Modelling Effects of Halo Breakup on Fusion

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Theories of breakup and fusion of two-body projectiles are examined, to see how complete and incomplete fusion may be predicted separately. A proposal is made for an 'optical decoherence model' which uses semigroup evolution equations to describe the decoherence effects of the imaginary parts of optical potentials without also losing flux.

§1. Fusion and irreversibility

The fusion of two nuclei around the Coulomb barrier provides a fascinating testing ground for theories of quantum tunneling leading to an irreversible fusion of the nuclei into a compound nucleus. In the fusion of a halo nuclei with a stable target, the breakup of the halo can occur before or during the tunneling process, leading to a competition between fusion and breakup, both of which remove flux from the elastic channel.

It is clear that compound nucleus formation is almost certainly irreversible, since the compound nucleus is very unlikely to decay to exactly the entrance channel, but considerable debate exists concerning the irreversibility (or otherwise) of breakup. We might think, because of the large phase space available for breakup, that it should not be reversible for the same reasons as for fusion. Another way of saying this is that there should be a loss of phase coherence between some sets of channels: a *decoherence* between the initial elastic and final breakup channels. Yet another standpoint is that breakup might deplete flux from the elastic channel, generating an effective *absorptive* component of the optical potential that is much larger than any *real* polarisation potential. This is true for fusion channels, we agree, but still we wish to solve an explicit few-body dynamical model to calculate the effects of breakup.

The subject of *decoherence* is of wide interest in the foundations of quantum mechanics. It is generally accepted, in what is now called the *Decoherent Histories approach*,[1)] that decoherence arises from couplings to the many degrees of freedom in the environment of a quantum system. While decoherence might not occur exactly in strict quantum mechanics without 'observations', many systems evolve over time into superpositions of states that are decoherent for all practical purposes. In halo fusion, we accept that the fusion channels definitely act like such an environment, but debate whether breakup channels are also an environment to induce decoherence in this sense.

A completely microscopic model of the interaction between a two-body halo projectile with a target should ideally begin with real-valued (effective) interactions between the fragments and the target. Complete quantum theories of breakup, therefore, would all be reversible, and generate just the real and absorptive potentials for the elastic channel that are physically appropriate.

In practice, however, we have to use 'semi-microscopic' or 'few-body' theories that start with effective interactions or optical potentials with *imaginary* parts, because the fragments may excite and/or fuse with the target in a detail that is outside the scope of the model. These imaginary parts describe the irreversible step of fragment fusion with the target, and hence produce phase decoherence as well as a loss of flux. Using an imaginary part of an effective interaction in a few body model, therefore, already makes assumptions about irreversibility, because absorption cannot be undone. But are these the correct and adequate assumptions to make? Does the few-body wave function calculated with these fragment-target optical potentials tell us all we need to know about the halo system, its breakup, and its complete and incomplete fusion?

These questions have added urgency in the theory of halo fusion (and in the breakup of many-body projectiles generally), because of the need for finding a theory which can describe both complete and incomplete fusion. We need to know not just those integrated cross sections, but the phase space distributions of the surviving fragment(s).

There is some confusion about the definition of fusion.[2)] Theorists[3),4)] usually define complete fusion as the capture all projectile fragments, and incomplete as the capture of only some fragments. Experimentalists,[5),6)] however, tend to define complete fusion as the capture all projectile *charge*, and incomplete as the capture only some of the charge. For ^{6}Li and ^{7}Li, for instance, the definitions would agree, but for halo projectiles, the classification depends on the fate of the neutrons that are difficult to detect.

We may, as theorists, assume that future experiments will be able to determine the destination of individual neutrons, and therefore look for a theory that can describe the evolution of *all* the fragments. In this scenario, halo nuclei (and other two-cluster systems) will be good testing grounds for such a theory.

In this contribution we examine the different ways in which the fragment-target imaginary parts have been included in scattering calculations of halo breakup and fusion, and present a new proposal for modelling their role that should yield further information about surviving subsystems.

