^\dagger$, the three last terms in our equation cancel, and we are left with conservation of the norm, i.e.

$$|||\langle|\psi|\psi\rangle|| = M\int dx\, |\psi\rangle^\dagger|\psi\rangle = 1 \tag{5.120}$$

if ψ itself is normalised. ■

There are a number of advantages possessed by the QSD equation when it is compared to the BE-LF. Firstly, and most unambiguously, there is a computational advantage, the BE-LF deals with an $(N \times N)$-dimensional matrix, as such it uses a large amount of computer memory and has a relatively long computational time. The QSD equation considers a $(N \times 1)$-dimensional vector, so individual runs of this equation will take up considerably less time and computer memory. However, to obtain an ensemble result, the equation must be run a number of times, which makes it less computationally efficient if the system being considered is very complex (i.e. if N is large). There is also an ontological advantage; while the BE-LF describes the behaviour of an entire *ensemble* of systems, the QSD equation models the evolution of one single system, as such it can be used to understand the dynamics of *individual* processes. This means that the QSD equation may be able to describe the process of detection in a non-arbitrary manner. In particular, it makes a realistic interpretation of the wave function possible. In this picture a quantum system evolves according to (5.114), becoming entangled with 'macroscopic' objects and

localising as a as a result of this interaction. Even the appearance of macroscopic objects is understood within this picture, a quantum system which cannot feasibly be separated from environmental interaction will localise when its time evolution is mapped using (5.114).

Thus the QSD equation, rather than merely being a computational convenience, is suggested as a possible foundation for a new theory — one which would be an *extension* of quantum mechanics.

In order for equation (5.114) to provide a viable alternative interpretation of quantum mechanics, it must be able to obtain the standard results of that theory. In particular, it must be possible to derive the BE-LF (5.113) from the QSD equation (5.114). The next section will show that this is indeed the case.

Derivation of ρ from the QSD equation

We are using stochastic calculus (see appendix B), so because of Itô's theorem, differentials must be calculated up to the second order, that is, to find $\dot{\rho}$ we must evaluate

$$d\rho = \mathrm{M}\left(|d\psi\rangle\langle\psi| + |\psi\rangle\langle d\psi| + |d\psi\rangle\langle d\psi|\right). \tag{5.121}$$

Substitution of equation (5.114) into each individual term of the Itô form (5.121) gives

$$\begin{aligned}
&\mathrm{M}|d\psi\rangle\langle\psi| \\
&= \mathrm{M}\left(-\frac{i}{\hbar}H|\psi\rangle dt - \frac{1}{2}\sum_j\left(L_j^\dagger L_j + l_j^* l_j - 2l_j^* L_j\right)|\psi\rangle dt + \sum_j\Big(L_j - l_j\Big)|\psi\rangle d\xi_j\right)\langle\psi| \\
&= -\frac{i}{\hbar}H\mathrm{M}\Big(|\psi\rangle\langle\psi|\Big)dt - \frac{1}{2}\sum_j\left(L_jL_j^\dagger + l_j^*l_j - 2l_j^*L_j\right)\mathrm{M}\Big(|\psi\rangle\langle\psi|\Big)dt + \sum_j\Big(L_j - l_j\Big)\mathrm{M}\Big(d\xi_j|\psi\rangle\langle\psi|\Big) \\
&= -\frac{i}{\hbar}H\rho dt - \frac{1}{2}\sum_j\left(L_jL_j^\dagger + l_j^*l_j - 2l_j^*L_j\right)\rho dt
\end{aligned} \tag{5.122}$$

and since $|d\psi\rangle\langle\psi| = \Big(|\psi\rangle\langle d\psi|\Big)^\dagger$, we can immediately write

$$\mathrm{M}|\psi\rangle\langle d\psi| = \frac{i}{\hbar}\rho H dt - \frac{1}{2}\rho\sum_j\left(L_j^\dagger L_j + l_jl_j^* - 2l_jL_j^\dagger\right)dt. \tag{5.123}$$

The last term of (5.121) evaluates to

$$
\begin{aligned}
&\mathrm{M}\left(|d\psi\rangle\langle d\psi|\right)\\
=&\mathrm{M}\left(-\frac{i}{\hbar}H|\psi\rangle dt-\frac{1}{2}\sum_j\left(L_j^\dagger L_j+l_j^*l_j-2l_j^*L_j\right)|\psi\rangle dt+\sum_j\Big(L_j-l_j\Big)|\psi\rangle d\xi_j\right)\\
&\quad\times\left(\frac{i}{\hbar}\langle\psi|Hdt-\frac{1}{2}\langle\psi|\sum_k\left(L_kL_k^\dagger+l_kl_k^*-2l_kL_k^\dagger\right)dt+\langle\psi|\sum_k\left(L_k^\dagger-l_k^*\right)d\xi_k^*\right)\\
=&-\frac{i}{\hbar}H\mathrm{M}\Big(|\psi\rangle\langle\psi|d\xi^*\Big)\sum_j\Big(L_j^\dagger-l_j^*\Big)dt+\frac{i}{\hbar}\sum_j\Big(L_j-l_j\Big)\mathrm{M}\Big(|\psi\rangle\langle\psi|d\xi_j\Big)Hdt\\
&\quad-\frac{1}{2}\sum_j\left(L_j^\dagger L_j+l_j^*l_j-2l_j^*L_j\right)\mathrm{M}\Big(|\psi\rangle\langle\psi|d\xi_j^*\Big)\sum_j\left(L_j^\dagger-l_j^*\right)dt\\
&\quad+\sum_j\Big(L_j-l_j\Big)\mathrm{M}\Big(|\psi\rangle\langle\psi|d\xi_j\Big)\sum_k\left(L_kL_k^\dagger+l_kl_k^*-2l_kL_k^\dagger\right)dt\\
&\quad+\sum_{j,k}\Big(L_j-l_j\Big)\mathrm{M}\Big(d\xi_jd\xi_k^*|\psi\rangle\langle\psi|\Big)\left(L_k^\dagger-l_k^*\right)+O(dt^2)\\
&=+\sum_j\Big(L_j-l_j\Big)\rho\left(L_j^\dagger-l_j^*\right),
\end{aligned}
\tag{5.124}
$$

where we have made use of the fact that $\mathrm{M}(d\xi_j)=0$ from (5.116), and made the assumption that dt is small, so $dt^2\to 0$. We can now use these three results, together with the Itô form (5.121), to find $\dot{\rho}$:

$$
\begin{aligned}
\dot{\rho} &= \frac{d\rho}{dt}\\
&= \frac{1}{dt}\mathrm{M}\left(|d\psi\rangle\langle\psi|+|\psi\rangle\langle d\psi|+|d\psi\rangle\langle d\psi|\right)\\
&= -\frac{i}{\hbar}H\rho-\frac{1}{2}\sum_j\left(L_jL_j^\dagger+l_j^*l_j-2l_j^*L_j\right)\rho+\frac{i}{\hbar}\rho H-\frac{1}{2}\rho\sum_j\left(L_j^\dagger L_j+l_jl_j^*-2l_jL_j^\dagger\right)\\
&\quad+\sum_j\left(L_j\rho L_j^\dagger-l_j\rho L_j^\dagger-L_jl_j^*\rho+l_jl_j^*\rho\right)\\
&= -\frac{i}{\hbar}[H,\rho]+\sum_j\left(L_j\rho L_j^\dagger-\frac{1}{2}L_j^\dagger L_j\rho-\frac{1}{2}\rho L_j^\dagger L_j\right)
\end{aligned}
\tag{5.125}
$$

which is the Bloch equation in Lindblad form (5.113).

So the QSD equation can reproduce the ensemble results of quantum mechanics. Is there a possibility that it might model detection?

Can the QSD equation model detection?

The following discussion is based upon the localization theorems discovered by Gisin and Percival [189].

Let $|\psi\rangle$ be the state vector of a system, where, for simplicity we shall set the Hamiltonian $H = 0$, and consider only a single environment operator L. In this case, the QSD equation reduces to

$$|d\psi\rangle = \left(-\frac{1}{2}L^\dagger L + l^* L - \frac{1}{2}l^* l\right)|\psi\rangle dt + (L - l)|\psi\rangle d\xi. \tag{5.126}$$

It is possible to divide the state vector into orthogonal subspaces or *channels* These channels are labelled by their projectors P_k, defined such that

$$P_k P_l = P_l P_k = \delta_{kl} P_l \tag{5.127}$$

$$\sum_k P_k = 1. \tag{5.128}$$

Defining the quantum expectation of the projector P

$$\langle\psi|P|\psi\rangle = \langle P\rangle_\psi = \langle P\rangle = p, \tag{5.129}$$

which is the probability of the system represented by $|\psi\rangle$ being in the channel P, the extent to which a state $|\psi\rangle$ is delocalised in the subspace of P or of its complementary projector $I - P$ can be measured by the *QMS deviation*[24]

$$(\Delta p)^2 = \langle P^2\rangle - \langle P\rangle^2 = \langle P\rangle\langle I - P\rangle = p(1-p) \tag{5.130}$$

for a single channel projector[25].

Now, the localisation of a system with respect to the channels P and $I - P$ occurs when the QMS deviation *decreases* with time [189]. From the QMS deviation (5.130) we find that the system will localise if

$$\mathrm{M}d\,(\Delta p)^2 = \mathrm{M}d\left(p - p^2\right) = \mathrm{M}\,(1-2p)\,dp - \mathrm{M}\,(dp)^2 \leq 0 \tag{5.131}$$

Now, we make the assumption that L only operates in the subspace of the projector P, so that

$$L = PL = LP = PLP. \tag{5.132}$$

[24]The quantum mean square deviation, is essentially just a mean square deviation defined for quantum operators.

[25]A single channel projector is similar to an *or* gate — it has one of two possible outcomes *yes* or *no*.

We can use Itô calculus to find the value of dp

$$\begin{aligned}
dp =& d\langle\psi|P|\psi\rangle \\
=& \langle\psi|P|d\psi\rangle + \langle d\psi|P|\psi\rangle + \langle d\psi|P|d\psi\rangle \\
=& \langle\psi|P\left(\langle L^\dagger\rangle L - \frac{1}{2}L^\dagger L - \frac{1}{2}\langle L^\dagger\rangle\langle L\rangle\right)|\psi\rangle dt + \langle\psi|P\left(L - \langle L\rangle\right)|\psi\rangle d\xi \\
&+ \langle\psi|\left(\langle L\rangle L^\dagger - \frac{1}{2}LL^\dagger - \frac{1}{2}\langle L\rangle\langle L^\dagger\rangle\right)P|\psi\rangle dt + \langle\psi|\left(L^\dagger - \langle L^\dagger\rangle\right)P|\psi\rangle d\xi^* \\
&+ \langle\psi|\left(\langle L\rangle L^\dagger - \frac{1}{2}LL^\dagger - \frac{1}{2}\langle L\rangle\langle L^\dagger\rangle\right)P\left(\langle L^\dagger\rangle L - \frac{1}{2}L^\dagger L - \frac{1}{2}\langle L^\dagger\rangle\langle L\rangle\right)|\psi\rangle dtdt \\
&+ \langle\psi|\left(L^\dagger - \langle L^\dagger\rangle\right)P\left(L - \langle L\rangle\right)|\psi\rangle d\xi d\xi^* \\
=& \left(\langle L^\dagger\rangle\langle L\rangle - \frac{1}{2}\langle L^\dagger L\rangle - \frac{1}{2}\langle L^\dagger\rangle\langle L\rangle P + \langle L\rangle\langle L^\dagger\rangle - \frac{1}{2}\langle LL^\dagger\rangle - \frac{1}{2}\langle L\rangle\langle L^\dagger\rangle P\right)dt \\
&+ \langle\psi|P\left(L - \langle L\rangle\right)|\psi\rangle d\xi + \langle\psi|\left(L^\dagger - \langle L^\dagger\rangle\right)P|\psi\rangle d\xi^* + \left(\langle L^\dagger L\rangle - 2\langle L^\dagger\rangle\langle L\rangle + \langle L^\dagger\rangle\langle L\rangle\langle P\rangle\right)dt \\
=& \langle\psi|L - \langle L\rangle P|\psi\rangle d\xi + \langle\psi|L^\dagger - \langle L^\dagger\rangle P|\psi\rangle d\xi^* \\
=& \left(1 - \langle P\rangle\right)\langle L\rangle d\xi + \left(1 - \langle P\rangle\right)\langle L^\dagger\rangle d\xi^*. \qquad (5.133)
\end{aligned}$$

Now, since Itô processes are Markovian they are non-anticipating, hence

$$\mathrm{M}X d\xi = 0 \qquad (5.134)$$

for any X. Using this relation in (5.133) we immediately find that

$$\mathrm{M}\left(1 - 2p\right)dp = 0 \qquad (5.135)$$

and that

$$\begin{aligned}
\mathrm{M}\left(dp\right)^2 =& (1 - \langle P\rangle)^2\langle L\rangle^2 dt + (1 - \langle P\rangle)^2\langle L^\dagger\rangle^2 dt \\
=& (1-p)^2 dt\left(\langle L\rangle^2 + \langle L^\dagger\rangle^2\right) \\
=& \left|\langle L\rangle\right|^2\left(1-p\right)^2 dt \geq 0 \qquad (5.136)
\end{aligned}$$

So, referring to (5.131), we find that the system will localise into the channel P or its complement $I - P$. It is straightforward to extend this formalism to more complicated scenarios. Complex entangled states such as Schrödinger's cat quickly resolve themselves into one alive or dead cat in this formalism. Thus the QSD equation can model detection processes. Because detection is a real process, physically incorporated into the QSD equation, the contextuality of quantum systems is therefore directly and simply modelled by this extension to the Schrödinger equation. A recent paper [182] examines the effect of the modified dynamics upon perception.

However, while even the likes of John Bell suggested that spontaneous localisation theories are prime candidates for a new realistic interpretation of quantum mechanics [59], there remain a number of problems with these theories [59, 255].

A regularly leveled criticism is that of the problem of 'tails' in the theory; collapse of the wavefunction only ever occurs *for all practical purposes* [30]. Small terms always remain in this formalism which can interfere and lead to quantum-type behaviour, although this is very unlikely. These tails imply that if we are to adopt this proposal as a replacement for standard quantum theory, then the notion of a definite state must be replaced with a weaker concept of 'almost collapsed'. It is even likely that experiments can be devised to test these theories based upon this notion [182].

Another problem which presents itself is that of the *ad hoc* nature of these theories. The above discussion made a somewhat arbitrary distinction between the system and its surrounding environment; the system is modelled by the Hamiltonian term (which was simplified still more by setting it to zero), and the environment is modelled using the Lindblad operators. However, it has still obtained a reasonable result. To describe detection in full it will be necessary to consider a more complex interaction, keeping the Hamiltonian terms, and ultimately perhaps constructing an equation that does not make ad-hoc distinctions at all, but models all of reality.

Percival has attempted to remove this arbitrary division [329, 330, 331, 334] with the development of a modified theory of spontaneous localisation, named Primary State Diffusion (PSD). Essentially Percival derives a new time evolution equation, which is similar to the QSD equation but without the Schrödinger term. Thus the new operators in Percival's modified equation [330] represent the dynamics of the Schrödinger evolution as well as their diffusion. While this theory is a start in the right direction, it is far from completed, and appears in our framework to be rather arbitrary. The interested reader is referred to the papers cited above.

A less mentioned problem is faced by all realistic interpretations of quantum mechanics; they imply a preferred positional reference frame (PRF), and therefore are in disagreement with relativity which forbids such a phenomenon [283]. This is due to the fact that realistic theories must adopt a realistic approach to measurement; the collapse of the wavefunction must be a real physical process in a realistic interpretation, and hence some explanation must be provided of the way in which a spatially separated, entangled quantum system (such as an EPR state) collapses preferentially depending upon who measures its state first (a concept which is forbidden by relativity). However, this is not necessarily a testable contradiction, rather, it is more a contradiction of what might be called the Einsteinian dogma. There is no immediate reason why there cannot be a hidden PRF which exhibits actual Lorentzian length contraction and time dilation effects [62]. In section 6.4.4 we shall return to this issue, with the understanding of the Universe as a quantum foam provided by Process Physics, where a PRF in the form of the foam

itself becomes necessary. Thus, a number of different arguments all seem to suggest that that a PRF is a necessary consequence of a realistic, process driven understanding of the Universe. A number of experimental results have been derived, all suggesting that such a PRF does in fact exist.

For all of these reasons, the QHFT high level modelling of the simple iterative system discussed in section 4.1.1 makes use of a QSD type addition to a standard time evolution process modelling stable structures in that system. In this theory, the process of measurement, or *objectification* results from the interplay between space and these emergent stable structures; space and matter are phenomena emergent from the same underlying process, and if matter becomes too delocalised then space acts to restore its locality, objectifying the matter, and in some cases causing a measurement to occur. However, even though it is possible to write such an equation, very little understanding results from such a step. It is necessary to develop high level models based upon this equation and to extract predictions from them. Thus one model of the complex Universe is not enough. In order to understand its full complexity we must create a number of models capable of examining specific instances of its behaviour. These will not be deducibly relatable, rather they will be observationally dependent upon one another; each theory being observationally emergent from its lower level counterpart.

5.6.2 Towards an Operational and Realistic Framework

In a work that owes its roots to von Neumann and Birkhoff [445], Aerts and coworkers have developed an operational, realistic generalisation of the quantum formalism that explicitly takes the contextuality of quantum mechanical systems into account [8, 15, 10, 18, 24]. This section provides an abbreviated discussion of one of the more recent papers on the subject [24].

