Modelling lubricated revolute joints in multibody mechanical systems

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Abstract: This work deals with the modelling of lubricated revolute joints in multibody mechanical systems. In most machines and mechanisms, the joints are designed to operate with some lubricant fluid. The high pressures generated in the lubricant fluid act to keep the journal and the bearing apart. Moreover, the thin film formed by lubricant reduces friction and wear, provides load capacity and adds damping to dissipate undesirable mechanical vibrations. In the dynamic analysis of journal–bearings, the hydrodynamic forces, which include both squeeze and wedge effects, produced by the lubricant fluid oppose the journal motion. These forces are obtained by integrating the pressure distribution evaluated with the aid of Reynolds' equation written for the dynamic regime. The hydrodynamic forces are nonlinear functions of journal centre position and velocity relative to the bearing centre. In a simple way, the hydrodynamic forces built up by the lubricant fluid are evaluated from the state of variable of the system and included into the equations of motion of the mechanical system. Results for an elementary slider–crank mechanism, in which a lubricated revolute joint connects the connecting rod and slider, are used to discuss the assumptions and procedures adopted.

Keywords: lubricated joints, dynamic journal–bearings, multibody mechanical systems

NOTATION

$\mathbf{A}$	rotational transformation matrix
c	radial clearance size
$\boldsymbol{e}$	eccentricity vector
e	absolute eccentricity
F	external load applied to the journal–bearing
$\boldsymbol{g}$	generalized force vector
h	fluid film thickness
L	length of the journal–bearing
$\mathbf{M}$	system mass matrix
m	moment
p	fluid pressure
P_i	centre of the bearing
P_j	centre of the journal
$\ddot{\boldsymbol{q}}$	vector that contains the state of accelerations
R_{B}	radius of the bearing
R_{J}	radius of the journal
$\boldsymbol{r}$	unit vector along the eccentricity direction
r	radial direction
$\boldsymbol{r}^{\mathrm{P}}$	vector of global position of point P
$\boldsymbol{s}^{\mathrm{P}}$	vector of global coordinates of point P
$\boldsymbol{s}'^{\mathrm{P}}$	vector of local coordinates of point P
t	tangential direction
U	relative tangential velocity
XY	two-dimensional global coordinate system
α	Baumgarte stabilization coefficient
β	Baumgarte stabilization coefficient
ε	eccentricity ratio
$\dot{\varepsilon}$	time rate of eccentricity ratio
$\boldsymbol{\Phi}_{\mathbf{q}}$	Jacobian matrix
$\boldsymbol{\gamma}$	right-hand side vector of acceleration equations
γ	angle of the line between the eccentricity vector and x-axis
$\dot{\gamma}$	first derivative of the γ with respect to time
$\boldsymbol{\lambda}$	vector of Lagrange multipliers
μ	dynamic fluid viscosity
θ	angular position
ω	relative angular velocity
$\xi\eta$	two-dimensional body-fixed coordinate system

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1 INTRODUCTION

The present work deals with the modelling of lubricated revolute joints in multibody mechanical systems.

In actual joints, clearance, friction and impact are always present in mechanical systems, mainly when there is no fluid lubricant. These phenomena can significantly change the dynamic response of the mechanical systems in so far as the impact causes noise, increases the level of vibrations, reduces the fatigue life and results in loss of precision. An overview literature on this topic up to 1980 is given by Haines [**1**]. More recently, some works have addressed the dynamic analysis of multibody mechanical systems with revolute clearance joints [**2–4**]. When the clearance joints are considered as dry, i.e., without lubricant, impacts take place and consequently the dynamic behaviour of the mechanical systems can be drastically modified.

However, in most machines and mechanisms, the joints are designed to operate with some lubricant fluid. The high pressures generated in the lubricant fluid act to keep the journal and the bearing apart. Moreover, the thin film formed by lubricant reduces friction and wear, provides load capacity, and adds damping to dissipate undesirable mechanical vibrations. Consequently, proper modelling of lubricated revolute joints in multibody systems is required to achieve better understanding of the dynamic performance of the machines. This aspect gains paramount importance due to the demand for the proper design of the journal–bearings in many industrial applications. Ravn [**2**], Schwab [**3**], and Alshaer and Lankarani [**5**] are amongst the few researchers who have incorporated the effect of the lubricant fluid in the dynamic study of multibody mechanical systems.

In general, multibody systems use journal–bearings in which the load varies in both magnitude and direction, which results in dynamically loaded journal–bearings. Typical examples of dynamically loaded journal–bearings include the crankshaft bearings in combustion engines, and high-speed turbine bearings supporting dynamic loads caused by an unbalanced rotor.

