

Modelling of deformations in  
Dynamic Soil-Structure  
Interaction Problems

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# Modelling of deformations in Dynamic Soil-Structure Interaction problems\*

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## Abstract

Use of geotechnical centrifuges has established as a powerful technique in the understanding of soil-structure interaction problems subjected to earthquake loading. In the VELACS conference, Arulanandan and Scott (1993), several numerical predictors have attempted to predict the dynamic behaviour of several bench mark problems which were simulated in the centrifuge tests in more than one geotechnical centrifuges. The results of the centrifuge tests conducted at different centrifuge centres were reasonable and provided a large data base against which numerical predictions may be compared. The Class A numerical predictions carried out without prior knowledge of the centrifuge test data were less satisfactory. Of all the predictions the finite element codes which employed fully coupled formulations for the solid and fluid phase performed well. There were some differences in the prediction of excess pore pressure for some of the bench-mark problems. Also attenuation in the local acceleration where there is rise in excess pore pressure was predicted by only some codes. A class C prediction carried out with prior knowledge of experimental data, on the quay wall with saturated back-fill (model no.II), Madabhushi and Zeng (1993), could simulate the experimental data accurately in terms of accelerations, excess pore pressures as well as the deformations.

One of the important prediction that is sought by the practising geotechnical earthquake engineers would be the deformation that will be suffered by the particular civil engineering structure under consideration. In this paper an attempt will be made on estimating the deformations in soil-structure interaction problems using a finite element code called SWANDYNE, Chan (1988). The formulation of the fully coupled code and the improvements brought in the predictions by knowledge of the experimental conditions, Madabhushi and Zeng (1993), Chan et al (1994), Madabhushi (1994), will be presented. The improvements to the prediction of the quay wall problem with saturated and dry back fill following the work of Madabhushi and Zeng (1993), will be presented Madabhushi and Zeng (1996). Also the deformations predicted in the problem of a tower structure founded on saturated sand bed, Madabhushi (1994), will be presented. Also more recent work on the liquefaction under saturated embankments, Madabhushi et al (1996) and its implications in the finite element modelling of this problem will be discussed.

## 1 Introduction

Dynamic centrifuge modelling has established as a powerful technique for studying the earthquake loading on geotechnical structures. The experimental data obtained from the centrifuge tests conducted at different centrifuge centres were used to compare the predictions made by various numerical procedures during the VELACS conference, Arulanandan and Scott (1993). The scatter in the experimental data from various centrifuge centres was attributed partly to the difference in earthquake input motion generated at different centres. On the other hand, numerical

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The practising geotechnical earthquake engineers are interested in the deformation that will be suffered by a civil engineering structure under consideration during a 'design earthquake'. While accelerations and excess pore pressures are important the prediction of deformation and ultimate settlement is more important from a design point of view. In this paper an attempt will be made on estimating the deformations in soil-structure interaction problems using a finite element code called SWANDYNE, Chan (1988). The formulation of the fully coupled code and the improvements brought in the predictions with the knowledge of the centrifuge model conditions, Madabhushi (1993), Chan et al (1994), Madabhushi et al (1994), will be presented. The improvements to the prediction of the quay wall problem with saturated and dry back fill subsequent to the work described by Madabhushi and Zeng (1993), will be presented, Madabhushi and Zeng (1996). Also the deformations predicted in the problem of a tower structure situated on a saturated sand bed, Madabhushi (1994), will be presented. Also more recent work on the liquefaction under saturated embankments Madabhushi et al (1996) and its implications in the finite element modelling of this problem will be discussed. In the next two sections a brief presentation of the numerical formulation and the constitutive model using will be attempted.

## 2 Numerical formulation

The dynamic equations of motion for a saturated porous medium subjected to cyclic loading may be written following Biot (1956) as

$$[K]\{\bar{u}_n\} + [M]\{\ddot{\bar{u}}_n\} - [Q]\{\bar{p}_n\} = \{f_n^s\} \quad \dots(1)$$

where  $[K]$  is the global stiffness matrix,  $[M]$  is the global mass matrix,  $[Q]$  is the coupling matrix and  $\{f_n^s\}$  is the force matrix for solid phase and  $\bar{u}, \bar{p}$  are global displacement and pore pressure matrices. For the fluid phase we can write,