The *operational axiomatic approach* describes a physical entity S, using

1. a set of states, $\Sigma \subset p, q, r, \ldots$,
2. a set of properties $\mathcal{L} \subset a, b, c, \ldots$
3. a relationship of 'actuality' between the states and the properties which expresses that the property $a \in \mathcal{L}$ is actual if the entity is in state $p \in \Sigma$. This involves introducing some function $\kappa : \mathcal{L} \rightarrow \mathcal{P}(\Sigma)$ which is such that $\kappa(a)$ is the set of all states of the entity S that make property a actual, called the Cartan map.

The triple $(\Sigma, \mathcal{L}, \kappa)$ is called a *state property space*, and is the basic mathematical structure of this approach.

This space can be operationally founded by supposing that for each property $a \in \mathcal{L}$ there is a yes/no experiment α that tests it. Then, a state $p \in \Sigma$ is contained in $\kappa(a)$ if and only if the outcome for the yes/no experiment α is yes with certainty. On the other hand, if $p \notin \kappa(a)$ then the outcome of experiment α can be either yes or no.

Six axioms are then introduced which allow for the formulation of a link with either classical or quantum mechanics: State Property Determination, Atomicity, Orthocomplementation, Covering Law, Weak Modularity, and Plane Transitivity. The authors claim that a theory which satisfies the six axioms can give rise to either a classical or a quantum theory, or even a mixture of both. This is because the six axioms are satisfied by a set of projection operators over either a classical state space, or a complex Hilbert space. However, one form of system, that of a *separated quantum system* is shown to elude this axiomatic description. Specifically, the axioms of covering law, and weak modularity fail for entangled, spatially separated quantum systems.

The authors point out that this problem is also inherent in standard quantum mechanics. This is correct, it is one aspect of the quantum measurement problem revisited in a new guise. There is no problem with the description of separated quantum systems until an act of measurement occurs. As was discussed in section 5.1.4, if the entities are entangled (and hence not separated in the standard sense — see section 5.1.1) then inconsistent predictions can be obtained from the quantum formalism. Specifically, the contextuality of these measurements must be taken into account. With a deeper understanding of the foundations underlying quantum theory, these problems could be avoided, something that both the spontaneous localisation theories discussed above, and the operational approach discussed here are moving towards. While neither of these approaches are fully implemented, they are both very promising, and share many aspects. The focus in this work will centre upon spontaneous localisation theories as the QSD approach has been incorporated into Process Physics (this will be discussed in section 6.4.1). However, a number of interesting results have been obtained from the operational realistic approach of Aerts and coworkers which have influenced the current work and so will now be discussed. A closer comparison of the two approaches is warranted, but will be left to the future.

Firstly, the cause of quantum probabilities is clarified. Probability theory, as formalised originally by Kolmogorov [250] was originally aimed at describing uncertainty to do with our lack of knowledge about some system. Quantum statistics are not associated with a lack of knowledge, their behaviour has been widely recognised as different [3, 20, 17]. In some cases the probability distribution necessary to describe the behaviour of a system can be decidedly non-Kolmogorovorian. For example, it has been shown that in situations where one moves from a state of indecision to a decided state, and where the change of state is context-dependent the probability distribution necessary to describe the system is non-Kolmogorovian [14]. In fact, as there is no real separation between the contextual systems examined by the group at CLEA and contextual systems in general, it is likely that a large number of contextually dependent systems will not satisfy Kolmogorovian statistics. If we adopt a model of the world where experimenters are interacting with and controlling their world, and where a system exhibits some sort

of contextual dependence upon their actions, it is possible to suggest that many different forms of random behaviour (specifically distributions) might be found. Given that only one concrete example of this phenomenon exists at present, which is reasonably well modelled by the quantum formalism, we might ask if quantum-type models can be extended beyond their traditional scope. As was discussed in section 5.2, the group at CLEA has followed just such an approach, suggesting that phenomena such as decision making, concept formation and voting can be modelled by some version of the quantum formalism. Thus, the quantum formalism is far more widely applicable than is usually considered to be the case.

Secondly, the group at CLEA does not believe that General Relativity is a good starting point for the development of a more complete theory of all physical phenomena [24]. The apparent nonlocality of the wavefunction leads to a suggestion that quantum phenomena are 'not inside space', even after the apparent localisation due to measurement. They claim that

> *Reality is much bigger than those parts of it that are contained inside space. Space should be interpreted as a structure that has emerged together with the macroscopic material entities that have emerged from the microscopic quantum entities, and it has emerged as 'their' space, meaning the 'space' in which these macroscopic entities exist and interact, as an emergent structure.*
>
> Aerts et al., p21, [24]

and propose a new philosophical view of reality, *creation discovery view* [12], which holds a lot of promise. This view involves a concept of potential *happenings*, which are aspects of experience that might be acted upon, and actualised *creations*, or experiences which are created, controlled, and acted upon by that participant. This is sometimes referred to as potentialities and actualities [176], where a process of *context-driven actualisation of potential* (CAP) has been used in the development of an interesting theory of evolution.

Thus, the group at CLEA appears to be formulating a very broad process oriented approach which rejects the static spacetime structure of modern physics, where an experimentalist is merely a passive observer, rather than an active participant in reality. There are therefore many similarities between the philosophy of Process Physics, and the approach adopted by CLEA. However, while the focus at CLEA appears to centre upon operational descriptions of phenomena, the emphasis of Process Physics is to achieve a dynamical understanding of reality. For this reason, it is believed that emergent complex behaviour fits more securely into the Process Physics approach, even though there is work underway to develop dynamical models of evolution and morphogenesis within the framework of CAP [25, 176].

Chapter 6

Quantum Field Theoretic Descriptions of Emergent Phenomena

The last chapter has discussed a number of techniques used in the analysis of different quantum theories, showing that the problems these theories solve are not specific to physics alone. It seems that high end complexity shares characteristics with quantum systems. In particular, quantum theories are well equipped to describe phenomena such as contextuality, hidden symmetries and their associated NG modes, and hierarchical levels of behaviour. This chapter will make these ideas explicit, discussing the way in which each of these phenomena can be related, and understood within one overarching methodology.

It is necessary to make the caveat at this point that despite a number of years of effort, this work is still in its infancy. As more work is performed in this area it is expected that it will be possible to generate more models within this new methodology.

6.1 Generating Complex Emergent Behaviour

QFT offers a new way in which the concept of emergent complexity might be understood, and conceivably generated in other fields. If it is possible to identify a hidden symmetry in a system then there is a very real sense in which the resultant NG-modes of behaviour can be thought of as emergent.

Before the identification of a broken symmetry, the fields describing the behaviour of a system are to some extent arbitrary, but after the identification of NG-modes, and the redescription of the system in terms of those modes, the fields describing the system are more intimately connected with its dynamics. While traditionally, the broken symmetries of physical systems are discovered within a system of interest, and therefore their associated NG-modes cannot be thought of as emergent but rather discovered, there is no reason why a symmetry could not be dynamically broken under the influence of some time evolution equation. If this were case then the NG-modes would be the emergent result of that dynamical symmetry breaking (DSB), and the resultant NG-modes defined as emergent. This provides us with a new theoretical basis for the discussion of dynamically emergent behaviour. Such models already exist in physics, in fact, SSB was first introduced in this DSB form. Consider for example the DSB models of BCS superconductivity [52] and the Nambu and Jona-Lasino (NJL) [298, 299] models of fermion mass generation

which posit, instead of 'elementary' Higgs bosons which eat up emergent NG-modes, a vacuum condensate from which such particles emerge.

In the BCS theory, for example, the gauge invariance of electromagnetism is spontaneously broken in a dynamical manner by pairs of electrons that condense to form a bound state in the ground state of a metal. In the NJL model an interaction term between fermions is used to generate masses for chiral fermions by means of DSB mechanism inspired by the BCS theory. The 'quasi-Higgs' phenomena produced by these mechanisms are not arbitrary correlates of a choice of dynamics, rather there is a sense in which they are understood to dynamically emerge. The symmetry breaking of these systems is driven by their underlying dynamics, through a consideration of the context (*i.e.* the condensate) of the relevant fields [279, 240]. NG-modes that emerge from such a mechanism are a clear contenders for emergent phenomena, for this reason we shall create a new classification of emergent behaviour *NG-emergence*.

In addition to the above modelling of emergent behaviour, NG-emergence also offers an avenue for the *generation* of complex emergent behaviour; with the correct choice of dynamical model, emergent NG-modes could be generated. Such a model is presented in section 6.5.

6.1.1 Generating Interaction among Emergent Phenomena

The relationship between gauge principles and interaction is another feature of QFT's which might be used[1] to introduce interaction between the emergent NG-modes. After the identification of such emergent NG-modes, their localisation to some coordinate frame should serve to introduce interaction between them, thus, with an appropriate choice of dynamics it is possible that a dynamically interacting emergent system might be created.

6.1.2 Generating Hierarchical Behaviour

The above process might be carried out more than once, which would lead to the generation of emergent hierarchical behaviour. We thus see a methodology that might be used not only to understand, but to actually generate complex emergent behaviour.

It is worth noting that there are at least two different scenarios that could be utilized in the generation of this desired complex hierarchical behaviour:

1. As was mentioned in section 5.3.3, the dynamics of a system might themselves contain hierarchical information in the form of $L_{strong} + L_{weak}$ type modelling which was discussed in section 5.3.3. In this case a hierarchical understanding of the system can be developed, very similar to the hierarchical form of the Standard Model of particle physics. Such a form of dynamics could be envisaged as important in implementing a model of morphogenesis (see section 6.6) however there is an artificial sense to such an understanding. In particular, such a model would not be

[1] Or rather abused when we consider the more customary use in physics.

displaying emergent hierarchical behaviour, rather this behaviour would have to be understood as being built into the model. For this reason, such a scenario, while interesting for the purposes of modelling emergent hierarchical behaviour could not truly be said to be generating emergent hierarchical behaviour in any interesting sense.

2. The generative approach that is being proposed, where a system is chosen so that emergent NG-modes result, which are then gauged in order to generate interaction between them. With an interesting choice of dynamics this process might be recursive to some number of levels, with the new fields themselves satisfying new symmetries which are again degenerate and so on. The resultant hierarchical behaviour should be considered more genuinely emergent since it would have arisen spontaneously from the dynamics of the model, rather than being 'pre-programmed' at the level of the Lagrangian.

With this in mind, we shall now formulate a principle which could be used to generate complex emergent behaviour, exhibiting characteristics such as hierarchical structure and contextual dependence upon its supposed environment.

6.2 The Recursive Gauge Principle (RGP)

Assume that we are interested in a system describable by a set of fields ϕ the dynamics of which are given by some Action S. A dynamical sense of interaction between the fields which respects this basic dynamic could be instantiated through an appropriate application of local internal symmetry principles. This interaction, and its consequent behaviour, form what should be considered as a set of second order phenomena. But it is likely that these can be described by a set of fields themselves; the process can then be repeated. As long as there is behaviour emergent from this process it would make sense to apply it.

Obviously, the choice of fields will be important throughout this process. It will be important that this choice be dynamically driven, or the description will lapse into arbitrary theorising. *Action Sequencing*, discussed in section 4.3, provides one such dynamically driven method. In particular, in the GCM, the use of symmetry breaking techniques to find the relevant modes of description (*i.e.* the NG-modes) suggests that there is a form of NG-emergence arising in the system. NG-emergence thus provides one way in which to dynamically determine which fields should be used in the description of such a system.

In order to model, and to generate complex emergent behaviour, a Recursive Gauge Principle (RGP) will now be proposed,[2] it consists of what might be seen as two steps which might be recursively applied:

[2]This principle was first proposed in [244].

$$
\begin{array}{ccc}
\text{Hadronic Local fields} & \rightarrow & S[\bar{N}, N \ldots \pi, \rho, \omega \ldots] \\
\uparrow & & \\
\text{Bilocal fields} & \rightarrow & S[B, \bar{D}, D] \\
\uparrow & & \\
\text{GCM} & \rightarrow & S_{GCM}[A, \bar{q}, q] \\
\uparrow & & \\
\text{QCD} & \rightarrow & S[A, \bar{q}, q]
\end{array}
$$

Figure 6.1: An illustration of the different descriptive levels and their dynamics represented as an action over the relevant fields, as is derived across levels in the Global Colour Model (GCM). See section 4.3 for a more detailed discussion of some of the mathematical apparatus involved, and the references therein for a full description of the model.

1. First, a new set of field variables ψ are identified which represent the new emergent variables. These variables are dynamically determined from the behaviour of the original system.
2. A new action S_ψ is then derived from the original action S_ϕ. This derivation usually consists of a number of intermediate steps, where the original fields ϕ are one by one replaced and the effect upon the action investigated.

In its original application (the process of action sequencing discussed in section 4.3.2), this process was used to consistently jump four descriptive levels, from QCD up to a very accurate model of hadronic processes, thus it consistently describes the dynamics of what might be considered a number of different hierarchical levels. This process also provides a good understanding of interactions, both within levels where both the relevant fields and the action describing their dynamics are specified, and across levels where, for example, the way in which quark and gluon dynamics affect hadronic processes is specified. These hierarchical aspects of the model are briefly sketched in figure 6.2, for more details see section 4.3.

The context of a system of interest is modelled within this methodology through two different mechanisms:

1. The form of Lagrangian chosen will to some extent reflect the environment of the system. This is because Lagrangians typically incorporate a potential term which often takes the form of the environment into account. For example, a Lagrangian will take a different form in the case of a ball rolling down a hill compared to the case in which it is subjected to a magnetic influence. An example of this phenomenon will be discussed in section 6.5.
2. If mechanisms such as DSB are used then a condensate will become relevant to the description of the system; its dynamics will not be isolatable from this condensate, which may take a highly non-trivial form depending upon the system of interest.

Obviously, it will not be possible to use the same formalism in the description of

every complex hierarchical system; an essentially new analysis must be undertaken for each new system, but the same methodology should have applicability across a number of different systems. However, it will be unlikely that one action will smoothly transform across as many levels as occurs in QCD. Physics is perhaps the discipline most amenable to mathematical analysis as the systems that it examines are among the most separable and hence our traditional modes of analysis can be used more readily. The contextuality of physical systems is arguably the weakest; most of them are well solved by a classical reductive analysis, it was only at the boundaries of physics that it became necessary to introduce the new quantum formalism. In contrast, biological systems for example are inherently contextual, and as such, far less separable. However there are enough parallels to make this avenue of investigation worthwhile. We can thus formulate a new principle, or procedure that might be used to discover, and even to create, systems capable of displaying complex emergent behaviour:

The Recursive Gauge Principle (RGP): Given a mathematical model which exhibits some sort of first order emergence, new physically determined fields can be assigned to the emergent objects of the theory. The dynamics of the new fields can then be determined using some appropriate form of action sequencing. If we find any symmetries in the configuration, as well as the dynamics of the fields, then we might localise them, and use this to determine any extra interaction between the fields. Finally, we examine the behaviour of the new system to determine if there is any new emergent behaviour, repeating the above steps if this is the case.

This is obviously no small task. However, this procedure provides a framework which offers a number of new avenues by which our theoretical understanding of emergent systems might be enhanced. The most difficult aspect of such a procedure is the identification of the emergent objects within any model. This emergence will quite possibly have contextual and observational characteristics which will add to the complication of the analysis, however it will also add to the complexity of the final system and should not therefore be ignored.

This principle is not expected to be capable of describing all forms of complex system, rather it offers one technique of what is expected to be a new complex systems toolbox.

Of particular importance to the current work, the RGP offers a procedure by which we might hope to actually *generate* complex emergent behaviour, an application that would be particularly useful in fields such as ALife. If the RGP could be coupled with a method that consistently creates emergent fields then the mathematical description so obtained might be used to generate increasingly complex systems. It will be suggested that NG-emergence provides just such a mechanism.

It should be pointed out that while the bulk of this work has been devoted to claiming that our reductive techniques are failing, we now appear to be applying just such a

technique and claiming that it can generate high end complexity. In fact, it can be argued that the proposed techniques do not necessarily fall into the reductive paradigm as presented in this work, and in fact depending on the system, a number of different models are necessary to generate a complete understanding of its dynamics. We shall return to this point in section 7.2.3 after the coming discussion of example applications of these principles.

The remainder of this chapter will discuss a number of applications of these principles. First, claims are often made that the dynamics of cellular automata exhibit emergence and can be used in the generation of high end complexity. One of the most theoretically well understood models will be discussed and while its dynamics is indeed interesting and shows promise, it appears to be missing some vital ingredient which would allow for a direct application of the RGP. In contrast, Process Physics offers the first example of this phenomenon, and we shall return to a discussion of that system, showing that the generated complex behaviour resulting from this system can in fact be understood in terms of the RGP. We shall then turn to a new application of the principle, showing that it can in fact be applied to systems which have thus far defied our reductive techniques, thereby increasing our understanding of complex emergent behaviour.