In the dynamic analysis of journal–bearings, the hydrodynamic forces produced by the lubricant fluid oppose the journal motion. These forces are obtained by integrating the pressure distribution evaluated with the aid of Reynolds' equation written for the dynamic regime. The hydrodynamic forces are nonlinear functions of journal centre position and velocity relative to the bearing centre. In a dynamic regime the journal centre has an orbit situated within a circle radius equal to the radial clearance. Thus, a lubricated revolute joint does not impose kinematic constraints like an ideal revolute joint; instead it deals with force constraints.

For dynamically loaded journal–bearings the classic analysis problem is predicting the motion of the journal centre under arbitrary and known loading. In contrast, in the present analysis the time variable parameters are known from the dynamic analysis and the instantaneous force on the journal–bearing is calculated.

In a simple way, the forces built up by the lubricant fluid are evaluated from the variables of the system and included in the equations of motion of the mechanical system. To carry out the dynamic analysis of the multibody system with lubricated revolute joints, an effective model is presented in this work. Both squeeze and wedge hydrodynamic effects are included in the dynamically loaded journal–bearings.

It should be mentioned that the methodology presented here uses the superposition principle for load capacity due to wedge effect entrainment and the squeeze film effect separately. This is only approximate, and has been adopted in the paper in order to arrive at an analytical rather a numerical solution. Furthermore, the methodology is only applicable to long bearings, and that is not the case in most engine bearings, where a finite width bearing is used, but it is useful for many other crank–slider mechanisms. It should be highlighted that in the methodology used in the present work, the applied loads and bearing reactions do not cause any deformation of the bearing bushing or shell. Thus, a hydrodynamic regime of lubrication is assumed at all times. In many modern mechanisms and at high transient loads these conditions are not always met. Thus, a more representative elastohydrodynamic solution may be required, such as in thin shell engine bearings of modern vehicles. This is the price paid for computational efficiency.

Results for a planar slider–crank mechanism in which a lubricated revolute joint connects the connecting rod and slider bodies are used to discuss the assumptions and procedures adopted.

2 MULTIBODY SYSTEMS FORMULATION

The equations of motion for planar multibody mechanical systems are written as a coupled set of differential and algebraic equations, which can be expressed as the form [**6**]

$$\begin{bmatrix} \mathbf{M} & \mathbf{\Phi}_{\mathbf{q}}^{\mathrm{T}} \\ \mathbf{\Phi}_{\mathbf{q}} & \mathbf{0} \end{bmatrix} \begin{Bmatrix} \ddot{\boldsymbol{q}} \\ \boldsymbol{\lambda} \end{Bmatrix} = \begin{Bmatrix} \boldsymbol{g} \\ \boldsymbol{\gamma} \end{Bmatrix} \tag{1}$$

where $\mathbf{M}$ is the system mass matrix, the vector $\ddot{\boldsymbol{q}}$ contains the generalized state accelerations, $\boldsymbol{\Phi_q}$ is the Jacobian of constraint equations, the vector $\boldsymbol{\lambda}$ holds the Lagrange multipliers, $\boldsymbol{\gamma}$ is the vector of quadratic velocity terms and $\boldsymbol{g}$ is the vector of generalized forces which contains the external applied forces as well as the forces developed in the lubricated revolute clearance joint.

A set of additional conditions together with the kinematic constraints defines the initial positions and velocities required to start the dynamic simulation. These initial conditions are obtained from kinematic simulation of the mechanical system in which all of the joints are assumed to be ideal or perfect joints. The subsequent initial conditions for each time step in the simulation are obtained from the results of the previous time step.

In order to stabilize or keep under control the constraint violation, equation (1) is solved using the Baumgarte stabilization method [**7**]. The integration process is performed using a predictor–corrector algorithm with variable step and order [**8**].

3 HYDRODYNAMIC FORCES IN DYNAMIC JOURNAL–BEARINGS

In most machines and mechanisms, the joints are designed to operate with some lubricant fluid. The high pressures generated in the lubricant fluid act to keep the journal and the bearing apart. Moreover, the thin film formed by lubricant reduces friction and wear, provides load capacity and adds damping to dissipate undesirable mechanical vibrations. In general, multibody mechanical systems use journal–bearings in which the load varies in both magnitude and direction, which results in dynamically loaded journal–bearings. When the load acting on the journal–bearing is not constant in direction and/or module, the journal centre describes a trajectory within the bearing boundaries. Typical examples of dynamically loaded journal–bearings include the crankshaft bearings in combustion engines, and high-speed turbines bearings supporting dynamic loads caused by rotor unbalanced. Figure 1 provides a general configuration of a dynamically loaded journal–bearing and notation.

