Modelling Multiple Regimes in the Business Cycle

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Abstract

The interest in business cycle asymmetry has been steadily increasing over the last fifteen years. Most research has focused on the different behaviour of macroeconomic variables during expansions and contractions, which by now is well documented. Recent evidence suggests that such a two-phase characterization of the business cycle might be too restrictive. In particular, it might be worthwhile to decompose the recovery phase in a high-growth phase (immediately following the trough of a cycle) and a subsequent moderate-growth phase. In this paper, the issue of multiple regimes is addressed using Smooth Transition AutoRegressive [STAR] models. A possible limitation of STAR models as they are currently used is that essentially they deal with only two regimes. We propose a generalization of the STAR model such that more than two regimes can be accommodated. It is demonstrated that the class of Multiple Regime STAR [MRSTAR] models can be obtained from the two-regime model in an elegant way. The main properties of the MRSTAR model and several issues which might be relevant for empirical specification are discussed in detail. In particular, a Lagrange Multiplier-type test is derived which can be used to determine the appropriate number of regimes. Application of the new model class to US real GNP and US unemployment rate provides evidence in favor of the existence of multiple business cycle phases.

Keywords: Business cycle asymmetry, Multiple regimes, Smooth transition autoregression, Lagrange Multiplier test.

JEL classification: E32, C22, C12

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1 Introduction

The notion of business cycle asymmetry has been around for quite some time. For example, Keynes (1936, p. 314) already observed that 'the substitution of a downward for an upward tendency often takes place suddenly and violently, whereas there is, as a rule, no such sharp turning point when an upward is substituted for a downward tendency'. Following Burns and Mitchell (1946), conventional wisdom has long held that 'contractions are shorter and more violent than expansions'. Starting with Neftçi (1984), interest in the subject of business cycle asymmetry has been revived and many macroeconomic series (output and (un)employment series in particular) have been examined for asymmetry properties using a variety of different statistical procedures. Neftçi (1984), Falk (1986), Sichel (1989), and Rothman (1991), among many others, test for asymmetry between expansions and contractions by considering the probabilities of transitions from one regime to the other. Various nonlinear time series models have also been employed to render some insight into the differing dynamics over the business cycle. Regime-switching models have been particularly popular in this line of research. Typically, these models consist of a set of linear models of which, at each point in time, only one or a linear combination of the models is active to describe the behaviour of a time series, where the activity depends on the regime at that particular moment.

Within the class of regime-switching models, two main categories can be distinguished, depending on whether the regimes are determined exogenously, by an unobservable state variable, or endogenously, by a directly observable variable. The most prominent member of the first class of models is the Markov-Switching autoregressive model, which has been applied to modelling business cycle asymmetry by Hamilton (1989), Boldin (1996), and Diebold and Rudebusch (1996), among others. From the second class of models, the (Self-Exciting) Threshold AutoRegressive [(SE)TAR] model (see Beaudry and Koop (1993), Tiao and Tsay (1994), Potter (1995), Peel and Speight (1996), and Clements and Krolzig (1996)) and the Smooth Transition AutoRegressive [STAR] model have been most frequently applied (see see Teräsvirta and Anderson (1992), Skalin and Teräsvirta (1996), and Jansen and Oh (1996)).

It is now well understood that recessions are different from booms, and there seem to be possibilities for even further refinement. Ramsey and Rothman (1996) and Sichel (1993) discuss concepts such as 'deepness', 'steepness' and 'sharpness', which all relate to different aspects of asymmetry. A cycle is said to exhibit steepness if the slope of the expansion phase differs from the slope of the contraction phase. Sichel (1993) argues that most research has focused exclusively on the possibility of steepness, neglecting other forms of asymmetry. The evidence presented by Sichel (1993) suggests however that deepness might be a more important characteristic of macroeconomic variables. Deepness occurs when the distance from the mean of the cycle to the peak is not equal to the distance from the mean to the trough. Sharpness focuses on the relative curvature around peaks and troughs. In general, peaks are thought to be more 'round' when compared to troughs, see Emery and Koenig (1992) and McQueen and Thorley (1993) for some evidence in favor of this premise.

Intuitively, if a macroeconomic variable exhibits different types of asymmetry simultaneously, the distinction between expansion and contraction might not be sufficient to characterize the behaviour over the business cycle. Sichel (1994) observes that real GNP tends to grow faster immediately following a trough than in the rest of the expansion phase. Wynne and Balke (1992) and Emery and Koenig (1992) present additional evidence in favor of this 'bounce-back' effect. This suggests the possibility of three business cycle phases - contractions, high-growth recoveries which immediately follow troughs of the cycle and subsequent moderate growth phases.

The nonlinear time series models mentioned above mainly focus on two regimes, i.e., expansions and contractions. The Markov-Switching and SETAR models can be extended to multiple regimes, at least conceptually. For example, Boldin (1996) presents a three-regime Markov Switching model in which the expansion regime is split into separate regimes for the post-trough rapid recovery periods and the moderate growth periods for the remainder of the expansion. In a similar vein, Pesaran and Potter (1997) and Koop *et al.* (1996) use principles of SETAR models to construct a 'floor and ceiling' model which allows for three regimes corresponding to low, normal, and high growth rates of output, respectively. Tiao and Tsay (1994) develop a four-regime SETAR model for US real GNP in which the regimes are labeled worsening/improving recession/expansion, thus allowing for a wide variety in dynamics in different phases of the business cycle. In contrast, extending the number of possible regimes in STAR models does not seem to be straightforward. Therefore, the objective of our paper is to explore how STAR models can be modified to allow for more than two regimes, with the purpose of examining whether a multiple regime STAR model can be used to describe the behavior of post-war US real GNP and US unemployment.

The outline of our paper is as follows. In section 2 we discuss the STAR model and a simple but elegant way to generalize this model to accommodate more than two regimes. In Section 2.2 we give a theoretical account of this Multiple Regime STAR [MRSTAR] model, while in Section 2.3 we focus on a simple example to demonstrate the main features of the MRSTAR model. In Section 3 we discuss some of the issues which are involved in specifying these models. Emphasis in that subsection is put on developing a test statistic which can be used to test a two-regime model against a multiple regime alternative. In Section 4 we discuss previous research on modelling business cycle asymmetry in somewhat more detail and apply the models to characterize the behavior of the growth rate of post-war US real GNP and the US unemployment rate. Finally, Section 5 contains some discussion.

2 Extending the STAR model

In this section we describe an extension of the STAR model to allow for more than two regimes. We start with a brief description of the basic STAR model. For a more elaborate discussion of these models we refer to Granger and Teräsvirta (1993) and Teräsvirta (1994). We next argue that, irrespective of the particular transition function which is used, this basic STAR model essentially allows for only two regimes. To overcome this limitation, the class of Multiple Regime STAR [MRSTAR] models is introduced. The potential usefulness of this class of models is illustrated by a simple example.

2.1 The basic STAR model

Consider the following STAR model for a univariate time series y_t,

$$y_t = \phi_1' y_t^{(p)}(1 - F(\tilde{y}_t^{(p)}; \gamma, \alpha, c)) + \phi_2' y_t^{(p)} F(\tilde{y}_t^{(p)}; \gamma, \alpha, c) + \varepsilon_t, \tag{1}$$

where $y_t^{(p)} = (1, \tilde{y}_t^{(p)})'$, $\tilde{y}_t^{(p)} = (y_{t-1}, \ldots, y_{t-p})'$, $\phi_i = (\phi_{i0}, \phi_{i1}, \ldots, \phi_{ip})'$, $i = 1, 2$, and ε_t is a white noise error process with mean zero and variance σ^2. The so-called transition function $F(y_t^{(p)}; \gamma, \alpha, c)$ is a continuous function, bounded between zero and one. One of the most often applied choices for $F(y_t^{(p)}; \gamma, \alpha, c)$, which is also central in this paper, is the logistic function[1],

$$F(\tilde{y}_t^{(p)}; \gamma, \alpha, c) = (1 + \exp\{-\gamma(\alpha' \tilde{y}_t^{(p)} - c)\})^{-1}, \quad \gamma > 0, \tag{2}$$

where $\alpha = (\alpha_1, \ldots, \alpha_p)'$, while γ and c are scalars. Note that α needs to be normalized in an appropriate way in order to achieve identification of the model, e.g., $\alpha_1 = 1$. The resulting model is called the Logistic STAR [LSTAR] model.

The way the model is written in (1) highlights the basic characteristic of the LSTAR model, which is that at any given point in time, the evolution of y_t is determined by a weighted average of two different linear AutoRegressive [AR] models. The weights assigned to the two models depend on the (recent) history of the time series itself. For small (large) values of $\alpha' \tilde{y}_t^{(p)}$, $F(\tilde{y}_t^{(p)}; \gamma, \alpha, c)$ is approximately equal to zero (one) and, consequently, almost all weight is put on the first (second) model. The parameter γ determines the speed at which these weights change as $\alpha' \tilde{y}_t^{(p)}$ increases; the higher γ, the faster this change is. If $\gamma \to 0$, the weights become constant (and equal to .5) and the model becomes linear, while if $\gamma \to \infty$, the logistic function approaches a Heaviside function, taking the value 0 for $\alpha' \tilde{y}_t^{(p)} < c$ and 1 for $\alpha' \tilde{y}_t^{(p)} > c$. In that case, the LSTAR model reduces to a two-regime SETAR model, see Tong (1990) for an extensive discussion.

[1] Chan and Tong (1986) first proposed the STAR model as a generalization of the two-regime SETAR model, to alleviate the problem of estimating the threshold c. They suggested to use the standard normal cumulative distribution function as transition function. The logistic function has become the standard choice, probably because of the existence of an explicit analytical form, which greatly facilitates estimation of the model.

Teräsvirta (1994) outlines a specification procedure for STAR models. Because this will be part of the specification procedure for multiple regime STAR models to be discussed below, we briefly sketch the different steps in this procedure here. After estimating a suitable linear AR model for y_t, linearity is tested against the alternative of a two-regime STAR model (1) using the tests developed by Luukkonen *et al.* (1988). The testing problem suffers from what has become known as the 'Davies-problem', i.e., the model is not identified under the null hypothesis of linearity, which can be formulated as $H_0 : \gamma = 0$. This problem of nuisance parameters which are not identified under the null hypothesis was first considered in some depth by Davies (1977,1987) and occurs in many testing problems, see Hansen (1996) for a recent account. The tests of Luukkonen *et al.* (1988) are based on replacing the transition function in (1) by a suitable approximation which leads to a reparameterized model in which higher order powers of the regressors $y_{t-j},\ j = 1, \ldots, p$, appear and the identification problem is no longer present. Linearity is tested by testing the joint significance of the coefficients corresponding to these auxiliary regressors. For details we refer to Luukkonen *et al.* (1988).

It is convenient to carry out the linearity test for fixed α, i.e., with the transition variable(s) specified in advance. This allows to select the most appropriate transition variable(s) prior to estimation of the STAR model. Concerning α, it is usually assumed that only a single lagged value y_{t-d} acts as transition variable, i.e., $\alpha = (0, \ldots, 0, 1, 0, \ldots)'$, where the 1 is the d-th element of α. An alternative which might be of interest is when a lagged first difference Δy_{t-d} is taken to be the threshold variable, i.e., $\alpha = (0, \ldots, 0, 1, -1, 0, \ldots, 0)'$. Following Enders and Granger (1996), the resulting model might be called a Momentum STAR [MSTAR] model, as the regime is determined by the direction in which the time series is moving, i.e., by its momentum. The choice of α for which linearity is rejected most convincingly is considered to render the most appropriate one. If linearity is rejected for certain α, the remaining parameters in the STAR model can be estimated by nonlinear least squares[2] [NLS], see Teräsvirta (1994) for a discussion of the issues involved. The final stage of building a STAR model is to subject the estimated model to some diagnostic tests to check whether it adequately captures the main features of the data. Eitrheim and Teräsvirta (1996) develop appropriate test statistics for serial correlation, constancy of parameters and remaining nonlinearity.

[2]To be precise, the specification procedure of Teräsvirta (1994) first proceeds by applying a sequence of nested tests to decide whether a logistic or exponential type transition funtion (given in (3) below) is most appropriate. We omit details here because we focus on models with logistic transition functions to introduce the multiple regime models. The same principles discussed below apply to models with different transition functions as well.

2.2 A multiple regime STAR model

The LSTAR model seems particularly well suited to describe asymmetry of the type that is frequently encountered in macroeconomic time series. For example, the model has been successfully applied by Teräsvirta and Anderson (1992) and Teräsvirta *et al.* (1994) to characterize the different dynamics of industrial production indices in a number of OECD countries during expansions and recessions. As argued in the introduction, sometimes more than two regimes might be required to adequately decribe the behaviour of a particular time series.