6.3 An Example System — Solitons in Cellular Automata

One obvious candidate for a mathematical model exhibiting emergence is the *soliton*. This is a solitary wave which results from a balance between nonlinearity and dispersion in a well understood set of equations. Specifically, solitons preserve their shape and speed in collision with one another, making them very stable emergent structures. Solitons have been invoked to explain many emergent phenomena including: Jupiter's long lived[3] "giant red spot", energy storage and transfer in proteins (the Davydov soliton), and, the propagation of short laser pulses in optical fibres over long distances with negligible shape change [88]. Even three dimensional spherical soliton-type solutions, light bullets, have now been identified.[4]

Because of their stability, solitons make ideal candidates for the first order emergent behaviour that we are searching for; the identification of a soliton allows for its behaviour to be described by a dynamically determined set of fields, and the stability of solitons during interactions suggests that it may even be possible to generate a high level action describing their dynamics. Most importantly for the current discussion, there is a direct way in which they are already being used in ALife; discrete soliton equations have already been implemented in a number of cellular automata [282], including implementations of

[3] At 'its' timescale, obviously it should not be considered long lived compared with, say, the age of the Universe (which is important at a higher hierarchical level).

[4] Strictly speaking, light bullets are not solitons, as they lose energy during collisions.

the reasonably complex Fermi-Pasta-Ulam dynamics [114] which exhibit 'particles' attracting, repelling, and bouncing off of each other.[5] Often, the 'particles' come together in what might be seen as an interaction, forming complex structures before separating after a period of time. This model is particularly interesting with respect to the programme outlined above as it exhibits all of the dynamics required in order to implement the proposed method; stable structures which exhibit a number of interesting interactions emerge from a simple set of rules. A dynamically driven change of variables is a feasible goal. If such a substitution can be achieved, then the RGP might possibly be applied, and more new interaction and emergent behaviour generated.

The question of how to model such structures is vital if they are to be posited in a RGP-type modelling. One clue is provided by the work which suggests that a certain limiting procedure can be used to recover a class of filter CA machines [401, 402] from continuous differential equations [436]. These *filter* systems include the well-known parity rule filter [317], and the following filter used in the limiting procedure itself. This is defined as a two valued (0 and 1) CA, in 1 + 1 D (one space and one time dimension), where the value of the jth cell at time t, is written as u_j^t and defined

$$u_j^{t+1} = \begin{cases} 1 & \text{if } u_j^t = 0 \text{ and } \sum_{i=-\infty}^{j-1} u_i^t > \sum_{i=-\infty}^{j-1} u_i^{t+1} \\ 0 & \text{otherwise,} \end{cases} \tag{6.1}$$

where $u_j^t = 0$ for $|j| >> 1$. At time t, this rule results in a CA consisting of an infinite sequence of 0's and 1's, which contains only a finite number of 1's. If the above rule is a bit arcane, the same CA results from applying the following procedure to find the state of the CA at time $t + 1$:

1. Move every 1 only once.
2. Exchange the leftmost 1 with its nearest right 0.
3. Exchange the next leftmost 1 from the remainder of the 1's with its nearest right 0.
4. Repeat this procedure until all of the 1's have been moved.

A very interesting aspect of this CA is the fact that every possible state consists entirely of solitons; every sequence of 1's is stable, even if the 1's merge a later separation will occur and the original sequences re-obtained. An example of this behaviour is shown in figure 6.2, more discussion, and another example can be found in [436].

A limiting procedure describing the dynamics of this system has been found [436] which starts from a well known nonlinear wave equation, the one dimensional Korteweg–de Vries (KdV) equation,

$$\frac{\partial}{\partial t} a(x,t) = \frac{\partial^3}{\partial x^3} a(x,t) + a(x,t) \frac{\partial}{\partial x} a(x,t) \tag{6.2}$$

[5] See Pawel Siwak's page on iterons (http://www.cie.put.poznan.pl/Tutorials/Iterons/index-main.html) for an introduction to these models.

$$
\begin{aligned}
(t): &\quad \cdots 01100000111110001001011000000000000000000\cdots \\
(t+1): &\quad \cdots 00011000000111110100100110000000000000000\cdots \\
(t+2): &\quad \cdots 00000110000000001111111001100000000000000\cdots \\
(t+3): &\quad \cdots 00000001100000000000000011001111111100000000\cdots \\
(t+4): &\quad \cdots 00000000011000000000000001100000001111111 0\cdots \\
&\qquad\qquad \vdots
\end{aligned}
$$

Figure 6.2: An example of the time evolution of the filter type soliton described by equation (6.1). Successive time steps are illustrated down the list.

which describes the amplitude a of a wave at a time and spatial location (x, t), and hence can be considered a nonlinear equation describing the dynamics of the field $a(x, t)$. The KdV equation is well known for its general soliton solutions, is shown via the limiting procedure to be somewhat equivalent to the above CA. The procedure starts by taking the Lotka–Volterra (LV) equation (which is commonly used to describe the oscillations in the density of a population),

$$\frac{d}{dt}b_j(t) = b_j(t)\left[b_{j+1}(t) - b_{j-1}(t)\right] \tag{6.3}$$

and showing that if we substitute

$$b_j(t) = 1 + (1/6)\epsilon^2 a\left((j + 2t)\epsilon, \epsilon^3 t/3\right) \tag{6.4}$$

into (6.3) then the KdV equation is regained when the limit $\epsilon \to 0$ is taken. Thus, the LV equation is a well known discretization of the KdV equation.

To get from the LV equation to the CA requires a few intermediary steps. First, the differential-difference form of the LV equation is a continuous limit of the following difference-difference equation:

$$\frac{c_j^{t+1}}{c_j^t} = \frac{1 + \delta c_{j-1}^t}{1 + \delta c_{j+1}^{t+1}}, \quad (j, t \in \mathbf{Z}), \tag{6.5}$$

a fact which can be verified by setting $c_j^t = b_j(-\delta t)$ where $\delta \to 0$. Now, setting $c_j^t = \exp(d_j^t)$ gives

$$d_j^{t+1} - d_j^t = \ln\left(\frac{1 + \delta exp(d_{j-1}^t)}{1 + \delta \exp(d_{j+1}^{t+1})}\right), \tag{6.6}$$

introducing a positive parameter $\epsilon = -(ln\delta) - 1$ (*i.e.* $\delta = e^{-1/\epsilon}$), we notice that if we set $d_j^t = e_j^t/\epsilon$, then we can define the new function on some variable X as

$$F(X) \equiv \lim_{\epsilon \to +0} \epsilon \ln\left(1 + e^{X/\epsilon}\right) = \max[0, X]. \tag{6.7}$$

Using this relationship, equation (6.6) becomes, in the limit $\epsilon \to +0$

$$e_j^{t+1} - e_j^t = -F(e_{j+1}^{t+1} - 1) + F(e_{j-1}^t - 1). \tag{6.8}$$

Finally, defining a new variable f such that $f_x^{-y+x} = e_y^x$, we obtain

$$\begin{aligned} f_{j+1}^{j+1} - f_j^t &= -F(f_{j+1}^t - 1) + F(f_j^{t+1} - 1) \\ &\equiv -(\Delta_j - \Delta_t)F(f_j^t - 1), \end{aligned} \tag{6.9}$$

where $\Delta_j X_j^t \equiv X_{j+1}^t$, and $\Delta_t X_j^t \equiv X_j^{t+1} - X_j^t$. If the values of f_j^t are restricted to integers, then equation (6.9) describes a filter type CA, but it is possible to show that the particular CA described by this equation is essentially the same as the one described by equation (6.1). To do this, we rewrite (6.1) as

$$u_j^{t+1} = \min\left(1 - u_j^t, \sum_{i=-\infty}^{j-1} u_i^t - \sum_{i=-\infty}^{j-1} u_i^{t+1}\right) \tag{6.10}$$

$$= \sum_{i=-\infty}^{j-1} u_i^t - \sum_{i=-\infty}^{j-1} u_i^{t+1} - \max\left(0, \sum_{i=-\infty}^{j-1} u_i^t - \sum_{i=-\infty}^{j-1} u_i^{t+1} + u_j^t - 1\right) \tag{6.11}$$

finally with the introduction of $S_j^t = \sum_{i=-\infty}^{j-1} u_i^t$ we obtain:

$$S_{j+1}^{t+1} - S_j^t = -F(S_{j+1}^t - S_j^{t+1} - 1), \tag{6.12}$$

an equation that is equivalent to (6.9) if $f_j^t = S_{j+1}^t - S_j^{t+1} = (\Delta_j - \Delta_t)S_j^t$.

Thus, the filter CA described by equation (6.9) is, in some sense, equivalent to the relevant forms of the KdV and the LV equations. The route from the KdV equation to the CA is illustrated in figure 6.3 which has been largely adapted from [436] (more details about the limiting procedure can be found there). This same procedure has been used to show similar equivalence relationships between CA systems and a number of other physical systems, see the original paper [436] for details.

However, this jump in descriptive levels is not dynamically motivated, rather, a set of almost arbitrary substitutions are carried out in the attempt to derive one set of equations from the other. Thus, although this argument jumps a number of descriptive levels, we cannot see this system as implementing the RGP in its present formulation, however, there is a sense of emergence in this CA system, the solitonic behaviour is not specified,

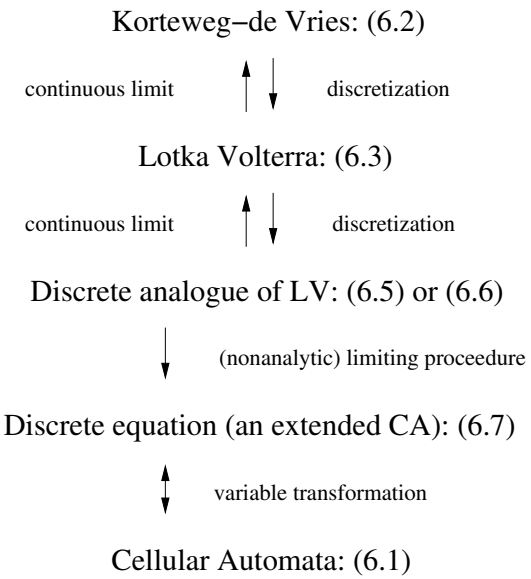


Figure 6.3: The route by which the filter CA can be derived from the KdV equation. Note the apparent similarity to figure 6.2. More explanation appears in the text.

rather it emerges out of the rules of its implementation. This leads to the possibility that we may be able to find a dynamically driven sequence of variable changes. This idea has been examined, but has not as of yet yielded any direct results. One difficulty has been an inability to find a least action type formalism which could be used in the extraction of NG-modes. As NG-emergence is at present the only known method for extracting dynamically driven emergent behaviour and hence utilising action sequencing type methods, the aim of deriving such a dynamically driven modelling of such CA systems has been reserved for future work.

6.4 Emergent Behaviour in Process Physics

Process Physics can be considered the first application of the principles developed in this work, in fact, these principles were developed from an attempt to understand the emergent behaviour exhibited by the systems that fall under the rubric of Process Physics. This section will discuss the forms taken by the emergent behaviour of the simple low level system, bringing the concepts developed in this work into sharp focus. The remainder of this chapter will be devoted to finding extensions of this methodology into very complex systems. We shall start by introducing in more detail the high level QHFT model that was briefly mentioned in section 4.2.2.

The simple iterative network model that was discussed in section 4.1 is not easy to analyse computationally. Instead a higher level model has been proposed, a Quantum Homotopic Field Theory (QHFT) which describes the behaviour of the stable structures which emerge in the low level system.

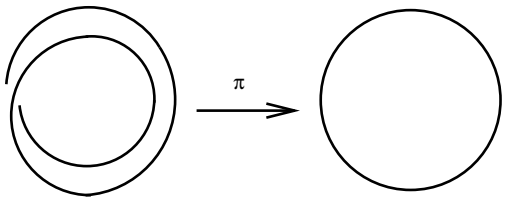


Figure 6.4: A circle S^1 that has been twisted and folded onto itself can be directly mapped onto another circle. Similar mappings can be achieved in higher dimensions.

A nonanalytic (in the Baas sense) link between the two models has been proposed by Cahill [91]. This makes use of the theory of nonlinear elastic deformations [308], suggesting that if we relax a configuration of emergent S^3 structures and their associated topological defects, then there is a relationship between the energy obtained, and the energy associated with *Skyrmions*, which are solitonic structures originally proposed as models of nucleonic structure [278, 277, 186]. A Skyrmion (a form of topological soliton) can be considered a mapping $\pi : S^3 \rightarrow S^3$, where a particular *winding number* is in this case taken to represent an integer number of 'folds' $\mathcal{Z} = \pm 1, \pm 2, ...$, which must be achieved by stable *topological defects* (TD) not of an S^3 form but embeddable under the map to the basic S^3 structures.[6] For Skyrmions, the winding number is [278, 277]

$$\mathcal{Z} = \frac{1}{24\pi^2} \int \sum \epsilon_{ijk} Tr(\partial_i U U^{-1} \partial_j U U^{-1} \partial_k U U^{-1}) \tag{6.13}$$

where U is an element of $SU(2)$.

However, Skyrmionic models do not capture the true complexity of this system. In particular they do not allow for the constant process of creation and decay that is observed in the low level model as structures emerge, interlink and gradually disappear as they become incapable of forming new links (or overlink, see the discussion at the end of section 4.1).

In order to incorporate this extra behaviour, a more complex *Quantum Homotopic Field Theory* (QHFT) has been proposed which represents the entire system of embeddings with the symbol $\Psi[t] = \psi[\ldots, \pi_{\alpha\beta}, \ldots; t]$, where $\pi_{\alpha\beta}$ represents one homotopic map $\pi : \alpha \rightarrow \beta$, which might be considered an extension of the Skyrme type embeddings discussed above.

A homotopic map is defined in this work as one particular instance of a mapping from within one homotopy group, which takes for example, a TD structure onto some other structure (but most often a S^3 hypersphere).

Homotopy groups are used in algebraic topology to classify topological spaces. They classify into equivalence *homotopy classes* the many different ways to (continuously) map

[6] Consider as a low dimensional example a circle which is folded and looped over itself as depicted in figure 6.4. In this work it is considered to be a TD with winding number 2.

an n-dimensional sphere into some other given space. Two mappings are homotopic if one can be continuously deformed into the other:

Homotopy: Two continuous maps $f_0 : X \to Y$, and $f_1 : X \to Y$ between topological spaces are said to be homotopic if there is a continuous map $F : X \times [0,1] \to Y$ such that $F(x,0) = f_0(x)$ and $F(x,1) = f_1(x)$.

This is an ideal tool to be used in the QHFT since we are interested in understanding how different structures emergent from the system can be embedded one into the other, specifically how defects can be embedded into S^3 structures. A homotopy group thus provides a notion of equivalence between two embeddings of S^3 and TD structures onto similar such structures. While mappings of defects onto defects are not to be ignored, they are most certainly very complex, and will not be discussed here. Indeed the straightforward homotopic theory of spheres is itself not totally understood, however, a number of relevant results[7] listing the stable mappings of spheres onto spheres exist [435, 353, 354]. Using the standard notation $\pi_m(X)$ to denote the set of homotopy classes of maps from an m-dimensional sphere to the space X, the following results which are relevant to the QHFT exist:

1. When $m > 0$, the set of all homotopic mappings $\pi_m(X)$ forms a group, called the m-th homotopy group of the space X.
2. $\pi_k(S^k) = Z$ for any $k \geq 1$.
   That is, there is always an integer Z which can be thought of as a *winding number* of a map from the k-sphere to itself, and any sphere of dimension greater than or equal to one can be wrapped about a sphere of the same dimension Z times. (This is the Hopf theorem [273].)
3. $\pi_m(S^k) = \{0\}$ whenever $m < k$.
   $\{0\}$ stands for the set of one element, so this result implies that there is only one way in which some sphere of arbitrary dimension can be wrapped about a sphere of a higher dimension.
4. Mapping a higher dimensional sphere onto a lower dimensional one is a very complicated process, generally specific to the particular spheres of interest. If we consider the ways in which higher dimensional spheres may be mapped onto three dimensional ones, *i.e.* S^3 structures (which would be a first approximation of the ways in which TD structures can be mapped onto basic space structures represented by S^3) then there are a number of relevant results:

[7] For a more introductory account to homotopy groups and algebraic topology in general see [273], the week 102 listing by Baez in *This weeks finds in Mathematical Physics* (http://math.ucr.edu/home/baez/week102.html) contains a reasonably straightforward introduction to these concepts as does the wikipedia entry on the topological properties of the 3-sphere (http://en.wikipedia.org/wiki/3-sphere#Topological_properties).

k	0	1	2	3	4	5	6	7	8	9	10	...
$\pi_k(S^3)$	0	0	0	Z	$Z/2$	$Z/2$	$Z/12$	$Z/2$	$Z/2$	$Z/3$	$Z/15$	...

...	11	12	13	14	15	16
...	$Z/2$	$Z/2 \oplus Z/2$	$Z/12 \oplus Z/2$	$Z/84 \oplus Z/2 \oplus Z/2$	$Z/2 \oplus Z/2$	$Z/6$

Table 6.1: The homotopy groups of S^3 (taken from [435]). The operator $\oplus$ represents that the homotopy group is the direct sum (equivalently, Cartesian product) of the cyclic groups of those orders.

(a) $\pi_4(S^3) = Z/2$, where $Z/2$ is the group with two elements, usually written 0 and 1, with addition mod 2.

(b) $\pi_5(S^3) = Z/2$.

(c) $\pi_6(S^3) = Z/12$, where $Z/12$ gives a similar set of 12 elements.

Table 6.1 depicts the relevant groups up to S^{16}.