$$[H]\{\bar{p}_n\} + [G]\{\ddot{\bar{u}}_n\} + [Q]^T \{\dot{\bar{u}}_n\} + [S]\{\bar{p}_n\} = \{f_n^f\} \quad \dots(2)$$

where  $[H]$  is the permeability matrix,  $[G]$  is the dynamic seepage force matrix,  $[S]$  is the compressibility matrix and  $\{f_n^f\}$  is the force matrix for the fluid phase. The general procedure adopted is to solve Eqs.1 and 2 at any one time station using the finite element method and extrapolating to the next time station using previous initial conditions and employing a standard finite differences technique. The unconditionally stable generalised Newmark method (Bathe and Wilson, 1978) is used in SWANDYNE. In this method Eqs. 1 and 2 are rewritten as

$$([M] + \frac{1}{2}[K]\theta_2\Delta t^2)\Delta\ddot{\bar{u}}_n - ([Q]\theta_1\Delta t)\Delta\dot{\bar{p}}_n = [f_{n+1}^s] \quad \dots(3)$$

$$([Q]^T\theta_1\Delta t)\Delta\ddot{\bar{u}}_n + ([S] + [H]\theta_1\Delta t)\Delta\dot{\bar{p}}_n = [f_{n+1}^f] \quad \dots(4)$$

Eqs. 3 and 4 are coupled by matrix  $[Q]$  and can be solved using a staggered approach. A single field variable is obtained using the extrapolated value of the other. For example displacement ' $\bar{u}$ ' and pore pressure ' $\bar{p}$ ' can be chosen as the field variables and the displacement can be obtained using the extrapolated values of the value of the pore pressure.

### 3 Constitutive model

The behaviour of soil under earthquake loading is complex. It is essential that the constitutive model used is able to capture the important features of the soil behaviour under cyclic loading such as permanent deformation, dilatancy, hysteresis and damping, etc. The analyses presented in this paper were all carried out using the model developed by Pastor and Zienkiewicz (1986) called the P-Z mark III model.

The P-Z mark III model is a generalised plasticity-bounding surface model with non associative flow rule. The model is described by means of potential surfaces given by

$$G\{p', q, p_g\} = \left\{ q - M_g p' \left( 1 + \frac{1}{\alpha_g} \right) \left( 1 - \left( \frac{p'}{p_g} \right)^\alpha \right) \right\} \dots (5)$$

where  $p'$  is the mean confining stress,  $q$  is the deviatoric shear stress,  $M_g$  is slope of the Critical State line,  $\alpha_g$  is a constant and  $p_g$  is a size parameter. The shapes of yielding surfaces and potential surfaces follow the same family of curves given by above equation. For the analysis the parameter  $M_g$  is obtained from the effective angle of friction  $\phi'$  of the soil and Lode's angle  $\theta$  by Mohr-Coulomb relation;

$$M_g = \frac{6 \sin \phi' \sin 3\theta}{3 - \sin \phi' \sin 3\theta} \dots (6)$$

$M_g$  is determined by assuming that  $\sin \phi'$  is constant and by considering  $M_g = M_{gc}$  when  $\theta = \pi/6$ .  $M_{gc}$  is obtained from the triaxial compression tests. The dilatancy of sands is approximated as suggested by Nova and Wood (1982);

$$d = (1 + \alpha_g)(M_g - \eta) \dots (7)$$

where  $\eta$  is the stress ratio ( $q/p'$ ). The direction of plastic flow is defined by means of a unit normal  $n_g$  given by,

$$\{n_g\} = \left( \frac{1}{\sqrt{1+d^2}} \right) \{d, s\}^T \text{ for loading,} \dots (8)$$

$$\{n_g\} = \left( \frac{1}{\sqrt{1+d^2}} \right) \{abs(d), -s\}^T \text{ for unloading,} \dots (9)$$

where  $s=+1$  during compression and  $s=-1$  during extension. The typical parameters that are required for this constitutive model are presented in Table 1 with typical values for medium dense sand.

### 4 Special considerations in the numerical modelling of the centrifuge tests

Numerical modelling of centrifuge tests requires certain special considerations apart from careful calibration of the constitutive model parameters using triaxial test data. Some of the special considerations which lead to better predictions by the finite element analysis are discussed below.