The notation in (1) shows that the set of linear AR models of which the STAR model is composed contains only two elements. Hence, it is immediately clear that the STAR model cannot accommodate more than two regimes, irrespective of what form the transition function takes. It has been suggested that a three regime model is obtained by using the exponential function,

$$F(\tilde{y}_t^{(p)}, \gamma, \alpha, c) = 1 - \exp(-\gamma\{\alpha' \tilde{y}_t^{(p)} - c\}^2), \quad \gamma > 0, \tag{3}$$

as transition function in (1). According to Teräsvirta and Anderson (1992), this exponential transition function gives rise to a model which allows expansions and contractions to have different dynamics than the 'middle ground', similar to the 'floor and ceiling' model of Pesaran and Potter (1997). However, it is obvious that the models in the two outer regimes, associated with very small and large values of $\alpha' \tilde{y}_t^{(p)}$ (and, hence, corresponding with the expansions and contractions), are restricted to be the same, so that effectively there still are only two distinct regimes. Furthermore, this Exponential STAR [ESTAR] model does not nest the SETAR model as a special case, because for both $\gamma \to 0$ and $\gamma \to \infty$, the model becomes linear. The latter can be remedied by using the 'quadratic logistic' function

$$F(\tilde{y}_t^{(p)}; \gamma, \alpha, c_1, c_2) = (1 + \exp\{-\gamma(\alpha' \tilde{y}_t^{(p)} - c_1)(\alpha' \tilde{y}_t^{(p)} - c_2)\})^{-1}, \ \ \gamma > 0\ , \tag{4}$$

as proposed by Jansen and Teräsvirta (1996). In this case, if $\gamma \to 0$, the model becomes linear, while if $\gamma \to \infty$, the function $F(\tilde{y}_t^{(p)}; \gamma, \alpha, c_1, c_2)$ is equal to 1 for $\alpha' \tilde{y}_t^{(p)} < c_1$ and $\alpha' \tilde{y}_t^{(p)} > c_2$, and equal to 0 in between. Hence, the STAR model with this particular transition function nests a three regime SETAR model, although the models in the outer regimes are still restricted to be the same.

In this paper we propose an alternative way to extend the basic STAR model to allow for more than two, genuinely different, regimes. Building upon the notation used in (1), we suggest to 'encapsulate' two different LSTAR models as follows,

$$\begin{aligned} y_t \;\; = \;\; & [\phi_1' y_t^{(p)}(1 - F_1(\tilde{y}_t^{(p)}; \gamma_1, \alpha_1, c_1)) + \phi_2' y_t^{(p)} F_1(\tilde{y}_t^{(p)}; \gamma_1, \alpha_1, c_1)][1 - F_2(\tilde{y}_t^{(p)}; \gamma_2, \alpha_2, c_2)] \\ & [\phi_3' y_t^{(p)}(1 - F_1(\tilde{y}_t^{(p)}; \gamma_1, \alpha_1, c_1)) + \phi_4' y_t^{(p)} F_1(\tilde{y}_t^{(p)}; \gamma_1, \alpha_1, c_1)] F_2(\tilde{y}_t^{(p)}; \gamma_2, \alpha_2, c_2) + \varepsilon_t, \end{aligned} \tag{5}$$

where both transition functions F_1 and F_2 are taken to be logistic functions as in (2). As both functions can vary between zero and one, (5) defines a model with four distinct regimes, each corresponding to a particular combination of extreme values of the transition functions. We call the model given in (5) a Multiple Regime STAR [MRSTAR] model. The MRSTAR model considered here allows for a maximum of four different regimes, but it will be obvious that, in the notation of (5), extension to 2^k regimes with $k > 2$ is straightforward, at least conceptually. Needless to say a model with three regimes can also be obtained from (5), by imposing appropriate restrictions on the parameters of the autoregressive models which prevail in the different regimes. If in fact $\alpha_1 = \alpha_2 \equiv \alpha$, i.e., a single linear combination of the past of y_t governs the transitions between all regimes, it will be intuitively clear that it is not sensible to allow for four different regimes. For example, if $c_1 < c_2$, F_1 changes from zero to one prior to F_2 for increasing values of $\alpha' \tilde{y}_t^{(p)}$ and, consequently, the product $(1 - F_1)F_2$ will be equal to zero almost everywhere, especially if γ_1 and γ_2 are large. Hence, it makes sense to exclude the model corresponding to this particular regime by imposing the restriction $\phi_3 = 0$.

Note that the MRSTAR model nests several other nonlinear time series models. For example, an artificial neural network [ANN] model, see Kuan and White (1994), is obtained by imposing the restrictions $\phi_{ij} = 0,\ i = 1, \ldots, 4,\ j = 1, \ldots, p$ and $\phi_{40} = \phi_{20} + \phi_{30} - \phi_{10}$. The last restriction ensures that the interaction term $\phi_{40}^* F_1 F_2$, where $\phi_{40}^* = \phi_{10} - \phi_{20} - \phi_{30} + \phi_{40}$ drops out of the model, which now can be rewritten as

$$y_t = \phi_{10}^* + \phi_{20}^* F_1(\tilde{y}_t^{(p)}; \gamma_1, \alpha_1, c_1) + \phi_{30}^* F_2(\tilde{y}_t^{(p)}; \gamma_2, \alpha_2, c_2) + \varepsilon_t. \tag{6}$$

where $\phi_{10}^* = \phi_1$, $\phi_{20}^* = \phi_{20} - \phi_{10}$ and $\phi_{30}^* = \phi_{30} - \phi_{10}$.

Additionally, the MRSTAR model (5) might be extended to a 'semi-multivariate' model by including exogenous variables as regressors or transition variables. Granger and Teräsvirta (1993) discuss incorporating exogenous variables x_{it} in the STAR model (1) to obtain the Smooth Transition Regression [STR] model, see also Teräsvirta (1996) for a more recent survey. Likewise, the MRSTAR model can be extended to a Multiple Regime STR [MRSTR] model by defining $z_t = (1, \tilde{z}_t')'$, $\tilde{z}_t = (y_{t-1}, \ldots, y_{t-p}, x_{1t}, \ldots, x_{kt})'$, and substituting z_t for $y_t^{(p)}$ in (5), i.e.,

$$\begin{aligned} y_t \ = \ & [\phi_1' z_t(1 - F_1(\tilde{z}_t; \gamma_1, \alpha_1, c_1)) + \phi_2' z_t F_1(\tilde{z}_t; \gamma_1, \alpha_1, c_1)][1 - F_2(\tilde{z}_t; \gamma_2, \alpha_2, c_2)] \\ & [\phi_3' z_t(1 - F_1(\tilde{z}_t; \gamma_1, \alpha_1, c_1)) + \phi_4' z_t F_1(\tilde{z}_t; \gamma_1, \alpha_1, c_1)] F_2(\tilde{z}_t; \gamma_2, \alpha_2, c_2) + \varepsilon_t, \end{aligned} \tag{7}$$

where now the vectors $\phi_i,\ i = 1, \ldots$ and $\alpha_i,\ i = 1, 2$ are of length $m+1$ and m, respectively, with $m = p + k$. In particular, Lin and Teräsvirta (1994) argue that polynomials of time are allowed as transition variables in STAR models; even though these are nonstationary variables, no problems occur because the transition function is bounded between zero and one. Lütkepohl *et al.* (1995) and Wolters *et al.* (1996) apply this idea to model time-varying parameters in German money demand. In the MRSTR model time trends might be used as transition variables as well. This opens the interesting possibility to model nonlinearity and time-varying parameters simultaneously. A possible application in business cycle research might be to examine whether the properties of expansions and contractions are time-invariant. For example, Lin and Teräsvirta (1994) demonstrate that the properties of the index of industrial production in the Netherlands have changed after the first oil crises in 1975. Sichel (1991) claims that expansions have become longer after World War II and have started to exhibit duration dependence, while recessions have become shorter and duration dependence has disappeared. This issue is beyond the scope of this paper and is left for further research.

The MRST(A)R model also nests the class of Nested TAR [NeTAR] models recently proposed by Astatkie *et al.* (1997) as an extension of conventional TAR models to allow for multiple sources of nonlinearity. A NeTAR model is obtained from (7) (or (5)) if the parameters γ_1 and γ_2 both tend to infinity (or, equivalently, the logistic functions are replaced by Heaviside functions), such that the different regimes are separated by sharply determined borders. Astatkie *et al.* (1997) describe a sequential specification procedure for NeTAR models.

2.3 A simple example

In this subsection we focus on a simple example of a four regime MRSTAR model to highlight some features of the model. We set $p = 2$, require all second order AR coefficients as well as the intercepts to be equal to zero, and set $\alpha_1 = (1, -1)'$, $\alpha_2 = (0, 1)'$. The resulting model then is written as

$$\begin{aligned} y_t \;=\; & [\phi_1 y_{t-1}(1 - F_1(\Delta y_{t-1}; \gamma_1, c_1)) + \phi_2 y_{t-1} F_1(\Delta y_{t-1}; \gamma_1, c_1)][1 - F_2(y_{t-2}; \gamma_2, c_2)] \\ & [\phi_3 y_{t-1}(1 - F_1(\Delta y_{t-1}; \gamma_1, c_1)) + \phi_4 y_{t-1} F_1(\Delta y_{t-1}; \gamma_1, , c_1)] F_2(y_{t-2}; \gamma_2, c_2) + \varepsilon_t. \qquad (8) \end{aligned}$$

For each combination of the transition variables $(\Delta y_{t-1}, y_{t-2})$ the resulting model is a weighted average of the four AR(1) models associated with the four extreme regimes. Figure 1 shows the weights given to each of these four models in the (y_{t-1}, y_{t-2}) plane, with $\gamma_1 = \gamma_2 = 2.5$ and $c_1 = c_2 = 0$. For $(\Delta y_{t-1}, y_{t-2}) = (0, 0)$ or equivalently $(y_{t-1}, y_{t-2}) = (0, 0)$, all models are given equal weight. Along the lines $y_{t-1} = y_{t-2}$ and $y_{t-2} = 0$, which might be interpreted as representing the borders between the different regimes, the models receive equal weight pairwise. For example, along $y_{t-1} = y_{t-2}$, the models in the first and third regime receive equal weight, while the same holds for the models in the second and fourth regime (where the subscript of the autoregressive parameters is used to identify the regime number). Moving into a particular regime increases the weight of the corresponding model.

- insert Figure 1 around here -

To illustrate the possible dynamics which can be generated by the MRSTAR model, Figure 2 shows some time series generated by the sample model (8). Two hundred pseudo-random numbers are drawn from the standard normal distribution to obtain a sequence of errors ε_t, while the necessary initial values y_{-1} and y_0 are set equal to zero. The thresholds c_1, c_2 and the parameters γ_1 and γ_2 are set equal to the values given above. In the upper panel of Figure 2, the autoregressive parameters are set as follows; $\phi_1 = \phi_2 = 0.3$ and $\phi_3 = \phi_4 = 0.9$. Hence, the model reduces to a basic LSTAR model (1) with y_{t-2} as transition variable. In all panels of Figure 2, a realization of an AR(1) model $y_t = \phi y_{t-1} + \varepsilon_t$ with autoregressive parameter $\phi = 0.6$, using the same errors ε_t, is also plotted for comparison. Although the time series generated by the LSTAR model has the same 'average' autoregressive parameter as the linear AR(1) model, the behavior is markedly different: for positive values of y_{t-2}, the tendency of the series to return to its attractor (which is equal to zero) is much smaller than for negative values of the transition variable. The middle panel of Figure 2 shows the AR(1) series together with a realization of the MRSTAR model with $\phi_1 = \phi_3 = 0.3$ and $\phi_2 = \phi_4 = 0.9$. The resulting model is a momentum STAR [MSTAR] model, as the autoregressive parameters only depend on the direction in which the series is moving. In our example, the memory of the series is longer for upward than for downward movements. The main difference between the AR and MSTAR models occurs in the peaks, the upward (downward) peaks being more (less) pronounced in the nonlinear model. Finally, the lower panel of Figure 2 shows the AR(1) series together with a realization of the MRSTAR model (8) with the autoregressive parameters taken to be the averages of the parameters in the LSTAR and MSTAR models, i.e., ϕ_1 through ϕ_4 are set equal to 0.3, 0.6, 0.6, and 0.9, respectively. Obviously, the resulting time series combines the properties of the LSTAR and MSTAR models: persistence is strongest for positive and increasing values, intermediate for positive and decreasing values, and negative and increasing values, and smallest for negative and decreasing values of the time series.

- insert Figure 2 around here -

3 Specification of MRSTAR models

We suggest a 'specific-to-general' approach to specify MRSTAR models, i.e., to build up the number of regimes by iterating between testing for the desirability of additional regimes and estimating multiple regime models[3]. The reason for preferring this approach rather than for example applying model selection criteria is that the MRSTAR model given in (5) is not identified, as the parameters in the different transition functions can be interchanged

[3] See Teräsvirta and Lin (1993) for a similar approach to determine the appropriate number of hidden units in ANN models.

without altering the model. The use of model selection criteria requires the estimation of all candidate models. If models with too many regimes are considered, estimation routines may fail.