Thus, while there are a number of mappings from high dimensional spheres to S^3, this number is not infinite which suggests that sets of stable embeddings might be taken to represent some aspects of the 'fundamental particles' or structures of this theory. The classification of these mappings and their possible consequences for the QHFT is a task well beyond the scope of this work, and must wait upon other investigations. However, it is possible to construct a general expression for the QHFT using these homotopy groups. We denote individual maps from within these groups as $\pi_{\alpha\beta}$, and note for future work that the set of all possible such maps is not infinite.

To construct the QHFT, we take as a configuration space all possible homotopic maps, denoted C_π, and describe the current state of the system as $\Psi[t]$. The time evolution of this functional is then represented using a QSD-type extension to the Schrödinger equation:

$$\Psi[t+\Delta t] = \Psi[t] - iH\Psi[t]\Delta t - \frac{1}{2}\sum_j (L_j^\dagger L_j + l_j^* l_j - 2l_j^* L_j)\Psi[t]dt + \sum_j (L_j - l_j)\Psi[t]d\xi_j \tag{6.14}$$

where the last two terms of equation (6.14) are those that were introduced during the discussion of the QSD equation in section 5.6.1. We recall that the l_j are defined

$$l_j \equiv \langle\Psi|L_j|\Psi\rangle \Longrightarrow l_j^* \equiv \langle\Psi|L_j^\dagger|\Psi\rangle, \tag{6.15}$$

and the $d\xi_j$ are independent, complex differential random variables which represent a complex normalized Wiener process (see appendix B.2 for a brief introduction), hence, if

M represents a mean taken over the relevant probability distribution,

$$\begin{aligned}
&\mathrm{M}\left(\mathrm{Re}\left(d\xi_j\right)\mathrm{Re}\left(d\xi_k\right)\right) = \mathrm{M}\left(\mathrm{Im}\left(d\xi_j\right)\mathrm{Im}\left(d\xi_k\right)\right) = \delta_{jk}dt \\
&\mathrm{M}\left(\mathrm{Re}(d\xi_j)\mathrm{Im}(d\xi_k)\right) = 0 \\
&\mathrm{M}(d\xi_j) = 0.
\end{aligned} \tag{6.16}$$

The properties and behaviour of the QHFT, as well as the implications of this model will now be discussed.

6.4.1 QSD in Process Physics

The above high level QHFT modelling (6.14) of the simple iterative system (4.1) contains QSD-type terms. The Hamiltonian term acts in the standard quantum mechanical way, it causes structures to diffuse, spreading throughout the system. Embeddings become fuzzy under the influence of this term. In contrast, the QSD terms act to localise the system when the spatial extension of the matter-type structures, S_M, violates by an amount that is deemed too high, the space-type structures, S_S.

One pleasing aspect of this formalism concerns the problem of 'tails' in spontaneous localisation models that was discussed in section 5.6.1. When a QSD-type model of objectification is incorporated into Process Physics, tails are expected; any localisation of S_M within the S_S, will of necessity never be complete, the embeddings of S_M int S_S will always be fuzzy. Thus, within this new model, the spontaneous localisation theories attain a new validity. This status is improved further by a consideration of the problem of the Preferred Reference Frame (PRF) faced by realistic collapse theories. This will be discussed in section 6.4.4.

6.4.2 The Structure of Space, Time and Matter

> *. . . the hidden measurement hypothesis has as a consequence that the 'locus' of a quantum entity is created by the position measurement itself and does not exist before the measurement has been performed. Nonlocality has to be interpreted as nonspatiality, and space cannot be seen as the theatre of all of reality. Reality is much bigger than those parts of it that are contained inside space. Space should be interpreted as a structure that has emerged together with the macroscopic material entities that have emerged from the microscopic quantum entities, and it has emerged as 'their' space, meaning the 'space' in which these macroscopic entities exist and interact, as an emergent structure.*
>
> Aerts *et al.*, p21, [24]

The understanding of Space, Time and Matter that results from both the low, and the high level models used in Process Physics is very different from that of the current understanding that is achieved by standard physics.

First, the phenomena of both space and matter are seen to be patterns emergent from the same underlying behaviour of the system; they are not two separate phenomena that must be unified, rather they might be seen as two sides of the same coin. Matter is understood in both systems as a more complex manifestation of the same behaviour that gives rise to Space. Rather than the current understanding, which sees space as a static four-dimensional manifold, time as a dimension of that manifold, and matter as fields upon that manifold, Process Physics emphasises the dynamical aspect of reality. The system must be understood in its holistic entirety, rather than from the standard object based approach; space, time and matter are intimately connected in Process Physics. Since both the high and the low level models postulate at the fundamental level a dynamical update equation (respectively (6.14) and (4.1)) which is driven by a random term, there is contingency built into the very basis of their resultant behaviour. The past can be understood as an ordered history of what has already occurred, and the present is understood as the system as it currently stands, the future on the other hand has not yet occurred, it is contingent, depending on the historical context of the system. The currently understood laws of physics are believed to emerge from the dynamics of these systems, they are patterns of behaviour, rather than built in axioms.

Space and Matter are both understood in these models as manifestations of a *quantum foam* [98, 454] in which structures are constantly being created, evolving, connecting with other structure and then decaying. This foam is dynamic, its structures can shift relative to one another. Of particular interest, any stable structures in the system will remain, despite the constant churning of the foam, their relative positions shifting and rearranging as the substrate in which they exist itself shifts and rearranges.

6.4.3 Nambu–Goldstone Modes

The concept of NG-emergence is present in this system. In section 4.3.3, the derivation of the Process Physics iterator equation, (4.1), proceeds from a simple bilocal action (4.32). The derivation of this action depends critically upon the concepts of symmetry breaking and NG-modes, in fact, as was discussed in section 5.3.5 the extraction of the relevant dynamically motivated bilocal fields depends upon finding the dominant configuration, or constituent quark effect.

Thus, NG-modes play a critical role in the extraction of an appropriate iteration equation (4.1) for the low level model. We might surmise that NG-emergence lies behind the interesting behaviour exhibited by the low level model. The fact that many of the relevant variables in this description are discarded in the extraction of (4.1), and yet a sense of observational emergence appears to emerge from the system (see section 3.2.2), suggests that if NG-emergence is indeed driving the emergent behaviour of the system then it is a reasonably robust phenomena. This is to be expected. As it has been formulated here NG-emergence requires only a dynamical breaking of a continuous symmetry so it

should be a very generic phenomenon.

NG-modes exist in the QHFT by virtue of its use of the Skyrmion concept. The Skyrme action

$$S = \int d^4x \left\{ \frac{f^2}{2} Tr(\partial_\mu U U^1 \partial^\mu U U^1) + \frac{1}{32a^2} Tr[\partial_\mu U U^1, \partial^\nu U U^1]^2 \right\} \tag{6.17}$$

when it is extended with the Wess–Zumino term in order to properly incorporate colour actually contains NG-modes, specifically, the U fields

$$U(x) = \exp[\frac{2i}{F_\pi} \lambda^a \pi^a(x)] \tag{6.18}$$

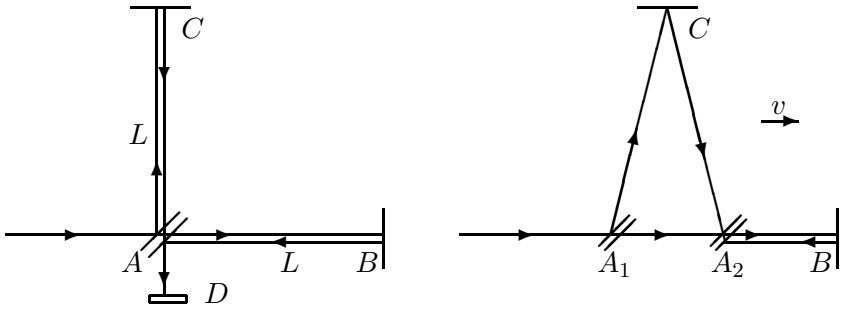
contain eight NG-modes $\pi^a(x)$ in addition to the eight $SU(3)$ generators λ^a [279]. This provides a vital clue as to the eventual form of the QHFT; we expect that NG-modes will play an important role in the dynamics of this theory.

6.4.4 Relativity in Process Physics

> *Embedding quantum theory into the Minkowski space-time is not an impossible task, but all the available options demand some rather severe sacrifices. . . . the common thread that runs through all of these proposals is that no results are to be had at a low price. Indeed, the cost exacted by those theories which retain Lorentz invariance is so high that one might rationally prefer to reject Relativity as the ultimate account of space-time structure.*
>
> Maudlin, p220, [283]

Given the above understanding of Space and Matter as different components of a quantum foam, a Lorentzian interpretation of the theories of relativity [62] appears to be appropriate. In this interpretation, the phenomena of Lorentz contraction and time dilation are considered to be real physical phenomena, resulting from an underlying dynamics of objects as they move through a preferred reference frame (PRF). Process Physics offers an insight into what that dynamics might be, namely the way in which the phenomena of space and of matter actually interact within the quantum foam. If this is the case, then a re-examination of the Michelson-Morley experiment (MM) is mandated, and was performed [107]. The results of that paper will be discussed here.

The MM experiment is depicted in figure 6.5. It shows a standard MM apparatus moving at speed v through the quantum foam. The two arms of the apparatus are constructed to have the same lengths when they are physically parallel to each other, and for convenience they are taken to have length L when at rest in the quantum foam. Since the quantum foam is taken to describe both space and matter, it is likely that different results would be obtained from MM experiments depending upon whether the apparatus is in a vacuum or filled with a gas (such as air as was the case with the original



(a) (b)

Figure 6.5: Schematic diagrams of the Micheslon Interferometer, with beamsplitter/mirror at A and mirrors at B and C, on equal length arms when parallel, from A. D is a quantum detector (not drawn in (b)) that causes localisation of the photon state by a collapse process. In (a) the interferometer is at rest in the quantum foam. In (b) the interferometer is moving with speed v relative to the quantum foam in the direction indicated. Interference fringes are observed at the quantum detector D. If the interferometer is rotated in the plane through 90^o, the roles of arms AC and AB are interchanged, and during the rotation shifts of the fringes are seen in the case of absolute motion, but only if the apparatus operates in a dielectric. By counting fringe changes the speed v may be determined.

experiments). For this reason, the speed of photons sent into the apparatus will be taken as dependent upon the medium inside the arms of the interferometer.

Lorentz contraction implies that the arm AB parallel to the direction of motion through the quantum foam is shortened to

$$L_{\parallel} = L\sqrt{1 - \frac{v^2}{c^2}}. \tag{6.19}$$

Individual photon states sent into the apparatus will then travel at speed $V = c/n$ relative to the quantum foam, where n is the refractive index of the gas and c is the speed of light, in vacuum, relative to the quantum foam. Let the time taken for ψ_1 to travel from $A \to B$ be t_{AB} and that from $B \to A$ be t_{BA}. In moving from the beamsplitter at A to B, the photon state ψ_1 must travel an extra distance because the mirror B travels a distance vt_{AB} in this time, thus the total distance that must be traversed is

$$Vt_{AB} = L_{\parallel} + vt_{AB}. \tag{6.20}$$

Similarly, on returning from B to A, the photon state ψ_1 must travel the distance

$$Vt_{BA} = L_{\parallel} - vt_{BA}. \tag{6.21}$$

Hence the total time t_{ABA} taken for ψ_1 to travel from $A \rightarrow B \rightarrow A$ is given by

$$t_{ABA} = t_{AB} + t_{BA} = \frac{L_\parallel}{V-v} + \frac{L_\parallel}{V+v} \tag{6.22}$$

$$= \frac{L_\parallel(V+v) + L_\parallel(V-v)}{V^2 - v^2} \tag{6.23}$$

$$= \frac{2L_\parallel V\sqrt{1-\dfrac{v^2}{c^2}}}{V^2-v^2}. \tag{6.24}$$

Now, assuming that the time taken for the photon state ψ_2 to travel from $A \rightarrow C$ is t_{AC}, but that the apparatus travels a distance vt_{AC} in that time, we can use the Pythagoras theorem:

$$(Vt_{AC})^2 = L^2 + (vt_{AC})^2 \tag{6.25}$$

which gives

$$t_{AC} = \frac{L}{\sqrt{V^2-v^2}}, \tag{6.26}$$

and including the return trip ($C \rightarrow A, t_{CA} = t_{AC}, t_{ACA} = t_{AC} + t_{CA}$) results in

$$t_{ACA} = \frac{2L}{\sqrt{V^2-v^2}}, \tag{6.27}$$

giving finally for the time difference for the two arms

$$\Delta t = \frac{2LV\sqrt{1-\dfrac{v^2}{c^2}}}{V^2-v^2} - \frac{2L}{\sqrt{V^2-v^2}}. \tag{6.28}$$

Now trivially $\Delta t = 0$ if $v = 0$, but also $\Delta t = 0$ when $v \neq 0$ but only if $V = c$. This then would result in a null result on rotating the apparatus. Hence the null result of the Michelson-Morley apparatus is only for the special case of photons travelling in vacuum for which $V = c$, as confirmed by the modern vacuum interferometer experiment of Brillet and Hall [84], which in-effect confirms (6.19). However if the apparatus is immersed in a gas then $V < c$ and a non-null effect is expected on rotating the apparatus, since now $\Delta t \neq 0$. It is essential then, in analysing data, to correct for this refractive index effect. Putting $V = c/n$ in (6.28) we find, for $v << V$ and when $n \approx 1^+$, that

$$\Delta t = L(n^2-1)\frac{v^2}{c^3} + ... \tag{6.29}$$

In contrast, if the data is analysed not using the Lorentz contraction (6.19), then, as was obtained using the standard analysis, the estimated time difference is

$$\Delta t = \frac{2LV}{V^2 - v^2} - \frac{2L}{\sqrt{V^2 - v^2}}, \tag{6.30}$$

which again for $v << V$ and $n \approx 1^+$, gives

$$\Delta t = Ln^3 \frac{v^2}{c^3} + ... \approx L\frac{v^2}{c^3} \tag{6.31}$$

The value of Δt is deduced from analysing the fringe shifts, and then the speed v_{MM} (in previous Michelson-Morley analyses) has been extracted using (6.31), instead of the correct form (6.29). However it is very easy to correct for this oversight. From (6.29) and (6.31) we obtain a corrected speed of the apparatus through the quantum foam v_{QF},

$$v_{QF} = \frac{v_{MM}}{\sqrt{n^2 - 1}}. \tag{6.32}$$

Note that for air at STP $n = 1.00029$, while for helium at STP $n = 1.000036$, and so the correction factor of $1/\sqrt{n^2 - 1}$ can be large.

A number of results have been obtained from examining different experimental scenarios and applying this correction factor [91, 93], and results agree very well with all experiments performed to date. In fact, a null result was not even obtained from the original Michelson-Morley experiments, rather the extracted speed of the earth through the preferred reference frame (PRF) was much smaller than expected and therefore existing theories could not explain the result. With an application of the correction factor (6.32) the original MM result becomes far more significant. The above references provide an in depth analysis of these results, which will not be discussed here due to their general complexity.

Rather than the static, passive PRF of Newtonian physics, the PRF of Process Physics is dynamical and very active. It causes inappropriately extended matter to be localised, its configuration changes as space flows and changes its relative dynamics, it can be experimentally probed and understood, although the analysis of the quantum foam is very difficult and still in its infancy. Thus the PRF in this understanding provides a very active and dynamic context against which the behaviour of matter can start to be understood; and in this theory, an understanding of one phenomenon cannot be achieved without an associated understanding of the other. Thus, in moving beyond our object based methodology in the low level model, and moving to an understanding of space and time as parts of the same complex phenomenon (as well as accepting that a nonanalytic understanding of that same model is as far as our methods can be pushed in the move to the QHFT) we have lost much of the dogma surrounding the theories of relativity, but

maintained many of their relevant results. We see how a limit to the speed of light emerges dynamically as a limit upon the speed at which information can propagate against the PRF, Lorentz contraction is understood as a real physical process, as is time dilation, and even more of the phenomena of general relativity are seen to emerge with a new shift to a higher still model of the QHFT.

6.4.5 Gravity in Process Physics

Section 4.1 discussed the manner in which, as the structures of the low level relational model overlink, they become less active in the system, and eventually 'die' being replaced by new, more active structures as they come into being. The structures which overlink are precisely those with a higher dimensionality than the base level S^3 hyperstructures, but the matter-like TD structures are assumed to be stable, existing on top of the spatial structure of the system, therefore it is space which is closely associated with the matter-like TD structures which is most likely to overlink, and separate from the dynamics of the system.

This idea has been incorporated into the quantum foam model, in which matter effectively acts as a 'sink', destroying the quantum foam as it flows into the matter. The more matter, the more quickly these spatial structures are destroyed leaving more space for new foam to move into vacated areas, and itself suffer the same fate; gravity is thus understood to be caused by the inhomogeneous flow of the quantum foam towards matter (itself part of the quantum foam).

However, so far there is no derivation of this flow physics postulated from the behaviour of the quantum foam system to the QHFT, and again, we do not expect that such a derivation would be exact, or computational, if it were found (recall the Baas definition of observational emergence in section 3.2.2). Despite these difficulties of analysis, a high-level classical theory has been based upon the expectation that gravity is caused by the above inhomogeneous in-flow. Indeed, a number of old experimental results have been found to support the supposition that this phenomenon exists [91], and new experiments have been proposed [104, 106] which will test the predictions of this theory when compared to General Relativity. This section will briefly discuss the main components of the theory, an up to date review can be found in [93] and different aspects of its details can be found in any of [100, 101, 102, 103, 104, 105, 106].