Table 1 Typical parameters for P-Z model

Parameter	Symbol	Typical value
Critical State friction angle	$\phi'$	31.3°
Slope of the CSL for plastic strain vector	$M_g$	1.26
Slope of the CSL for loading vector	$M_l$	0.95
Dilatancy parameter	$\alpha_g$	0.45
Dilatancy parameter for plastic strain vector	$a_f$	0.45
Plastic modulus on loading	$H_{0 \text{ loading}}$	700 kPa
Plastic modulus on unloading	$H_{0 \text{ unloading}}$	6000 kPa
Plastic deformation during unloading	$\gamma_{Hu}$	2
Plastic deformation during reloading	$\gamma_{Dm}$	2
Shear hardening parameter 1	$\phi$	4.2
Shear hardening parameter 2	$\phi_l$	4.2

#### 4.1 Simulation of semi-infinite half space

In the centrifuge tests the semi-infinite extent of the soil strata is modelled by using either absorbing boundaries like duxseal (putty, clay-like material) or more recently using Equivalent Shear Beam containers, Schofield and Zeng (1996), Madabhushi et al(1994). In the later case the dynamic stiffness of the soil is accurately simulated by the special boundary. While carrying out the FE analysis the boundaries are rendered non-reflective by various schemes such as Smith-Cundall boundary or Lysmer-Kuhlemeyer boundary. An elegant scheme based on compound parabolic collectors was developed by Madabhushi, (1993). In this scheme the boundary is modelled with a compound parabolic shape which will transmit stress waves arriving at any angle less than the angle of receptance into a region where they undergo multiple reflections. This scheme is particularly suitable for transient analysis described in this paper.

#### 4.2 Initial velocity and displacement conditions

It is important to simulate the initial velocity and displacement conditions experienced by the centrifuge model. The input motion experienced by a centrifuge model during an earthquake is measured by means of a base accelerometer. The finite element method requires assignment of initial velocity and displacement of all the nodes at the beginning of the analysis. Chan et al (1994) describe a method of obtaining the initial velocity and displacement conditions using the method of FFT and IFFT's (Digital Fast Fourier Transform and its Inverse Transform) applied to the recorded base acceleration. This method is especially important while modelling the deformations and was used in all the finite element analyses described in this paper.

#### 4.3 Slip elements

Slip elements are required in the finite element mesh at locations where there is reduced friction thereby facilitating a slip. These are especially important in soil-structure interaction problems where the interface between the structure and the soil needs such elements. For example, in the quay wall problem described in sec.5.1 of

this paper, erroneous results are obtained if one does not provide the slip elements between the quay wall and the back-fill.

## 5 Prediction of deformation

The serviceability of a civil engineering structure following an earthquake depends on the deformations it suffers. These may take the form of ultimate crest settlement of an embankment, rotation of structures like quay-walls, tower buildings or bridge piers, etc. Prediction of deformations is very important from a earthquake geotechnical engineer designing a new structure. Hence a finite element code must predict the permanent deformations that will occur when a structure is subjected to earthquake loading. In the following sections three class C predictions for permanent deformations for different boundary value problems are presented. In all these analyses special consideration was given in modelling the non-reflecting boundaries, using correct initial velocity/displacement conditions and use of slip elements as explained in section 4.

### 5.1 Quay walls with dry and saturated back-fill

Centrifuge tests were carried out on model quay walls with dry and saturated back-fills, **Zeng** (1992). The schematic section of these models is shown in Fig.1 and the finite element discretisation is shown in Fig.2.