Hence we suggest that specification begins with specifying and estimating a basic LSTAR model (1), using the specification procedure of Teräsvirta (1994) as discussed in Section 2.2. The two-regime model should be tested against the alternative of a general MRSTAR as given in (5). The principle of approximating the transition function as applied by Luukkonen *et al.* (1988) to develop LM-type tests against STAR nonlinearity is used below to obtain a test against the MRSTAR alternative (5)[4]. For this purpose it is convenient to rewrite the model as follows,

$$\begin{aligned} y_t \;=\;& \phi_1^{*\prime} y_t^{(p)} + \phi_2^{*\prime} y_t^{(p)} F_1(\tilde{y}_t^{(p)};\gamma_1,\alpha_1,c_1) + \phi_3^{*\prime} y_t^{(p)} F_2(\tilde{y}_t^{(p)};\gamma_2,\alpha_2,c_2) + \\ & \phi_4^{*\prime} y_t^{(p)} F_1(\tilde{y}_t^{(p)};\gamma_1,\alpha_1,c_1) F_2(\tilde{y}_t^{(p)};\gamma_2,\alpha_2,c_2) + \varepsilon_t, \end{aligned} \tag{9}$$

where $\phi_1^* = \phi_1$, $\phi_2^* = \phi_2 - \phi_1$, $\phi_3^* = \phi_3 - \phi_1$ and $\phi_4^* = \phi_1 + \phi_4 - \phi_2 - \phi_3$. The two-regime model which has been estimated is assumed to have $F_1(\cdot)$ as transition function. Hence, we desire to test if the addition of the regimes determined by $F_2(\cdot)$ is appropriate. Subtracting 1/2 from the logistic function F_2 does not alter the model, while it allows to express the null hypothesis to be tested as $H_0 : \gamma_2 = 0$. We assume that the test is to be carried out for fixed α_2 (while α_1 is also assumed to have been fixed at an earlier stage). Although it is fairly straightforward to derive a test statistic for general α_2 assuming this parameter vector fixed facilitates the notation involved considerably. Moreover, this seems to be the most relevant case in a practical model building situation, as one might have an intuitive idea of the appropriate transition variables, or be interested in getting such an idea. As the model is not identified under the null hypothesis, a test statistic cannot be derived directly. We proceed by replacing the transition function $F_2(\tilde{y}_t^{(p)};\gamma_2,\alpha_2,c_2)$ in (9) by a third-order Taylor expansion around zero[5]. After rearranging terms, the model becomes

$$\begin{aligned} y_t \;=\;& \theta_1' y_t^{(p)} + \theta_2' y_t^{(p)} F_1(\tilde{y}_t^{(p)};\gamma_1,\alpha_1,c_1) + \\ & \beta_1' \tilde{y}_t^{(p)} (\alpha_2' \tilde{y}_t^{(p)}) + \beta_2' \tilde{y}_t^{(p)} F_1(\tilde{y}_t^{(p)};\gamma_1,\alpha_1,c_1)(\alpha_2' \tilde{y}_t^{(p)}) \\ & \beta_3' \tilde{y}_t^{(p)} (\alpha_2' \tilde{y}_t^{(p)})^2 + \beta_4' \tilde{y}_t^{(p)} F_1(\tilde{y}_t^{(p)};\gamma_1,\alpha_1,c_1)(\alpha_2' \tilde{y}_t^{(p)})^2 \\ & \beta_5' \tilde{y}_t^{(p)} (\alpha_2' \tilde{y}_t^{(p)})^3 + \beta_6' \tilde{y}_t^{(p)} F_1(\tilde{y}_t^{(p)};\gamma_1,\alpha_1,c_1)(\alpha_2' \tilde{y}_t^{(p)})^3 + e_t. \end{aligned} \tag{10}$$

[4]The test of Eitrheim and Teräsvirta (1996) for remaining nonlinearity can be regarded as a test against a restricted version of the MRSTAR model (5). To be precise, Eitrheim and Teräsvirta (1996) derive their test from an additive STAR model, which is obtained from (1) by adding an extra nonlinear term $\phi_3' y_t^{(p)} F_2(\tilde{y}_t^{(p)};\gamma_2,\alpha_2,c_2)$. Alternatively, this model can be obtained from (5) by imposing the restriction $\phi_1 + \phi_4 - \phi_2 - \phi_3 = 0$. Here we are primarily interested in specifying a multiple regime model, and therefore we do not want to impose such restrictions a priori. Note however that our test can also be interpreted and used as a diagnostic tool to evaluate estimated two-regime STAR models.

[5]Because we restrict attention to logistic transition functions, a first-order Taylor expansion would suffice. However, there might be certain alternatives against which the resulting test statistic has very little or no power, see Luukkonen *et al.* (1988) for details.

The null hypothesis can now be reformulated as $H_0^*: \beta_i = 0,\ i = 1, \ldots, 6$. Note that under the null hypothesis $\theta_1 = \phi_1^* = \phi_1$ $\theta_2 = \phi_2^* = \phi_2 - \phi_1$ and $e_t = \varepsilon_t$. Assuming the errors to be normally distributed, it follows that the conditional log-likelihood for observation t is given by

$$l_t = c - (1/2)\ln\sigma^2 - e_t^2/2\sigma^2, \tag{11}$$

where c is an irrelevant constant. As the information matrix is block diagonal, the error variance σ^2 can be assumed fixed. The remaining partial derivates evaluated under the null hypothesis are given by

$$\left.\frac{\partial l_t}{\partial \theta_1}\right|_{H_0} = (1/\sigma^2)\hat{e}_t y_t^{(p)}, \tag{12}$$

$$\left.\frac{\partial l_t}{\partial \theta_2}\right|_{H_0} = (1/\sigma^2)\hat{e}_t y_t^{(p)} F_1(\tilde{y}_t^{(p)}; \hat{\gamma}_1, \alpha_1, \hat{c}_1), \tag{13}$$

$$\left.\frac{\partial l_t}{\partial \gamma_1}\right|_{H_0} = (1/\sigma^2)\hat{e}_t \hat{\theta}_2' y_t^{(p)} \frac{\partial F_1(\tilde{y}_t^{(p)}; \hat{\gamma}_1, \alpha_1, \hat{c}_1)}{\partial \gamma_1}, \tag{14}$$

$$\left.\frac{\partial l_t}{\partial c_1}\right|_{H_0} = (1/\sigma^2)\hat{e}_t \hat{\theta}_2' y_t^{(p)} \frac{\partial F_1(\tilde{y}_t^{(p)}; \hat{\gamma}_1, \alpha_1, \hat{c}_1)}{\partial c_1}, \tag{15}$$

where

$$\frac{\partial F_1(\tilde{y}_t^{(p)}; \hat{\gamma}_1, \alpha_1, \hat{c}_1)}{\partial \gamma_1} = (1 + \exp\{-\hat{\gamma}_1(\alpha_1' \tilde{y}_t^{(p)} - \hat{c}_1)\})^{-2} \exp\{-\hat{\gamma}_1(\alpha_1' \tilde{y}_t^{(p)} - \hat{c}_1)\}(\alpha_1' \tilde{y}_t^{(p)} - \hat{c}_1) \tag{16}$$

$$\frac{\partial F_1(\tilde{y}_t^{(p)}; \hat{\gamma}_1, \alpha_1, \hat{c}_1)}{\partial c_1} = \hat{\gamma}_1(1 + \exp\{-\hat{\gamma}_1(\alpha_1' \tilde{y}_t^{(p)} - \hat{c}_1)\})^{-2} \exp\{-\hat{\gamma}_1(\alpha_1' \tilde{y}_t^{(p)} - \hat{c}_1)\}. \tag{17}$$

The partial derivatives (16) and (17) will be denoted as $\hat{F}_{\gamma_1}(t)$ and $\hat{F}_{c_1}(t)$, respectively, while we also use the short-hand notation $\hat{F}_1(t)$ to denote $F_1(\tilde{y}_t^{(p)}; \hat{\gamma}_1, \alpha_1, \hat{c}_1)$.

The above suggests that an LM-type test statistic to test H_0^* can be computed in a few steps as follows:

1. Estimate the two regime LSTAR model (1) with (2) by nonlinear least squares and compute the sum of squared residuals under the null hypothesis, SSR_0.

2. Regress the residuals $\hat{e}_t$ on $\{y_t^{(p)}, y_t^{(p)}\hat{F}_1(t), \hat{\theta}_2' y_t^{(p)}\hat{F}_{\gamma_1}(t), \hat{\theta}_2' y_t^{(p)}\hat{F}_{c_1}(t)\}$ and the auxiliary regressors $\{y_t^{(p)}(\alpha_2'\tilde{y}_t^{(p)})^i, y_t^{(p)}\hat{F}_1(t)(\alpha_2'\tilde{y}_t^{(p)})^i,\ i = 1, 2, 3\}$ and compute the sum of squared residuals under the alternative, SSR_1.

3. Compute the LM-type test statistic as

$$LM_{MR} = \frac{SSR_0 - SSR_1/6p}{SSR_1/(T - (2p+2) - 6p)}. \tag{18}$$

In the second step, the estimates of the autoregressive parameters in the LSTAR model are used to obtain an estimate of θ_2, i.e., $\hat{\theta}_2 = \hat{\phi}_2 - \hat{\phi}_1$, which is consistent under the null hypothesis. Under the null hypothesis, the statistic LM_{MR} is F distributed with $6p$ and

$T-(2p+2)-6p$ degrees of freedom. As usual, the F version of the test statistic is preferable to the χ^2 variant in small samples because its size and power properties are better. The remarks made by Eitrheim and Teräsvirta (1996) concerning potential numerical problems are relevant for our test as well. If $\hat{\gamma}_1$ is large, such that the transition between the two regimes in the model under the null hypothesis is fast, the partial derivatives of the transition function F_1 with respect to γ_1 and c_1, as given in (16) and (17), approach zero functions (except for $F_{c_1}(t)$ at the point $\alpha_1'\tilde{y}_t^{(p)}$). Hence, the moment matrix of the regressors in the auxiliary regression becomes near-singular. However, as the terms in the auxiliary regression involving these partial derivatives are likely to be very small for all $t = 1, \ldots, T$, they contain very little information. It is therefore suggested to simply omit these terms under such circumstances, which will not harm the test statistic.

If the LM-type test (18) rejects the two-regime model in favor of the four-regime alternative, one might proceed with estimation of the alternative model. Estimation of the general MRSTAR model as given in (5) might pose a problem because the model is not identified. However, if α_1 and α_2 are fixed, estimation is fairly straightforward and can be done by nonlinear least squares. Once the general model has been estimated, restrictions on the autoregressive parameters, to test for example equality of models in two different regimes, can be tested using likelihood ratio tests. Diagnostic tests for serial correlation, constancy of parameters and remaining nonlinearity can be developed along the same lines as in Eitrheim and Teräsvirta (1996).

4 Multiple regimes in the business cycle?

Business cycle asymmetry has been investigated mainly by examining output series, such as gross national or domestic product and industrial production, and (un)employment series. We follow this practice here and explore whether multiple regimes in the business cycle can be identified by applying MRSTAR models to US real GNP in Section 4.1 and US unemployment in Section 4.2.

4.1 An MRSTAR model for US GNP

Tests for asymmetry in US real GNP have provided mixed results. In particular, the evidence obtained from nonparametric procedures has not been very compelling. For example, Falk (1986) cannot reject symmetry when examining US real GNP for steepness, see also DeLong and Summers (1986) and Sichel (1993). Similarly, Brock and Sayers (1988) only marginally reject linearity, while Sichel (1993) finds only moderate evidence for deepness. An exception to the rule is Brunner (1992), who obtains fairly strong indications for asymmetry in GNP, which might be associated with an increase in variance during contractions. This is confirmed by Emery and Koenig (1992) who suggest that the variance of leading and coincident indexes increases as the contraction proceeds.

The application of parametric nonlinear time series models has been more successful. Hamilton (1989) and Durland and McCurdy (1994) for example find that a two-state Markov Switching model for the growth rate of post-war quarterly US real GNP outperforms linear models. Boldin (1996) examines the stability of this model and demonstrates that the model is not robust to extension of the sample period. Tiao and Tsay (1994), Potter (1995) and Clements and Krolzig (1996) all estimate a two-regime SETAR model consisting of AR(2) models (although Potter (1995) adds an additional fifth lag). The growth rate two periods lagged is used as the transition variable, while the threshold is either fixed at zero (Potter (1995)) or estimated to be equal to or close to zero (Tiao and Tsay (1994), Clements and Krolzig (1996)). Hence, a distinction is made between periods of positive and negative growth. A common feature of all these estimated models is that the dynamics in recessions are very different from those during expansions. In particular, the SETAR models of Tiao and Tsay (1994), Potter (1995) and Clements and Krolzig (1996), which are estimated on data from 1948 until 1990, all contain a large negative coefficient on the second lag in the lower regime, suggesting that US GNP moves quickly out of recessions. Notably, Clements and Krolzig (1996) find much less evidence of this property when they re-estimate their model on a recent vintage of data, ranging from 1960 until 1996. Beaudry and Koop (1993) estimate a two-regime TAR model in which the 'current depth of recession', which measures deviations from past highs in the level of real GNP, is the threshold variable. This variable is discussed in more detail below.