§2. Where are the imaginary potentials?

Let us take an historical look at the different ways of including imaginary potentials in the calculations for the scattering of cluster projectiles. We talk here of *core* and *valence* clusters as the heavy and light fragments respectively.

2.1. *Core imaginary potentials*

The simplest approach is to have no imaginary potentials but to restrict the impact parameters to outside a grazing radius. Almost equivalently, we could have short-range potentials in only the coordinate of the core relative to target, as suggested by Yabana et al.[7] In such an approach, 'complete fusion' not calculated, and 'incomplete core fusion' could be defined as the absorption from this core imaginary potential.

The disadvantage is that the valence-target interaction is completely transparent, and may need very many partial waves[7] to describe that motion, as, for example, transfers to bound and resonant target states have to be described within the model.

2.2. *Valence imaginary potentials*

Alternatively, models could include the short-range imaginary parts in only valence-target potential, as in Esbensen and Bertsch.[8] In this case again, 'complete fusion' not calculated, but now we can obtain 'incomplete valence fusion' as the absorption from this valence imaginary potential.

2.3. *Projectile imaginary potential*

A third option is to have short-range potentials in simply the coordinate of *whole projectile* centre of mass relative to target. This has the advantage that we can now calculate separate fusion cross sections from the absorption through the ground state and through the breakup channels, since there are no off-diagonal fusion contributions. Hagino et al.[4] have used this approach, and have gone on to suggest identifying 'complete fusion' as absorption from ground state, and 'incomplete fusion' as absorption from breakup states. We have also used this approach,[9] in order to test other questions concerning continuum-continuum couplings, but in fact we know that the identification of fusion using projectile energy levels is not the true separation of complete and incomplete fusion.

2.4. *Imaginary potentials for all fragments*

A more physically justified approach is to include the short-range imaginary potentials in the coordinates of *both* projectile fragments relative to target.[2] This is more realistic from the viewpoint of few-body dynamics, as capture of the c.m. of the projectile is not necessarily connected to the capture of the fragments. This approach has the disadvantage, however, of having no way of separating complete from incomplete fusion. We can only calculate 'total fusion' as the total absorption by any imaginary part, and identify this as the sum of complete and incomplete fusions.

We have used this approach for ^{6,7}Li breakup on targets of ^{59}Co and ^{209}Pb,[2] and compared the numerical results with those using projectile imaginary potentials. We saw that there are many events where one of the fragments of ^{6}Li is captured, but the c.m. of the projectile does not reach the absorption (fusion) region.

§3. Complete and incomplete fusion

We come back to the need for a theoretical model of few-body dynamics that is able to distinguish complete from incomplete fusion. To do this, it is clear that we need to follow correlations after breakup, so we need either time-dependent[7] or CDCC[2] calculations as a starting point. The need is still, after absorption of one fragment, to follow the evolution of remaining part(s), in order to see whether it escapes (yielding incomplete fusion) or fuses with the target (yielding complete fusion).

An preliminary estimate of this may be obtained by following classical trajectories of the fragments. Three-body classical trajectory model has recently be solved[10] for ^{6}Li and ^{7}Li breakup on ^{209}Bi, to find whether no, one or two fragments are 'captured' by colliding with the target. It is then reasonable to identify these three outcomes with elastic breakup, incomplete fusion, and complete fusion, respectively. The paper[10] remarks at its end that "CDCC would give a more realistic picture", but, for reasons given below, this will still not be sufficient.

