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Modelling of water demand in distribution networks

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Abstract

The allocation of water demand to nodes is compared with uniformly distributed demand along a pipeline, and it is shown that the nodal approach produces an upper bound or unsafe solution for pressures in the distribution network. Although the differences are likely to be minor for computer models with many nodes, the simplest examples show differences of up to 25% in head loss between the two approaches. Terminology and concepts from structural engineering are useful in this comparison. The results are particularly significant to simplified models using independently derived values of pipe friction factor.

Keywords: water distribution, Darcy-Weisbach head loss, pipe network analysis.

1. Introduction

When water distribution networks are modelled by computer or by hand calculation, it is usual to group the demand and to apply this at nodes, rather than to model every individual house connection along each pipeline. Often to simplify the model, the number of nodes is kept low, so the pattern of demand in the model can represent a significant approximation to the real situation. Twort et al. (2000) outline the benefits of manual analysis using a 'skeleton layout' for preliminary planning. They remind readers that 'the accuracy of any computer model is not greater than the accuracy with which nodal demands can be estimated'. The Haestad publication (Walski et al. 2001) refers to the grouping of water demand at nodes as being a possible source of error, but which produces relatively minor differences between computer predictions and actual performance.

The effect of the approximation of allocating demand to nodes is considered and discussed below, using concepts from structural engineering which are likely to be part of the general civil engineering training of many water engineers. These will first be outlined.

2. Structural engineering concepts

A topic studied in structural engineering is the plastic analysis of frames, with the upper bound ('unsafe') and lower bound ('safe') theorems. Heyman (1974) explains that all three conditions of equilibrium, mechanism and yield are necessary and sufficient to determine the true collapse load factor for a frame. When only two out of the three conditions are satisfied, the solution may be either an upper bound or a lower bound to the true solution, as summarised in Table 1. Clearly an upper bound solution for the collapse load is an unsafe estimate since this overestimates the true load capacity of the frame.

Texts such as Williams and Todd (2000) explain that elastic design methods are based on the safe or lower bound theorem, but that plastic analysis principally makes use of the unsafe theorem. In the latter approach, various possible mechanisms are formulated, all representing upper bounds to the true collapse load, and the mechanism giving the lowest load factor is deduced to be the critical case. It may then be checked whether this satisfies all three conditions and is the true collapse load, or whether in fact the true collapse mechanism has been overlooked. So there is an awareness here of whether a calculated result is an upper or lower bound to the actual solution.

The initial study of structural engineering also involves considering the effects of point loads and uniformly distributed loads on beams. Plenty of examples are contained in introductory texts such as Smith (2001). Clearly the representation of loads in this way involves the idealisation of actual loading cases.

3. Application to hydraulic modelling

The concept of uniformly distributed loads, and of upper bound solutions, that are familiar in structural engineering, may prove to be useful ideas when considering a water distribution network. Demand may be considered as uniformly distributed along pipelines, for comparison with results obtained from point demands applied at nodes. The various ways of idealising the system to apply nodal demands may be compared, and it will be shown that these represent upper bound or unsafe estimates in relation to the true solution.

Consider a pipeline AB as part of a network. The demand may be taken to be uniformly distributed between A and B, along the length L of the pipeline. In an urban situation this would closely represent the reality of many house connections along the length of a distribution main.

For simplicity, consider that the flow at node B is zero, and the incoming flow rate at A is equal to Q, as shown in Fig.1. The uniformly distributed demand is therefore Q/L where L is the length of the pipeline AB. Frictional head losses will be evaluated using the Darcy-Weisbach friction formula, in the form

$$h_f = \frac{\lambda L V^2}{2gD} = \frac{8\lambda L Q^2}{g\pi^2 D^5} = KQ^2 \qquad (1)$$

where g (m/s^2) is the acceleration due to gravity, D (m) is the internal pipe diameter, L (m) is the length of pipeline with flow at a mean velocity of V (m/s) and a volumetric flow rate Q (m^3/s), and K (s^2/m^5) is pipeline resistance, all with S.I. units as shown in brackets. The dimensionless pipe friction factor λ will be considered to be constant with flow rate, as in the rough turbulent region, but results may also be derived using alternative formulae such as Hazen-Williams for the transitional turbulent region.