Whereas most attention focuses on the distinction between contractions and expansions, some indications for the existence of multiple regimes has been obtained as well. For example, Sichel (1994) demonstrates that growth in real GDP is larger immediately following a business cycle trough than in later parts of the expansion, suggesting that the business cycle consists of three distinct phases: contractions, high-growth recoveries, and moderate-growth expansions. Wynne and Balke (1992) and Balke and Wynne (1996) also document this 'bounce-back' effect in industrial production. Furthermore, they examine the relationship between growth in the first twelve months following a trough and the decline of the preceding contraction and show that in general deep recessions are followed by strong recoveries. Emery and Koenig (1992) also find that the mean growth rate in leading and coincident indexes is larger (in absolute value) in early (late) stages of the expansion (contraction). The three-regime Markov Switching model estimated by Boldin (1996), the 'floor-and-ceiling' model of Pesaran and Potter (1997), and the four-regime SETAR model of Tiao and Tsay (1994) explicitly model the existence of a strong-recovery regime as these models include a regime in which output is growing fast (following a recession).

Compared to the previous studies mentioned above, we use a relatively long span of data, which ranges from 1947:I to 1995:II. The data, which are at 1987 prices, are seasonally adjusted and are taken from the Citibase database. The growth rate y_t is graphed in the upper panel of Figure 3. The solid circles indicate NBER-dated peaks and troughs. The lower graph of this Figure shows the mean growth rates during contractions and different phases of expansions. It is seen that in the first four quarters following a trough, growth is considerably higher than during the rest of this expansion, confirming the observation of Sichel (1994).

- insert Figure 3 around here -

Following the approaches of previous authors, we use an AR(2) model as basis for our model building exercise. The estimated model over the period 1947:IV-1995:II is

$$\underset{}{y_t =} \underset{(0.091)}{0.430} + \underset{(0.073)}{0.345}\, y_{t-1} + \underset{(0.073)}{0.095}\, y_{t-2} + \varepsilon_t, \tag{19}$$

$\hat{\sigma}_\varepsilon = 0.917$, SK = 0.01(0.48), EK = 1.40(0.00), JB = 15.58(0.00), ARCH(1) = 3.03(0.08), ARCH(4) = 9.27(0.06), LB(8) = 5.05(0.41), LB(12) = 14.00(0.12), AIC = −0.142, BIC = −0.091,

where standard errors are given in parentheses below the parameter estimates, $\hat{\sigma}_\varepsilon$ is the residual standard deviation, SK is skewness, EK excess kurtosis, JB the Jarque-Bera test of normality of the residuals, ARCH is the LM test of no AutoRegressive Conditional Heteroscedasticity [ARCH], LB is the Ljung-Box test of no autocorrelation, and AIC and BIC are the Akaike and Schwarz order selection criteria, respectively. The figures in parentheses following the test statistics are p-values.

Normality of the residuals is rejected due to the considerable excess kurtosis. Closer inspection of the residuals reveals that this may be caused by large residuals in the first quarter of 1950 and the second quarter of 1980. These observations may also cause the ARCH tests to reject homoscedasticity. On the other hand, the LM test for ARCH is known to have power against alternatives other than ARCH as well, and, hence, it may also be that the significant values of this test statistic are caused by neglected nonlinearity.

This final conjecture is investigated further by applying the LM-type tests of Luukkonen *et al.* (1988) to test for the possibility of STAR-type nonlinearity. We only report results of their test S_2 which is obtained by replacing the transition function in (1) by a third-order Taylor approximation (similar as in going from (9) to (10)) as well as the 'economy'-version S_3, which is obtained from S_2 by omitting redundant terms and which therefore might have better power properties. Apart from lagged growth rates and changes therein, we also consider a measure of the current depth of recession [CDR] as possible transition variable, following Beaudry and Koop (1993). We define CDR_t as

$$CDR_t = \max_{j\geq 1}\{x_{t-j}\} - x_t, \tag{20}$$

with x_t the log of US real GNP. Note that this definition differs slightly from the one of Beaudry and Koop (1993), who take the maximum of past and *current* GNP. Hence, their

CDR measure is equal to zero if real GNP is at an all time high, and greater than zero otherwise. Since using such a truncated variable as transition variable in STAR models is not very convenient, we only consider the maximum up to time t[6].

If α is left unspecified, the test rejects the null hypothesis of linearity quite convincingly; the p-value of the S_2 and S_3 tests are equal to 0.029 and 0.057, respectively. However, if α is fixed in order to get an impression of the most appropriate transition variable(s), the evidence for nonlinearity, in particular from the S_2 test, disappears somewhat[7]. As shown in Table 1, the p-values of the tests seem to suggest that y_{t-2}, Δy_{t-1}, Δy_{t-2}, CDR_{t-1}, and CDR_{t-2} might be considered as transition variable.

- insert Table 1 around here -

We decide to estimate an LSTAR model with CDR_{t-2} as transition variable, because the p-value of the S_3 test is lowest for this variable. The parameters in this LSTAR model are estimated as

$$\begin{aligned} y_t &= [\underset{(0.138)}{0.160} + \underset{(0.090)}{0.346}\, y_{t-1} + \underset{(0.108)}{0.282}\, y_{t-2}] \times [1 - F(CDR_{t-2})] \\ &\quad [\underset{(0.163)}{0.665} + \underset{(0.121)}{0.308}\, y_{t-1} + \underset{(0.148)}{0.048}\, y_{t-2}] \times F(CDR_{t-2}) + \varepsilon_t \qquad (21) \\ F(CDR_{t-2}) &= (1 + \exp[-\underset{(-)}{200.0}(CDR_{t-2} - \underset{(0.135)}{0.281})/\sigma_{CDR_{t-2}}])^{-1} \qquad (22) \end{aligned}$$

$\hat{\sigma}_\varepsilon = 0.899$, SK $= -0.17(0.16)$, EK $= 1.19(0.00)$, JB $= 12.21(0.00)$, ARCH(1) $= 2.74(0.09)$, ARCH(4) $= 7.09(0.13)$, $LM_{SI}(4) = 1.39(0.24)$, $LM_{SI}(8) = 1.48(0.17)$, $LM_{C1} = 1.12(0.35)$, $LM_{C2} = 1.01(0.44)$, $LM_{C3} = 0.87(0.62)$, AIC $= -0.129$, BIC $= 0.008$,

where $LM_{SI}(q)$ denotes the LM-type test for q-th order serial correlation in the residuals and LM_{Ci}, $i = 1, 2, 3$ denote LM-type tests for parameter constancy. Both sets of diagnostic checks are developed in Eitrheim and Teräsvirta (1996), to which we refer for details.

The exponent in the transition function is scaled by the standard deviation of the transition variable in order to make γ scale-free. The sum of the autoregressive coefficients is considerably larger in the regime where $F(CDR_{t-2})$ is equal to zero, which corresponds to expansions. This confirms the findings of Beaudry and Koop (1993) and Potter (1995), among others, that contractions are less persistent than expansions. Also note the large

[6] Note that CDR_t resembles the growth rate y_t quite closely. Given that real GNP is upward trending, $\max_{j\geq 1} x_{t-j}$ will be equal to x_{t-1} most of the time. In that case CDR_t equals $-y_t$. To be more precise, it is straightforward to show that $CDR_t = \max(CDR_{t-1}, 0) - y_t$. Hence, during expansions (i.e., when $CDR_{t-1} > 0$) CDR_t and y_t coincide, while during contractions they might differ. The correlation between CDR_t and y_t equals -0.8, which confirms their similarity.

[7] Jansen and Oh (1996) also report that tests for STAR-type nonlinearity do not reject the null hypothesis of linearity. Similarly, Hansen (1996) shows that tests for threshold-type nonlinearity do not provide very convincing evidence in favor of a threshold model.

constant in the upper regime, which might be taken as an additional indication of a quick recovery following contractions, cf. Sichel (1994) and Wynne and Balke (1992).

Apart from the diagnostic checks reported below the LSTAR model (21), we also applied the LM-type test against the MRSTAR alternative, as derived in Section 2.3, as well as the LM-type tests for remaining nonlinearity of Eitrheim and Teräsvirta (1996). Table 2 shows the p-values of the different tests for various choices of transition variables in the second transition function. The same Table also reports on results of the same tests when the additional transition function is replaced by only a first-order Taylor expansion, which should, in theory at least, be sufficient if only the logistic function is considered. It can be seen from the entries in this Table 2 that there is some evidence for the necessity of considering a more elaborate nonlinear model than the fitted standard LSTAR model, especially if the change in the growth rate lagged one period is taken to be the transition variable in the second transition function.

- insert Table 2 around here -

Hence we proceed with estimating a four regime MRSTAR model, with CDR_{t-2} and Δy_{t-1} as transition variables in the two logistic functions. After deleting some of the variables having insignificant coefficients from the different regimes and restricting the threshold for CDR_{t-2} to be equal to zero (as this also turned out to be insignificant), the parameters in the final simplified model are estimated as

$$
\begin{aligned}
y_t = \; & [(\underset{(0.122)}{0.471} + \underset{(0.113)}{0.527}\, y_{t-1}) \times (1 - F(\Delta y_{t-1})) \\
& (\underset{(0.762)}{-0.467} + \underset{(0.335)}{0.709}\, y_{t-1}) \times F(\Delta y_{t-1})] \times [1 - F(CDR_{t-2})] \\
& +[(\underset{(0.272)}{+0.349} - \underset{(0.349)}{0.522}\, y_{t-1} + \underset{(0.418)}{1.000}\, y_{t-2}) \times (1 - F(\Delta y_{t-1})) \\
& (\underset{(0.394)}{+0.041} + \underset{(0.230)}{0.678}\, y_{t-1} - \underset{(0.235)}{0.263}\, y_{t-2}) \times F(\Delta y_{t-1})] \times F(CDR_{t-2})
\end{aligned} \tag{23}
$$

$$
F(\Delta y_{t-1}) = (1 + \exp[-\underset{(6.598)}{6.461}\, (\Delta y_{t-1} - \underset{(0.254)}{0.484})/\sigma_{\Delta y_{t-1}}])^{-1} \tag{24}
$$

$$
F(CDR_{t-2}) = (1 + \exp[-\underset{(-)}{75.862} CDR_{t-2}/\sigma_{CDR_{t-2}}])^{-1} \tag{25}
$$

$\hat{\sigma}_\varepsilon = 0.875$, SK $= -0.20(0.13)$, EK $= 0.82(0.01)$, JB $= 6.61(0.04)$, ARCH(1) $= 1.16(0.28)$, ARCH(4) $= 4.01(0.40)$, AIC $= -0.130$, BIC $= 0.091$.

Figure 4 shows the two logistic transition functions (25) and (24) against Δy_{t-1} and CDR_{t-2}, respectively. Each circle represents an observation. For the function governed by the indicator of the current depth of the recession, the transition from zero to one is almost instantaneous at zero. The transition in the other function starts roughly at zero change in the growth rate of real GNP, while the transition is completed at a change in the growth rate of one percentage point.

- insert Figure 4 around here -

The model thus distinguishes between four different regimes, depending on whether the level of real GNP is below or above its previous high and whether growth is accelerating or not, which suggests the following interpretation of the four regimes.

- $\Delta y_{t-1} < 0, CDR_{t-2} < 0$. The economy is in expansion (recall that CDR_t as defined in (20) measures the distance in the level of real GNP relative to the previous all time high), but growth is declining.
- $\Delta y_{t-1} > 1, CDR_{t-2} < 0$. The economy is in a strengthening expansion, as growth is accelerating.
- $\Delta y_{t-1} < 0, CDR_{t-2} > 0$. The economy is in a worsening contraction.
- $\Delta y_{t-1} > 1, CDR_{t-2} > 0$. The economy is in a contraction, but is improving given the positive change in growth.

The fourth regime more or less corresponds with the recovery phase identified by Sichel (1994), in which growth is strong immediately following a trough.

Figure 5 shows the distribution of the observations across the different regimes. When we take model (23), it is seen that the bulk of the observations is in regime 1, while the remaining observations are evenly distributed over the other three regimes.

- insert Figure 5 around here -

Figure 6 shows the classification according to regime in a slightly different way. The graph shows again the quarterly growth rates of US real GNP, augmented with symbols which represent the different regimes. Observation t is classified as belonging to a certain regime if the weight given to the corresponding model for prediction of the next observation is larger than 0.5.

- insert Figure 6 around here -

4.2 US Unemployment Rate

In general, evidence for asymmetry in the unemployment rate has been somewhat more convincing than for output series. Neftçi (1984) suggests that increases in the aggregate unemployment rate are steeper than decreases. Sichel (1989) identifies a mistake in Neftçi's analysis, and is not able to reject symmetry with a corrected procedure. Rothman (1991) considers industrial sector unemployment rates and does find indications of steepness. Neftçi (1993) shows that conventional linear models are able to replicate the observed patterns in the unemployment rate only with very small probability. Escribano and Jorda (1996) also reject linearity for these disaggregate unemployment rates using tests against STAR-type nonlinearity. Peel and Speight (1996) succesfully estimate SETAR models for (logistically transformed) unemployment rates. Rothman (1997) estimates several nonlinear models for the aggregate unemployment rate and examines their usefulness for (long-term) forecasting. He finds that several nonlinear models perform superior to a linear model. A particular interesting result for our purposes is that Sichel (1993) shows that the US unemployment rate exhibits both steepness and deepness characteristics, which is taken as a first indication of the possible existence of multiple regimes. Similarly, McQueen and Thorley (1993) focus on growth rates surrounding peaks and troughs and obtain fairly strong evidence in favor of sharpness in the unemployment rate by using a three-state Markov process to characterize the change in the unemployment rate.