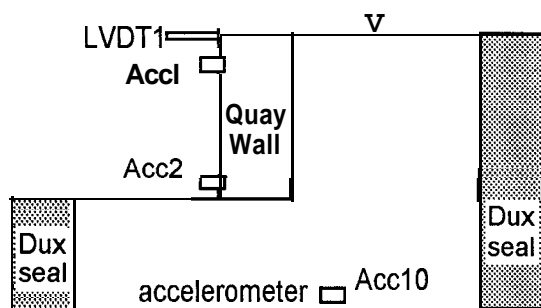


Fig. 1 Schematic section of quay wall

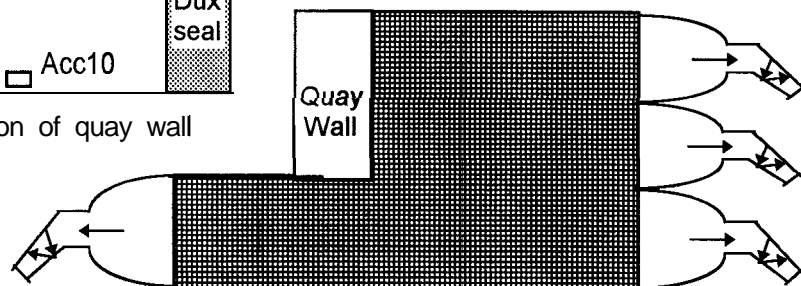


Fig.2 FE discretisation of the soil and the CPC boundary scheme

The rotation of the quay wall was determined by measuring the lateral movement using an LVDT in the centrifuge test. Also accelerations were measured on the wall. FE analyses were carried out using the SWANDYNE code and the results from these analyses are compared to the centrifuge test data. In Fig.3 the response of the quay wall to the base acceleration recorded by Acc10 is presented for the dry backfill case. Both the acceleration time histories and the lateral displacement time history match satisfactorily with the centrifuge test data. The finite element analysis is able to capture the sliding or threshold acceleration when the wall moves away from the backfill as well as the acceleration peaks when the wall moves into the backfill. In Fig.4 the deformations predicted by the FE analyses for the case of saturated backfill are presented. The vertical settlement of the quay wall was not recorded in the centrifuge test. However, the predicted vertical settlement is shown