In a broad sense, dynamically loaded journal–bearings can be classified into two groups, namely squeeze-film action and wedge-film action [**9**]. The first group refers to the situations in which the journal does not rotate about its centre, rather, the journal moves along some path inside the bearing. The second group deals with cases in which there is journal rotation. In a journal–bearing, squeeze-film action is dominant when the relative rotational velocity is small compared with the relative radial velocity. When the relative rotational velocity between the two elements is high, the wedge-film effect must also be considered.

Reynolds' equation is used to evaluate the forces developed by the fluid film pressure field. Pinkus and Sternlicht [**9**], amongst others, have presented a detailed derivation of the Reynolds' equation, which can be deduced either from the Navier–Stokes equations or from first principles. The Reynolds equation contains viscosity, density and film thickness as parameters. Furthermore, the derivation of this equation is based upon several premises, namely, the flow is laminar, the fluid is Newtonian and incompressible, the pressure across the film thickness is constant, the journal and bearing axis are parallel, the fluid inertia is negligible, there is no slip at the bearing surface, the bearing and journal surfaces are rigid, and no oil supply groove is considered. Under these assumptions, the isothermal generalized Reynolds' equation can be written as

$$\frac{\partial}{\partial X}\left(\frac{h^3}{\mu}\frac{\partial p}{\partial X}\right)+\frac{\partial}{\partial Z}\left(\frac{h^3}{\mu}\frac{\partial p}{\partial X}\right)=6U\frac{\partial h}{\partial X}+12\frac{\mathrm{d}h}{\mathrm{d}t} \tag{2}$$

The two terms on the right-hand side of equation (2) represent the two different effects of pressure generation in the lubricant film, that is, wedge-film action and squeeze-film action, respectively.

It is known that equation (2) is one non-homogeneous partial differential of elliptical type. The exact solution of the Reynolds' equation is difficult to obtain, and in general requires considerable numerical effort. However, it is possible to solve the equation approximately where the first or second term on the left-hand side is treated as zero. These solutions correspond to those for an infinitely short and infinitely long journal–bearing, respectively.

For an infinitely long journal–bearing, a constant fluid pressure and negligible leakage in the Z-direction are assumed. In many cases it is possible to treat a journal–bearing as infinitely long and consider only the middle point of it. This approach is valid for length-to-diameter (L/D) ratios greater than 2 [**10**].

Thus, the Reynolds equation for an infinitely long journal–bearing can be written as

$$\frac{\partial}{\partial X}\left(\frac{h^3}{\mu}\frac{\partial p}{\partial X}\right)=6U\frac{\partial h}{\partial X}+12\frac{\mathrm{d}h}{\mathrm{d}t} \tag{3}$$

The pressure distribution in the fluid is given by [**9**]

$$p=6\mu\left(\frac{R_J}{c}\right)^2\left\{\begin{array}{l}\dfrac{(\omega-2\dot{\gamma})(2+\varepsilon\cos\theta)\varepsilon\sin\theta}{(2+\varepsilon^2)(1+\varepsilon\cos\theta)^2}\\ \quad+\dfrac{\dot{\varepsilon}}{\varepsilon}\left[\dfrac{1}{(1+\varepsilon\cos\theta)^2}-\dfrac{1}{(1+\varepsilon)^2}\right]\end{array}\right\} \tag{4}$$

Equation (4) enables the calculation of the pressure field in a hydrodynamic loaded journal–bearing as a function of the dynamic parameters ω, ε, $\dot{\varepsilon}$ and $\dot{\gamma}$.

It is convenient to determine the force components of the resultant pressure along and perpendicular to the line of centres. These force components can be obtained by integration of the pressure field either around the entire surface, 2π, or around a half-surface, π, that is, equation (4) is integrated only over the positive region by setting the pressure in the remaining portion equation to zero. These boundary conditions associated with the pressure field correspond to Sommerfeld's and Gümbel's conditions, respectively. Sommerfeld's conditions (complete film) do not take into account the cavitation phenomenon and, consequently, the

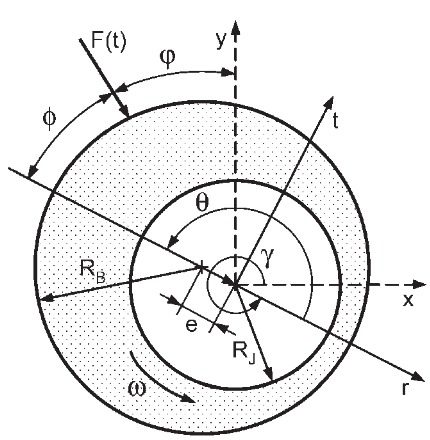

Fig. 1 Dynamically loaded journal–bearing

existence of negative pressures for $\pi < \theta < 2\pi$. This case is unreal due to the fluid incapacity to sustain sub-ambient pressures. Gümbel's conditions (rupture film) preconize the existence of a zero pressure zone between π and 2π. This situation, however, configures fluid film discontinuity at $\theta = 2\pi$.