The unemployment rate we consider in this section represents the percentage of US males aged 20 and over without a job, and is constructed by taking the ratio of the series LHMU and LHMC from *Citibase*. The series is sampled at a monthly frequency and covers the period January 1948 until July 1996. The same series is analyzed by Hansen (1997) using SETAR models[8]. The series is graphed in Figure 7, where circles indicate individual peaks and troughs as dated by the NBER[9].

- insert Figure 7 around here -

The cyclical behaviour of the unemployment rate can be characterized as steep increases during recessions, followed by slow(er) declines during expansions. The existence of a recovery phase would imply that the decline in the unemployment rate is larger in months immediately following a trough than in later stages of the expansion. This in fact can be observed from Figure 7. The decline in the unemployment rate appears to start somewhat slow after a high (which corresponds with a trough in the business cycle), accelerates after roughly six months, and then declines again. The mean growth rates in quarters surrounding peaks and troughs is also shown in Figure 8, which confirms the above observations.

- insert Figure 8 around here -

It is also clear that, especially after 1970, the unemployment rate does not return to previous lows during expansions. This might be interpreted as (circumstantial) evidence

[8]Our analysis cannot easily be compared with Hansen (1997) though, because he considers the series only from January 1959 onwards and uses a different method to detrend the series.

[9]These peaks and troughs differ from the reference business cycle turning points, as the unemployment rate is, on average, leading at peaks and lagging at troughs.

for a rise in the natural rate of unemployment. This natural rate has been a heavily debated concept, and no consensus has been reached how to properly account for it, see for example Staiger *et al.* (1997) and other papers in the same issue of the *Journal of Economic Perspectives* for some recent viewpoints. As this is not the main subject of our analysis, we adopt a fairly simplistic approach and linearly detrend the unemployment rate to obtain the cyclical component, cf. Escribano and Jorda (1996) and Rothman (1997). Obviously, the method of detrending might influence any subsequent analysis, but this point is beyond the scope of this paper[10].

Both the Akaike and Schwarz criteria indicate that an fifth-order AR model is appropriate for the detrended series, which is estimated as follows,

$$\begin{array}{lllllll} y_t = & \underset{(0.043)}{0.153} & + \underset{(0.041)}{1.093}\, y_{t-1} & + \underset{(0.062)}{0.112}\, y_{t-2} & - \underset{(0.062)}{0.081}\, y_{t-3} & - \underset{(0.062)}{0.025}\, y_{t-4} & - \underset{(0.041)}{0.123}\, y_{t-5} + \varepsilon_t, \end{array} \quad (26)$$

$\hat{\sigma}_\varepsilon = 0.201$, SK = 0.99(0.00), EK = 7.68(0.00), JB = 1518.16(0.00), ARCH(1) = 10.63(0.00), ARCH(4) = 13.03(0.01), LB(8) = 7.84(0.17), LB(12) = 31.82(0.00), AIC = −3.198, BIC = −3.153.

The model appears to show all kinds of shortcomings, as the residuals suffer from skewness, excess kurtosis, heteroskedasticity and serial correlation. We next calculate the tests against STAR nonlinearity. As we are concerned with the behaviour of the unemployment rate over the business cycle, we are interested in medium-term movements. The month-to-month unemployment rate exhibits considerable short-term fluctuations, especially in the last months of expansions, as shown by Figure 7. In essence this makes the monthly rate unsuitable as indicator of the business cycle regime, see Birchenhall *et al* (1996) and Neftçi (1984) for more elaborate discussions of this point. For that reason, we concentrate on the use of lagged quarterly unemployment rates as potential transition variable, as well as changes in this medium-term rate[11]. We denote as x_t the average unemployment rate during the quarter up to and including month t, i.e., $x_t = (y_t + y_{t-1} + y_{t-2})/3$. Table 3 displays p-values of the LM-type tests, which indicate that linearity can be confidently rejected. For sake of completeness, test results with monthly rates as transition variable are also reported. We (somewhat arbitrarily) select the change in the quarterly unemployment rate lagged one month as transition variable. Interestingly, the test sequence which is employed in the specification procedure for STAR models of Teräsvirta (1994) indicates that an exponential or 'quadratic logistic' function (as given in (3) and (4), respectively) might be most appropriate. For reasons discussed earlier, we do not want to restrict a multiple regime model a priori, and hence proceed with estimating a LSTAR model.

[10] See Falk (1986) and Canova (1994) for more discussion on the influence of detrending procedures on analysis of the cyclical component.

[11] Note that Hansen (1997) uses even longer differences of the unemployment rate as threshold variable.

- insert Table 3 around here -

The estimation of the two-regime LSTAR model confirms the results of the LM-type tests: depending on the starting values for the parameters, either a model with a threshold of approximately -0.34% (and a fast transition between the two regimes) or a model with a threshold of approximately 0.58% (and a fairly gradual transition) is obtained. The complete estimation results are not shown here to save space but are available on request from the corresponding author. The p-values corresponding to the tests for remaining nonlinearity of Eitrheim and Teräsvirta (1996) and the LM-type tests against an MRSTAR alternative developed above are shown in Table 4. Evidently, both models are not able to capture all the nonlinearity in the dynamics of the unemployment rate series. Note that the tests reject the null hypothesis most convincingly if the same variable, the quarterly unemployment rate lagged one period, is used in the second transition function as well, see the rows labelled x_{t-1} in Table 4.

- insert Table 4 around here -

Hence, an MRSTAR model cf. (5) is estimated with the autoregressive parameters in the third regime, ϕ_3, restricted to zero.

$$
\begin{aligned}
y_t \;=\; & [(\underset{(0.399)}{0.053} + \underset{(0.427)}{1.182}\, y_{t-1} + \underset{(0.259)}{0.001}\, y_{t-2} - \underset{(0.201)}{0.344}\, y_{t-3} + \underset{(0.361)}{0.162}\, y_{t-4} - \underset{(0.139)}{0.068}\, y_{t-5}) \\
& \times(1 - F_1(\Delta x_{t-1})) + \\
& (-\underset{(0.018)}{0.053} + \underset{(0.113)}{0.763}\, y_{t-1} + \underset{(0.092)}{0.204}\, y_{t-2} + \underset{(0.092)}{0.120}\, y_{t-3} + \underset{(0.126)}{0.027}\, y_{t-4} - \underset{(0.065)}{0.141}\, y_{t-5}) \\
& \times F_1(\Delta x_{t-1})] \times [1 - F_2(\Delta x_{t-1})] \\
& +(\underset{(0.069)}{0.073} + \underset{(0.116)}{1.212}\, y_{t-1} + \underset{(0.155)}{0.056}\, y_{t-2} - \underset{(0.158)}{0.296}\, y_{t-3} + \underset{(0.170)}{0.039}\, y_{t-4} - \underset{(0.103)}{0.049}\, y_{t-5}) \\
& \times F_1(\Delta x_{t-1}) \times F_2(\Delta x_{t-1}) \qquad (27)
\end{aligned}
$$

$$
F_1(\Delta x_{t-1}) = (1 + \exp[-\underset{(48.857)}{25.303}\,(\Delta x_{t-1} + \underset{(0.065)}{0.381})/\sigma_{\Delta x_{t-1}}])^{-1} \qquad (28)
$$

$$
F_2(\Delta x_{t-1}) = (1 + \exp[-\underset{(0.162)}{3.490}\,(\Delta x_{t-1} - \underset{(0.007)}{0.308})/\sigma_{\Delta x_{t-1}}])^{-1} \qquad (29)
$$

$\hat{\sigma}_\varepsilon = 0.192$, SK $= -0.14(0.09)$, EK $= 2.25(0.00)$, JB $= 123.51(0.00)$, ARCH(1) $= 2.34(0.13)$, ARCH(4) $= 18.22(0.00)$, AIC $= -3.227$, BIC $= -3.061$.

The estimates of the thresholds in the two transition functions indicate that a three regime classification of the unemployment rate is appropriate, with the regimes 1, 2, and 3 corresponding to recovery, moderate and contraction phases, respectively (since unemployment is countercyclical, a rise in the unemployment rate corresponds with a contraction of the economy). The estimates of the parameters γ show that the transition from the recovery to moderate regime is almost instantaneous a around a change in the unemployment rate of -0.38% in the previous quarter. The transition from moderate to contraction is more gradual, centered around a change of 0.31% in the previous quarter. The two transition functions are shown in Figure 9.

- insert Figure 9 around here -

Note that the shapes of the transition functions do not necessarily contradict the results of McQueen and Thorley (1993), who obtain evidence supporting the hypothesis that peaks are more round than troughs. They however focus on the probabilities of transition from the contraction to expansion phase and vice versa, which is not comparable with the transition functions in the MRSTAR model. To obtain an impression of the implications of the MRSTAR model for such transition probabilities one would have to consider the long-term properties of the model, for example by means of impulse response functions.

Finally, Figure 10 shows the classification of the observations in the different regimes for a selected time period. It can be observed that the regimes correspond fairly well with division of a cycle in a contraction regime, a fast-growth recovery regime immediately following a trough and a moderate-growth regime during the rest of the expansion.

- insert Figure 10 around here -

5 Concluding remarks

In this paper we have explored possibilities to extend the basic STAR model to allow for more than two regimes. We have shown that this can be done by writing the model such that the different models which constitute the STAR model appear explicitly. A (specific-to-general) specification procedure was proposed and a new LM test for nonlinearity was developed, which can be used to test for the presence of multiple regimes. Alternatively, this test might be used as a diagnostic tool in order to test the adequacy of a fitted STAR model, complementing the tests of Eitrheim and Teräsvirta (1996). The applications of the MRSTAR model to post-war US real GNP and US unemployment rate demonstrate that a multiple regime characterization of the business cycle might indeed be useful.

This paper offers some possibilities for further research. Second, the effect of outliers on the detection of regimes seems to be of interest, as one does not want to spuriously fit a model which contains additional regimes only to capture some aberrant observations. It appears that a robust estimation method for STAR models needs to be developed to achieve proper protection against the influence of such anomalous observations. Alternative ways to compare different STAR models, possibly with a different number of regimes might also be explored. Perhaps it is possible to use the ideas of Koop *et al.* (1996) and use impulse response analysis as a model selection device, or as a diagnostic check on the added value of additional regimes. Alternatively, the techniques of Hess and Iwata (1997) might be used to examine explicitly whether the switching-regime models are capable of replicating basic stylized facts such as amplitude and duration of expansions and contractions. Finally, it might be worthwhile to extend the application to US real GNP to a multivariate model, again following the ideas of Koop *et al.* (1996), or to model nonlinearity and time-varying parameters simultaneously. All these issues are left for further research.

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Table 1: LM-type tests for STAR nonlinearity in US GNP[1]

Transition Variable	Test	d = 1	2	3	4	5	6
y_{t-d}	S_2	0.211	0.120	0.646	0.602	0.242	0.376
	S_3	0.330	0.053	0.256	0.258	0.235	0.248
Δy_{t-d}	S_2	0.089	0.065	0.982	0.819	0.291	0.220
	S_3	0.074	0.248	0.971	0.840	0.287	0.460
CDR_{t-d}	S_2	0.023	0.083	0.157	0.758	0.835	0.664
	S_3	0.022	0.014	0.123	0.498	0.645	0.564
ΔCDR_{t-d}	S_2	0.777	0.059	0.714	0.712	0.296	0.587
	S_3	0.649	0.159	0.745	0.544	0.067	0.356

[1] p-values for LM-type tests for smooth transition nonlinearity in quarterly growth rate of US real GNP. CDR_t measures the current depth of a recession, $CDR_t = \max_{j\geq 1}\{x_{t-j}\} - x_t$ with x_t the log of US GNP.

Table 2: LM-type tests for multiple regimes in US GNP[1]

Transition Variable	ET_1	ET_3	$LM_{MR,1}$	$LM_{MR,3}$
y_{t-1}	0.35	0.26	0.27	0.53
y_{t-2}	0.35	0.06	0.16	0.15
Δy_{t-1}	0.08	0.06	0.01	0.05
CDR_{t-1}	0.18	0.06	0.23	0.07
CDR_{t-2}	0.18	0.32	0.12	0.61
ΔCDR_{t-1}	0.56	0.56	0.22	0.41

[1] The entries in columns ET_1 and ET_3 are p-values for the LM-type tests for remaining nonlinearity of Eitrheim and Teräsvirta (1996), based on first- and third-order Taylor approximation of the second transition function, respectively. The entries in columns $LM_{MR,1}$ and $LM_{MR,3}$ are p-values for the tests of a basic LSTAR model against an MRSTAR alternative as developed in Section 2.3, also using first and third-order Taylor approximations, respectively.