The theory is based upon asking how the gravitation acceleration, g changes dynamically depending upon the position within the quantum foam at which the acceleration is measured. This can be modelled using a fluid-type model, in fact, a similar model has already been formulated which is consistent with General Relativity [241, 242, 280]. Given the fluid-type nature of the model, we can expect effects such as acceleration, vorticity and turbulence, but the nature of the quantum foam *i.e.* the fact that space can actually disappear as it overlinks implies that this system will have sink-like phenomena

where space breaks down. This will occur in parts of the system which exhibit a higher than normal rate of linking in the low level models, that is, places with a high density of matter. In effect, space flows into massive objects, overlinks and disappears. This is the phenomena of gravity.

The acceleration of one element of space is given by the Euler form

$$g \equiv \frac{\partial v}{\partial t} + (v.\nabla)v = \frac{dv}{dt}, \tag{6.33}$$

where $v(r,t)$ is a vector 'flow' field which to low order and neglecting vorticity,[8] is modelled by the equation

$$\frac{\partial}{\partial t}(\nabla.v) + \nabla.((v.\nabla)v) + C(v) = -4\pi G\rho \tag{6.35}$$

where

$$C(v) = \frac{\alpha}{8}((trD)^2 - tr(D^2)) \tag{6.36}$$

and

$$D_{ij} = \frac{1}{2}\left(\frac{\partial v_i}{\partial x_j} + \frac{\partial v_j}{\partial x_i}\right) \tag{6.37}$$

Note that $tr(D) = \nabla v$. The new theory of gravity thus involves two constants, which cannot at present be derived, but must be fitted with experimental data:

1. Newtons gravitational constant G, which now is seen to essentially determine the rate at which matter dissipates the quantum foam.
2. A new dimensionless 'gravitational constant', α, which determines the self-interaction of the quantum foam, and turns out to be in remarkable agreement with the fine structure constant [100].

Newton's original explanation of the phenomena of gravity was also in terms of the gravitational acceleration vector field $g(r,t)$:

$$\nabla.g = -4\pi G\rho \tag{6.38}$$

where $\rho(r,t)$ is the matter density. However there is an alternative formulation [99, 92] in terms of the above vector flow field $v(r,t)$ which gives

$$\nabla.\left(\frac{\partial v}{\partial t} + (v.\nabla)v\right) = -4\pi G\rho \tag{6.39}$$

[8] A far more complicated equation has been derived for the case of vorticity [100]:

$$\frac{dD_{ij}}{dt} + \frac{\delta_{ij}}{3}tr(D^2) + \frac{trD}{2}\left(D_{ij} - \frac{\delta_{ij}}{3}trD\right) + \frac{\delta_{ij}}{3}\frac{\alpha}{8}\left((trD)^2 - tr(D^2)\right) + (\Omega D - D\Omega)_{ij}$$
$$= -4\pi G\rho\left(\frac{\delta_{ij}}{3} + \frac{v_R^i v_R^j}{2c^2} + \dots\right), \quad i,j = 1,2,3. \tag{6.34}$$

see that paper for details.

Note that for the case of the Solar system, with the mass concentrated in one object, namely the Sun, $C(v) = 0$ and therefore the in-flow field (6.35) satisfies the original Newtonian approximation for gravitational in-flow (6.39). However, this formulation is by no means the most general. In any case which does not exhibit spherical symmetry, the term $C(v) = 0$ becomes quite significant, a situation which can lead to profound differences between the predictions of the two theories. It is interesting to note that all of the original tests of both of the Newtonian and General Relativistic theories of gravity were for the simpler spherically symmetric case, and those situations which do not meet this criterion are in fact still in disagreement with experimental observations. The missing mass of the Universe is just such a scenario, dark matter has essentially been proposed because spiral galaxies do not behave as expected.

A number of results have been obtained from this new formulation of gravity:

- In this theory, black holes have a well defined physical dynamics; they arise in situations where the gravitational inflow is extreme [103].
- The problems concerning dark matter find a particularly elegant solution in this framework. This is because of the extra $C(v)$ term in equation (6.35), which does not exist in the Newtonian approximation of gravity (6.39).
- It has recently been shown [105] through a generalisation of the Schrödinger equation, that the gravitational acceleration of mass is equal to that of space, *i.e.* $g(mass) = g(space)$, which would be expected from the quantum foam perspective as it is space that is flowing, with matter essentially fixed to spatial structures.

It is interesting to note that even at this high level of description, with a number of simplifying assumptions, this model still exhibits many of the criteria that have been identified in this work with high-end complexity. The system that is extracted from this theory is highly dynamic, and contextual; the local inflow and PRF experienced by an individual depends in a gross manner upon the global nature of the system. Indeed, since all elements of this system are moving relative to one another, it becomes impossible to extract global information; local gravitational flows can be discussed, but only with reference to some larger, almost fixed frame.

6.4.6 Thermodynamics in Process Physics

This work has not concentrated upon thermodynamic theories and conceptions of complexity in any detail. In particular, the connections between open dissipative structures operating far from equilibrium (often exhibiting some form of SOC behaviour), and the theoretical constructs of Process Physics have not been discussed. This is due to the high level nature of thermodynamic arguments; while it is expected that concepts such as the second law of thermodynamics are important at this level, such behaviour is very much expected to emerge from the lower level processing of the systems discussed in this work. Indeed if such behaviour was not generated then this would be seen as a devastating

failure of the theory. However, there is every reason to suppose that such behaviour does in fact emerge from such a system. Indeed, one of the original motivating factors in the development of Process Physics was the apparent contradiction between the second law of thermodynamics, and the time reversible equations of motion that apply to classical and quantum systems, a problem that is sometimes referred to (for its historical discoverer) as Loschmidt's paradox. This problem has attracted considerable attention over the decades, appearing particularly relevant in debates about the missing arrow of time, and the apparent contradiction between the spacetime of physical theory and our biased experience of the present moment alone [345]. While the fluctuation theorem [151], gives one apparent solution to this paradox, this is achieved by *assuming* time reversal symmetry, rather than questioning the assumptions behind time reversal symmetry. Process Physics provides another resolution, but this is achieved by taking the opposite stance; time reversal symmetry is lost in Process Physics which assigns an ontological status *only* to the present. In Process Physics, the past is effectively a history (that we can recall, but not relive) while the future is yet to occur (we can attempt to predict it, but our predictions may be wrong). This is reflected in the quantum equations of Process Physics which lose their reversibility due to the actions of the noise terms. Thus, there need be no conflict between these two sets of theories under the Process Physics framework. Indeed, thermodynamics is expected to be derivable from the higher level QHFT once this theory is more fully developed, a problem that is reserved for future work. With a more adequate ontological picture of the way in which thermodynamic concepts arise, this framework would then go some distance in explaining the importance of entropic concepts in higher level systems, such as those arising in biology.

6.4.7 Observational Emergence in Process Physics

There are a number of different descriptions used in the discussion of the system of space, time and matter that is represented by the term of Process Physics:

1. The simple iterative model of a network of connected nodes as is represented by equation (4.1).
2. The QHFT which ignores much of the flux of the low level system, focussing only upon modelling the dynamics of the stable structures that emerge from that system.
3. It is possible to extract both standard quantum and relativistic behaviour from this system in the face of a number of simplifications and approximations.
4. Additionally, the modes of analysis used to extract concepts such as 'stable structure' from the low level system *e.g.* the argument utilising equation (4.3) which leads to the identification of the S^3 structures as most stable in the system, and therefore presumably most prevalent.

Despite the fact that arguments can be presented linking the different models of this system, there is no analytical link between these different modes of analysis. All modes are

necessary in order to develop a full understanding of this system. Thus, this system appears to satisfy the Baas conception of observational emergence discussed in section 3.2.2, and in fact this was shown to be the case in section 4.2. This result is also reminiscent of Rosen's definition of complexity which was examined in section 1.1.1 and the observer driven models of complexity discussed in section 1.5.1; if different models are necessary to understand the behaviour of this system then we might conclude that it is displaying complex behaviour, and possibly behaviour at the high end of the complexity scale (see section 1.5).

6.5 Proof of Concept — Sympatric Speciation

Speciation, the process whereby an existing species splits into two or more new ones, provides an example of emergent behaviour requiring explanation. There are many different definitions of a speciation event, and it is unlikely that the same mechanism will be behind all events [285, 142]. This process is generally separated into two different categories [285, 286, 355]:

Allopatric speciation occurs when some subpopulation of a single species becomes geographically isolated from the main population. Each population then evolves separately under different environment conditions.

Sympatric speciation involves a situation where the original population remains in a single geographical location, but for some reason the population splits into two different subgroups which eventually form two different species.

While allopatric speciation simply involves standard evolutionary mechanisms (*i.e.* the physical separation of populations allows for evolutionary drift which results in new reproductive boundaries forming between originally compatible member organisms of the two populations) coupled with natural selection, and is therefore well understood, this is not so in the case of sympatric speciation.

Although a number of models exist (see [155, 152, 157, 251, 217, 427, 428] and the references within for a good selection of models) the original mechanism underlying the generation of new choice is not well understood. What leads to the generation of a viable mutant?

A strong connection has been made between mating choice and speciation in the above models:

> *In sexual populations with random mating, the continual production of intermediate phenotypes from two incipient branches prevents evolutionary branching. In contrast, when mating is assortative for the ecological characters under study, evolutionary branching is possible in sexual populations and can lead to speciation.*
>
> Dobeli *et al.*, pS77, [156]

which implies that after the appearance of a viable mutant, population models can be used; once mating is assortative something like a species can already be considered to have emerged. While it is likely that a different biological mechanism will lie behind each appearance of a new viable mutant, and a biological understanding of the mechanism generated for each individual case,[9] our mathematical models of such a mechanism are not at the same state of development.[10] Throughout this work, we have seen that reductive analysis cannot generally *generate* new behaviour, or in this case objects (*i.e.* new viable phenotypes), how then can we generally model the emergence of such new mutants? Specifically, if we are interested in generating OEE in ALife modelling, what kind of formalism could lead to new, emergent species?

It is possible that the understanding of NG-emergent behaviour developed above can be used to create a new model of sympatric speciation which can explain this initial splitting of a species. The initial model will now be presented.

If we consider one species, then the phenotype of any organisms belonging to that species satisfies a symmetry as follows. We define a new term, *Species Equivalent* (SE) to express the fact that the phenotype of any organism belonging to the species is SE to that of any other organism of that species. Species equivalence of a phenotype is defined in a very broad sense for the purposes of the current model. For example, if we consider the phenotypes of two humans we see that they are considered to belong to the one species, and hence are SE, despite the fact that phenotypically they might have different eye, hair and skin colour *etc.* For the purposes of the model, we discard this extra individual specific information $p_{specific}$ and consider just the species relevant portion p_{SR}, of the phenotype, where the total phenotype is defined as:

$$p_T = (p_{SR}, p_{specific}). \tag{6.40}$$

Within the one species S, all genotypes give the same SR phenotype,

$$\{g_1, g_2, \ldots g_N\} \rightarrow p_{SR}, \tag{6.41}$$

hence there is a sense of symmetry in this system. This can be represented mathematically by extending the model to incorporate a sense of *niche*. Specifically, a niche is defined

[9] A number of biological arguments explaining this phenomenon in the case of specific examples exist [380, 65].

[10] A notable exception to this is provided by Stewart and coworkers [427, 428] who proposes a model of speciation within the framework of symmetric bifurcation theory (*i.e.* nonlinear dynamical systems theory). This work is interesting, but does not incorporate the role of the gene, concentrating instead upon a phenotypic description, and for this reason it is felt that the current model, while at present less well developed, holds more promise for our long term mathematical understanding of such systems. As the current model also works upon two levels of a hierarchy (genotype→phenotype) it is also felt to be applicable to hierarchical systems in general, not just the present example system of speciation, a possibility that will be pursued in future work.

rather broadly in this model as a potential which represents all the relevant environmental factors (or contexts) giving rise to one species. If niche is defined broadly enough then there will only ever be one species in one niche (we will delay a discussion of this idea until the end of this section). Thus, at this point the concepts of genotype, phenotype, species and niche have been naturally incorporated into this model.

Now, if we represent an arbitrary system as a set of fields $p = p_1, \ldots p_N$ where N represents the number of species in the system then we might start to understand sympatric speciation as a process which occurs when an environment, represented by a potential, dynamically changes to a situation of degeneracy. In this situation, DSB might be understood to have occurred, and according to our discussion of section 6.1, NG-emergence will be understood to have occurred. In this case, the description of the system in terms of one phenotype will become inadequate, and a shift to the dynamically motivated NG-modes required. The number of emergent modes will depend upon the number of broken symmetries; the number of ways in which the environment can be understood to have changed, generating new niches.

The representation of ecological models by a Lagrangian formulation is by no means new. A paper by Webb [449] shows that an entire class of ecological models can be directly represented as Actions through an appropriate application of Hamilton's principle. For example, the logistic growth equation representing the rate of change of the number n of organisms of phenotype p:

$$\frac{dn}{dt} = rn\left(1 - \frac{n}{k}\right) \tag{6.42}$$

where r and k are both constants, representing reproductive rate and carrying capacity respectively, is obtained via Hamilton's principle from the following action:

$$S = \int dt \left[\frac{1}{2}\left(\frac{\dot{n}}{n}\right)^2 + \frac{1}{2}r^2\left(1 - \frac{n}{k}\right)^2\right] \tag{6.43}$$

which gives a potential function of

$$V(x) = -\frac{1}{2}r^2\left(1 - e^x\right)^2 \tag{6.44}$$

if we introduce the variable $x = ln(\frac{n}{k})$ in order to directly represent equation (6.43) in the form $S = \int dt(\frac{1}{2}\dot{x}^2 - V(x))$. Equation (6.44) has the form represented in figure 6.6, which we can see has no minimum and cannot be directly used in the proposed technique. However, Webb has shown that a class of second order rate equations of the form

$$\frac{d}{dt}production = environmental\ forces, \tag{6.45}$$

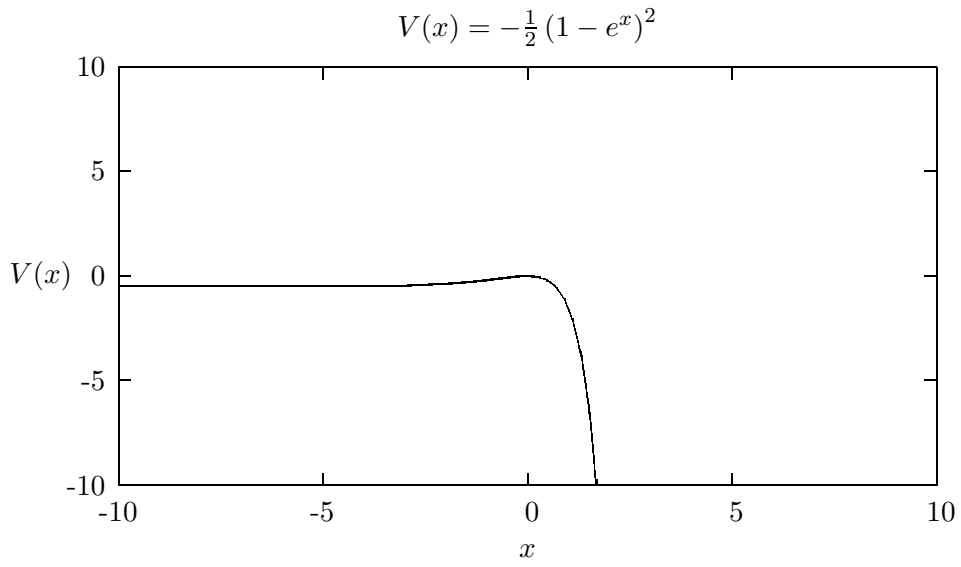


Figure 6.6: The potential function that leads directly to the logistic equation. Notice that this function has no minimum, and will not therefore be used in the application of NG-emergent behaviour.

where

$$production = \frac{d}{dt} biomass, \tag{6.46}$$

can naturally result from the correct application Hamilton's principle to ecological models. Thus, it is possible to incorporate whichever environmental forces are considered relevant to the model of interest and interesting behaviour up to at least the second order can result from a direct application of Hamilton's principle. Actions that lead to interesting behaviour can then be utilised in a field theoretic model, describing the rate of production of phenotypes within a specified niche. A broad range of actions are possible, and an investigation into their varying characteristics has only been instigated at the present moment of time, however, we know from our discussion of section 5.3.4, that the most general low order, low energy Lagrangian which will generate NG-modes has the form of (5.95). If models of this form are chosen then NG-modes will emerge from the system under a correct choice of dynamically evolving potential and a clear model of sympatric speciation will have been found.