There is already a history of attempts to develop theories of incomplete fusion, also called partial fusion, inelastic breakup, or stripping. Eikonal scattering theory is capable of predicting total projectile 'stripping' as well as 'diffraction dissociation' (see Hansen and Tostevin[11] for a recent review), but not the momentum distributions of the surviving fragment. Other workers have developed integral expressions (exact and approximate) for the integrated incomplete-fusion cross section. A survey and comparison of these was given by Ichimura[12] in 1989, who reviewed three-body models by Austern et al.,[13] approximations based on post-form DWBA by Ichimura, Austern and Vincent,[15] as well as an imaginary potential integral suggested by Hussein and McVoy,[14] and the theory of elastic-breakup-fusion from Udagawa and Tamura.[16] Hussein's group has continued these investigations with a more recent series of papers,[3],[17] and Baur and Trautman[18] have usefully employed a 'surface approximation' to calculate inelastic breakup.

We now propose a new theoretical model, whose results should in the future be compared with the methods summarised by Ichimura. The main argument is that we do need more information than is present in a full few-body calculation of dynamics of elastic scattering and breakup, because we still need, after absorption of one fragment, to follow evolution of remaining parts. This information is not in the CDCC wave function, since the complete three-body state undergoes depletion as soon as *any one* fragment is absorbed by its imaginary interaction with the target. A dynamical approach using a density-matrix semigroup theory, outlined here, separates the *decoherence* from the *absorption* caused by the imaginary potentials.

§4. A density matrix model

4.1. *Schrödinger time evolution*

Given an initial time t_0 state $\psi(x, t_0)$, the initial density matrix is

$$\rho(x, x', t_0) = \psi(x, t_0)\psi(x', t_0)^* \ ,$$

or, as an operator,

$$\rho(t_0) = |\psi(t_0)\rangle\langle\psi(t_0)| \ .$$

From the Schrödinger equation for $\psi(x, t)$ with Hamiltonian $H = H_0 - iW$, the Schrödinger time evolution of the density matrix is

$$\partial\rho/\partial t = -i[H, \rho]$$

$$= -i[H_0, \rho] - W\rho - \rho W \ .$$

With Schrödinger density evolution, there is a continuing pure state $\rho(t) = |\psi(t)\rangle\langle\psi(t)|$ from $i\hbar\partial\psi(x,t)/\partial t = H\psi(x,t)$. The effect of the imaginary potential in Schrödinger evolution is to reduce the off-diagonal terms of the density matrix, and this is the density-matrix representation of decoherence.

Consider $H_0 = 0$ in a toy model, with

$$\partial\rho(x,x',t)/\partial t = -W(x)\rho(x,x',t) - \rho(x,x',t)W(x') \ ,$$

which has solution

$$\rho(x,x',t) = e^{-W(x)t}\rho(x,x',0)e^{-W(x')t}. \tag{4·1}$$

Here the off-diagonal terms are indeed reduced, but diagonal terms also! Is there a way of producing decoherence by reduction of the off-diagonal terms, but still not loss of flux, by having no reduction of the diagonal terms $\rho(x,x,t)$?

4.2. *Semigroup decoherence model*

We may consider the more general case of *semigroup* time evolution. Semigroups are a set of groups in which inverse elements do not necessary exist, and hence are appropriate for describing irreversible time evolution. Lindblad[19)] proved that the general trace-preserving completely-positive master equation is

$$\partial\rho/\partial t = -i[H,\rho] + \sum_m (2L_m\rho L_m^\dagger - L_m^\dagger L_m\rho - \rho L_m^\dagger L_m)/2 \ , \tag{4·2}$$

where L_m are any bounded operators, now often called ‘Lindblads’.

Let us choose a Lindblad form to give same decoherence effect on the off-diagonal terms of the density matrix as the imaginary optical potential above. Consider for example the Lindblad

$$L_1 = \sqrt{2W(x)} \ .$$

In the $H_0 = 0$ simplified case, the evolution equation (4·2) is

$$\rho'(t) = 2\sqrt{W}\rho\sqrt{W} - W\rho - \rho W \ ,$$

which has the exact solution

$$\begin{aligned}\rho(x,x',t) &= \rho(x,x',0)\exp(-[\sqrt{W(x')} - \sqrt{W(x)}]^2 t) \\ &= e^{-W(x)t}\rho(x,x',0)e^{-W(x')t} \times e^{+2\sqrt{W(x)W(x')}t}.\end{aligned} \tag{4·3}$$