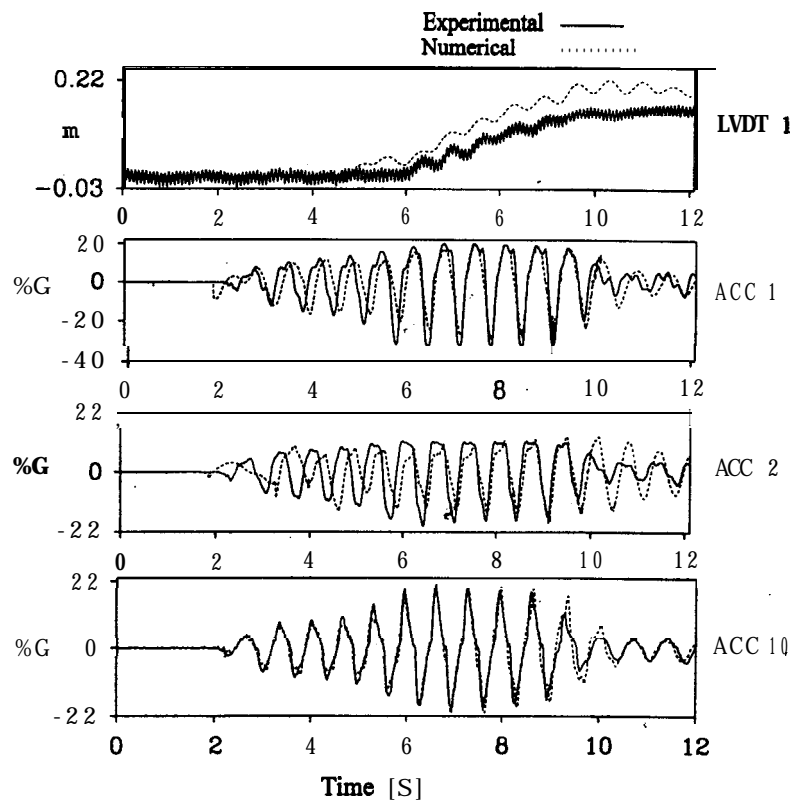


Fig.3 Deformation and response of the quay wall with dry backfill

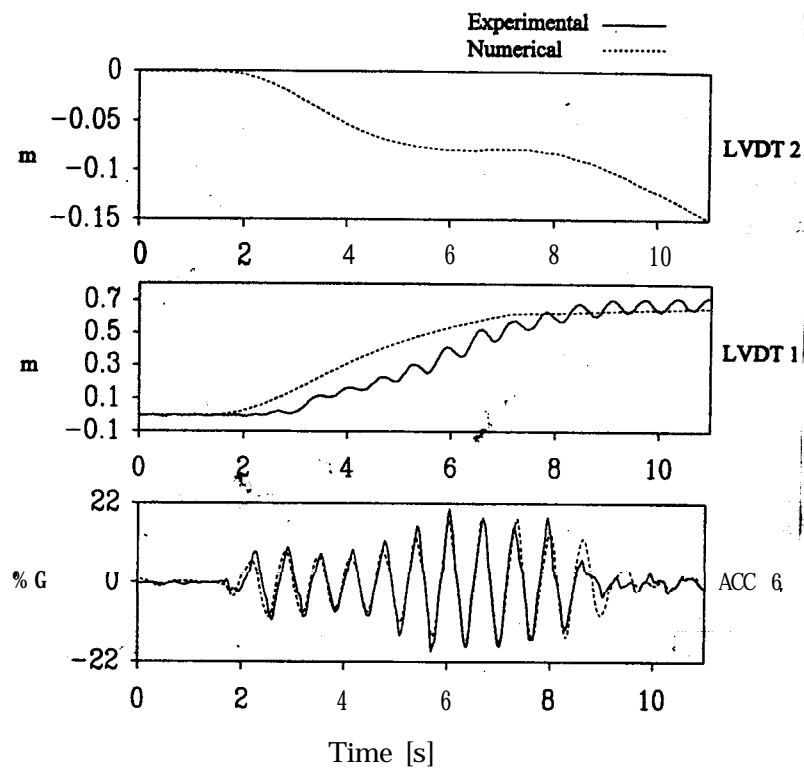


Fig.4 Deformation of the quay wall with saturated backfill

in this figure. The ultimate lateral movement of the wall predicted by the FE analysis compares satisfactorily with the experimentally measured movement.

### 5.2 Saturated embankment with embedded block structure

Steedman (1986) reported centrifuge tests on saturated embankments with a block structure located on the crest. A sectional view of this model is presented in Fig.5. Finite element analyses were carried out by Chan et al (1994) to predict the dynamic behaviour of the embankment and the embedded block. The deformations predicted by the FE analysis are compared to the centrifuge data as seen in Fig.6.

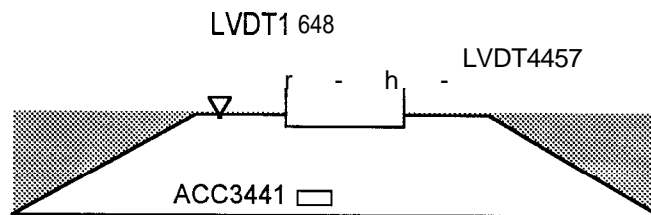


Fig.5 A schematic section of a saturated embankment with embedded block structure

In this figure we can see that the FE analyses is able to capture the vertical settlement very satisfactorily. The horizontal displacement is captured in terms of magnitude but the decoupling of the block owing to the partial liquefaction of the embankment and subsequent recoupling towards the end of the earthquake is not satisfactorily captured in the finite element analysis.

### 5.3 Tower structures on saturated sand beds

Finite element analyses were carried out to predict the dynamic behaviour of a tower structure located on a saturated sand bed and subjected to earthquake loading. Centrifuge tests on this problem were carried out by Madabhushi (1992). The schematic diagram of the centrifuge model is shown in Fig.7. The comparison of centrifuge test data and the numerical predictions from the FE analysis is presented in Fig.8.