We might very quickly construct a toy model of this phenomenon using the above arguments. First we consider a system which has one phenotype field p, thus the symmetric map (6.41) holds and construct a very simple Lagrangian describing its interaction in a simple Mexican hat potential:

$$\mathcal{L} = (\partial_\mu p^* \partial_\mu p) - \mu^2 p^2 - \lambda p^4 \tag{6.47}$$

where we have written the Lagrangian in terms of the vector field $p = (g_1, g_2, \dots)$ which represents the symmetrical phenotype in the state space of all possible genotypes, and where

$$V(p^2) = \frac{\mu^2}{2}(p^2) + \frac{1}{4}(p^2)^2, \tag{6.48}$$

where μ^2 is a key indicator of the state of the environment. Recall from section 5.3.3 that $\mu^2 > 0$ results in a potential with a unique minimum, whereas $\mu^2 < 0$ results in the Mexican hat potential illustrated in figure 5.4(c)). Assuming that under the influence of some environmental factor $\mu^2 \to -\mu^2$, the potential will become degenerate and symmetry breaking occurs, we are left with a number of new phenotypes equivalent to the number of broken symmetries, or genotypes which do not result in the same p_{SR} as was previously the case.

The discussion of Webb's work above suggests that with a more advanced model, the time dynamics of the new species might be extracted, and interesting high level behaviour predicted. For the present we note that with this new understanding of the underlying mechanism behind speciation (namely a change in a species niche) and the associated new species, there is no reason why some of the more traditional preferential mating models cannot now be utilised. However, an incorporation of this dynamics into the model is expected to be possible, and will be pursued in future work.

It is interesting to consider some of the implications of the new modelling technique for our understanding of ecological systems.

Firstly, according to this model a niche is defined as a unique set of resource plus locality characteristics associated with some environment that leads to a particular species. In this sense, the term guild [359] might perhaps be more accurately used, as this is defined as a group of species that exploits the same class of environmental resources in a similar way. For example the carrion eaters niche is occupied by the Tasmanian devil in Tasmania (Aust.), jackals and wild dogs in Africa, and vultures in the Americas. However, as the current purpose of this discussion is to identify a particular set of environmental factors that are utilised by some species in a particular ecosystem, the term niche was chosen. In the current usage, the occupation of a niche is precisely what leads to the identification of a species.

Thus, in this model the concept of niche is definitionally linked with that of species. Instead of the more traditional, object based approach which sees an abiotic landscape and then places organisms upon it, this framework inextricably links species to environment; they can no longer be separated, or decoupled in the model, which is a pleasing result since this is also the case in reality. Thus, genotype, phenotype and environment must be considered holistically in this model, something that we were led to believe was the case in the earlier chapters of this work. A change in the behaviour of one can have profound consequences for the rest of the system.

Finally, although this modelling is very simple, it is expected that a large variety of models can be obtained from it, through the utilisation of different fields and their interactions, both with each other, and with the environment.

6.6 Further Work — Biological Development

A key future goal is to use the concepts developed in this work to create a model of the process of biological development. This process can be understood as consisting of two sub-processes, morphogenesis (the development of shape) and differentiation (the creation of different cell fates depending upon spatial position). These processes lead to the development (or growth) of an organism when understood in a dynamical sense. The above model of speciation is precisely a model of differentiation (but for a species rather than an organism) and hence is expected to be readily extensible to this process. The process of morphogenesis can be modelled by Turing style rate differences [440], and hence should be applicable if a set of hierarchically organised structures can be generated.

Such a model would place DNA firmly within the behaviour of a cell, and the cell firmly within an organism in a manner similar to the previous section where an organism from a species was placed within an environment. Thus we start to see a way in which the hierarchical picture of biological systems that we have been searching for might start to emerge.

Another result of such a model would be to throw new light upon the apparently neverending debates about nature versus nurture [341]. This debate springs from an inappropriate emphasis upon either the environment of an organism or its genes as being most important when asking questions about how a mature organism will behave. In fact, both the organism and the environment must be considered in tandem before such questions can be asked, recall that the phenotype is a product of the genotype *and* the environment. The above modelling technique incorporates this combined nature of the phenotype, but not in a way that is often expected. Since the influence of the environment upon a system is incorporated by such models through the use of quantum type contextuality we must expect that the combined effect of genes and environment will not be additive, but this should be expected when we consider the behaviour of biological systems. A small change in the environment might lead to a very profound change in the realised phenotype, and knocking out an apparently unimportant gene can have the same large effect. Equally, apparently large changes can lead to no visible changes in the phenotype. Despite the apparently straightforward nature of these assertions, it is still very common to see claims that a developed organism is the result of some percentage nature and the remaining nurture, a quantum model of morphogenesis would provide a straightforward reason why such claims are nonsensical.

However, this model has been very difficult to develop. The complex interrelationships involved, the manner in which they change in time, and the feedback involved are all very

difficult concepts to treat mathematically. In particular, finding simple toy models is not a simple task (consider for example the problem associated with finding a Lagrangian describing the dynamics of cascade reactions resulting from the 'reading' of a strand of DNA) and very little work has been performed along these lines by other researchers. With the current understanding, we now see a way to progress our understanding of this very important problem, something that will be pursued in future work.

Chapter 7

Conclusions: An Emergent Methodology

This work has examined our current methodologies, finding that they generally lack the ability to both understand and to generate interesting complex behaviour. Generally this manifests itself in:

- A lack of contextual dependence.
- A lack of generated hierarchical structure.
- A lack of evolutive open ended behaviour.
- An insistence upon objects with predefined modes of interaction, which inhibits the possibility of new emergent interaction.

Thus, the methodologies being used almost rule out phenomena such as observational emergence and high-end complexity. This work has sought to extend our methodologies, developing a less object-based modelling methodology which shows some promise in our search for an understanding of high-end complexity, and in particular the aim to actually generate such behaviour in fields such as ALife.

The Process Physics system introduced in sections 4.1 and 6.4 shows signs of generating high-end complexity and so has been examined with the intension of applying its behaviour to a more general set of phenomena than fundamental physics, namely systems which have thus far generally defied our reductive methodologies. Initial results suggest that such an extension may be possible. This chapter will re-examine some of the key ideas developed in this work, and specify the form of the new emergent methodology that has been developed.

7.1 Defining Emergence

Emergence is not a phenomenon that is easy to define. This work has adopted the Baas definition of *observational emergence* (discussed in section 3.2.2) as a criterion dividing interesting (observational) emergence from that which is less so (computationally emergent phenomena). This criterion can be used somewhat in the identification of where different systems fall onto the complexity scale, but it will not be the only one. Baas himself expects that it will be possible to further subdivide the categorisations of observational versus deducible emergence [43]. Interestingly, Baas classifies Gödel's result about the

foundations of mathematics among observationally emergent systems, which is in agreement with our discussion from section 1.2.2, as well as an early claim in Process Physics that the limits of logic must be incorporated into our modelling of reality [98, 91]. Here we see that this claim can be incorporated into the hypothesis that observational emergence is a *necessary* condition for the generation of high end complexity; a system which does not exhibit such behaviour will not display open ended behaviour with increasing levels of complexity and modes of interaction. Instead it will generally fall prey to object based modelling and to failures of emergence such as those exemplified by the ADH. Returnig to the Rosen definition, we can re-express the same concept in yet another way; more than one model is required for a complete understanding of a complex system.

7.2 Defining Complexity

It is perhaps unrealistic to expect a definition of complexity which lists characteristic properties and their behaviour. This work has adopted the stance that complexity exhibits a broad range of characteristics, in diverse phenomena, and that for this reason a scale is more appropriate in the definition of complexity than a dividing line between the simple and the complex. While much progress has been made in the identification and analysis of systems at what might be identified as the low end of this scale, very few attempts have been directed towards the generic modelling high-end complexity. This thesis has attempted to achieve just such an extension to our understanding of these important systems. Searching for the reasons behind the success of the models of Process Physics has led to the claim that quantum theories are in fact far more widely applicable than is traditionally thought to be the case, and that they show particular promise when it comes to the modelling of high-end complexity. This is because:

- The nonseparable nature of these systems, *i.e.* their contextual nature, is a phenomenon that has been well incorporated into quantum models of reality.
- The concepts of symmetry, and its associated spontaneous breaking, is well modelled, and a framework exists by which new behaviour, or more appropriate descriptions of the relevant behaviour can be extracted, namely Goldstone's theorem, and its extension to the concept of NG-emergence.
- The concept of observational emergence is well incorporated into QFTs via the process of action sequencing, where dynamically driven changes of variables are used to extract high-level behaviour from a system of interest. Not all of these jumps need be computational, rather they must be driven by the dynamics of the system, and can take observational forms. For example, approximations are used in the GCM.
- QFT's also have a hierarchical structure very similar to that of the low level model and the QHFT in Process Physics (which is not surprising considering that all of these theories are based upon similar formalism). Objects are not present in

these theories in the same way as in more standard reductive models. Instead, all components of the theories are interelated in a very complex way and cannot realistically be considered in isolation. This will be discussed more in section 7.2.3.

Perhaps the most interesting aspect of current physical modelling is the way in which a number of different theories must be used in order to discuss many of the systems of interest to physics. Consider the Standard Model, which itself consists of a number of subtheories, and has not at present been unified with General Relativity.

Process Physics itself consists of a number of different theories, and although there are indications that these theories overlap in a hierarchical manner, they do not directly map one onto the other. However, this non-deducible relationship among theories is not the disadvantage it might seem, rather, if high-end complexity is to be understood as an inability to understand its dynamics using one description alone, then there is every reason to suppose that this framework can in fact be extended to systems exhibiting high-end complexity. In particular, the observational emergence exhibited by the different theoretical descriptions within Process Physics is very promising as it suggests that this system provides a framework for generating such behaviour more generally. Thus, although this work has not weighed into the debate surrounding the attempt to define complexity, a significant new methodology has been provided which can be applied to a broad range of systems.

7.2.1 Modelling the Contexts of Emergent Behaviour

> *The concept of individuals unencumbered by explicit relations is paramount in ordinary language and scientific theories, for they stand as the subjects of subject-predicate propositions. In actual discourse, however, the individuals are always understood within certain situations or implicit relations, and the contexts make them constituents rather than bare elements. The implicit relations introduce a holistic sense that mitigates atomism by making the atoms relatable.*
>
> Auyang, p47, [42]

A number of techniques capable of modelling the contextuality of complex systems have been discussed in this work:

- The quantum formalism itself provides a direct link to contextuality, and as was discussed in section 5.2, extends directly to other systems which are not generally considered quantum mechanical, but can often be considered as complex.
- As discussed in section 5.5, a change in the potential function of a system can result in vastly different dynamics. This is because the potential is intimately linked to the vacuum, or ground state of a system.

- NG-emergence can be linked with an environment by virtue of its dependence upon the relevant potential of the system; as was shown in section 6.5 a change in the system may strongly affect its emergent behaviour.

Of particular interest, in section 3.1.9, it was noticed that the possibility often arises to discuss systems exhibiting high-end complexity in a hierarchical manner. In particular, the contextuality of such systems can often be modelled using a jump to a higher hierarchical level. Consider for example the process of biological development that was discussed in that section. The amazing contextuality exhibited by genetic regulatory cascades as different reactions are turned on or off, is often well explained with a shift to a cellular description, the apparent mystery of metabolic or cell biological reactions becomes straightforward by jumping to a new hierarchical level. Some of the techniques discussed in this work provide ways in which such a jump might be carried out in the examination of such systems. For example, action sequencing provides just such a tool. If two levels in a system of interest are describable using a least action formalism, then there is every reason to believe that a jump in levels of description can be carried out. Just such a jump was performed in section 6.5, there is every indication that this technique can be widely applied.

7.2.2 Moving Beyond an Object-Based Methodology

Section 1.4 briefly discussed the problems that arise in an object-based methodology; despite its many successes in the modelling of separable and non-contextual systems, the extension of this methodology to complex and contextual systems remains problematic. One of the reasons why the techniques presented here have proved more successful in the modelling of high-end complexity than many of the standard techniques commonly used is that they have largely moved beyond an object-based methodology.

- The low level model of Process Physics (4.1) models the time evolution (or processing) of relationships, between objects which are themselves understood as sets of relationships. Thus, the emphasis of this model has been shifted from objects to the time evolution of relationships.
- The high level QHFT (6.14) the emphasis has again shifted from objects to relationships, with the relevant variables constituting the mappings of TD and S^3 structures into other such structures.
- In the Gravity Model discussed in section 6.4.5, phenomena such as acceleration and gravitational attraction are understood not with reference to objects, but to the flows of 'space'. It is not objects that cause gravitational effects, they merely ride along 'on top of' spatial flows.
- The Proposed Model of Sympatric Speciation sees species not as pre-defined objects, but as phenomena that spontaneously emerge depending upon the context in which a system finds itself, in this case the environment.

Thus, each of the models discussed in this work utilise the same 'trick' in formulating a theory that can model or generate emergent behaviour; although objects are used in the modelling each model emphasises the relationships between the objects rather than the objects themselves. In fact, it is all but impossible to leave object-based modelling behind, our language and associated mathematical techniques are all based primarily upon objects, even our minds have evolved to identify and to categorise objects and to base our models of the world around us upon them [340].

In moving beyond the standard emphasis upon objects, these models move from a static to a more dynamic approach; new behaviour can emerge from such a methodology, something that is all but forbidden when an emphasis is placed upon existing structures. For example, the agent based models in ALife traditionally use agents (or objects) which have predefined modes of interaction, in such a system new channels of interaction are all but forbidden by the very choice of model, something that is emphasised by the ADH discussed in section 2.2.2.

Some of our phyical theories have already started to move beyond this object based methodology, however, there is often a large amount of confusion surrounding these models which can be thought of as arising from an invalid insistence upon object-based modelling despite the fact that such modelling is often almost incompatible with the very nature of the model. This phenomenon can be seen in the general insistence upon objects such as particles in quantum experiments, and the associated mystery that is encountered when one must define a trajectory followed by those particles. Adopting a less object-based approach such as the spontaneous localisation theories discussed in section 5.6.1, where the wavefunction gains an ontological status, but is contextually acted upon by the environment in which it finds itself allows for a much clearer understanding of quantum behaviour. When this understanding is placed within the broader set of models identified as Process Physics a very consistent view of the Universe starts to emerge, although such a picture is very different from that of standard physics. It is the claim of this work that the less object-based, or more relational, models of Process Physics provide a better basis for the understanding of high end complexity, although the Rosen-type understanding of complex behaviour as requiring more than one model for a full description suggests that standard physics will still provide a very appropriate view of some aspects of reality, just not all of them. Indeed, the early failure of relational models of the Universe (such as that proposed by Leibniz [260]) can be understood to arise from their very complexity, such models are hard to build, and often harder still to understand, if a simpler object-based model is available then it is likely to be preferable. However, we must be wary of the overapplication of Occam's razor; sometimes the simplest explanation is not the most preferable, and in fact an inappropriate search for simplicity might rule out much of the behaviour that we seek to understand. Such a problem is particularly relevant when we start to consider the contexts of theories themselves A theory which at one level appears

to be unnecessarily complicated might become remarkably simple when considered in a larger context. It might also serve to simplify the description of a number of phenomena which have always remained mysterious to simpler theories. Process Physics provides a particularly good example of this phenomenon.

7.2.3 Reductive Paradigms?

It must be claimed that while chapter 1 claimed that reductive techniques are coming to their limit, and cannot be expected to retain their general validity, the new proposed methodology makes use of a number of techniques developed within what might be regarded as that same paradigm. This is not necessarily the case. The general failure to find an object-based interpretation of quantum mechanics suggests that this theory is already pushing beyond the boundary of reductive analysis.

Object-based modelling is in some sense synonymous with reductive analysis. Indeed, it is very difficult adopt a reductive approach with less object-based methodologies. Consider the use of the fields in QFT and the way in which an appropriate description at the QFT level often precludes an understanding of these systems as individualistic groupings of particles interacting in some way, rather, QFT describes objects with reference to their context, in this case the vacuum. If a reductive analysis is attempted, such as the commonly used perturbation expansion, then ever increasing levels of interaction and description start to emerge. An electron in QFT cannot be understood in this picture without reference to a hierarchy of photons and electrons, themselves in some sense 'inside' the original electron. However, no sensible theorist believes that this reflects reality, rather this might be seen as an example of a phenomenon where an insistence upon object-based methodology leads to a rather bizarre result. With a proper recognition of the holistic nature of QFT comes a more reasonable understanding of what the perturbation expansion implies, namely that the context of an electron must be properly incorporated into models of its behaviour.

In a similar manner, many of the phenomena discussed in this work can only be understood through the adoption of a less reductive methodology. We might begin to suspect that in addition to a complexity scale, there is an associated scale of theories with an ever decreasing number of reductive characteristics. This idea will be examined in future work.

7.3 Contextuality and Observer Dependence

This work has discussed a number of ways in which contextuality can be incorporated into our modelling without recourse to fully subjective models. With a proper understanding of contextuality, many of the problems raised by the proponents of subjective theories of the world [455, 144, 296, 431, 384, 385] become relatively insignificant; such subjective interpretations become necessary only when context is not properly incorporated into our methodology. With an insistence upon objective measurements and passive observers we

effectively lose realism and are forced into such subjective theories in order to try and recover an ability to explain contextual phenomena. If however, context is incorporated into our methodology at the outset, then realism is maintained and our theories become much richer, capable of describing more phenomena than is currently the case.

7.4 In Conclusion

Finally, it should be noted that within the set of theories denoted as Process Physics, much of the physical universe is explained, but only through the adoption of a stance that might be considered quite unpalatable to many physicists:

- A hierarchy of non-deducibly connected theories is necessary in order to model the dynamics of the Universe at a fundamental level.
- Notions such as objectivity are lost, but realism is very much recovered, although only through taking phenomena such as contextuality, nonlocality and observer-driven phenomena very seriously in the modelling.
- A shift from the standard object-based, reductive methodology is necessary, which often makes the modelling of any systems considered far more challenging.