The properties of this semigroup evolution are that if one of the $W(x)$ and $W(x')$ is zero, then this evolution gives the same result as in Eq. (4·1). When $x = x'$ we have $\rho(x,x',t) = \rho(x,x',0)$, so that there is no diagonal decoherence or loss of any flux. If $W(x) = W(x')$, there is again no decoherence where $W(x)$ is constant. If, by contrast, $W(x) \neq W(x')$, then $\rho(x,x',t) \to 0$, so that we do have off-diagonal decoherence where $W(x)$ varies.

4.3. *Optical decoherence model (ODM)*

It is therefore proposed that we try to solve the time evolution of composite projectile with the semigroup master equation:

$$\begin{aligned}\rho'(t) &= -i[H_0,\rho] + 2\sqrt{W}\rho\sqrt{W} - W\rho - \rho W \\ &= -i[H,\rho] + 2\sqrt{W}\rho\sqrt{W}\ . \end{aligned} \tag{4·4}$$

The imaginary potential $-iW(x) = -iW_1(x_1) - iW_2(x_2)$, for each fragment-target interaction of a two-body projectile, gives decoherence, but *no* loss of flux as $\mathrm{Tr}(\rho) = 1$ always. After one fragment has fused, for example, this dynamical model can still follow time evolution to give the flux and momenta of the other fragments. The fused fragment does not disappear, but remains trapped in one of the regions of the imaginary potentials.

4.4. *Potential scattering*

Consider one-body scattering from a potential with an imaginary part $-iW(x)$. Let $|u(t)\rangle$ be the time-evolving solution of the normal Schrödinger with $H = H_0 - iW$, namely $i\hbar\partial|u(t)\rangle/\partial t = H|u(t)\rangle$. We may expand the density matrix as

$$\rho(t) = \sum_{s,s'} c_{ss'}(t)|s\rangle\langle s'| + c_u(t)|u(t)\rangle\langle u(t)|\ , \tag{4·5}$$

where the $|s\rangle\langle s'|$ projection operators are a complete set defined where $W(s) \neq 0 \neq W(s')$, and before the collision $c_{ss'}(t_0) = 0$ and $c_u(t_0) = 1$. If normal scattering theory results are to be regained, we will want to prove that $c_u(t) = 1$ at all times t. Define $N_u(t) = \langle u(t)|u(t)\rangle$ as the time-dependent square norm of $u(t)$.

Substituting the expansion (4·5) into the master Eq. (4·4) we find the coefficient evolution equation:

$$\begin{aligned}&\sum_{s,s'} \dot{c}_{ss'}(t)|s\rangle\langle s'| + \dot{c}_u(t)|u(t)\rangle\langle u(t)| = c_u(t)\sqrt{W}|u(t)\rangle\langle u(t)|\sqrt{W} \\ &+ \sum_{s,s'} c_{ss'}(t)\left\{-i[H_0|s\rangle\langle s'| - |s\rangle\langle s'|H_0] - (\sqrt{W(s)} - \sqrt{W(s')})^2|s\rangle\langle s'|\right\}\ . \end{aligned} \tag{4·6}$$

From this, the diagonal terms vary as

$$\dot{c}_{ss}(t) = 2W(s)|u(s,t)|^2\ ,$$

whose integral $\sum_s \dot{c}_{ss}(t) = 2\int W(s)|u(s,t)|^2 ds$ gives the rate of change $dN_u(t)/dt$ of the square norm of $u(t)$, and hence agrees with the conservation of flux $\mathrm{Tr}(\rho) = 1$.