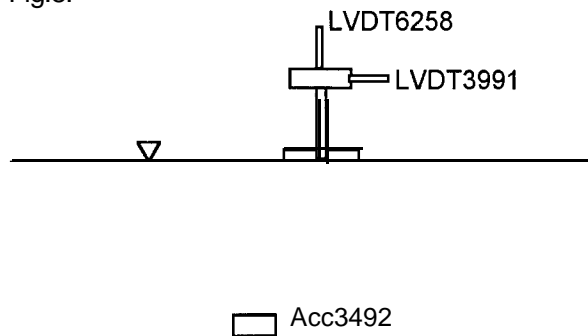
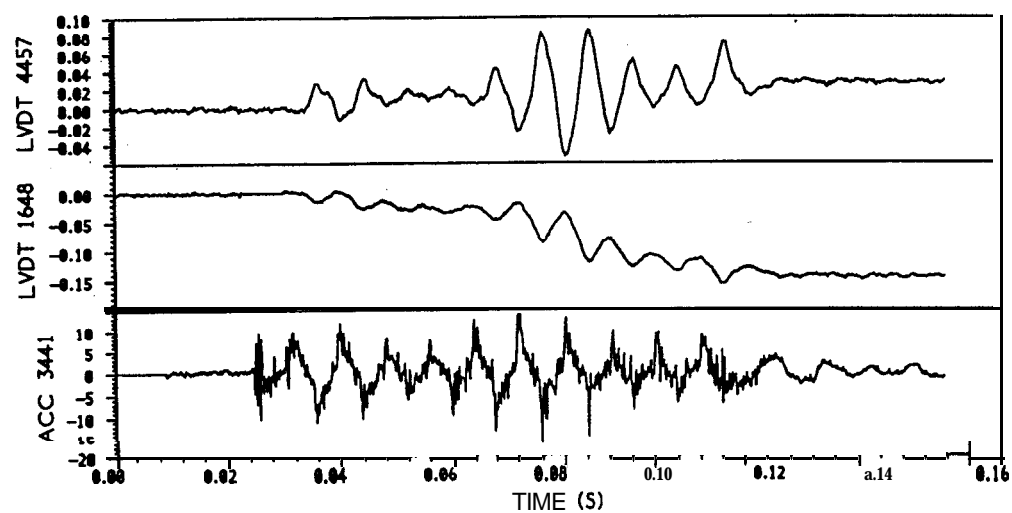


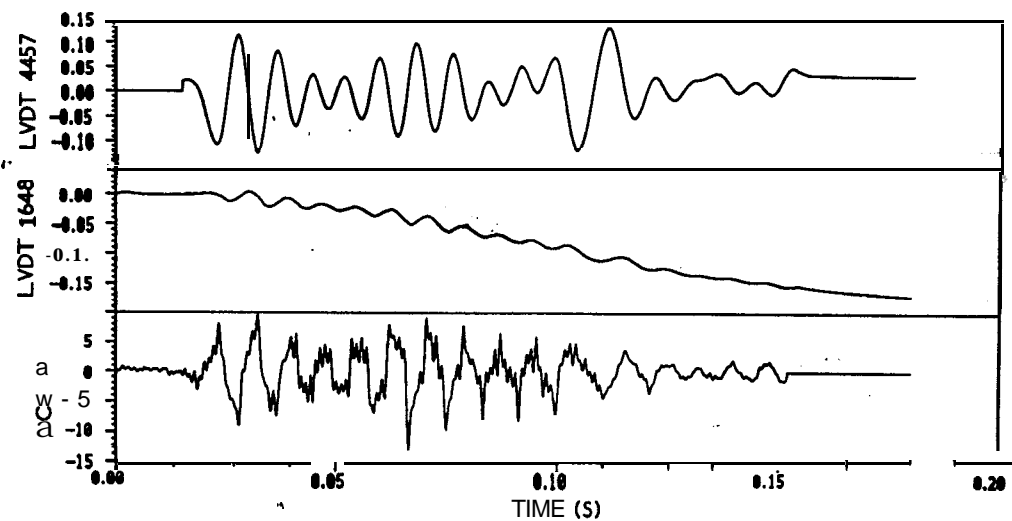
Fig.7 Schematic section of the tower-soil interaction problem

In Fig.8 the comparison of accelerometer time histories, excess pore pressure time histories and the deformation time histories is presented for all the instruments used in the experiment. In this figure particular attention is drawn to the top two plots showing the two displacement measuring LVDT's and their corresponding predictions. From this figure we can see that both the horizontal displacement and the vertical settlement are captured very well by the numerical analysis.





