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Modelling of crankshaft roller burnishing

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ABSTRACT

Roller burnishing is a surface treatment inducing compressive residual stresses with a view to enhancing endurance. A cyclic elastoplastic contact problem is thus raised, for which a model based on Simplified Analysis of Inelastic Structures is proposed, enabling fast determination of the residual stresses induced at minimum cost. Model results are compared with experimental data and results obtained by another numerical method. A parametric study has been undertaken to determine the incidence of certain of the parameters involved in the process.

INTRODUCTION

The crankshaft is a major component of an engine, by means of which the to-and-fro motion of a coupler is transformed in rotary motion. This process involves intensive service loads, particularly concentrated in linkage areas where they can exceed endurance limits. This is prevented by reinforcing the linkage areas, using, for example, prestressing techniques. Roller burnishing is the most economic and efficient way of creating residual compression stresses in order to optimize endurance.

The roller burnishing operation consists in rotating a roller over the crankshaft journal, at the same time applying a pressure, the latter being the key parameter in the process. The appropriate pressure value, determined by extensive tests and measurements, effectively induces compressive residual stresses favouring fatigue resistance, but it has not been possible to optimize it. The proposed model could be used for this purpose as well as for calculation of the residual stresses. In addition, since the geometrical characteristics of the two parts in contact are known, the corresponding contact area can be determined and its influence analyzed with a view to improving efficiency by optimizing these geometrical characteristics.

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MODELLING OF ROLLER BURNISHING

Modelling context : Simplified Analysis of Inelastic Structures

Roller burnishing raises a 3-D cyclic elastoplastic contact problem. It can consequently only be dealt with by numerical methods, which are conventionally step-by-step methods. Since a moving load is involved, the step-by-step elastoplastic calculations will have to be augmented by a series of incremental calculations simulating roller motion by moving the load at each increment. It then become obvious that the numerical computation and time requirements of such a method would be extremely demanding. So it would be better to find another approach providing a very fast solution, even if results remain approximate. The model proposed is based on this type of method, using Simplified Analysis of Inelastic Structures (S.A.I.S.). Unlike conventional methods, it requires few purely elastic calculations. However, before describing the model itself, we should like to briefly recall the basic principles of this analysis technique.

Basic principles of Simplified Analysis of Inelastic Structures

We shall consider only linear kinematically hardening material (L.K.H) obeying the Von Mises criterion. The structure, occupying a volume V and a surface ∂V is subjected at each instant t to surface forces $F_i^d(t)$ on $\partial_{F_i} V$, mass forces $\underline{X}^d(t)$ in V, initial strains $\underline{E}^I(t)$ in V and imposed displacements $U_j^d(t)$ on $\partial_{U_j} V$.

Elastic response It is represented by $\underline{\Sigma}^{el}(t)$, $\vec{\underline{U}}^{el}(t)$ and $\underline{E}^{el}(t)$ respectively, under stress, displacement and strain, with :

$$\underline{E}^{el}(t) = \underline{M}\,\underline{\Sigma}^{el}(t) + \underline{E}^I(t) \tag{1}$$

where $\underline{M}$ is the elastic compliance matrix. An equivalent response can be obtained using an operator, denoted **ELAS** which may be either numerical or analytical :

$$\{\underline{\Sigma}^{el}, \vec{\underline{U}}^{el}, \underline{E}^{el}\} : \mathbf{ELAS}\,[\,V, \partial_{F_i} V, \partial_{U_j} V \,;\, \underline{X}^d, F_i^d, U_j^d, \underline{E}^I \,;\, \underline{M}\,] \tag{2}$$

Real response and elastoplastic behaviour It is expressed as $\underline{\Sigma}(t)$, $\vec{\underline{U}}(t)$ and $\underline{E}(t)$ respectively under stress, displacement and strain. We use the conventional decomposition of total strain into an elastic part and a plastic part. In the stress deviator space $\underline{S}(t) = \text{dev}\,\underline{\Sigma}(t)$, the Von Mises criterion is written :

$$C(\underline{S}(t)) : 1/2\,(\underline{S}(t) - C\,\underline{E}^p(t)) : (\underline{S}(t) - C\,\underline{E}^p(t)) - k_0^2 \leq 0 \tag{3}$$

where k_0 is the elastic limit in shear, C the hardening modulus and $\underline{E}^p(t)$ the plastic strain.