The fact that this modelling methodology appears to make sense within the larger context of complex systems, and to be extensible to such systems leaves me with the belief that this added complexity in the fundamental modelling of the Universe is justified with the added scope that might be achieved in what it can explain. Indeed, in adopting a more complex initial set of models, many more phenomena appear to be justifiably brought into the realms of physics,[1] although at the price of so altering physics that it may no longer be considered the same field.

[1]This leaves us feeling rather ironically that perhaps there was a broader reality to the ADH; if our models are not sufficiently complex to begin with then they will not explain all that we seek to understand.

Appendix A

Notation

This appendix lists the most important notation used throughout this work in an effort to draw together the disparate list of symbols and equations into one 'quick-reference' guide. Symbols are listed, along with a reference to their relevant appearances in the text.

A.1 Observational Emergence

This was introduced as a part of the Baas definition of emergence, in sections 2.2.1, and 3.2.2.

S^n:	nth-order structures	equations (2.11),(2.13),(2.15),(2.16)
$P = O^n(S^n)$:	nth-order properties	equations (2.12),(2.14)
Int^n:	nth-order properties	equations (2.12),(2.16)

Second order emergent properties are then defined as $P \in Obs^2(S^2)$ from the first order set $(S_i^1, O^1(S_i), Int^1)$. They are then classified as computable/deducible or observational depending upon whether there is a computational procedure for obtaining the properties.

Note that a change of notation was used in section 4.2.2 ($S^n \rightarrow \Xi^n$) to avoid confusion with the hypersphere notation.

A.2 Quantum Mechanics

Quantum mechanical concepts are largely discussed in sections 4.3 and 5.1.

ψ:	wave function(al)	section 4.3.1
H:	Hamiltonian	equation (4.10)
$\langle\psi(\mathbf{x})\|\hat{O}\|\psi(\mathbf{x})\rangle$:	quantum expectation value	equation (4.12)
$\dot{\rho_r}$:	reduced density matrix	equation (5.113)
L_j:	Lindblad operator	equations (5.113),(5.114),(6.14)
l_j:	$l_j \equiv \langle\psi\|L_j\|\psi\rangle$	equation (5.114)
M:	Mean over a probability distribution	(5.116)
$P(\hat{a}, \hat{b})$:	expectation value	equation (5.1)
$d\xi_j$:	stochastic noise term	equations (5.114),(6.14)

A.3 Quantum Field Theory

As per most modern physics texts, this work uses the Einstein convention to sum over repeated Greek indicies. All equations are written in the Euclidean metric.

$L[\phi]$:	Lagrangian of the field ϕ	equation (4.15)
$\mathcal{L}[\phi]$:	Lagrangian Density of the field ϕ	equation (4.15)
$S[\phi]$:	Action of the field ϕ	equation (4.15)
$\mathcal{G}$:	correlation function	equations (4.13),(4.16)
Z:	generating function	equation (4.14)
$V(\phi)$:	potential function of the field ϕ	sections 5.3.3 and 5.3.4

$\int D\phi$ is a functional integration, not an ordinary integral. It is an infinite product of integrals, taken over all possible $d\phi$.

A.4 Process Physics

A.4.1 Low level relational model

This model is discussed in chapter 4.

B_{ij}:	real valued matrix entry linking the node i to the node j	equation (4.1)
ω_{ij}:	additive noise term	equation (4.1)
α:	tuning parameter	equation (4.1)
D:	Defect	equation (4.2.2)
TD:	Topological Defect (stable defect)	equation (4.2.2)

A.4.2 Quantum Homotopic Field Theory (QHFT)

This model is discussed in section 6.4.

$\mathcal{Z}$:	winding number	equation (6.13)
$\pi_{\alpha\beta}$:	homotopic map $\pi : \alpha \rightarrow \beta$	section 6.4
$\pi_m(X)$:	the set of homotopy classes of maps from an m-sphere onto the space X	section 6.4
$\Psi[t]$:	system of homotopic embeddings	equation (6.14)
L_j:	Lindblad operator	equations (5.114),(6.14)
$d\xi_j$:	stochastic noise term	equations (5.114),(6.14)
v_{QF}:	speed through the quantum foam	equation (6.32)

A.4.3 Gravity Model

This model is discussed in section 6.4.5.

$v(r,t)$:	vector flow field	equation (6.35)
G:	Newton's gravitational constant	equation (6.35)
α:	quantum self-interaction 'fine structure constant'	equation (6.36)

A.4.4 Model of Sympatric Speciation

This model is discussed in section 6.5.

p_T:	total phenotype	equation (6.40)
p_{SR}:	species relevant portion of a phenotype	equation (6.40)
$p_{specific}$:	specific portion of a phenotype	equation (6.40)

A.5 Abbreviations used in the text

This table lists abbreviations used in the text, together with the page numbers consiered most relevant for definitions and explanations of the concepts involved.

ADH:	Ansatz for Dynamical Hierarchies	p43
ALife:	Artificial Life	p27
BE–LF:	Bloch Equation in Lindblad form	p145,p202
DSB:	Dynamical Symmetry Breaking	p156
EPR:	Einstein Podolsky Rosen	p92
GCM:	Global Colour Model	p77,p84
NG-mode:	Nambu–Goldstone mode	pp129–134,p156
OEE:	Open Ended Evolution	p34
PRF:	Preferred Reference Frame	p171
QCD:	Quantum Chromodynamics	p83
QED:	Quantum Electrodynamics	p82
QFT:	Quantum Field Theory	p79
QHFT:	Quantum Homotopic Field Theory	p165
QSD:	Quantum State Diffusion	p145
RGP:	Recursive Gauge Principle	p160
SE:	Species Equivalent	p180
SOC:	Self Organised Criticality	p69,p140
SSB:	Spontaneous Symmetry Breaking	p127

Appendix B

Itô calculus — a brief summary

Our discussion of spontaneous localisation theories deals with stochastic processes. This makes it necessary to use Itô calculus, the relevant elements of which shall be briefly summarised in this appendix. More details can be found in [35].

B.1 The Markov property

Markov formulated the theory of stochastic processes in 1906. In his investigation of connected experiments, Markov formulated the principle that the 'future' does not depend upon the 'past' when we know the 'present'. When we extend this principle to stochastic processes we get the Markov property:

> If the state of a system at a particular time t_p (the present) is known, then additional information regarding the behaviour of the system at times $t < t_p$ (the past) has no effect on our knowledge of the probable development of the system at time $t > t_p$ (in the future).

Essentially this property means that the system has no 'memory'.

B.2 Wiener processes

Also called Brownian motion, a Wiener process is a continuous stochastic process that has independent increments, distributed as Normal random variables. Formally, a Wiener process $\left\{d\xi(t), t \in [0,\infty]\right\}$ is a stochastic process if:

1. $d\xi$ has independent increments $d\xi(t) - d\xi(s), s < t$ (i.e. it is Markovian).
2. The increment $d\xi(t) - d\xi(s), 0 < s < t < \infty$, is Gaussian (Normal) with zero mean and variance, σ_t^2.
3. $d\xi(0) = 0$ almost surely (or with probability one).

This definition quickly leads to the following multiplication rules for Wiener processes [35]:

$$\mathrm{M}\left(\mathrm{Re}\left(d\xi_j\right)\mathrm{Re}\left(d\xi_k\right)\right) = \mathrm{M}\left(\mathrm{Im}\left(d\xi_j\right)\mathrm{Im}\left(d\xi_k\right)\right) = \delta_{jk}dt \tag{B.1}$$

$$\mathrm{M}\left(\mathrm{Re}(d\xi_j)\mathrm{Im}(d\xi_k)\right) = 0 \tag{B.2}$$

$$\mathrm{M}(d\xi_j) = 0 \tag{B.3}$$

where δ_{ij} is a Kroneker delta.

B.3 Itô's theorem

The Itô form discussed in the text arises from Itô's lemma, differentials up to the second order must be evaluated because of (B.1) :

> Let $U(t,X)$ be a continuous nonrandom function with continuous partial derivatives $\frac{\partial U}{\partial t}$, $\frac{\partial U}{\partial X}$ and $\frac{\partial^2 U}{\partial X^2}$. Suppose that $X(t), t \in [0,T]$ is a process with stochastic differential
>
> $$dX(t) = A(X_t)dt + B(X_t)d\xi(t) \tag{B.4}$$
>
> and let $Y(t) = U[t, X(t)]$. Then the process $Y(t)$ also has a differential on $[0,T]$, given by
>
> $$\begin{aligned} dY(t) &= \left\{ \frac{\partial}{\partial t}[U(t,X)] + \left[\frac{\partial}{\partial X} U(t,X) \right] A(X_t) + \frac{1}{2}\frac{\partial^2}{\partial X^2} U(t,X_t)[B(X)]^2 \right\} dt \\ &\quad + \frac{\partial}{\partial X} U(t,X_t) B(X_t) d\xi(t) \\ &= \frac{\partial U}{\partial t}dt + \frac{\partial U}{\partial X}dX(t) + \frac{1}{2}\frac{\partial^2 U}{\partial X^2}[B(X_t)]^2 dt \end{aligned} \tag{B.5}$$
>
> where $[B(X_t)]^2 \equiv (dX_t)^2$.

Appendix C

Generalized Master Equations

C.1 Solution by Generalized Master Equations

Generalised master equations describe a system, $\mathcal{S}$, which includes both a quantum system under investigation, and the relevant detectors in interaction with a large and complex environment, $\mathcal{E}$. Together, the system and its environment form a closed and interacting system, $\mathcal{S} \oplus \mathcal{E}$. *Projection operators* serve as crude models of this possibly very complex interaction. Choosing the right projection operators makes it possible to ignore the effects of the environment, and consider the system alone.[1]

To find the time evolution of density operators we use the Schrödinger equation in Dirac form, which gives

$$\begin{aligned} \partial_t \rho &= \sum_j P(j) \Big[\Big(\partial_t |\psi_j\rangle \Big) \langle \psi_j| + |\psi_j\rangle \Big(\partial_t \langle \psi_j| \Big) \Big] \\ &= \frac{1}{i\hbar} \left(H\rho - \rho H \right) \end{aligned} \tag{C.1}$$

or,

$$\partial_t \rho = -i\hbar \left[H, \rho \right] \equiv -i\mathcal{L}\rho \tag{C.2}$$

which is the quantum Liouville equation.[2]

We can write the Hamiltonian of the Liouvillian equation (C.2) in three separate parts, one representing the system $\mathcal{S}$, one the environment $\mathcal{E}$ and the last part representing the interaction between these two systems

$$H = H_{\mathcal{S}} + H_{\mathcal{E}} + H_{\mathcal{S}\oplus\mathcal{E}}. \tag{C.4}$$

[1] The following discussion can be found in Haake [207], who also considers subsystems of open systems and some applications of this approach, for our discussion we need only consider a subsystem embedded in a closed system.

[2] The origin of this name lies in classical statistical mechanics, where the time evolution of the Liouville density $f(\mathbf{q}, \mathbf{p}, t)$ satisfies

$$\frac{df}{dt} = \frac{\partial f}{\partial t} + \sum_k \left(\frac{\partial f}{\partial q_k} \frac{dq_k}{dt} - \frac{\partial f}{\partial p_k} \frac{dp_k}{dt} \right) \equiv \frac{\partial f}{\partial t} + [f, H]_{\mathrm{PB}} = 0, \tag{C.3}$$

which is reminiscent of (C.2). The quantum Liouville equation actually takes the form of an infinite series expansion, some of the details of this expansion follow, but more details can be found in [335].

Now, write the observables of the system $\mathcal{S}(\mathcal{E})$ in terms of the operators

$$S_1, S_2, \ldots (E_1, E_2, \ldots) \tag{C.5}$$

which act on the Hilbert space $\mathcal{H}_{\mathcal{S}}(\mathcal{H}_{\mathcal{E}})$. The Hilbert space $\mathcal{H}_{\mathcal{S}\oplus\mathcal{E}}$ of the component systems can be written as a direct product

$$\mathcal{H}_{\mathcal{S}\oplus\mathcal{E}} = \mathcal{H}_{\mathcal{S}} \otimes \mathcal{H}_{\mathcal{E}}. \tag{C.6}$$

This means that if the interaction term in (C.4) is turned off then each of the systems, $\mathcal{S}$ and $\mathcal{E}$ can be considered separately.

Now, the expectation value of the observables of the total combined system and its surrounding environment, $\mathcal{S} \oplus \mathcal{E}$, can be written

$$\langle E_i S_j \rangle = \mathrm{Tr}\{\rho E_i S_j\}, \tag{C.7}$$

where E_i describes environmental operators and S_j relevant system ones.

We assume that only the expectation values of the system $\mathcal{S}$ are interesting. (The accuracy of this assumption shall be discussed shortly.) That is, we are only interested in

$$\langle S_1 S_2 \ldots S_n \rangle = \mathrm{Tr} S_1 S_2 \ldots S_n \rho(t). \tag{C.8}$$

Now, because the Hilbert space $\mathcal{H}_{\mathcal{S}\oplus\mathcal{E}}$ can be written as a direct product, we can carry out the trace operation in two steps, i.e.

$$\mathrm{Tr} = \mathrm{Tr}_{\mathcal{S}} \mathrm{Tr}_{\mathcal{E}}, \tag{C.9}$$

which means that the expectation values of the observables of the system $\mathcal{S}$ can be written in terms of a *reduced density matrix*, ρ_r, which describes only the open system $\mathcal{S}$,

$$\rho_r \equiv \mathrm{Tr}_{\mathcal{E}} \rho(t). \tag{C.10}$$

This gives a new way of calculating the expectation value of the observables of the system

$$\begin{aligned} \langle S_1 S_2 \ldots S_n \rangle &= \mathrm{Tr}_{\mathcal{S}} \mathrm{Tr}_{\mathcal{E}} S_1 S_2 \ldots S_n \rho(t) \\ &= \mathrm{Tr}_{\mathcal{S}} S_1 S_2 \ldots S_n \rho_r(t), \end{aligned} \tag{C.11}$$

suggesting that we can construct an equation of motion for system alone, using the reduced density matrix to eliminate the irrelevant parts (i.e. those connected with the environment) from the Liouville equation (C.2). This is done by decomposing the full

density matrix $\rho(t)$ into two parts using the projector P:

$$\rho(t) = P\rho(t) + (1-P)\rho(t), \tag{C.12}$$

where P is defined such that

$$P = P_{\text{ref}}\text{Tr}_{\mathcal{E}}, \text{ and} \tag{C.13}$$

$$\text{Tr}_{\mathcal{E}}P_{\text{ref}} = 1 \tag{C.14}$$

P_{ref} is a parameter which may be chosen in an arbitrary manner such that it satisfies the normalisation condition (C.14). It plays the role of a *reference state* of the environmental system $\mathcal{E}$. A good choice of P_{ref} will lead to a more physically relevant description of the system. We shall discuss this reference state in more detail shortly. Notice that because of the definition

$$\begin{aligned} P^2 &= P_{\text{ref}}\text{Tr}_{\mathcal{E}}P_{\text{ref}}\text{Tr}_{\mathcal{E}} \\ &= P_{\text{ref}}\text{Tr}_{\mathcal{E}} \\ &= P, \end{aligned} \tag{C.15}$$

and that

$$\begin{aligned} P\rho(t) &= P_{\text{ref}}\text{Tr}_{\mathcal{E}}\rho(t) \\ &= P_{\text{ref}}\rho_r. \end{aligned} \tag{C.16}$$

So all of the information that is relevant to the system $\mathcal{S}$, is represented by $P\rho(t)$, while the environment terms, and the interaction terms are given by $(1-P)\rho(t)$.