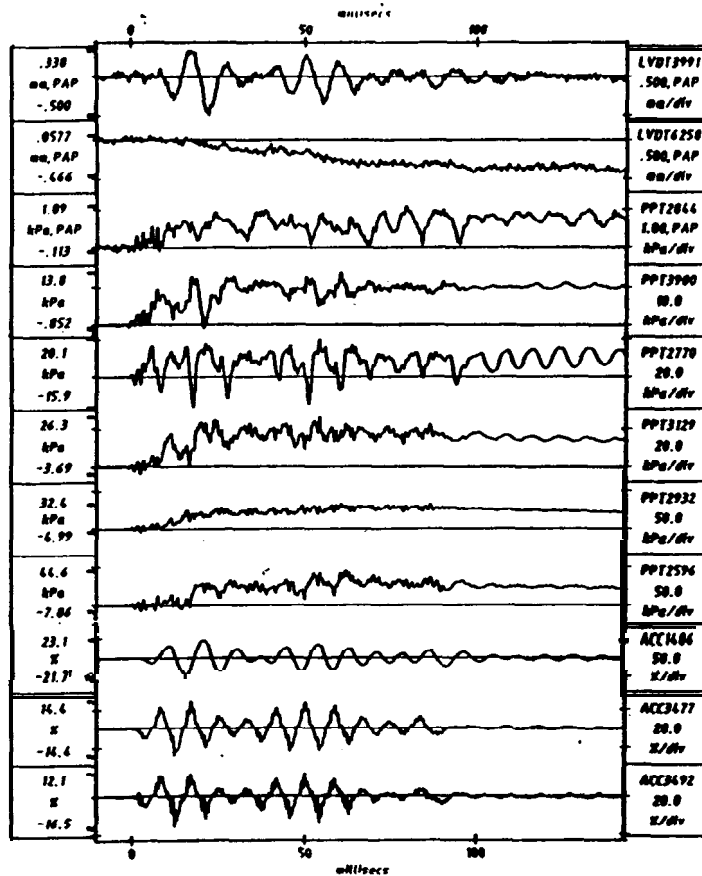
a) centrifuge test results



b) finite element predictions

Fig.6 Deformations of the embedded block structure following earthquake loading on the embankment

a) Centrifuge test results



b) Finite element results

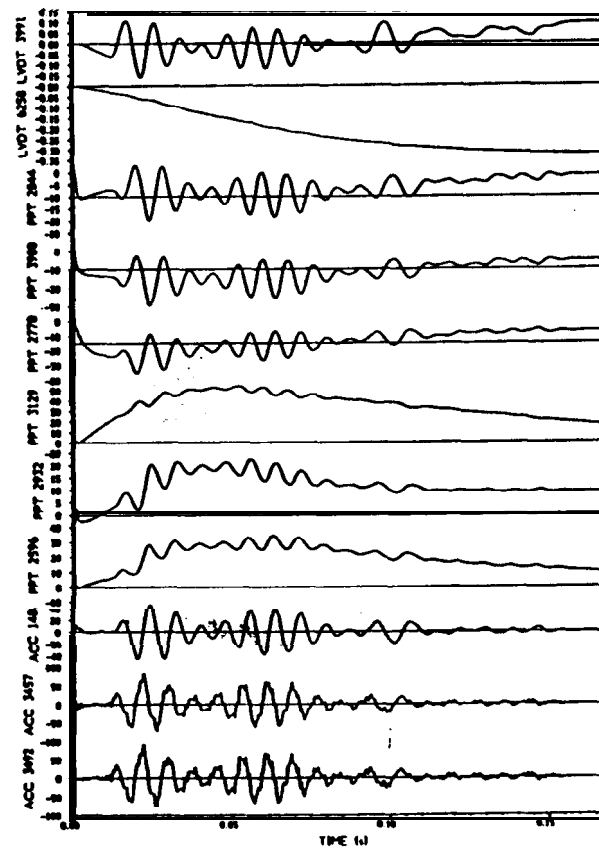


Fig.8 Comparison of the centrifuge test data and the finite element predictions for the tower-soil interaction problem

## 6 Numerical modelling of embankments on liquefiable soil deposits

Madabhushi et al (1996) report the centrifuge tests carried out on gravel embankments located on saturated sand beds. In these series of centrifuge tests it was observed that the gravel elements sink into the soil when it is liquefied following an earthquake loading. The post and pre test profiles clearly showed that the settlement was very large compared to a centrifuge test in which an equivalent embankment made out of solid metal. This may be due to the fact that as the soil liquefies the gravel elements at the base of the embankment sink in. This may be explained by means of a 'trap door' mechanism which opens when the foundation soil below the embankment is partially or fully liquefied. The post and pre test profiles are presented in Fig.9. The solid metal embankment could not settle as much owing to its large bearing area on the soil. This may be due to the fact that individual **sections** of the embankment are not free to move as the foundation liquefies. Only when the soil has liquefied 'enbloc' does the embankment settle in.

Numerical modelling of this problem using finite element method may pose some additional problems. The test results seem to suggest that there is a need to consider the discrete nature of the gravel elements and the continuum approach may lead to unconservative estimates of settlement.