Inelastic response We denote $\underline{R}(t)$, the residual stress field obtained by purely elastic unloading of the structure.

$$\underline{R}(t) = \underline{\Sigma}(t) - \underline{\Sigma}^{el}(t) \tag{4}$$

Now, $\underline{\Sigma}(t)$ and $\underline{\Sigma}^{el}(t)$ are statically admissible (S.A.) whith volume forces $\underline{X}^d(t)$ in V and surface forces $F_i^d(t)$ on $\partial_{F_i} V$. The residual stress field will consequently be

S.A. with $\underline{0}$ in V et 0 on ∂_{F_i} V. Similarly, the inelastic strain field :

$$\underline{E}^{ine}(t) = \underline{E}(t) - \underline{E}^{el}(t) \tag{5}$$

is kinematically admissible whith null data on ∂_{U_j} V and obtained by :

$$\underline{E}^{ine}(t) = \underline{M}\,\underline{R}(t) + \underline{E}^{p}(t) \tag{6}$$

or as for (1) and (2) :

$$\{\underline{R}, \vec{\underline{U}}^{ine}, \underline{E}^{ine}\} : \textbf{ELAS}\,[\,V, \partial_{F_i} V, \partial_{U_j} V\,;\, \underline{0}^d, 0_i^d, 0_j^d, \underline{E}^p\,;\, \underline{M}\,] \tag{7}$$

In other words, all that is required is a homogeneous elastic computation (null boundary data), taking field $\underline{E}^p$ as the initial strain data.

<u>Global behaviour</u> We define a new parameter, ometting variable t :

$$\underline{Y} = C\,\underline{E}^p - dev\underline{R} \tag{8}$$

$\underline{Y}$ are **<u>the structure transformed parameters</u>**. Using these parameters, (3) becomes :

$$C(\underline{S}^{el}) : 1/2\,(\underline{S}^{el} - \underline{Y}) : (\underline{S}^{el} - \underline{Y}) \leq k_0^2 \tag{9}$$

We shall henceforth be operating in the structure transformed parameter space. These parameters have to be plastically admissible (P.A.) at each instant, i.e. they have to belong to the convex surface centered on $\underline{S}^{el}$. The positions of $\underline{Y}$ are determined locally at all points in the structure. Once these are known, the inelastic strain $\underline{E}^{ine}$ can be expressed as a function of $\underline{Y}$ and $\underline{R}$, since, on the basis of (8), $\underline{E}^p$ can be suppressed, giving :

$$\underline{E}^p = C^{-1}(\underline{Y} + dev\underline{R}) \tag{10}$$

and using (6) :

$$\underline{E}^{ine} = \underline{M}'\,\underline{R} + C^{-1}\,\underline{Y} \tag{11}$$

whre $\underline{M}'$ is the modified elasticity operator :

$$\underline{M}' = \underline{M} + dev\,C^{-1}\,\underline{Id} \tag{12}$$

If P.A. $\underline{Y}$ is known, we can then obtain the residual stress field by means of the **ELAS** operator, with initial strain $C^{-1}\,\underline{Y}$ and modified elasticity matrix $\underline{M}'$, giving :

$$\{\underline{R}, \vec{\underline{U}}^{ine}, \underline{E}^{ine}\} : \textbf{ELAS}\,[\,V, \partial_{F_i} V, \partial_{U_j} V\,;\, \underline{0}^d, 0_i^d, 0_j^d, C^{-1}\,\underline{Y}\,;\, \underline{M}'\,] \tag{13}$$

Expressions (7) and (13) will be seen to be strictly equivalent, providing a basis for determination of overall structure response under stress and strain.
Since equation (10) gives the plastic strain field $\underline{E}^p$, the stress field $\underline{\Sigma}$ is obtained simply by superimposing the elastic and residual responses. Consequently, before

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going further, the transformed parameters have to be constructed [9, 10]. It is then a simple matter to describe structure response by means of a few elastic computations performed using **ELAS**. In particular, in the event of cyclic loading (as in roller burnishing), we propose to assess these transformed parameters on the basis of the initial internal condition ($\underline{Y}_0$, $\underline{E}_0^p$), using for this purpose elementary constructions.

Model

Problem context and assumptions We deal with a cyclic elastoplastic problem, where loading transmission is by roller bearings. This complex 3-D problem can be simplified by adopting some assumptions. Firstly, we suppose that the structure has already reached a stabilized condition. Our attention is consequently focussed on the effect of new loads. Seconly, The residual condition is assumed to be axisymmetric, and finally, the postulated loading is hertzian and periodic and is also considered as radial.