Inserting the decomposition (C.12) into the Liouville equation (C.2) gives the formal solution,

$$\dot{\rho} = -i\mathcal{L}(P\rho(t) + (1-P)\rho(t)), \tag{C.17}$$

from which two coupled equations can be obtained by acting on the left of (C.17) with P and $1-P$, to obtain the two coupled equations

$$P\dot{\rho}(t) = -iP\mathcal{L}P\rho(t) - iP\mathcal{L}(1-P)\rho(t) \text{ and,} \tag{C.18}$$

$$(1-P)\dot{\rho}(t) = -i(1-P)\mathcal{L}P\rho(t) - i(1-P)\mathcal{L}(1-P)\rho(t). \tag{C.19}$$

A solution of (C.19) which writes the environmental terms $(1-P)\rho(t)$ in terms of the system terms $P\rho(t)$ is:

$$(1-P)\rho(t) = \exp[-i(1-P)\mathcal{L}t](1-P)\rho(0) - i\int_0^t dt' \exp[-i(1-P)\mathcal{L}t'](1-P)\mathcal{L}P\rho(t-t'). \tag{C.20}$$

This is simple to verify since (C.19):

$$\begin{aligned}(1-P)\frac{d}{dt}\rho(t) &= -\exp[-i(1-P)\mathcal{L}t'](1-P)\mathcal{L}P\rho(t-t')\big|_{t'=0}^{t'=t} \\ &= \exp[-i(1-P)\mathcal{L}t](1-P)\mathcal{L}P\rho(t). \end{aligned} \tag{C.21}$$

Substituting (C.20) into (C.18) gives

$$\begin{aligned} P\dot{\rho}(t) &= -iP\mathcal{L}P\rho(t) \\ &\quad - iP\mathcal{L}\left(\exp\left[-i(1-P)\mathcal{L}t\right](1-P)\rho(0) - i\int_0^t dt' \exp[-i(1-P)\mathcal{L}t'](1-P)\mathcal{L}P\rho(t-t')\right) \\ &= -iP_{\rm ref}{\rm Tr}_{\mathcal{E}}\mathcal{L}P_{\rm ref}\rho_r(t) - iP_{\rm ref}{\rm Tr}_{\mathcal{E}}\mathcal{L}\exp\left[-i(1-P)\mathcal{L}t\right](1-P)\rho(0) \\ &\quad - P_{\rm ref}{\rm Tr}_{\mathcal{E}}\mathcal{L}\int_0^t dt' \exp[-i(1-P)\mathcal{L}t'](1-P)\mathcal{L}P_{\rm ref}\rho_r(t-t') \\ &= -i\mathcal{L}_{\rm eff}\rho_r(t) + \int_0^t dt' K(t')\rho(t-t') + I(t) \end{aligned} \tag{C.22}$$

which is the *generalised master equation* for the reduced density operator $\rho_r(t)$. Thus, if it is possible to solve the formal expressions

$$\mathcal{L}_{\rm eff} \equiv {\rm Tr}_{\mathcal{E}}\mathcal{L}P_{\rm ref}\rho_r(t) \tag{C.23}$$

$$K(t') \equiv -{\rm Tr}_{\mathcal{E}}\mathcal{L}\exp[-i(1-P)\mathcal{L}t'](1-P)\mathcal{L}P_{\rm ref} \tag{C.24}$$

$$I(t) \equiv -i{\rm Tr}_{\mathcal{E}}\mathcal{L}\exp\left[-i(1-P)\mathcal{L}t\right](1-P)\rho(0). \tag{C.25}$$

then the master equation can be solved.

This evaluation involves a perturbation expansion of the exponential term $\exp[-i(1-P)\mathcal{L}t]$ that arises in (C.23–4.23) in terms of the interaction Liouvillian $\mathcal{L}_{\mathcal{S}\oplus\mathcal{E}}$. If the interaction between the system and its surrounding environment is too strong, then these expansions will not converge, and the theory will be useless. It is important that a good choice of $\mathcal{S}$ and $\mathcal{E}$ is made such that this does not happen. Not much more can be said without a specific example, for this the reader is referred to [207].

The reference state

Any choice of $P_{\rm ref}$ which satisfies the normalisation condition (C.14) will satisfy the generalised master equation (C.22). However, a reasonable description of the physical process is only achieved with a good approximation of $P_{\rm ref}$. The formalism above does not tell us

how to choose P_{ref}, however, the influence of the environment upon the system expresses itself in terms of a set of correlation functions $\text{Tr}_{\mathcal{E}} B_1(t_1)B_2(t_2)\dots B_n(t_n)P_{\text{ref}}$, where the B_i are the observables of the environment $\mathcal{E}$, in the interaction Hamiltonian (see [207] for more details). The state in which these correlation functions are to be evaluated is the parameter P_{ref}, which means that a physically well-motivated choice of P_{ref} will ensure that we get a good low-order approximation to the expansions of the Liouvillan terms (C.23–4.23) in the master equation (C.22). Hence the selection of the reference state should be based upon the physics of the system $\mathcal{S} \oplus \mathcal{E}$.

C.2 A derivation of the Quantum State Diffusion equation

The quantum state diffusion equation (5.114) can be derived from the Bloch equation in Lindblad form (BE-LF) (5.113). What follows is an expansion of the derivation from [188].

Given a system with an N-dimensional state space, it is possible to express the density operator ρ as a mean M over a distribution of normalized pure-state projection operators

$$\rho = \text{M}|\psi\rangle\langle\psi|. \tag{C.26}$$

We require differential equations for $|\psi\rangle$ such that

$$\dot{\rho} = \frac{\text{d}\rho}{\text{d}t} \tag{C.27}$$

is determined by some differential equation for ρ, in this case the Bloch equation in Lindblad form [266]

$$\dot{\rho} = -\frac{i}{\hbar}\left[H, \rho\right] + \sum_j \left(L_j \rho L_j^\dagger - \frac{1}{2} L_j^\dagger L_j \rho - \frac{1}{2} \rho L_j^\dagger L_j \right). \tag{C.28}$$

Consider an open system described in the density matrix formulation by ρ, which starts in a pure state (hence ρ is diagonal) at time $t = 0$. Because the system is open, it will evolve in a non-deterministic fashion into a mixed state at time $t = t'$. However, there are no deterministic equations which can describe this process. In fact the evolution is highly probabilistic, which suggests that stochastic equations may be a more appropriate way in which to model it. We shall seek one such stochastic equation.

A variation $|\text{d}\psi\rangle$ in $|\psi\rangle$ over time step $\text{d}t$ can be given by the Itô form

$$|\text{d}\psi\rangle = |v\rangle\text{d}t + \sum_j |u_j\rangle\text{d}\xi_j \tag{C.29}$$

where $|v\rangle\text{d}t$ is a drift term and $\sum_j |u_j\rangle\text{d}\xi_j$ is a term which represents the differential stochastic fluctuations of the process. The $\text{d}\xi_j$ are the complex Wiener processes (5.116)

from appendix B which are complex (because quantum mechanics is complex), with equal and independent fluctuations in their real and imaginary parts. If we require that the state vector remain normalized throughout the evolution then the random fluctuations in the state must be orthogonal to the state,

$$\text{i.e.} \quad \langle\psi|u_j\rangle = 0 \tag{C.30}$$

a result that will often be used in what follows.

In Itô calculus, differentials up to the second power must be evaluated [35], which means that the time evolution of the state $|\psi\rangle$ is

$$\begin{aligned} \dot{\rho} &= \frac{\mathrm{d}\rho}{\mathrm{d}t} \\ &= \mathrm{M}\frac{1}{dt}\left(|\psi\rangle\langle\mathrm{d}\psi| + |\mathrm{d}\psi\rangle\langle\psi| + |\mathrm{d}\psi\rangle\langle\mathrm{d}\psi|\right). \end{aligned} \tag{C.31}$$

The means of $|\mathrm{d}\psi\rangle$ and $|\mathrm{d}\psi\rangle\langle\mathrm{d}\psi|$ can be calculated as follows:

$$\begin{aligned} \mathrm{M}|\mathrm{d}\psi\rangle &= \mathrm{M}\left(|v\rangle\mathrm{d}t + \sum_j |u_j\rangle\mathrm{d}\xi_j\right) \\ &= |v\rangle\mathrm{d}t + \sum_j |u_j\rangle\mathrm{M}\mathrm{d}\xi_j \\ &= |v\rangle\mathrm{d}t \end{aligned} \tag{C.32}$$

where we have used the mean properties of the Wiener processes (5.116), and

$$\begin{aligned} \mathrm{M}|\mathrm{d}\psi\rangle\langle\mathrm{d}\psi| &= \mathrm{M}\left(\left(|v\rangle\mathrm{d}t + \sum_j |u_j\rangle\mathrm{d}\xi_j\right)\left(\langle v|\mathrm{d}t + \sum_k \langle u_k|\mathrm{d}\xi_k^*\right)\right) \\ &= \mathrm{M}\left(|v\rangle\langle v|\mathrm{d}t^2 + \sum_j |u_j\rangle\langle v|\mathrm{d}\xi_j\mathrm{d}t + \sum_k |v\rangle\langle u_k|\mathrm{d}t\mathrm{d}\xi_k^* + \sum_{j,k} |u_j\rangle\langle u_k|\mathrm{d}\xi_j\mathrm{d}\xi_k^*\right) \\ &= |v\rangle\langle v|\mathrm{d}t^2 + \sum_j |u_j\rangle\langle v|\mathrm{M}(\mathrm{d}\xi_j)\mathrm{d}t + \sum_k |v\rangle\langle u_k|\mathrm{d}t\mathrm{M}(\mathrm{d}\xi_k^*) + \sum_{j,k} |u_j\rangle\langle u_k|\mathrm{M}(\mathrm{d}\xi_j\mathrm{d}\xi_k) \\ &= 2\sum_j |u_j\rangle\langle u_j|\mathrm{d}t \end{aligned} \tag{C.33}$$

again using (5.116), but also making the assumption that dt is small, and hence $\mathrm{d}t^2 \to 0$. Substituting these results for the means, (C.32) and (C.33), into the time evolution

equation for ρ, (C.31), gives

$$\dot{\rho} = \left(|\psi\rangle\langle v| + |v\rangle\langle\psi| + 2\sum_j |u_j\rangle\langle u_j| \right). \tag{C.34}$$

Recalling equation (C.30) we know that the stochastic terms are determined by the component of $\dot{\rho}$ in the space orthogonal to ψ, that is

$$\begin{aligned}
2\sum_j |u_j\rangle\langle u_j| &= \Big(I - \rho_\psi\Big)\dot{\rho}\Big(I - \rho_\psi\Big) \\
&= \Big(I - |\psi\rangle\langle\psi|\Big)\left(-\frac{i}{\hbar}[H,\rho] + \sum_j \left(L_j\rho L_j^\dagger - \frac{1}{2}L_j^\dagger L_j\rho - \frac{1}{2}\rho L_j^\dagger L_j\right)\right)\Big(I - |\psi\rangle\langle\psi|\Big) \\
&= \Big(I - |\psi\rangle\langle\psi|\Big)\left(-\frac{i}{\hbar}[H,\rho]\right)\Big(I - |\psi\rangle\langle\psi|\Big) \\
&\quad + \Big(I - |\psi\rangle\langle\psi|\Big)\sum_j L_j\rho L_j^\dagger\Big(I - |\psi\rangle\langle\psi|\Big) - \Big(I - |\psi\rangle\langle\psi|\Big)\sum_j \frac{1}{2}L_j^\dagger L_j\rho\Big(I - |\psi\rangle\langle\psi|\Big) \\
&\quad - \Big(I - |\psi\rangle\langle\psi|\Big)\sum_j \frac{1}{2}\rho L_j^\dagger L_j\Big(I - \rho|\psi\rangle\langle\psi|\Big) \\
&= \sum_j \Big(I - |\psi\rangle\langle\psi|\Big)L_j|\psi\rangle\langle\psi|L_j^\dagger\Big(I - |\psi\rangle\langle\psi|\Big) \qquad \text{(C.35)} \\
&= \sum_j \Big(L_j|\psi\rangle\langle\psi|L_j^\dagger - |\psi\rangle\langle\psi|L_j|\psi\rangle\langle\psi|L_j^\dagger \\
&\quad - L_j|\psi\rangle\langle\psi|L_j^\dagger|\psi\rangle\langle\psi| + |\psi\rangle\langle\psi|L_j|\psi\rangle\langle\psi|L_j^\dagger|\psi\rangle\langle\psi| \Big) \\
&= \sum_j \Big(L_j|\psi\rangle\langle\psi|L_j^\dagger - |\psi\rangle l_j\langle\psi|L_j^\dagger - L_j|\psi\rangle\langle\psi|l_j^* + |\psi\rangle l_j l_j^*\langle\psi| \Big) \\
&= \sum_j \Big(\Big(L_j - l_j\Big)|\psi\rangle \Big)\Big(\langle\psi|\left(L_j^\dagger - l_j^*\right)\Big), \qquad \text{(C.36)} \\
\text{where } l_j &\equiv \langle\psi|L_j|\psi\rangle \qquad \text{(C.37)}
\end{aligned}$$

((C.35) becomes evident when we realise that all of the terms (except for the survivor) are individually identical to zero upon expansion of ρ and multiplication of this term through the other terms.) Thus, the stochastic terms are

$$|u_j\rangle = (L_j - l_j)\,|\psi\rangle. \tag{C.38}$$

To find the drift terms, we multiply the time evolution of the density matrix equation (C.34) through on the left by $|\psi\rangle$, which gives (using equation (C.30))

$$\dot{\rho}|\psi\rangle = |\psi\rangle\langle v|\psi\rangle + |v\rangle, \tag{C.39}$$

or

$$\begin{aligned} |v\rangle &= \dot{\rho}|\psi\rangle - \langle v|\psi\rangle|\psi\rangle \\ &= \dot{\rho}|\psi\rangle - \left(\frac{1}{2}\langle\psi|\dot{\rho}|\psi\rangle + \mathrm{i}c\right)|\psi\rangle, \end{aligned} \tag{C.40}$$

where we have made use of the relation

$$\begin{aligned} \langle\psi|\dot{\rho}|\psi\rangle &= \langle\psi|\psi\rangle\langle v|\psi\rangle + \langle\psi|v\rangle\langle\psi|\psi\rangle + \sum_j \langle\psi|u_j\rangle\langle u_j|\psi\rangle \\ &= 2\mathrm{Re}\langle\psi|v\rangle, \end{aligned} \tag{C.41}$$

and where ic is a nonphysical, imaginary phase change constant which is chosen by convention in such a way that the usual Schrödinger equation is obtained in the absence of interaction with the environment. In this case c is set equal to zero.

Now we substitute the BE-LF (C.28) for $\dot{\rho}$ in (C.40) to find the drift term. First we calculate,

$$\begin{aligned} \dot{\rho}|\psi\rangle &= \left(-\frac{i}{\hbar}\left[H,\rho\right] + \sum_j \left(L_j\rho L_j^\dagger - \frac{1}{2}L_m^\dagger L_j\rho - \frac{1}{2}\rho L_j^\dagger L_j\right)\right)|\psi\rangle \\ &= \left(-\frac{i}{\hbar}\Big(H\mathrm{M}|\psi\rangle\langle\psi| - \mathrm{M}|\psi\rangle\langle\psi|H\Big)\right. \\ &\qquad \left. + \sum_j \left(L_j\mathrm{M}|\psi\rangle\langle\psi|L_j^\dagger - \frac{1}{2}L_j^\dagger L_j\mathrm{M}|\psi\rangle\langle\psi| - \frac{1}{2}\mathrm{M}|\psi\rangle\langle\psi|L_j^\dagger L_j\right)\right)|\psi\rangle \\ &= -\frac{i}{\hbar}\Big(H\mathrm{M}|\psi\rangle - \mathrm{M}|\psi\rangle\langle\psi|H|\psi\rangle\Big) \\ &\qquad + \sum_j \left(L_j\mathrm{M}|\psi\rangle\langle\psi|L_j^\dagger|\psi\rangle - \frac{1}{2}L_j^\dagger L_j\mathrm{M}|\psi\rangle\langle\psi|\psi\rangle - \frac{1}{2}\mathrm{M}|\psi\rangle\langle\psi|L_j^\dagger L_j|\psi\rangle\right) \\ &= -\frac{i}{\hbar}\Big(H|\psi\rangle - |\psi\rangle\langle\psi|H|\psi\rangle\Big) + \sum_j \left(L_j|\psi\rangle l_j^* - \frac{1}{2}L_j^\dagger L_j|\psi\rangle - \frac{1}{2}|\psi\rangle\langle\psi|L_j^\dagger L_j|\psi\rangle\right) \end{aligned} \tag{C.42}$$

and secondly

$$\begin{aligned}\langle\psi|\dot{\rho}|\psi\rangle|\psi\rangle &= \langle\psi|\left(-\frac{i}{\hbar}\left[H,\rho\right]+\sum_j\left(L_j\rho L_j^\dagger-\frac{1}{2}L_j^\dagger L_j\rho-\frac{1}{2}\rho L_j^\dagger L_j\right)\right)|\psi\rangle|\psi\rangle \\ &= -\frac{i}{\hbar}\Big(\langle\psi|H\mathrm{M}|\psi\rangle\langle\psi|\psi\rangle|\psi\rangle-\langle\psi|\mathrm{M}|\psi\rangle\langle\psi|H|\psi\rangle|\psi\rangle\Big) \\ &\quad+\sum_j\left(\langle\psi|L_1\mathrm{M}|\psi\rangle\langle\psi|L_j^\dagger|\psi\rangle|\psi\rangle-\frac{1}{2}\langle\psi|L_j^\dagger L_j\mathrm{M}|\psi\rangle\langle\psi|\psi\rangle|\psi\rangle-\frac{1}{2}\langle\psi|\mathrm{M}|\psi\rangle\langle\psi|L_jL_j^\dagger|\psi\rangle|\psi\rangle\right) \\ &= -\frac{2i}{\hbar}\langle\psi|H|\psi\rangle|\psi\rangle+\sum_j\Big(l_jl_j^*|\psi\rangle-\langle\psi|L_j^\dagger L_j|\psi\rangle|\psi\rangle\Big). \qquad \text{(C.43)}\end{aligned}$$

Substituting these results into (C.40) leads to a drift term

$$|v\rangle=-\frac{i}{\hbar}H|\psi\rangle+\frac{1}{2}\sum_j\Big(2l_j^*L_j-L_j^\dagger L_j-l_j^*l_j\Big)|\psi\rangle. \qquad \text{(C.44)}$$

Now, upon substitution of (C.38) and (C.44) into (C.29), we obtain the quantum state diffusion equation

$$|\mathrm{d}\psi\rangle=-\frac{i}{\hbar}|\psi\rangle\mathrm{d}t-\frac{1}{2}\sum_j(L_j^\dagger L_j+l_j^*l_j-2l_j^*L_j)|\psi\rangle\mathrm{d}t+\sum_j(L_j-l_j)|\psi\rangle\mathrm{d}\xi_j \qquad \text{(C.45)}$$

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