For the Sommerfeld's conditions (full film) the component force of the fluid film can be written as [**9**]

$$F_r = -\frac{12\pi\mu L R_J^3 \dot{\varepsilon}}{c^2(1-\varepsilon^2)^{3/2}} \tag{5}$$

$$F_t = \frac{12\pi\mu L R_J^3 \varepsilon(\omega - 2\dot{\gamma})}{c^2(2+\varepsilon^2)(1-\varepsilon^2)^{1/2}} \tag{6}$$

Equation (5) refers to the force from the squeeze-film action, and equation (6) refers to the force from the wedge-film effect. These equations reduce to the steady-state form for the full Sommerfeld solution when $2\dot{\gamma} = \dot{\varepsilon} = 0$ [**10**].

The component forces obtained from the integration only over the positive regions by setting the pressure in the remaining portions equal to zero involve finding the zero points, i.e., the angles at which the pressure begins and ends. This analysis involves a good deal of mathematical manipulation (for details, see Reference [**9**]). The component forces along eccentricity direction and normal to it are for $\dot{\varepsilon} > 0$ given by

$$F_r = -\frac{\mu L R_J^3}{c^2}\frac{6\dot{\varepsilon}}{(2+\varepsilon^2)(1-\varepsilon^2)^{3/2}} \times \left[4k\varepsilon^2 + (2+\varepsilon^2)\pi\frac{k+3}{k+3/2}\right] \tag{7}$$

$$F_t = \frac{\mu L R_J^3}{c^2}\frac{6\pi\varepsilon(\omega - 2\dot{\gamma})}{(2+\varepsilon^2)(1-\varepsilon^2)^{1/2}}\frac{k+3}{k+3/2} \tag{8}$$

and for $\dot{\varepsilon} < 0$

$$F_r = -\frac{\mu L R_J^3}{c^2}\frac{6\dot{\varepsilon}}{(2+\varepsilon^2)(1-\varepsilon^2)^{3/2}} \times \left[4k\varepsilon^2 - (2+\varepsilon^2)\pi\frac{k}{k+3/2}\right] \tag{9}$$

$$F_t = \frac{\mu L R_J^3}{c^2}\frac{6\pi\varepsilon(\omega - 2\dot{\gamma})}{(2+\varepsilon^2)(1-\varepsilon^2)^{1/2}}\frac{k}{k+3/2} \tag{10}$$

where the parameter k is defined as

$$k^2 = (1-\varepsilon^2)\left[\left(\frac{\omega - 2\dot{\gamma}}{2\dot{\varepsilon}}\right) + \frac{1}{\varepsilon^2}\right] \tag{11}$$

In the present work, μ is the dynamic fluid viscosity, L is the journal–bearing length, R_J is the journal radius, c is the radial clearance, ω is the relative angular velocity between the journal and the bearing, ε is the eccentricity ratio which is obtained from distance between the bearing and journal centres divided by the radial clearance and γ is the angle between the eccentricity direction and the x'-axis. The dot above expressions denotes the time derivative of the corresponding parameter.

Equations (5)–(10) for infinitely long journal–bearings present the connection between the journal centre motion and the fluid reaction force on the journal. The solution of these equations presents no problem since the journal centre motion is known from dynamic analysis.

The force components of the resulting pressure distribution along and perpendicular to the line of centres have to be projected onto the x and y directions. From Fig. 1 it is clear that

$$F_x = F_r \cos\gamma - F_t \sin\gamma \tag{12}$$

$$F_y = F_r \sin\gamma + F_t \cos\gamma \tag{13}$$

In classic design of journal–bearings the external forces are known and the motion of the journal centre inside the bearing boundaries is evaluated by solving the differential equations for the time-dependent variables. Yet, in the present analysis, instead of knowing the applied load, the relative journal–bearing motion characteristics are known and the fluid force from the pressure distribution in the lubricant is desired. Thus, since all the variables are known from dynamic analysis, the hydrodynamic forces given by equations (12) and (13) are introduced as generalized forces in the system's equations of motion.