At each point in the structure, the elastic stress condition involves two extrema $\underline{\Sigma}^{el}_{max}$ and $\underline{\Sigma}^{el}_{min}$. $\underline{\Sigma}^{el}_{max}$ is that induced by the 3-D (elliptic) contact of the roller on the crankshaft. Determination of this field requires, in the first place, a 3-D elastic computation. However, for the time being, we shall put this aside and consider the structure to be treated as a semi-infinite solid and then as a cylinder, for two fundamental reasons. Firstly, this would allow quasi-analytical representation of the 3-D character of the hertzian contact whilst remaining in a single plane. Secondly, from the process optimization standpoint, it would be easier to identify general trends in this context, since findings would also be valid for the real structure. After this, the real 3-D structure would have to be considered. This has been done, directly and on the basis of the plane computation on the solid [1], but is not presented in this paper.

Construction of the transformed parameters When stabilized, a structure consisting of a L.K.H. material conforms either to elastic or to plastic shakedown [4]. In the first place, the nature of such a state has to be determined, after which, with a few elementary considerations, the transformed parameters $\underline{Y}_\ell$ have to be constructed for this limit state. For the sake of simplification, we shall assume that the structure was initially in its "natural state", i.e. prior to the rolling process, residual stresses and plastic strains were null. The initial transformed parameters $\underline{Y}_0$ are null.

In the transformed parameter $\underline{Y}$ space, we can represent the extreme convexes $C(\underline{S}^{el}_{max})$ and $C(\underline{S}^{el}_{min})$ centered respectively on $\underline{S}^{el}_{max}$ and $\underline{S}^{el}_{min}$, whith (9). Simply knowing the two extreme convexes would enable us to determine whether the structure was in an elastic or plastic shakedown [7]. This is done by examining at all points the intersection C_ℓ of the two extreme convexes, or, by determining the distance d^{el} between the centers of the two extreme convexes [1].

a) Elastic shakedown : we have $d^{el} \leq 2k_0$ at all points in the structure. There are two possibilities :
i. $k_0 \leq d^{el} \leq 2k_0$ (fig. 1.b) : the initial transformed parameter $\underline{Y}_0$ is outside C_ℓ. The projection of $\underline{Y}_0$ on C_ℓ will give an estimation of $\underline{Y}_\ell$.
ii. $d^{el} \leq k_0$ (fig. 1.c)) : $\underline{Y}_0$ is in C_ℓ, which is also the case for $\underline{Y}_\ell$, although not neces-

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sarily in the same position. This factor consequently remains undetermined. It shall then be considered that the elastic state is maintained and that no plastic strain is incurred. $\underline{E}^p$ can henceforth be taken as being equal to the null tensor.

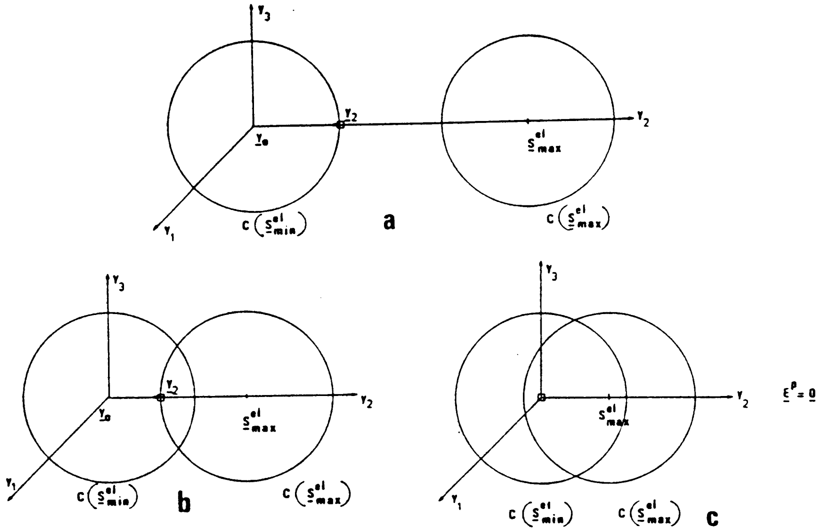

Figure 1 Construction of the transformed parameters

The structure can now be divided into two complementary zones, Ω_p and Ω_e respectively. The first zone Ω_p comprises all points where the transformed parameters have been determined ($k_0 \leq d^{el} \leq 2k_0$) but not the plastic strains. The modified elasticity matrix $\underline{M}'$ is used in this zone. In the second zone, the transformed parameters are not precisely known but the plastic strains are null. The elasticity matrix remains unchanged ($\underline{M}$).