For the situation in Fig. 1, with a flow rate varying linearly from Q at node A down to zero at node B, the total head loss h_{fAB} between A and B is evaluated as follows:

$$h_{fAB} = \frac{8\lambda}{g\pi^2 D^5}\int_0^L \left[\frac{Q(L-x)}{L}\right]^2 dx \qquad (2)$$

yielding $h_{fAB} = \dfrac{8\lambda L Q^2}{3g\pi^2 D^5} = \dfrac{KQ^2}{3}$ (3)

This value h_{fAB} in equation (3) will be taken to represent the true value.

Consider now the demand grouped at nodes A and B, as shown in Fig. 2. The head loss h_{f1} may be calculated for this idealisation as follows:

$$h_{f1} = \frac{8\lambda L}{g\pi^2 D^5}\left(\frac{Q}{2}\right)^2 = \frac{2\lambda L Q^2}{g\pi^2 D^5} = \frac{KQ^2}{4} \qquad (4)$$

So treating the pipeline AB as one length with the demand allocated equally to the end nodes, it is found by comparing equations (3) and (4) that the calculated head loss h_{f1} is related to the true value by $h_{f1} = \frac{3}{4} h_{fAB}$, and that the head loss is therefore underestimated by one quarter in this approximation.

If the pipeline AB is divided into two lengths, as shown in Fig.3, with the demand divided in two, and then split between the nodes as shown, the calculated head loss h_{f2} is as follows:

$$h_{f2} = \frac{8\lambda}{g\pi^2 D^5}\left(\frac{L}{2}\right)\left[\left(\frac{3Q}{4}\right)^2 + \left(\frac{Q}{4}\right)^2\right] = \frac{5\lambda LQ^2}{2g\pi^2 D^5} = \frac{15}{16}h_{fAB} \qquad (5)$$

So in this case the head loss is underestimated by one sixteenth of the true value.

A general expression may be deduced for such a pipeline split in this way into n lengths, that the head loss h_{fn} is an underestimate by $1/(2n)^2$ of the head loss that results from uniformly distributed demand.

4. Loop example

Similar analysis may be applied to loops that form parts of pipe networks. A simple symmetrical example shown in Fig.4 comprises a square WXYZ with sides of pipework length L and resistance K. With demand allocated equally to the four nodes W, X, Y and Z as shown, the head loss from the supply point at node W to the farthest node Y is given by:

$$h_f = K\left(\frac{3Q}{8}\right)^2 + K\left(\frac{Q}{8}\right)^2 = \frac{10KQ^2}{64} \qquad (6)$$

This example may be seen to be similar to the pipeline of Figure 3. If the demand is considered to be uniformly distributed around the square, the resulting head loss obtained may be deduced from equation (3) as:

$$h_{fWY} = \frac{2K(Q/2)^2}{3} = \frac{KQ^2}{6} \qquad (7)$$

Comparison of equations (6) and (7) shows that the nodal approach in (6) underestimates the head loss by one sixteenth, when compared with the uniformly distributed result.

5. Discussion of implications

Usually the objective of calculating head losses, is to ensure that at least a certain minimum acceptable pressure is provided to consumers, as one of the level of service criteria. Therefore it may be seen that the underestimation of head loss resulting from the various approximations allocating demand to nodes, will result in overestimates or upper bound solutions for the available pressure heads.

Dividing the network into a greater number of pipe lengths and nodes will reduce the inaccuracy, which is seen above to be inversely related to the square of the number of lengths into which the pipeline is divided.

The maximum error shown in the above calculations amounts to one quarter or 25% of the head loss for uniformly distributed demand. Such possible errors should be noticed when a very simplified model is used, perhaps to provide an overview of a complex situation.

The above comments apply particularly where the pipe friction values have been obtained independently of the model. If the pipe friction values in the model have been obtained by calibrating the model against measured values of flow and pressure, then the above effect will have been accounted for in the deduced friction values.

6. Conclusions

Concepts of uniformly distributed loads and upper bound solutions from structural engineering have been used in pipe network analysis to compare head losses resulting from various idealised situations.

It has been demonstrated that idealisations of pipe networks that place the demand at nodes are in effect upper bound or ‘unsafe’ solutions when used to estimate the minimum pressures available to consumers.

The maximum error demonstrated in head loss for a single pipe is an underestimate by 25%, compared with uniformly distributed demand.

Possible errors of this type arise when running very simplified models, and will be reduced by increasing the number of nodes in the model.

It is noted that the above comments apply where pipe friction factors have been independently derived, and not adjusted as part of the model calibration.