4 MODELLING LUBRICATED REVOLUTE JOINTS IN MULTIBODY SYSTEMS

In multibody mechanical systems, a lubricated revolute joint, the so-called journal–bearing, does not produce any kinematic constraint like the ideal joint. Instead, it acts in a similar way to a force element, producing time-dependent forces. These hydrodynamic forces are nonlinear functions of the time parameters, ω, ε, $\dot{\varepsilon}$, γ and $\dot{\gamma}$, which can be evaluated at any instant of time from the dynamic analysis of the mechanical system.

Thus, in order to evaluate the forces produced by the fluid lubricant on the journal–bearing, the different dynamic parameters on which these forces depend need to be evaluated.

Figure 2 shows a general configuration of a dynamically loaded journal–bearing in a multibody mechanical system. The two bodies i and j are connected by a lubricated revolute clearance joint, in which the gap between the bearing and the journal is filled with a fluid lubricant. The fluid lubricant introduces damping and stiffness to the system. Part of body i is the bearing and part of body j is the journal. The centre of mass of bodies i is O_i and the centre of mass of body j is denoted by O_j. The local coordinate systems of bodies i and j are attached to their centre of mass, while the global coordinate system is represented by the xy-coordinate. Point P_i indicates the centre of the bearing, and the centre of the journal is at point P_j.

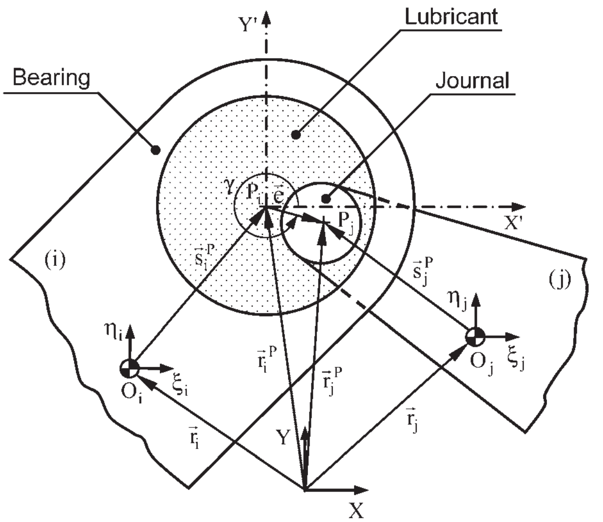

Fig. 2 Generic configuration of dynamically loaded journal–bearing in a multibody system

With regard to Fig. 2, the eccentricity vector $\boldsymbol{e}$, which connects the centres of the bearing and the journal, is calculated as

$$\boldsymbol{e} = \boldsymbol{r}_j^{\mathrm{P}} - \boldsymbol{r}_i^{\mathrm{P}} \tag{14}$$

where both $\boldsymbol{r}_j^{\mathrm{P}}$ and $\boldsymbol{r}_i^{\mathrm{P}}$ are described in global coordinates with respect to the inertial reference frame [**6**]

$$\boldsymbol{r}_k^{\mathrm{P}} = \boldsymbol{r}_k + \mathbf{A}_k \boldsymbol{s}_k^{\prime\mathrm{P}} \quad (k = i, j) \tag{15}$$

The rotational transformation matrix is given by

$$\mathbf{A}_k = \begin{bmatrix} \cos\phi_k & -\sin\phi_k \\ \sin\phi_k & \cos\phi_k \end{bmatrix} \quad (k = i, j) \tag{16}$$

Thus, equation (14) can be rewritten as

$$\boldsymbol{e} = \boldsymbol{r}_j^{\mathrm{P}} + \mathbf{A}_j \boldsymbol{s}_j^{\prime\mathrm{P}} - \boldsymbol{r}_i^{\mathrm{P}} - \mathbf{A}_i \boldsymbol{s}_i^{\prime\mathrm{P}} \tag{17}$$

The magnitude of the eccentricity vector can be evaluated as

$$e = \sqrt{\boldsymbol{e}^{\mathrm{T}}\boldsymbol{e}} \tag{18}$$

A unit vector, $\boldsymbol{r}$, along the eccentricity direction is defined as

$$\boldsymbol{r} = \frac{\boldsymbol{e}}{e} \tag{19}$$

The unit vector has the same direction as the line of centres of the bearing and the journal, here denoted as the radial direction, while the tangential direction is obtained by rotating vector $\boldsymbol{r}$ 90° in a counter-clockwise direction.