## 7 Conclusions

Dynamic centrifuge tests may be used to study boundary value problems and the experimental data from these tests can be used to compare the numerical predictions from different finite element codes. It is particularly important to predict the deformations in the soil-structure interaction problems subjected to earthquake loading.

In this paper the deformations during and immediately after an earthquake event for the cases of a quay wall with saturated and dry backfill, a saturated embankment with embedded block structure and a tower structure on saturated sand bed, are presented. Finite element analysis were carried out to model each of these boundary value problems using SWANDYNE code. For each of the soil-structure interaction problems studied the deformations predicted by the numerical analysis compared satisfactorily with the observed deformations.

Recent centrifuge test data from gravel embankments on liquefiable soil deposits suggested that the gravel elements are likely to sink into liquefied layer there by causing excessive settlement of the embankment. Numerical modelling of this problem may be difficult and the continuum approach will tend to give **unconservative** estimate of settlement of such embankments.

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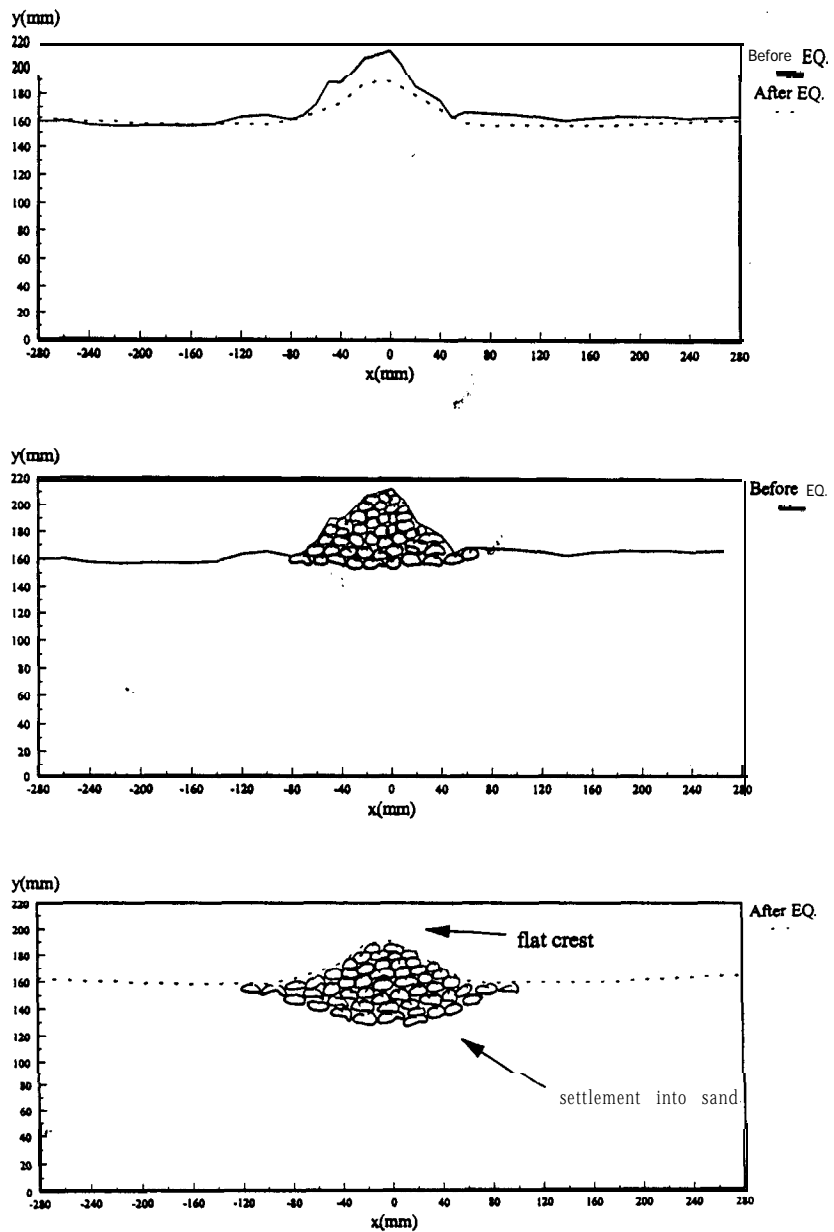


Fig.9 Settlement of a gravel embankment on liquefiable saturated sand bed

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