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Table 1. Structural design of frames by plastic analysis

Condition	Upper bound theorem (unsafe)	Lower bound theorem (safe)
Equilibrium	Satisfied	Satisfied
Mechanism	Satisfied	Not satisfied
Yield	Not satisfied	Satisfied

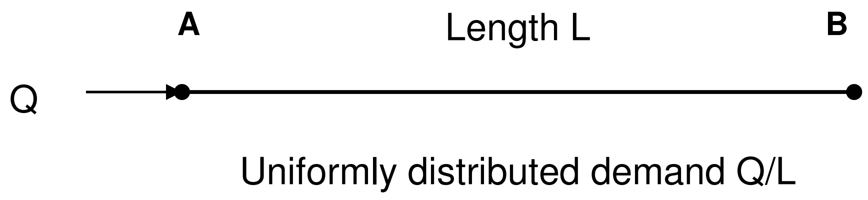

Fig. 1. Uniformly distributed demand along pipeline

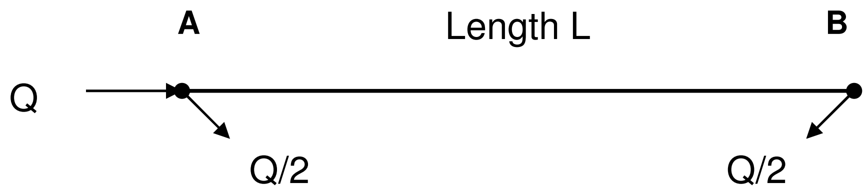

Fig. 2. Demand grouped at nodes with pipeline as one length

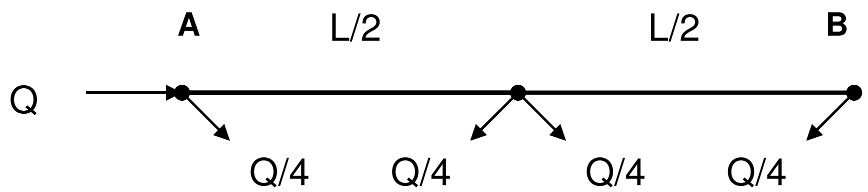

Fig. 3. Demand grouped at nodes with pipeline divided into two equal lengths

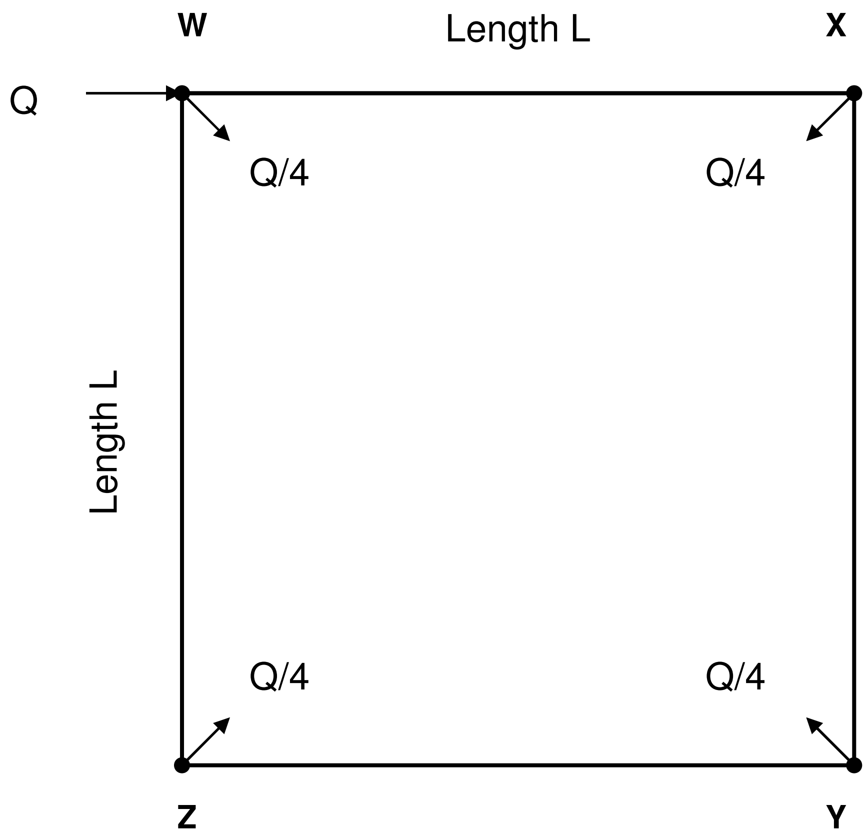

Fig. 4. Simple loop example with demand allocated equally to nodes