The parameter ε which defines the eccentricity ratio is obtained from distance between the bearing and journal centre divided by the radial clearance, that is

$$\varepsilon = \frac{e}{c} \tag{20}$$

The parameter $\dot{\varepsilon}$ can be obtained by differentiating equation (17), and dividing the result by radial clearance. Differentiating equation (17) results in

$$\dot{\boldsymbol{e}} = \dot{\boldsymbol{r}}_j^{\mathrm{P}} + \dot{\mathbf{A}}_j \boldsymbol{s}_j^{\prime\mathrm{P}} - \dot{\boldsymbol{r}}_i^{\mathrm{P}} - \dot{\mathbf{A}}_i \boldsymbol{s}_i^{\prime\mathrm{P}} \tag{21}$$

where the dot denotes the derivative with respect to the time. The time rate of eccentricity ratio is given by

$$\dot{\varepsilon} = \frac{\dot{e}}{c} \tag{22}$$

The line of centres between the bearing and the journal makes an angle γ with the x'-axis, as shown in Fig. 2. Since the unit radial vector $\boldsymbol{r}$ has the same direction as the line of centres, the angle γ can be defined as

$$\begin{bmatrix} \cos\gamma \\ \sin\gamma \end{bmatrix} = \begin{bmatrix} \mathrm{r}_x \\ \mathrm{r}_y \end{bmatrix} \tag{23}$$

Therefore,

$$\gamma = \tan^{-1}\frac{r_y}{r_x} \tag{24}$$

The parameter $\dot{\gamma}$ can be obtained by differentiating equation (24) with respect to the time, yielding,

$$\dot{\gamma} = \frac{e_x \dot{e}_y - \dot{e}_x e_y}{e^2} \tag{25}$$

The components of forces of the resulting pressure projected onto the x and y directions given by equations (12) and (13) act on the journal centre. Thus, these forces have to be transferred to the centres of mass of both the bearing and the journal. Concerning Fig. 3, the forces and moments that act on the centre of mass of journal body, O_j, are given by

$$\begin{bmatrix} f_j^x \\ f_j^y \\ m_j \end{bmatrix} = \begin{bmatrix} F_x \\ F_y \\ -(\xi_j^{\mathrm{P}} \sin\phi_j + \eta_j^{\mathrm{P}} \cos\phi_j) F_x \\ +(\xi_j^{\mathrm{P}} \cos\phi_j - \eta_j^{\mathrm{P}} \sin\phi_j) F_y \end{bmatrix} \tag{26}$$

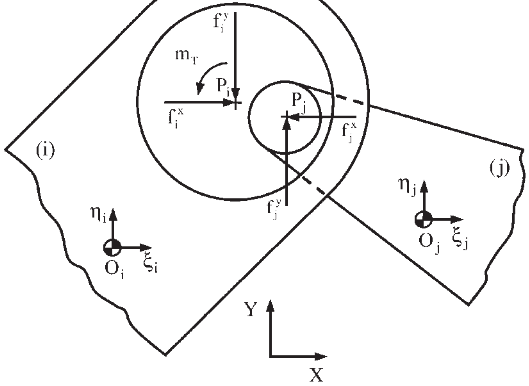

Fig. 3 Vectors of forces working at the journal and bearing

and for the centre of mass of bearing body at O_i,

$$\begin{bmatrix} f_i^x \\ f_i^y \\ m_i \end{bmatrix} = \begin{bmatrix} -F_x \\ -F_y \\ (\xi_i^{\mathrm{P}} \sin\phi_i + \eta_i^{\mathrm{P}} \cos\phi_i + e_y)F_x \\ -(\xi_i^{\mathrm{P}} \cos\phi_i - \eta_i^{\mathrm{P}} \sin\phi_i + e_x)F_y \end{bmatrix} \tag{27}$$

The transport moment produced by transferring the forces from the centre of journal to the centre of the bearing can be evaluated as

$$m_{\mathrm{T}} = e_y F_x - e_x F_y \tag{28}$$

5 APPLICATION EXAMPLE: SLIDER–CRANK MECHANISM

An elementary slider–crank mechanism is used to illustrate the efficiency and accuracy of the methodology presented throughout this work. Figure 4 depicts the kinematic configuration of the planar slider–crank mechanism, which consists of four bodies, including ground, two ideal revolute joints and one ideal translational joint. The body numbers and their corresponding coordinate systems are shown in Fig. 4.

A revolute clearance joint exists between the connecting rod and slider. This rotational joint is modelled with the hydrodynamic formulation presented in the previous sections. This joint is an example of a lubricated journal–bearing in which the load varies in both magnitude and direction. In order to keep the analysis simple and to illustrate the dynamic clearance joint behaviour, all the bodies are considered to be rigid and the inertia due to the driving motor is neglected. The dimensions and inertia properties of each body are listed in Table 1.