b) Plastic shakedown : In this case, there is at least one point in the structure where intersection C_ℓ is reduced to the blank data set ($d^{el} > 2k_0$). Then we have two cases :
i. If $d^{el} > 2k_0$ (fig. 1.a), the residual condition transformed $\underline{Y}_\ell$ parameters are determined by projection of $\underline{Y}_0$ on the convexity centered on the origin.
ii. If $k_0 \leq d^{el} \leq 2k_0$ (fig. 1.b) or $d^{el} < k_0$ (fig. 1.c) $\underline{Y}_\ell$ is constructed as in the elastic shakedown case previously discussed.
The structure will be divided into two zones Ω_p and Ω_e in similar fashion, with Ω_p comprising points such that $d^{el} > 2k_0$ or $k_0 \leq d^{el} \leq 2k_0$ and Ω_e comprising those where $d^{el} < k_0$ is confirmed.

Residual stresses and plastic strains Since the residual condition transformed parameters are known, only the homogeneous elastic computation (null boundary conditions) has still to be performed :
- with an initial strain $C^{-1}\,\underline{Y}$ and modified elasticity matrix $\underline{\mathbf{M}}'$ in zone Ω_p
- with a null initial strain and elasticity matrix $\underline{\mathbf{M}}$ in zone Ω_e.
In short, we simply apply the operator :

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$$\{ \underline{R}, \vec{\underline{U}}^{ine}, \underline{E}^{ine} \} : \textbf{ELAS} \, [\, V, \partial_{F_i} V, \partial_{U_j} V \, ; \underline{0}^d, 0_i^d, 0_j^d, \underline{E}^I \, ; \underline{M}_Y \,] \tag{14}$$

with, for Ω_p $\quad \underline{E}^I = C^{-1} \underline{Y} \quad ; \quad \underline{M}_Y = \underline{M}'$

and for Ω_e $\quad \underline{E}^I = \underline{0} \quad ; \quad \underline{M}_Y = \underline{M}$

Plastic strain is simply calculated with :

$$\text{on } \Omega_p \qquad \underline{E}^p = C^{-1} \underline{Y} + \text{dev} \, \underline{R}$$
$$\text{on } \Omega_e \qquad \underline{E}^p = \underline{0} \tag{15}$$

Criterion verification By their construction, the transformed parameters are P.A. However, in practice $\underline{Y}$ are recalculated by (8). After which it is checked that they are P.A. If this is the case, then determination of the residual condition is completed. If, on the other hand, points remain where the transformed parameters are not P.A., the condition reached will be taken as a new initial condition and the steps described repeated until the criterion is verified. Convergence is in fact rapidly obtained, requiring only one or two steps.

APPLICATION AND VALIDATION

Plane computation

Let us consider a semi-infinite solid, viewed from position Oxyz, where Ozx defines the plane surface of the solid. Roller motion takes place within this plane, parallel to the Oz axis. O is the contact center and Oy is perpendicular to the solid, enabling determination of the depth (figure 2).

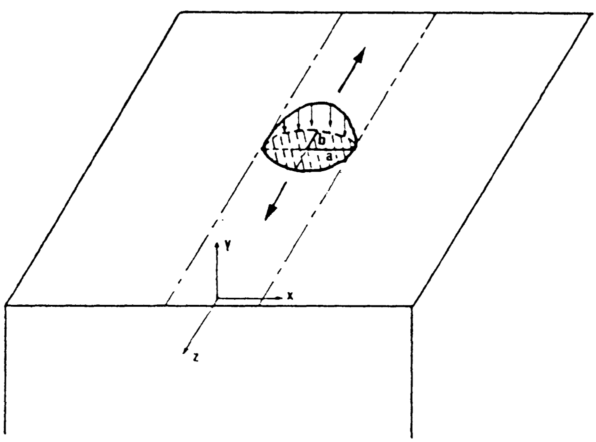

Figure 2 Contact and motion of a roller on a semi-infinite solid

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The contact area is an ellipse, the dimensions of which can be determined by application of the Hertz theory [8]. Using the same theory [5], the elastic stresses induced by this contact at any point in the xOy plane can be semi-analytically known. This residual state is the same in any plane parallel to xOy.
The roller in motion receives a load Q which it transfers to the solid as an ellipsoid pressure distribution. We shall call a and b, respectively, the large and small semiaxes of the contact ellipsis, $k = b/a$ is the ellipticity ratio, $c = \sqrt{ab}$ is the equivalent contact radius, p_0 is the maximum contact center pressure and $Q = 2\pi/3\ abp_0$ is the load received by the roller.
We represent on figure 3 only the isovalues of the normalized component R_{zz} of the residual stresses calculated by the model (σ_0 is the elastic limit).