Table 3: LM-type tests for STAR nonlinearity in US unemployment rate[1]

Transition Variable	Test	d = 1	2	3	4	5	6
y_{t-d}	S_2	0.439	0.686	0.712	0.566	0.657	0.437
	S_3	0.202	0.335	0.326	0.326	0.459	0.429
Δy_{t-d}	S_2	0.009	0.007	0.005	0.010	0.016	0.176
	S_3	0.019	0.025	0.018	0.011	0.007	0.220
x_{t-d}	S_2	0.677	0.705	0.675	0.404	0.493	0.629
	S_3	0.304	0.356	0.377	0.334	0.473	0.675
Δx_{t-d}	S_2	0.001	0.006	0.001	0.003	0.011	0.114
	S_3	0.010	0.008	0.001	0.001	0.000	0.121

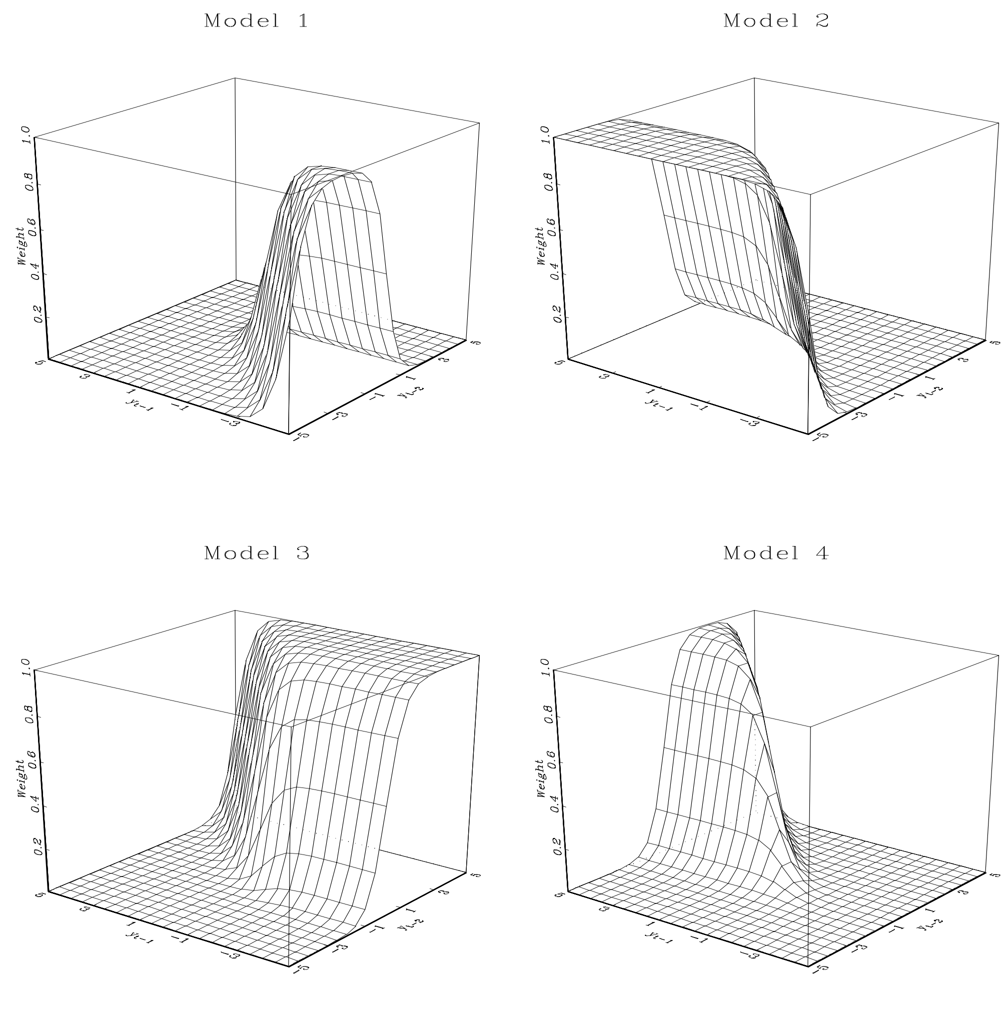
[1] p-values for LM-type test for smooth transition nonlinearity in monthly, linearly detrended, US unemployment rate. x_t is the average unemployment rate during the quarter up to and including month t, i.e., $x_t = (y_t + y_{t-1} + y_{t-2})/3$.

Table 4: LM-type tests for multiple regimes in US unemployment rate[1]

Transition Variable	Test: ET_1	ET_3	$LM_{MR,1}$	$LM_{MR,3}$	Test: ET_1	ET_3	$LM_{MR,1}$	$LM_{MR,3}$
x_{t-1}	0.32	0.74	0.63	0.82	0.18	0.48	0.59	0.10
x_{t-2}	0.31	0.75	0.63	0.84	0.18	0.40	0.59	0.08
x_{t-3}	0.37	0.74	0.66	0.82	0.20	0.36	0.59	0.08
Δx_{t-1}	0.14	0.02	0.08	0.00	0.01	0.15	0.09	0.05
Δx_{t-2}	0.07	0.08	0.14	0.26	0.32	0.30	0.27	0.37

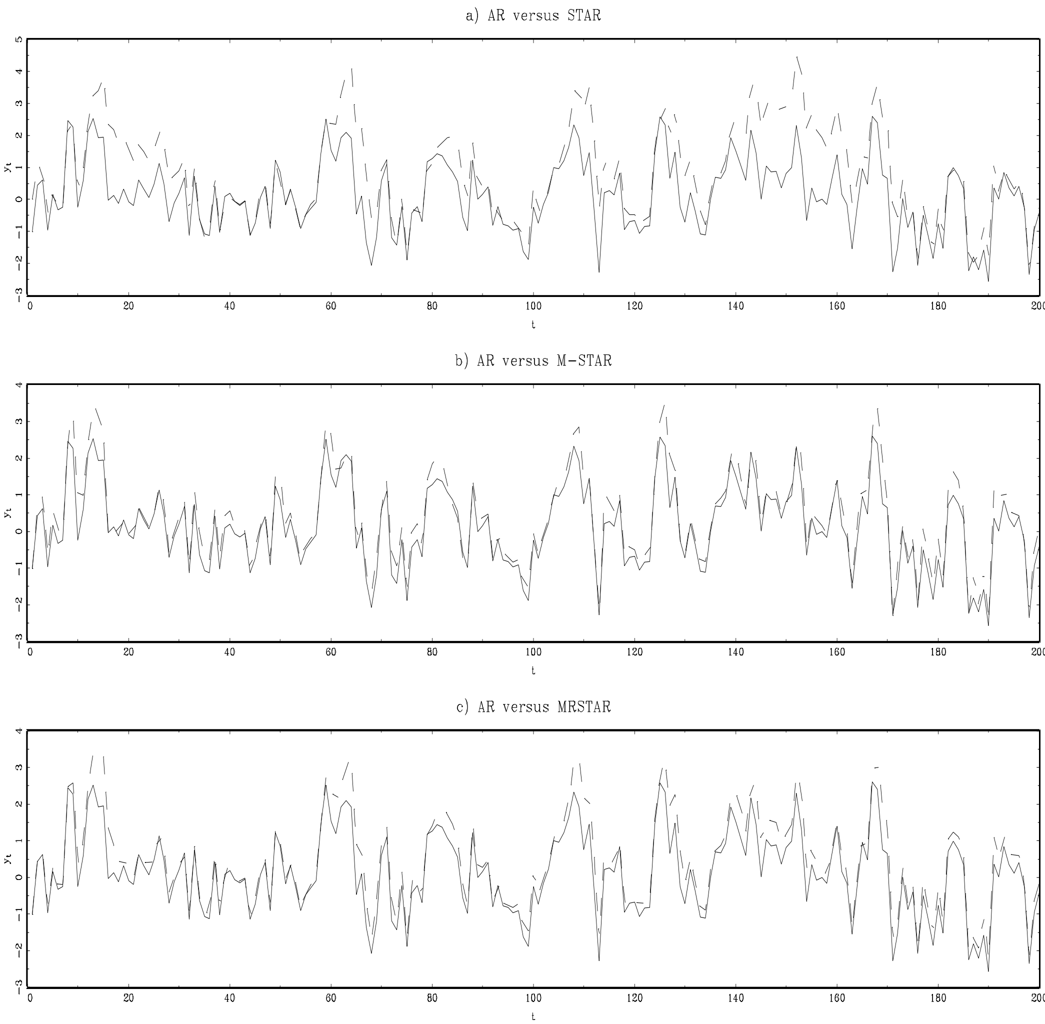
[1] The entries in columns ET_1 and ET_3 are p-values for the LM-type tests for remaining nonlinearity of Eitrheim and Teräsvirta (1996), based on first- and third-order Taylor approximation of the second transition function, respectively. The entries in columns $LM_{MR,1}$ and $LM_{MR,3}$ are p-values for the tests of a basic LSTAR model against an MRSTAR alternative as developed in Section 2.3, also using first and third-order Taylor approximations respectively. The left block of test results corresponds to an estimated two-regime LSTAR model with low value for the threshold, the right block corresponds to an estimated two-regime LSTAR model with high value for the threshold.

Figure 1: Weigths in MRSTAR Model



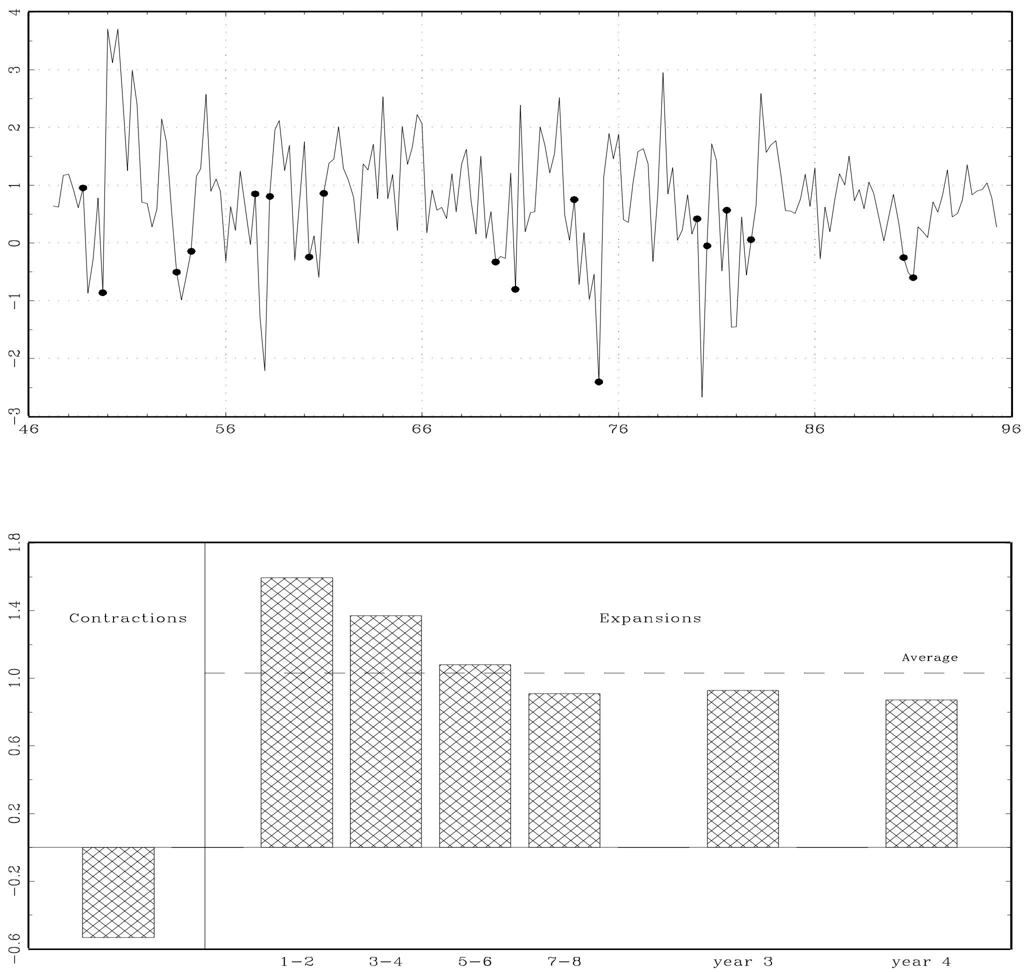
Note: Weights assigned to different AR models in the sample MRSTAR model (8).