Table 1 Governing properties for the slider–crank mechanism

Body no.	Length (m)	Mass (kg)	Moment of inertia (Kg m^2)
2	0.05	0.30	0.00001
3	0.12	0.21	0.00025
4	—	0.14	—

In the dynamic simulation the crank is the driving (motor) body and rotates with a constant angular velocity equal to 5000 rpm clockwise. The initial configuration corresponds to crank and connecting rod collinear and the position and velocity journal centre are taken to be zero. Initially the journal and bearing centres are coincident. The properties for the dynamic simulation are listed in Table 2.

In order to study the influence of the use of the hydrodynamic model in the dynamic behaviour of the slider–crank mechanism, a long time simulation was done. The time interval used corresponded to two complete crank rotations. The dynamic behaviour of the slider–crank mechanism was measured by quantifying the reaction force developed in the lubricated revolute joint and the crank torque necessary to maintain constant the crank angular velocity. In addition, the trajectory of the journal centre inside the bearing and the minimum fluid thickness are presented. The dynamic analysis performed with lubricated revolute joints is compared with a simulation in which all joints are considered to be ideal.

Figure 5 shows the reaction force developed in the lubricated revolute joint, that is, the resultant force due to the generation of the pressure field. The results are of the same order as those obtained with ideal joint. The smooth curve obtained for reaction force is propagated throughout the mechanical systems until the crank moment. The reaction moment on the crank represents the input power necessary to maintain constant the crank angular velocity. In a similar manner to the reaction force, the crank moment obtained with hydrodynamic model matched quite well with the crank moment obtained with ideal joint simulation.

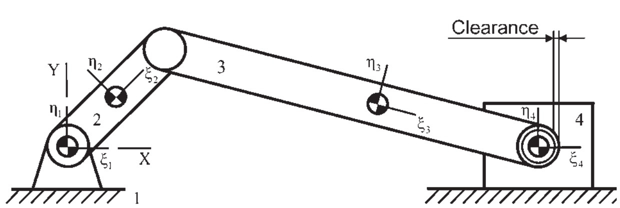

Fig. 4 Slider–crank mechanism with a lubricated revolute joint between the connecting rod and slider

Table 2 Parameters used in the dynamic simulation

Bearing radius	10.0 mm
Journal bearing	9.8 mm
Journal–bearing length	40.0 mm
Dynamic fluid viscosity	400 cP
Baumgarte coefficient, α	5
Baumgarte coefficient, β	5
Integration time step	0.00001 s
Integration tolerance	0.000001 s

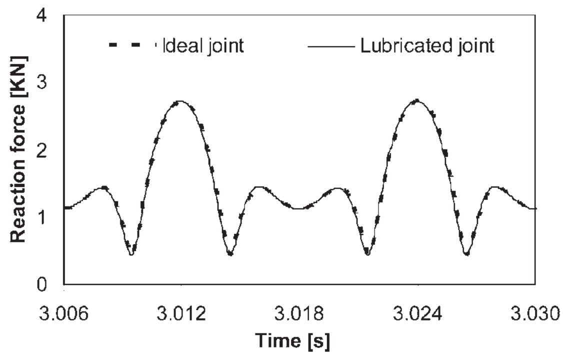

Fig. 5 Reaction force developed in the lubricated joint

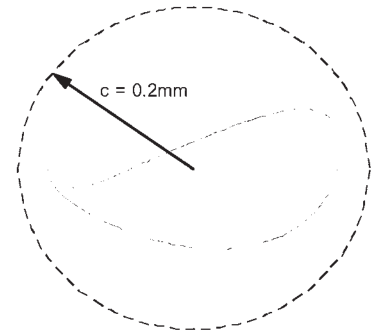

Fig. 7 Journal centre trajectory inside the bearing

Looking at the results for the reaction force and reaction moment (see Fig. 6), it is clear that they are basically the same as for the case with ideal joints. The first and the second crank rotations show the same results, which indicates that the system has reached the steady state. This can be confirmed by the orbit of the journal centre relative to the bearing centre, in which the journal moves far from the bearing wall, that is, there is always a minimum film lubricant in between the two bodies (see Fig. 7).

The practical criterion for determining whether or not a journal–bearing is operating satisfactorily is the value of the minimum oil film thickness, which is probably the most important quantity in the performance of the journal–bearings. It is not easy to state a unique value of minimum film thickness that can be assumed to be safe since a great deal depends on the manufacturing process, the alignment of the machine elements associated with the journal–bearings, the general operating conditions, including the environment of the machine, etc. In practical engineering design it is recommended that the minimum oil film thickness should be at least 0.00015 mm/mm of bearing diameter [**11**].