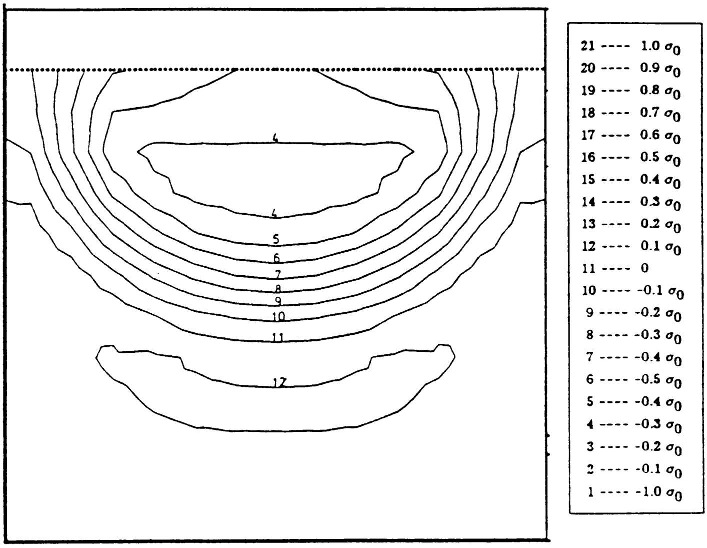

Figure 3 Isovalues of the roller burnishing residual stress R_{zz} component

Residual compression stresses were satisfactorily determined by means of the model. These stresses are high at the surface for components R_{zz} and R_{xx} and practically null for components R_{yy} and R_{xy}. However, maximum values correspond to through-thickness stressing. This result, also encountered with rollers running on rails, can be interpreted as characteristic of hertzian loading.

Axisymmetric calculation and validation

We consider now a cylinder to be roller burnished. The direction of roller motion on the cylinder is circumferential ($\theta\theta$). The contact area is an ellipse, with the longer axis in direction Oz, and the shorter axis orthoradial. Since the residual state is assumed to be axisymmetric, we shall concern only a meridian plane (rOz). Residual stresses in such a plane are practically identical to those observed in the xOy plane of the semi-infinite solid [1], justifying with hindsight the model design decision in this respect. This structure will be used as a model for experimental and numerical validation.

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Experimental study The model was validated by means of residual stress measurements carried out on a roller burnished cylinder using the incremental gap method. The experimental results are compared with calculated results on figures 4 and 5.