Figure 2: Realizations of MRSTAR Model



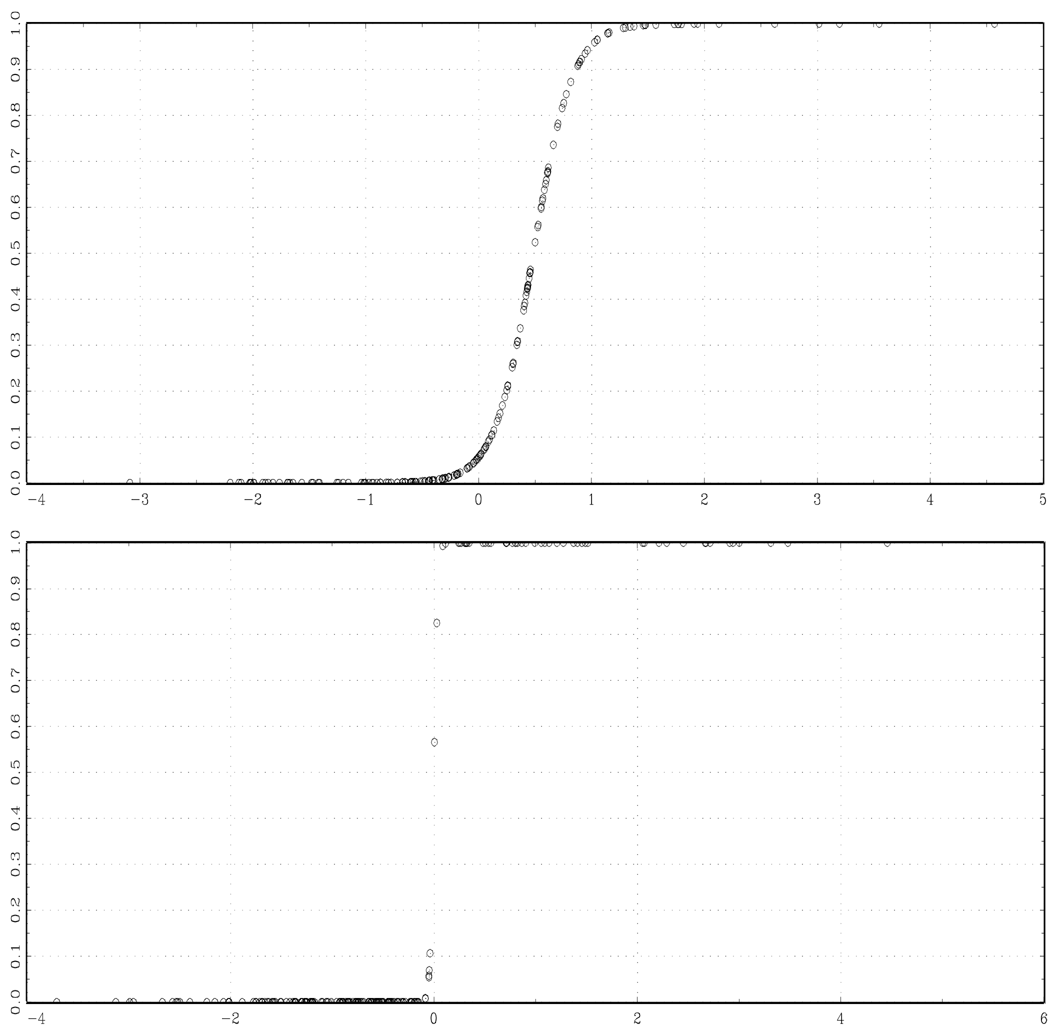
Note: Realizations of the sample MRSTAR model (8) with $\gamma_1 = \gamma_2 = 2.5$, $c_1 = c_2 = 0$, $\varepsilon_t \sim N(0,1)$ for different combinations of autoregressive parameters. Panel a): $\phi_1 = \phi_3 = 0.3$ and $\phi_2 = \phi_4 = 0.9$, Panel b): $\phi_1 = \phi_2 = 0.3$ and $\phi_3 = \phi_4 = 0.9$, Panel c): $\phi_1 = 0.3$, $\phi_2 = 0.6$, $\phi_3 = 0.6$, and $\phi_4 = 0.9$. The solid line is a realization of an AR(1) with autoregressive parameter 0.6, using the same errors ε_t.

Figure 3: US Real GNP, Quarterly Growth Rate



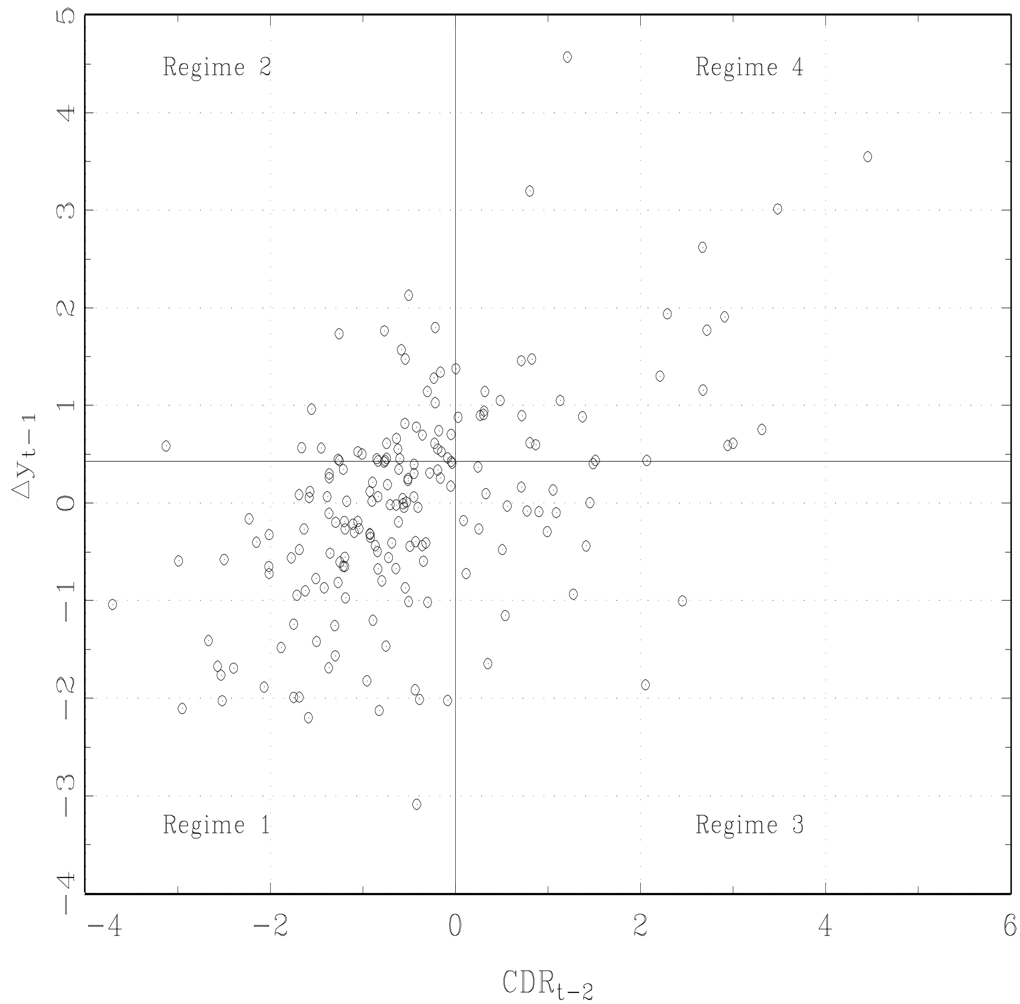
Note: The upper graph shows quarterly growth rates of US real GNP, 1947:II-1995:II. Solid circles indicate NBER peaks and troughs. The lower graph displays average growth over the business cycles.

Figure 4: US Real GNP, Transition Functions in MRSTAR Model



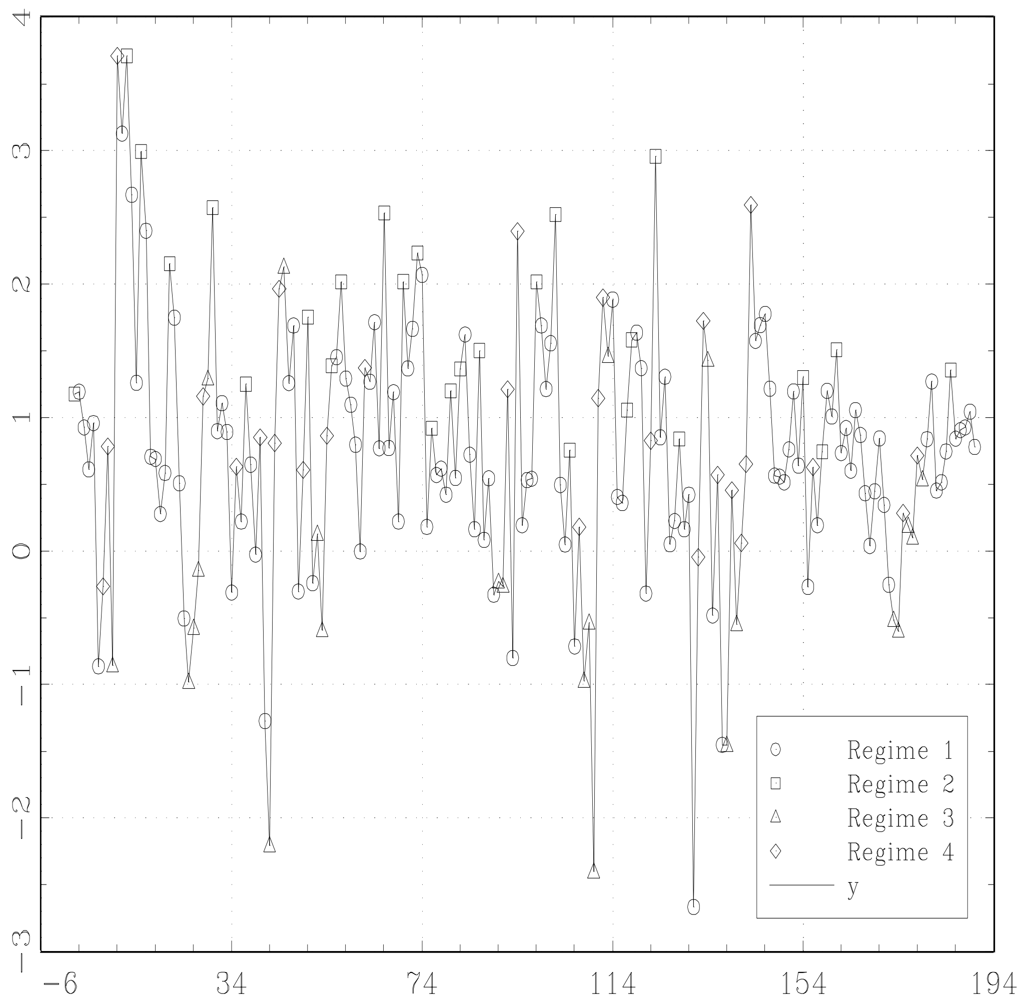
Note: Transition functions in MRSTAR model (23) for quarterly growth rates of US real GNP.
Upper graph: $F(\Delta y_{t-1}) = (1 + \exp[-6.461(\Delta y_{t-1} - 0.484)/\sigma_{\Delta y_{t-1}}])^{-1}$.
Lower graph: $F(CDR_{t-2}) = (1 + \exp[-75.862 CDR_{t-2}/\sigma_{CDR_{t-2}}])^{-1}$.

Figure 5: US Real GNP, Distribution of Observations



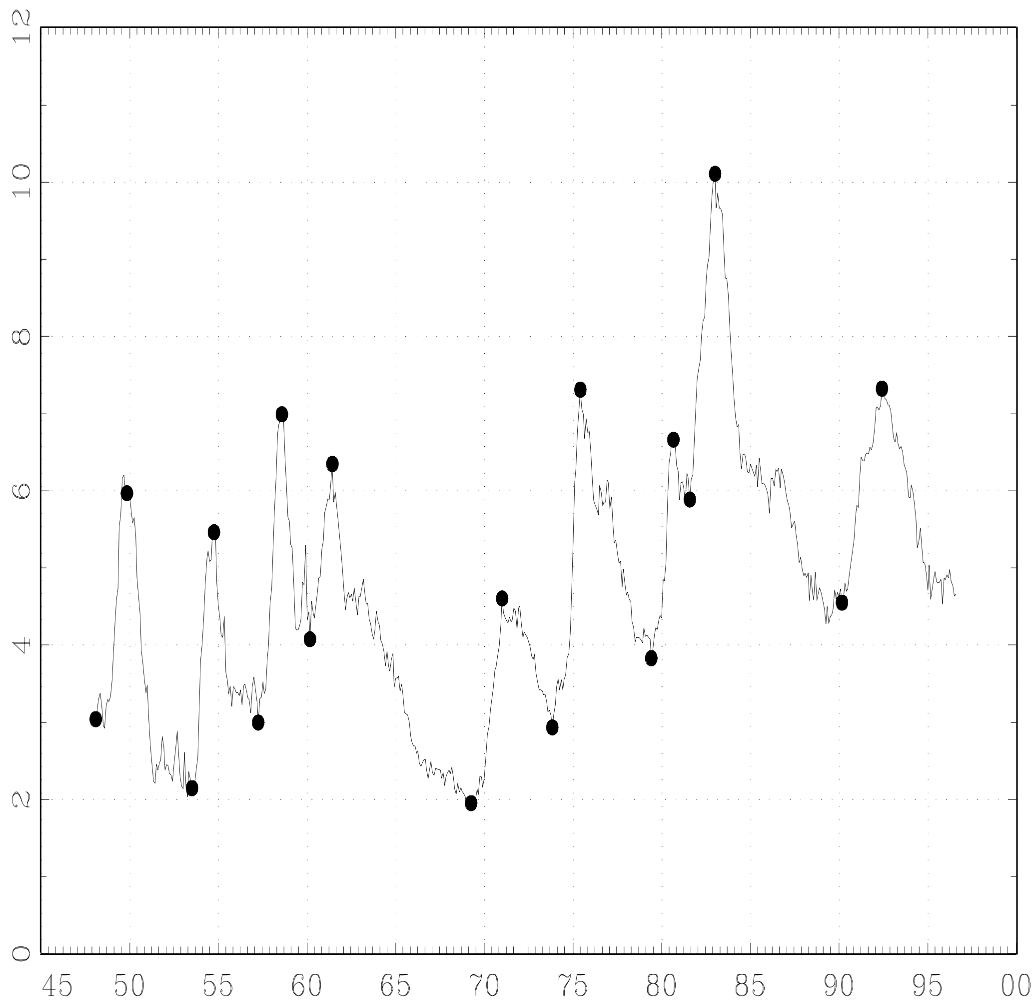
Note: Distribution of observations on quarterly growth rates of US real GNP over the different regimes.