If we can neglect the commutator $[H_0, |s\rangle\langle s'|]$ in Eq. (4·6), it leads to the result

$$\dot{c}_u(t)\left[N_u(t)^2 - \left\{\sum_s |\langle s|u\rangle|^2\right\}^2\right] = 0\ ,$$

which, if $u(t)$ extends outside the range of the imaginary potentials so that its norm $N_u(t) > \sum_s |\langle s|u\rangle|^2$, implies $\dot{c}_u(t) = 0$ and hence that $c_u(t) = 1$ always. This result, which should be confirmed by a complete (e.g. numerical) analysis, ensures that expectation values taken outside the range of the imaginary potentials will be the same as in conventional quantum mechanics.

Assuming that $c_u(t) \equiv 1$, we have the expansion

$$\rho(t) = \sum_{s,s'} c_{ss'}(t)|s\rangle\langle s'| + |u(t)\rangle\langle u(t)| \tag{4·7}$$

from which the (exact) time evolution of the $c_{ss'}(t)$ is given by

$$\begin{aligned}\sum_{s,s'} \dot{c}_{ss'}(t)|s\rangle\langle s'| &= \sqrt{W}|u(t)\rangle\langle u(t)|\sqrt{W} \\ &+ \sum_{s,s'} \dot{c}_{ss'}(t)\left\{-i[H_0|s\rangle\langle s'| - |s\rangle\langle s'|H_0] - (\sqrt{W(s)} - \sqrt{W(s')})^2|s\rangle\langle s'|\right\} .\end{aligned} \tag{4·8}$$

If we may again neglect the H_0 commutator, this becomes

$$\dot{c}_{ss'}(t) = \sqrt{W(s)}u(s,t)u(s',t)^*\sqrt{W(s')} - c_{ss'}(t)(\sqrt{W(s)} - \sqrt{W(s')})^2 . \tag{4·9}$$

Here we see that the terms $c_{ss'}(t)$ are generated by imaginary potentials when the incoming wavepacket overlaps with non-zero values of both $W(s)$ and $W(s')$, and that the off-diagonal terms are damped in proportion to $(\sqrt{W(s)} - \sqrt{W(s')})^2$, just as in Eq. (4·3).

4.5. *Summary of the ODM agenda*

This new approach is not yet worked out in detail. We should:

- Rederive stationary-state scattering theory for optical potentials, by evolving the density matrix rather than simply the wave function.
- Include the second (semigroup) term of the master equations (4·4) in, for example, model problems of reduced dimensions. Verify the identity of expectation values with those from Hamiltonian evolution.
- Determine the practicality of evolving three-body-model density matrices (where the number of spatial dimensions is squared!).
- If solving for the few-body $\rho(x_1..., x'_1..., t)$ is too difficult, we could try to produce a statistical ensemble of source terms for the set of wave function eigensolutions of the density matrix, which are the different decoherent outcomes of the remaining particle(s).

This approach is summarised by noting again that quantum mechanics gives approximate decoherence. A semi-group theory as above, with its exact decoherence over time, is therefore an approximation to quantum mechanics, and is equivalent[20] to averaging randomised unitary evolution. The present theory may hence be useful when solving approximate quantum models such as the few-body models of halo breakup that use optical potentials, and expectation values are needed for subsystems of the original projectile. As a semigroup theory, it is one of a more general class of reduction models recently reviewed.[21]

§5. Conclusions

Halo and cluster nuclei, with well-defined breakup and fusion modes, are good test-benches for theories of breakup and fusion. At least they soon should be, when experimentalists can measure the final destinations of all projectile fragments, including that of the neutrons.

There are still severe theoretical varieties and uncertainties in modelling incomplete and complete fusion in a clearly distinguishable manner. To produce predictions which discriminate between complete and incomplete fusion modes, this paper proposes an 'Optical Decoherence Model' (ODM), which uses semigroup evolution of density matrix in order to separate the decoherence and flux-loss effects of imaginary potentials. This should allow after us to follow, even after the absorption of one fragment, the evolution of the remaining parts.

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