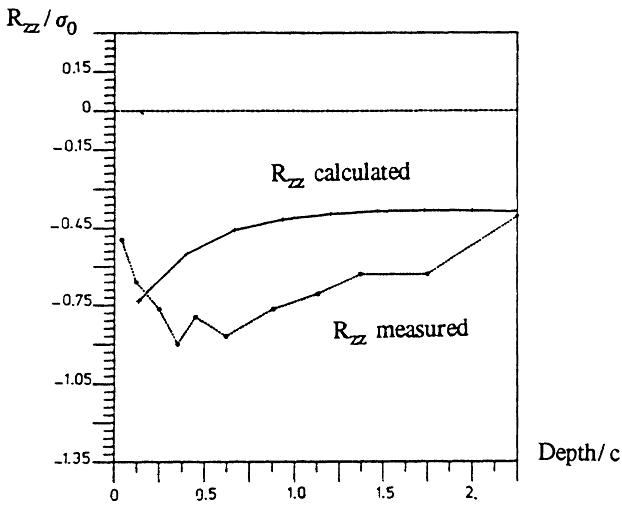

Figure 4 Measured and calculated R_{zz} through-thickness variations

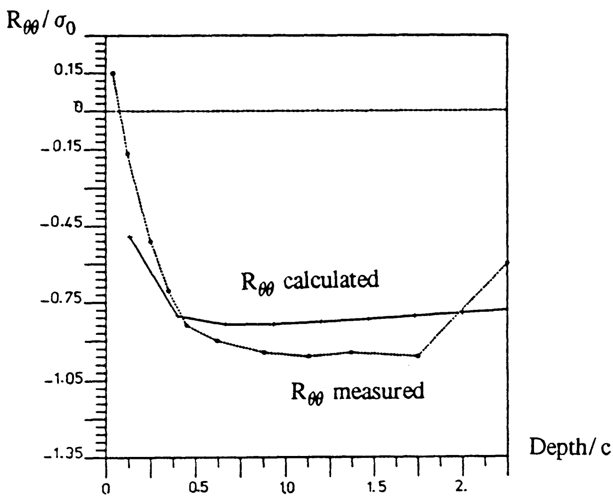

Figure 5 Measured and calculated $R_{\theta\theta}$ through-thickness variations

Disregarding the two surface measurements corresponding to test specimen surface treatment, the measured and model results may be said to be fairly close, considering uncertainties pertaining both to residual stress measurements and to the simplifications adopted for the model.

Comparison with another numerical method A comparison was also made with another numerical method based on the stationary algorithm (Dang Van et al. [2, 6]) where the Hertzian loading is extended in Fourier series. Both method and results are presented in [1]. Fairly similar results are obtained but with an overwhelming time factor in favour of the S.A.I.S. model (10 minutes as against 12 hours of CPU time on ALLIANT FX 40).

Parametric study

Influence of the maximum Hertz pressure Figure 6 shows the variations of residual stress component R_{zz} along the rolling axis versus maximum Hertzian stress and through-thickness depth in the solid. For low pressures, it is slight and rapidly saturates on the maximum. The depth of the latter, on the other hand, increases with p_0 and can exceed 2c. For high pressures, the compression stress zone is extensive, but residual tensile stresses may appear in the vicinity of the contact edge.

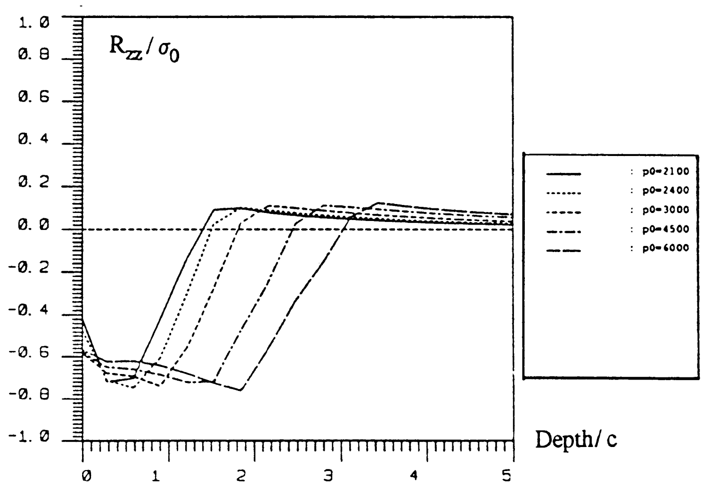

Figure 6 Variations of residual stress component R_{zz} through thickness and versus p_0

The R_{xx} component profile is less regular [1]. Two relative compression extrema are involved, located respectively on and below the surface. The second rapidly saturates on the maximum value but penetrates to a lower level. The amplitude of the surface extremum diminishes, highlighting the harmful effects of excessive roller burnishing pressure.

Influence of the ellipticity ratio The maximum residual stress, both for component R_{zz} and component R_{xx} [1] is higher for low ellipticity ratios. However, it should be noted that for k = 1, tensile stresses may appear in the vicinity of the contact edge.

CONCLUSION

This parametric study of roller burnishing was made possible, and at low cost, using S.A.I.S. The following trends were identified :

1. The maximum Hertzian stress, and consequently the load applied to the roller, has a favourable influence on the process, which was to be expected. However, residual stresses rapidly saturate on the maximum. In addition, a sharp increase in this pressure will cause tensile stresses to appear in the vicinity of the contact edge, which has harmful effects.
2. For the same contact area and pressure, it is advisable for the contact ellipse to be as expanded as possible, which indicates the shape of the roller to be adopted. Insofar as possible, shapes resulting in spherical type contact should be avoided.
3. The model must now be followed up with a fatigue behaviour analysis. This has been partially undertaken in [1]. It involves possible study of mechanical relaxation of residual stresses before the selection of a suitable fatigue criterion. Afterwich, it would be possible to revert to the initial roller burnishing problem with a view to optimizing the process.

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