The minimum film thickness is related to the eccentricity ratio (ε) and radial clearance (c) by the equation [**10**]

$$h_{\min} = c(1 - \varepsilon) \tag{29}$$

The value of the minimum oil film thickness defines the regime of lubrication present in the journal–bearing, namely thick-film lubrication, where the journal–bearing surfaces are totally separated by the lubricant, or thin-film lubrication, in which the field of pressure developed produces elastic deformation. The lubrication between two moving surfaces can shift from one of these two regimes to another, depending on the load, velocity, lubricant viscosity and roughness of the surfaces.

For the journal–bearing considered in the present work, the minimum or safe oil film thickness that ensures good operating conditions is of order of 0.003 mm. Figure 8 shows the minimum oil film thickness for the two crank rotations. The value of the minimum fluid film thickness is greater than the safe film thickness, meaning that the hydrodynamic lubrication is effective performed and completely separates the journal and bearing surfaces, avoiding the solid-to-solid contact.

It should be highlighted that the methodology used in this work applies to light and medium loaded bearings, where no deformation of contiguous solid surfaces occurs. For the case of deformation, the elastohydrodynamic regime of lubrication becomes important. The theory of elastohydrodynamics is quite complex and goes beyond the scope of the present work. For details, the interested reader is referred to Reference [**12**].

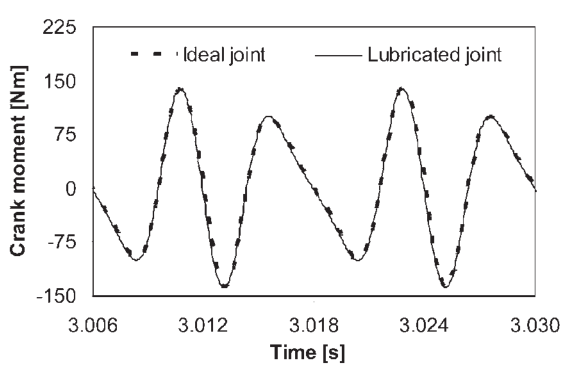

Fig. 6 Reaction or driving crank moment

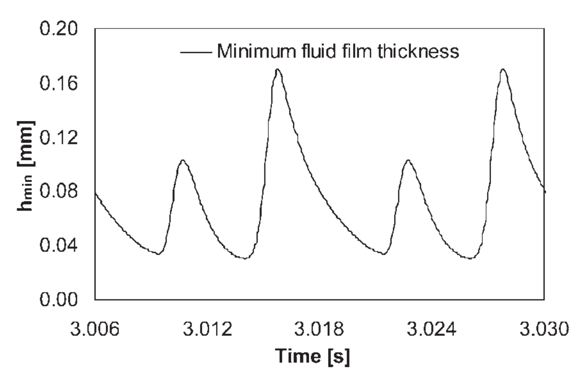

Fig. 8 Minimum oil film thickness in the journal–bearing

6 CONCLUSIONS

A general and comprehensive methodology for modelling lubricated revolute clearance joints in mechanical systems was presented throughout this work. In multibody mechanical systems the external force that acts on the journal–bearing can vary in both magnitude and direction, and often cyclically, which results in a dynamically loaded journal–bearing. Hence, the journal centre describes a trajectory inside the bearing boundaries. The hydrodynamic film thickness is formed simultaneously by squeeze-film and wedge-film actions.

The results for the hydrodynamic lubrication model matched quite well with those obtained with ideal joints since a minimum fluid film thickness was ensured and steady state had been reached, meaning that the use of lubricant at the machine joints is an effective way to ensure better performance. For instance, the crank moment required to maintain constant the crank angular velocity is very smooth, meaning that the global motion of the slider–crank mechanism with a lubricated joint is periodic.

Indeed, the lubricant acts like a nonlinear spring-damper in so far as the lubricated journal–bearing absorbs some of the energy produced by the slider when it accelerates or decelerates, which results in lower reaction moment compared with ideal joints. Indeed, the lubricant introduces effective stiffness and damping to the slider–crank mechanism.

The hydrodynamic model for lubricated revolute joints in multibody systems is numerically efficient and fast because the pressure distribution is not evaluated. Further, the methodology is easy and straightforward to implement in a computational code because resultant forces due to the fluid action are in explicit form.

The model presented throughout this paper can be used to predict the dynamic response of the machines and mechanisms having lubricated revolute clearance joints.

Numerical difficulties can be observed if either the fluid viscosity is very low or the radial clearance of the journal–bearing is too large, which leads to large eccentricities and consequently the system becomes stiff. However, for an appropriate choice of these parameters, the methodology presented in this work is quite useful in the design of mechanical systems with lubricated revolute joints.

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