Figure 6: US Real GNP, Classification of Observations



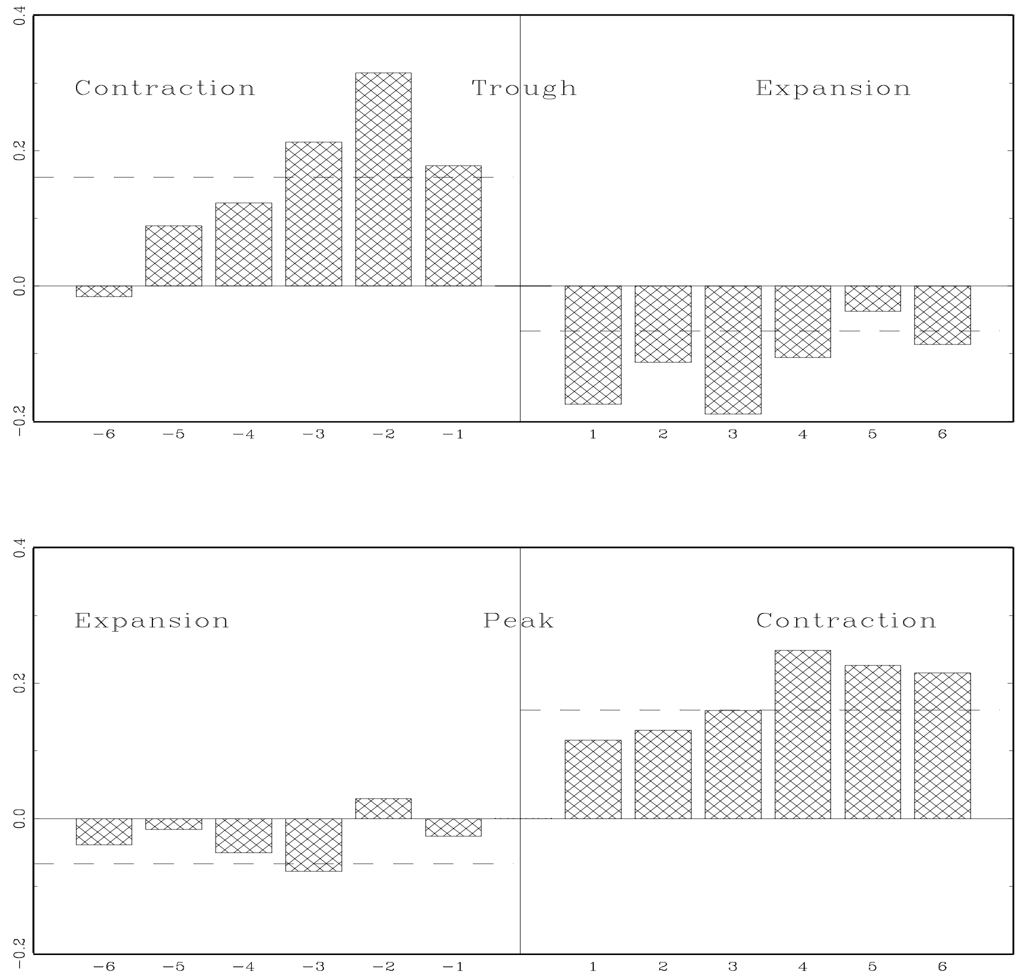
Note: Classification of observations on quarterly growth rates of US real GNP in different regimes. Open circles, cubes, triangles and diamonds indicate observations assigned to regime 1 ($F_1 < 0.5, F_2 < 0.5$), 2 ($F_1 > 0.5, F_2 < 0.5$), 3 ($F_1 < 0.5, F_2 > 0.5$), and 4 ($F_1 > 0.5, F_2 > 0.5$), respectively.

Figure 7: US Unemployment Rate



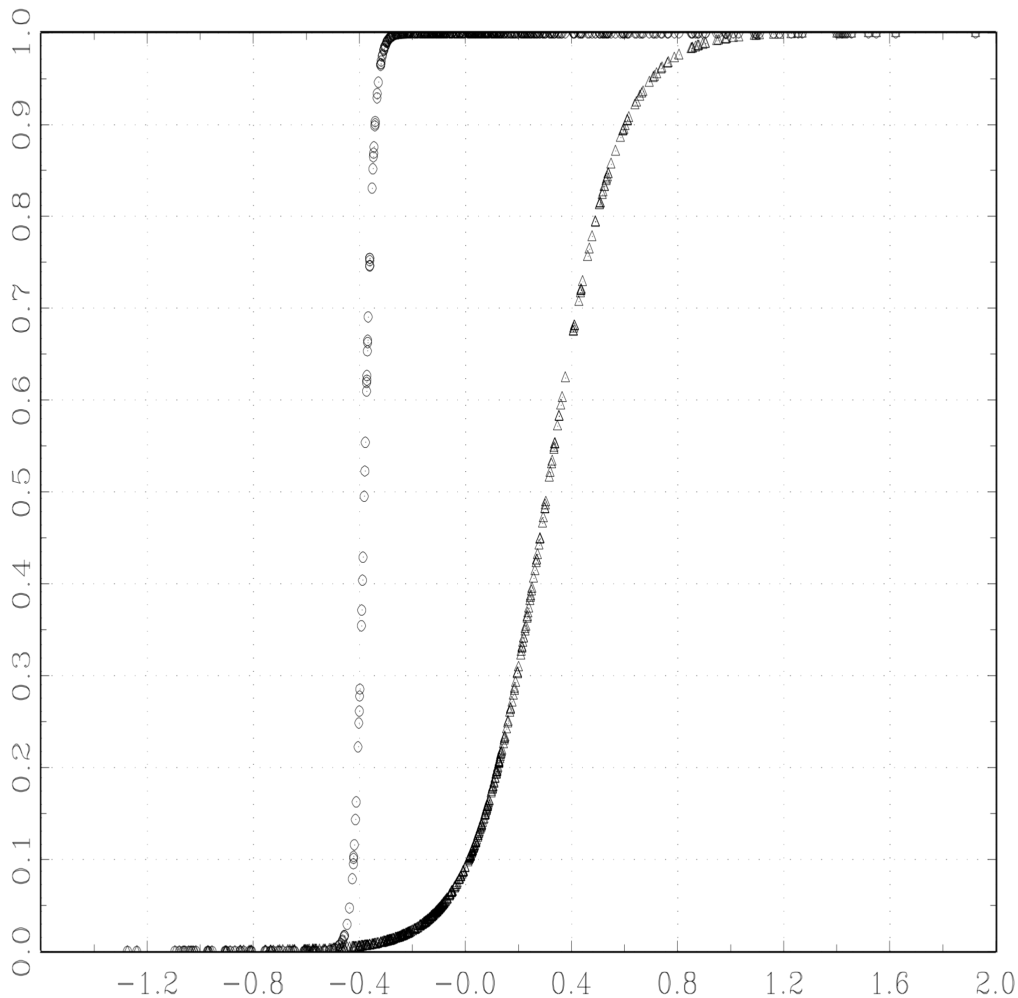
Note: Monthly US unemployment rate, males aged 20 and above, January 1948-July 1996. Solid circles indicate NBER unemployment peaks and troughs.

Figure 8: US Unemployment Rate, Mean growth rates surrounding peaks and troughs



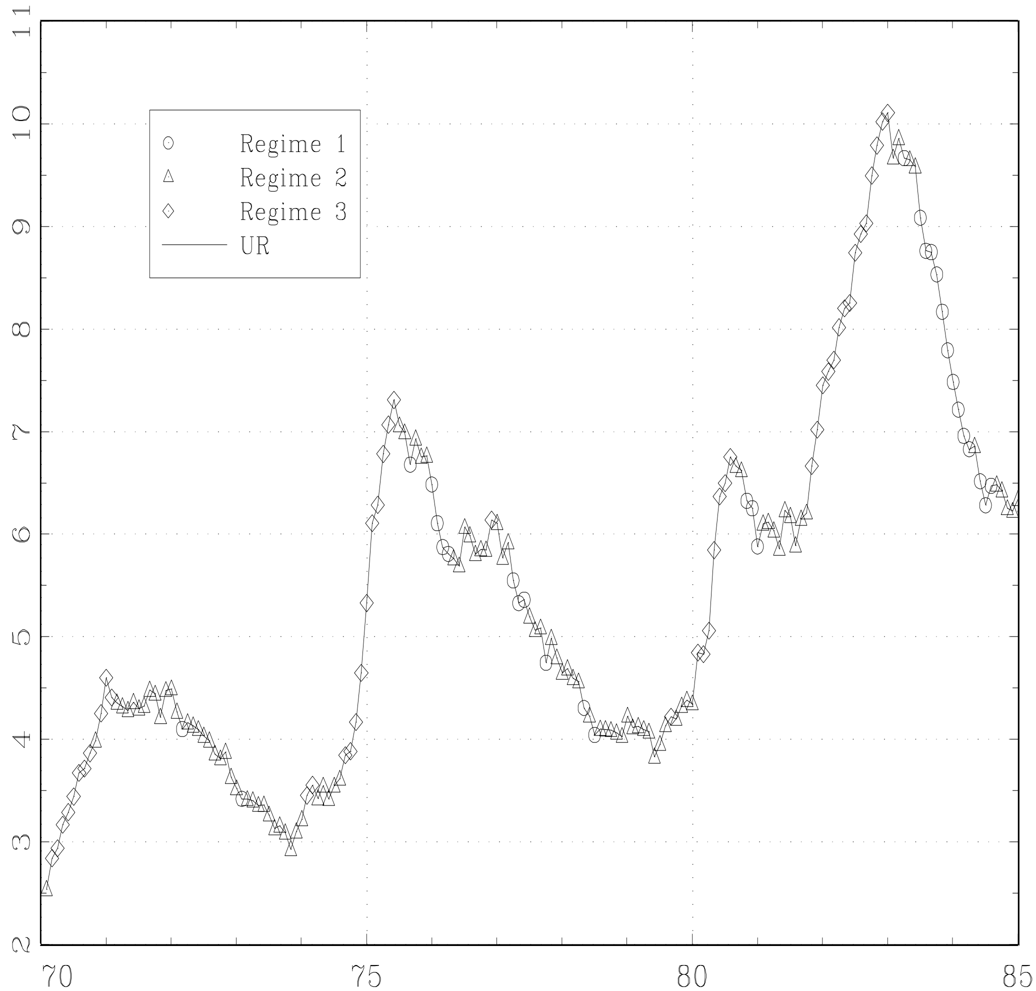
Note: Mean growth rates in US unemployment rate, males aged 20 and above, January 1948-July 1996, in quarters surrounding NBER unemployment peaks and troughs.

Figure 9: US Unemployment Rate, Transition Functions in MRSTAR Model



Note: Transition functions in MRSTAR model (27) for monthly US unemployment rate, males aged 20 and above, January 1948-July 1996. $F_1(\Delta x_{t-1}) = (1+\exp[-25.303(\Delta x_{t-1}+0.381)/\sigma_{\Delta x_{t-1}}])^{-1}$ (circles), $F_2(\Delta x_{t-1}) = (1+\exp[-3.812(\Delta x_{t-1}-0.308)/\sigma_{\Delta x_{t-1}}])^{-1}$ (triangles).

Figure 10: US Unemployment Rate, Classification of Observations



Note: Classification in different regimes of observations on detrended US unemployment rate, males aged 20 and above, January 1970-December 1984. Open circles, triangles and diamonds indicate observations assigned to lower ($F_1 < 0.5, F_2 < 0.5$), middle ($F_1 > 0.5, F_2 < 0.5$), and upper ($F_1 > 0.5, F_2 > 0.5$) regime, respectively.